MODELS FOR RELATIVISTIC STATISTICAL MECHANICS

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Since these are the last fifteen minutes (or the last three pages) of our $M \cap \Phi$ and presumably everybody is already tired of exact definitions, theorems lemmas and proofs, I shall not attempt to bore you with these. Neither will I go into all the intricacies of physical interpretation of relativistic statistical mechanics. I will rather concentrate on some more or less realistic models I have been considering with my students. These may turn out to be useful in future more realistic attempts at formulating relativistic statistical mechanics.

As a general reference, containing a complete bibliography which I found extremely helpful, I would like to recommend the recent report by Jürgen EHLERS: "Progress in Relativistic Statistical Mechanics, Thermodynamics and Continuum Mechanics", Talk at GR 7, Tel Aviv, June 1974.

I will be mostly interested in general-relativistic statistical mechanics (GRSM), will briefly mention the problems of special-relativistic statistical mechanics (SRSM), and would only like to note briefly that the best developed area of this field is relativistic kinetic theory (RKT), which is treated in detail in Ehlers' reviews.

1. Why Relativistic Statistical Mechanics? We might as well ask why not? In addition to its academic interest (even the question of how to define equilibrium becomes difficult if time is not uniquely determined) and its potential heuristic value in learning how to quantize gravitation, astrophysics poses some "practical" problems requiring concepts from GRSM (albeit on a cosmic scale one can hardly talk of an equilibrium SM). There are also problems in relativistic plasma theory which might require the use of curvilinear coordinates. Last, and not least, some of the techniques used (at least in our models) bear a strong resemblance to "gauge theory" methods in quantum field theory, and might lead to the development of useful mathematical tools for the latter.

2. GRSM Without SRSM? Since locally general relativity reduces to Minkowski space, one might ask whether it makes any sense to attempt a GRSM before SRSM is on a sound footing. Any straightforward attempt to formulate classical statistical mechanics in a Poincaré invariant manner runs into the difficulty with the famous "no-interaction" theorems encountered in relativistic Hamiltonian mechanics. Non-Hamiltonian approaches (e. g., Hakim's formulation) use measures on world lines and

do not lend themselves too easily to generalizations to curved spaces.

We will therefore attempt to discuss models in which GRSM is in a sense "locally nonrelativistic", but evolves on a curved Riemannian manifold, thus simulating a gas in an external prescribed gravitational field. No attempt has been made yet to attack the self-consistent problem of including the gravitational field among the observables (in particular, the connection coefficients, which seem to be more suitable for this than the components of the metric tensor). Another model under study, which is locally "relativistic", is modeled on relativistic field theory. Throughout, when I talk of observables and states I do not necessarily exclude classical statistical mechanics. The models are designed to be vague enough to accomodate classical observables (abelian algebras, states are measures) and quantum systems (operator algebras and their locally normal states, i. e., local density matrices). Detailed results will be published elsewhere.

3. Models. A. "Locally Nonrelativistic" GRSM. Due to the need to define equilibrium in terms of a unique "time" variable we shall consider an Einstein space with a synchronous comoving coordinate system, such that the metric has the form

$$ds^2 = dt^2 - g_{ik}dx^i dx^k \quad (i,k = 1, 2, 3). \qquad (1)$$

This choice of a single "time" may introduce some fictitious "potentials" which affect the definition of equilibrium; a more refined discussion of this point is necessary, as well as the notion of time evolution and equilibrium). The system under consideration "lives" on the 3-dimensional manifold $t = \text{const.}$, with the metric determined by the second term of the r.h.s. of Eq. (1). In the tangent space to each point of this manifold we consider an algebