The diffeomorphism constraint operator in loop quantum gravity

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Abstract. We construct the diffeomorphism constraint operator at finite triangulation from the basic holonomy- flux operators of Loop Quantum Gravity, and show that the action of its continuum limit provides an anomaly free representation of the Lie algebra of diffeomorphisms of the 3- manifold. Key features of our analysis include: (i) finite triangulation approximants to the curvature, F^i_{ab} of the Ashtekar- Barbero connection which involve not only small loop holonomies but also small surface fluxes as well as an explicit dependence on the edge labels of the spin network being acted on (ii) the dependence of the small loop underlying the holonomy on both the direction and magnitude of the shift vector field (iii) continuum constraint operators which do *not* have finite action on the kinematic Hilbert space, thus implementing a key lesson from recent studies of parameterised field theory by the authors. Features (i) and (ii) provide the first hints in LQG of a conceptual similarity with the so called “mu- bar” scheme of Loop Quantum Cosmology. We expect our work to be of use in the construction of an anomaly free quantum dynamics for LQG.

We highlight the main steps and results of our construction while suppressing most of the technical details. This work was done jointly with Alok Laddha.

1. Introductory Remarks:

One of the key open problems in LQG is that of a satisfactory definition of its quantum dynamics. The current definition of the Hamiltonian constraint operator is far from unique due to the many ad- hoc choices which go into its construction. The reason this happens is that the Hamiltonian constraint is a local expression constructed out of local fields whereas some of the basic operators of LQG are non- local and have discontinuous action. Specifically, the main source of ambiguities is that there is no known way to obtain the operator corresponding to the curvature of the Ashtekar- Barbero connection, $F^i_{ab}(x)$, from holonomy operators. While in the classical theory, $F^i_{ab}(x)$ can be obtained from a limit of holonomies around loops which shrink to the point x, the corresponding limit of operators in the quantum theory does not exist primarily because the Hilbert space structure of LQG is unable to adequately distinguish between a ‘small’ loop and a ‘smaller’ loop by virtue of its background independence.

Hence one proceeds as follows [1]. Fix a triangulation T of the spatial manifold. Choose finite triangulation approximants to the various local fields which make up the Hamiltonian constraint These approximants are constructed from the the basic holonomy- electric flux functions of the theory. Using these approximants define the finite triangulation Hamiltonian constraint $C_{ham,T}$. By construction $C_{ham,T}$ approaches the classical Hamiltonian constraint in the continuum limit of infinitely fine triangulation. Next, replace the holonomy- flux functions in $C_{ham,T}$ by their

Journal of Physics: Conference Series **360** (2012) 012009 doi:10.1088/1742-6596/360/1/012009

operator correspondents to obtain $\hat{C}_{ham,T}$. Finally, take the continuum limit of this operator, the idea being that while the individual (holonomy) operators do not have a continuum limit, the composite operator $\hat{C}_{ham,T}$ may have one.

Remarkably, a continuum limit exists [1, 2, 3] but, unfortunately, it depends on the choice of finite triangulation holonomy approximants to $F^i_{ab}(x)$. There are two aspects of this choice. One is the representation of $SU(2)$ chosen to evaluate the holonomy [4] and the second is the choice of little loops which are shrunk away to the point x. Typically the representation is chosen to be some fixed j (usually $j = \frac{1}{2}$) representation and the little loop is chosen by some subjective criterion of "simplicity". Each choice of representation and each choice of the diffeomorphism invariant way that the loop interacts with the state being acted upon yields a potentially inequivalent continuum Hamiltonian constraint operator.

Moreoever, simpler contexts suggest these choices are incorrect. In Parameterised Field Theory [9, 10], the representation of the 'holonomy' operator depends on the edge labels of state. In Loop Quantum Cosmology [11], the small loop depends on the triad operator. Given this state of affairs, notwithstanding the impressive pioneering nature of earlier works [5, 6, 7, 8, 16], it is imperative to improve upon the choice of approximants to F^i_{ab}. We seek insight into the correct choice by constructing the *diffeomorphism* constraint operator in such a way that it kills the standard diffeomorphism invariant distributions obtained by group averaging [12]. Note that the standard treatment does *not* construct this operator. The diffeomorphism constraint is imposed indirectly by the demand that states be invariant under the unitary action of *finite* diffeos. Indeed the generator of diffeomorphisms is not even defined on the kinematic Hilbert Space.

Here, we treat the diffeomorphism constraint in a manner similar to the Hamiltonian constraint through the following steps:
- Fix a triangulation T of the spatial manifold. Construct finite T approximants to relevant local fields from the holonomy- flux variables.
- From these, construct the diffeomorphism constraint $D_T(\vec{N})$ at finite triangulation, $\vec{N}$ denoting the shift vector.
- Replace the holonomy- flux functions by the corresponding operators, obtain $\hat{D}_T(\vec{N})$ and construct its continuum limit.

The above procedure necessitates a choice of finite T approximant to $F^i_{ab}(x)$ by virtue of the dependence of the diffeomorphism constraint on $F^i_{ab}(x)$. Our choice of finite T approximant to $F^i_{ab}(x)$ is guided by the requirement that at finite T:

$$\hat{D}_T(\vec{N}) = -i\hbar \frac{\hat{U}_{\phi(\vec{N},\delta)} - \mathbf{1}}{\delta}. \tag{1}$$

Here δ parameterizes the fineness of the triangulation (the continuum limit is defined as the limit $\delta \to 0$) and $\phi(\vec{N}, \delta)$ is the diffeomorphism which translates points along integral curves of the shift vector $\vec{N}$ by affine parameter length δ. This requirement has a fourfold justification:
- The expression (1) kills the standard diffeomorphism invariant states obtained by group averaging.
- If there was a generator, this form is natural.
- This form manifests in Parameterised Field Theory which is the only known field theoretic model in which all steps of the LQG program can be carried through without ambiguities.
- The continuum limit operator action obtained from (1) yields a faithful representation of its algebra on Lewandowski- Marolf habitat.

The plan of this article is as follows. In section 2 we sketch the main steps of our construction which yield the operator of equation (1). In section 3 we display our choice of curvature approximants for the simple case of the state being an edge holonomy in the $j = \frac{1}{2}$ representation. In section 4 we comment briefly on the continuum limit of the finite triangulation diffeomorphism constraint operator. Section 5 is devoted to a discussion of our results. Our presentation will be schematic and meant to convey the broad picture. A comprehensive account is already available [13] for the reader interested in the details.

2. Brief Sketch of Main Steps

The purpose of this section is to sketch the main steps in the construction so as to give the reader a rough global view of the logic; In this section we shall set $G = \hbar = c = 1$. We shall further simplify our presentation by choosing the Barbero- Immirizi parameter γ to be unity in this section.

The diffeomorphism constraint $D(\vec{N})$ is

$$D(\vec{N}) = \int_\Sigma \mathcal{L}_{\vec{N}} A^i_a \tilde{E}^a_i \quad (2)$$

$$= V(\vec{N}) - \mathcal{G}(N^c A^i_c) \quad (3)$$

where

$$V(\vec{N}) = \int_\Sigma N^a F^i_{ab} \tilde{E}^b_i \quad (4)$$

$$\mathcal{G}(N^c A^i_c) = \int_\Sigma N^c A^i_c \mathcal{D}_a \tilde{E}^a_i . \quad (5)$$

Here Σ is the 3- manifold, A^i_a is the Ashtekar Barbero connection, F^i_{ab} is its curvature, $\tilde{E}^a_i$ is the densitized triad and $\vec{N} \equiv N^a$ is the shift vector field.

Let $T(\delta)$ be a 1 parameter family of triangulations of Σ with the continuum limit being $\delta \rightarrow 0$ and let $D_{T(\delta)}, V_{T(\delta)}, \mathcal{G}_{T(\delta)}$ be finite triangulation approximants to the quantities $D, V, \mathcal{G}$ of the above equations. Thus $D_{T(\delta)}, V_{T(\delta)}, \mathcal{G}_{T(\delta)}$ are expressions which yield $D, V, \mathcal{G}$ in the continuum limit.

For simplicity consider a (non- gauge invariant) spin network state consisting of a single edge e with spin label j so that the state is just the m, n component of an edge holonomy $h^{(j)\;\;n}_{e\;m}$ of the (generalized) connection along the edge e in the representation j, the indices m, n taking values in the set $1, .., 2j + 1$. In what follows we shall supress some of these labels and denote the state simply by h_e.

From (1) of section 1, our desired result is:

$$(1 + i\delta \hat{D}_{T(\delta)}) h_e = h_{\phi(\delta, \vec{N}) \circ e}. \quad (6)$$

where, $\phi(\delta, \vec{N}) \circ e$ is the image of e by the diffeomorphism $\phi(\delta, \vec{N})$ which translates e by an amount δ along the integral curves of the shift vector field N^a (see Fig 1a).

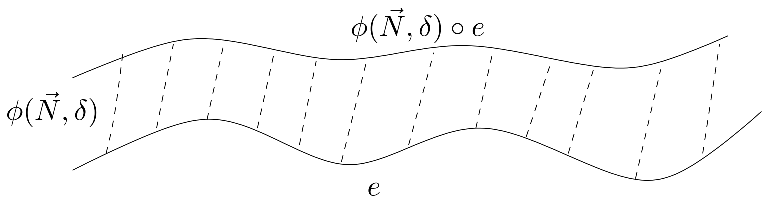

Fig 1a

We obtain the desired result through the following steps:
(i) First we set

$$(1 + i\delta \hat{D}_{T(\delta)}) := (1 + i\delta \hat{\mathcal{G}}_{T(\delta)})(1 + i\delta \hat{V}_{T(\delta)}). \quad (7)$$

(ii) Next, we show that

$$(1 + i\delta \hat{V}_{T(\delta)}) h_e = h_{\bar{e}(\vec{N}, \delta)}. \quad (8)$$

Here $\bar{e}(\vec{N}, \delta)$ has the same end points as e (as it must by virtue of the gauge invariance of V) and is obtained by joining the end points of $\phi(\delta, \vec{N}) \circ e$ to those of e by a pair of segments which are aligned with integral curves of N^a as shown in Fig 1b.

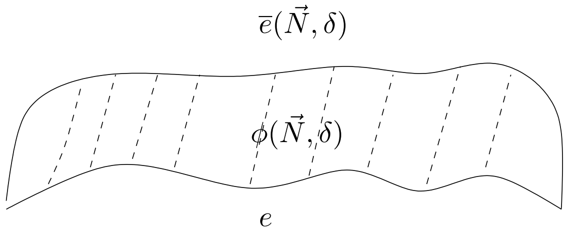

Fig-1b

(iii) Finally, we show that the Gauss Law term, $(1 + i\delta \hat{\mathcal{G}}_{T(\delta)})$ removes these two extra segments (see Fig 1c).

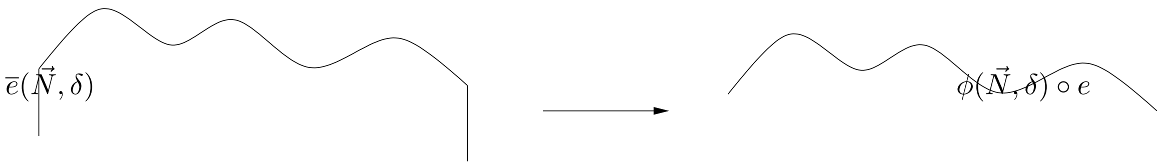

Fig-1c

The major part of the analysis concerns the derivation of the identity (8) in step (ii) above. We proceed as follows.

$V_{T(\delta)}$ is written as a sum over contributions $V_\triangle$ where $\triangle$ denotes a 3- cell of the triangulation *dual* to $T(\delta)$, and $V_\triangle$ is a finite triangulation approximant to the integral $\int_\triangle N^a F^i_{ab} \tilde{E}^b_i$. We order the triad operator to the right in $\hat{V}_\triangle$ so that only those 3- cells contribute which intersect e.

The triangulation $T(\delta)$ is adapted to the edge e so that its restriction to e defines a triangulation of e. Thus, there is a triangulation of e by 1- cells and vertices of $T(\delta)$ so that each of these vertices $v_I, I = 1, .., N$ is located at the centre of some 3- cell $\triangle = \triangle_I$. We define a finite triangulation approximant to F^i_{ab} in a such a way that the following identity holds:

$$(1 + i\delta \hat{V}_{\triangle_I}) h_e = h_{\bar{e}(\triangle_I)}. \tag{9}$$

$\forall\ I\ \in\ \{1, ..., N-1\}$.
Here $\bar{e}(\triangle_I)$ is obtained by moving the segment of e between v_I and v_{I+1} along the integral curves of N^a by an amount δ and joining this segment to the rest of e at the points v_I, v_{I+1} by a pair of segments which run along the integral curves of N^a as shown in Fig 1d.

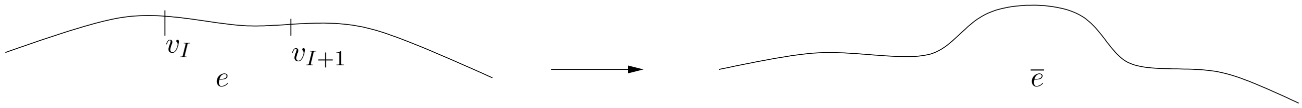

Fig-1d

Next, we show that the contributions from all the $\hat{V}_{\triangle_I}$ yield the edge $\bar{e}(\vec{N}, \delta)$ of Fig 1b. Recall that $V_{T(\delta)}$ is obtained by summing over all the cell contributions $V_\triangle$. However, summing over the action of all the $\hat{V}_{\triangle_I}$ on h_e only yields a sum over states of the type $h_{\bar{e}(\triangle_I)}$. In order to obtain the desired result, $h_{\bar{e}(\vec{N},\delta)}$, the sum over $\triangle$ is first converted to a *product* over $\triangle$ i.e. to leading order in δ, we show that

$$1 + i\delta \sum_\triangle V_\triangle \sim \prod_\triangle (1 + i\delta V_\triangle). \tag{10}$$

Hence, replacing the sum over the corresponding operators by the product provides an equally legitimate definition of $\hat{V}_{T(\delta)}$. The replacement then leads, modulo some details, to the following identity

$$\prod_{\triangle}(1+i\delta\hat{V}_{\triangle})h_e=\prod_{I=1}^{N-1}(1+i\delta V_{\triangle_I})h_e. \tag{11}$$

Our definition of $\hat{V}_{\triangle_I}$ is such that each factor in the product acts independently, the Ith factor acting only on the part of e between v_I and v_{I+1}. We are then able to show that the result (8) follows essentially through the mechanism which is illlustrated schematically in Fig 1e.[1]

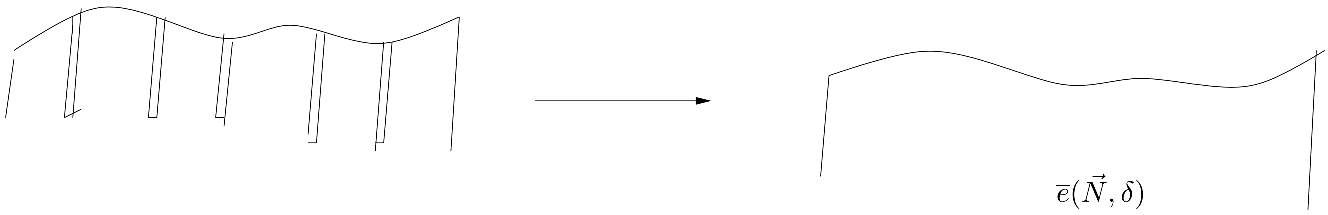

Fig-1e

To summarise, we obtain the desired result (6) through the sequence: Fig 1d→Fig 1e→Fig 1c.

3. Curvature Approximants

For simplicity let the state h_e be in the $j=\frac{1}{2}$ representation. We seek a finite triangulation approximant to F^i_{ab} such that equation (9) holds. Hence, restrict attention to the part e_I of the edge e between v_I and v_{I+1}. Let e_I be small enough that e_I is in an open patch with coordinates (x,y,z), such that x runs along e. Let γ_I be the 'rectangular' loop obtained by running along (but opposite to) e_I from v_{I+1} to v_I, then climbing up the integral curve of N^a by an amount δ, then running along the segment $\phi(\vec{N},\delta)\circ e_I$ and finally coming down the integral curve of N^a to v_{I+1} as shown in Figure 2. Let the part of γ_I excluding e_I be $\bar{e}_I$.

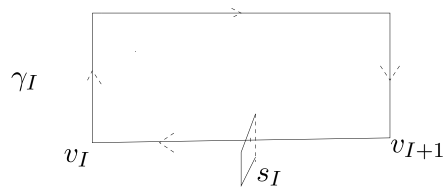

Figure 2

From the discussion in section 1, we want to approximate $V_{\triangle_I}=\int_{\triangle_I}N^aF^i_{ab}\tilde{E}^b_i$ in a way that leads to:

$$(1+i\delta V_{\triangle_I})h_{e_I}=h_{\bar{e}_I}. \tag{12}$$

$\tilde{E}^b_i$ is approximated by the electric flux through the surface S^b_I in the coordinate plane normal to the b- direction. We choose the location of these surfaces such that they are centred at the midpoint v of v_I,v_{I+1} and choose their size to be small enough that they intersect the edge γ_I only along e_I.

Clearly only the flux operator $\hat{E}_i(S_I)$ through the surface $S_I:=S^x_I$ contributes to equation (12) so that we may focus only on the contribution V^x_I to $V_{\triangle_I}$ where $V^x_I:=\int_{\triangle_I}N^aF^i_{ax}\tilde{E}^x_i$.

[1] The double lines in the figure indicate retraced paths.

Accounting for the various small parameters coming from the coordinate volume of $\triangle_I$ and the coordinate area of S_I it turns out that we obtain the approximant:

$$V_I^x \sim (N^a F_{ax}^i)_I E_i(S_I)\delta \tag{13}$$

where $(N^a F_{ax}^i)_I$ is an approximant to $N^a F_{ax}^i(v_I)$.

The usual way in which curvature approximants are handled in LQG corresponds to approximating $F_{ax}^i(v_I)$ by a small loop holonomy with $N^a(v_I)$ being an overall factor. Note that this is *not* feasible here due to the product structure $\prod_I(1+i\delta\hat{V}^x_{\triangle_I})$. If we did set $(N^a F_{ax}^i)_I = N^a(v_I)F_{ax}^i(v_I)$, we would have terms containing products of shifts at vertices along e, which, in the continuum limit of the triangulation would not be well defined. This leads us to conclude that *the shift must be part of the specification of the small loop.*

Hence we focus on $(N^a F_{ax}^i)_I$ as opposed to $(F_{ax}^i)_I$. The most straightforward approximant is then

$$N^a F_{ax}^i \approx \frac{-\mathrm{Tr}(h_{\gamma_I}\tau^i)}{\delta^2} \Rightarrow \quad i\delta V^x_{\triangle_I} \sim -i\mathrm{Tr}(h_{\gamma_I}\tau^i)\ \ E_i(S_I) \tag{14}$$

where the insertion τ_i is made at v. Using this choice we obtain

$$(1+i\delta\hat{V}^x_{\triangle_I})h_e \sim h_{\bar{e}_I} - [\mathrm{Tr}(h_{\gamma_I})-1]h_e. \tag{15}$$

Hence we want to modify our choice of approximant so that the second, unwanted term is removed. To this end note that $\hat{E}^i(S_I)\hat{E}_i(S_I)$ is proprtional to the square of the angular momentum operator for the group element h_e so that

$$\hat{E}^i(S_I)\hat{E}_i(S_I)h_e = \lambda h_e. \tag{16}$$

This implies that the addition of a term of the form '$[\mathrm{Tr}(h_{\gamma_I})-1]E^i$' to $(N^a F_{ax}^i)_I$ would yield the desired result. The additional term is of higher order in the small parameters δ and the coordinate area of the surface S_I and hence the modified expression is a valid curvature approximant.

Putting all the factors in, we obtain the final result:

$$(N^a F_{ax}^i)_I \ := \ \frac{1}{\delta^2}\{\ -Tr(h_{\gamma_I}\cdot\tau_i)\ -\ \frac{2i}{3}Tr(h_{\gamma_I}-1)\frac{E_i(S_I)}{\gamma l_p^2}\} \tag{17}$$

The above expression has two remarkable properties:
- the size and shape of the small loop γ_I depends on the shift
- the 2nd term has l_p^{-2} terms, leading one to speculate if (perhaps in contexts other that the diffeomorphism constraint) they could provide a seed for non-perturbative quantum corrections.

This concludes are our discussion of the curvature approximant for the $j=\frac{1}{2}$ case. It turns out that our considerations can be generalised, first, to the case of a single edge holonomy for any j (wherein one obtains higher order contributions in the small parameters δ and the area of S_I which are polynomial in the holonomy- flux operators) and thence to the case of an arbitrary spin network state. The latter generalization is particularly easy because of the dovetailing of the *product* form of the constraint with the fact that a general spin network is obtained as the *product* of edge holonomies (sewn together with intertwiners).

In summary, we have found a choice of curvature approximants such that equation (1) holds.

4. Continuum Limit

As sketched in previous sections we have found a choice of curvature approximants such that the diffeomorphism constraint at finite triangulation acts as $\hat{D}_T(\vec{N}) = -i\hbar\frac{\hat{U}_{\phi(\vec{N},\delta)}-\mathbf{1}}{\delta}$. Clearly, as expected, the $\delta \to 0$ continuum limit of $\hat{D}_T(\vec{N})$ is not a well defined operator on the kinematic Hilbert space due to the discontinuous action thereon of the finite diffeomorphism unitaries $\hat{U}_{\phi(\vec{N},\delta)}$ coupled with the factor of δ in the denominator. However it turns out that the continuum limit *is* well defined on a different representation space, namely, the Lewandowski- Marolf (LM) habitat [2].

We briefly describe how this happens. Recall that the LM habitat is a subspace of the algebraic dual to the space of finite linear combinations of spin network states. A basis element of the LM habitat is labelled by a diffeomorphism equivalence class of spin nets and a 'vertex smooth function'. Specifically, let $[s]$ denote the set of spin nets diffeomorphic to the spin net s. Let n be the number of vertices of s. Clearly n is the same for every element of $[s]$. Let $f(x_1,..,x_n)$ be a smooth,complex valued function on n copies of the spatial manifold. Then we denote by $\Psi_{[s],f}$ the basis element of the LM habitat associated with $[s], f$. From [2] and equation (1) it follows that

$$\hat{D}_T(\vec{N})\Psi_{[s],f} = \Psi_{[s],g_T} \tag{18}$$

where the vertex smooth function g_T is (modulo overall factors) just the difference between the evaluation of f at the points $(x_1,..x_n)$ and their diffeomorphic images i.e. modulo a multiplicative constant g_T is just $\frac{f(\phi^{-1}(\vec{N},\delta)x_1,.,\phi^{-1}(\vec{N},\delta)x_n)-f(x_1,.,x_n)}{\delta}$.

The continuum limit then just gives the new function

$$g = i\hbar\sum_{i=1}^{n} N^a(x_i)\frac{\partial f}{\partial x_i^a}. \tag{19}$$

It is then straightforward to show that the operator action $\hat{D}(\vec{N})\Psi_{[s],f} = \Psi_{[s],g}$ provides a representation of the constraint algebra as the Lie algebra of the group of spatial diffeomorphisms.

5. Conclusions

The diffeomorphism constraint $D(\vec{N})$ generates diffeomorphisms along the integral curves of the shift vector field $\vec{N}$. Hence, one expects the quantum constraint operator, $\hat{D}(\vec{N})$, to have a non- trivial action at *all* the (infintely many) points lying on those edges of a spin network state which are transverse to $\vec{N}$. In contrast, almost all operators of significance in LQG have a non-trivial action only at a *finite* number of points namely the vertices of the graph underlying the spin network state. Indeed, the necessity of an action at infinitely many points was thought to be an obstacle to the construction of the operator $\hat{D}(\vec{N})$ [14]. Our construction gets around this obstruction through the reformulation of the classical constraint at finite triangulation as a *product* over 3- cells of the triangulation described in section 2. This leads, in the quantum theory, to a *product* of bounded operators at finite triangulation rather than a *sum*. The product admits a satisfactory continuum limit whereas the sum does not. Thus, it is the passage to the product form which enables us to deal with the contributions from infinitely many points in the continuum limit.

A sensible product reformulation also seems to require that the shift vector $N^a(x)$ cannot appear as an overall factor multiplying the diffeomorphism constraint $D_a(x)$ at the point x because a product over all x of shift vectors at each point x is not an object which makes sense in the continuum limit. Hence it seems inevitable that the shift vector dependence in $D(\vec{N})$ at finite triangulation is taken care of by the incorporation of both its direction and magnitude in the specification of the small loop which underlies the holonomy approximant to the Ashtekar-Barbero curvature, F^i_{ab}. Indeed, what we are able to construct is the quantity $N^a F^i_{ab}$ at finite triangulation rather than F^i_{ab} itself. As a consequence, our construction of curvature approximant bears a great conceptual similarity to that of Loop Quantum Cosmology (LQC) when viewed in the following manner.

In isotropic LQC, the diffeomorphism constraint is satisfied identically and the Hamiltonian constraint reduces to its Euclidean part $H = \frac{\epsilon^{ijk} F_{abi} \tilde{E}^a_j \tilde{E}^b_k}{\sqrt{q}}$ where we have used standard notation for the densitized triad and the determinant of the 3- metric. Our work here suggests that rather than F^i_{ab} it is $\frac{\tilde{E}^a_j}{\sqrt{q}} F^i_{ab}$ which needs to be approximated at finite triangulation, and, that one should attempt to incorporate $\frac{\tilde{E}^a_j}{\sqrt{q}}$ as part of the specification of the small loop underlying the holonomy approximant. In the quantum theory such an attempt, if successful, would lead to the consideration of a loop whose size depends on the triad operator, thus exhibiting a close conceptual similarity to the "$\bar{\mu}$" scheme [11] for the Hamiltonian constraint in LQC.

Setting aside considerations of the Hamiltonian constraint, this work in itself reveals the necessity of a triad operator dependence in the construction of curvature approximants. This dependence is both explicit (as seen in the occurence of the electric flux terms in equations (17) as well as implicit in that, as mentioned in section 3, the expressions for the curvature approximants depend on the spin label j of the edge on which the curvature operator acts. Recall that the j label specifies the eigen values of the area operator which is built from the triads [15]. A similar dependence of "connection" type operators on conjugate "electric fluxes" was also seen to be crucial in recent work on Polymer Parameterised Field Theory (PPFT) [9, 10].

Apart from this "electric flux dependence", one of the key lessons of our work in PPFT [9] is the necessity of considering kinematically *singular* constraint operators in order to obtain a non- trivial representation of the constraint algebra. Here, too, the existence of a non- trivial representation of the quantum constraint algebra can be traced to the kinematically singular nature of the diffeomorphism constraint operator. That this operator is singular on $\mathcal{H}_{kin}$ is an obvious consequence of the factor of δ^{-1} in equation (1). It is this factor which leads to a non-trivial representation of the constraint algebra on the LM habitat alluded to in section 4. Had this factor been absent the action of the constraint operator would have yielded the difference of the evaluations of a vertex smooth function at points seperated by δ. This difference vanishes in the $\delta \to 0$ limit by virtue of the smoothness of the function. Instead, just as for PPFT [9], the factor of δ^{-1} converts this difference into a derivative in the continuum limit, thus yielding a non- trivial action of the diffeomorphism constraint operator on the habitat as well as a non-trivial representation of the algebra of diffeomorphism constraints thereon.

Our final goal is the construction of the Hamiltonian constraint operator in such a way as to obtain a non- trivial anomaly free representation of its algebra. Let us refer to the quantum commutator between a pair of Hamiltonian constraints as the Left Hand Side (LHS) and the quantum correspondent of the classical Poisson bracket between this pair as the Right Hand Side (RHS). The RHS is closely related to the diffeomorphism constraint operators studied here; the only difference being that the shift vector field in the RHS is operator valued. Earlier work by Thiemann [16], Lewandowski and Marolf [2] and Gambini, Lewandowski, Pullin and Marolf [17] showed that for density weight one Hamiltonian constraints, the algebra consistently trivialises i.e. the LHS and the RHS can be independently defined either with respect to the Uniform Rovelli- Smolin- Thiemann Topology on $\mathcal{H}_{kin}$ [16, 14] or on the LM habitat [2, 17] and, in both cases, both the RHS and the LHS vanish. Our work on PPFT [9] (as well as the 'rescaling by hand' in Reference [17]) suggests the use of higher density weight constraints to probe the existence of a *non- trivial* representation of the constraint algebra. Both these works also suggest that the current set of choices for curvature approximants are inappropriate. As emphasized in Reference [17] the current set of choices used in the LHS do not result in an RHS which can move vertices by diffeomorphisms. Since the choice of curvature approximants used in this work does result in the diffeomorphism constraint moving vertices around by diffeomorphisms, the considerations of this work should be of use for a better understanding of both the LHS as well as the RHS.

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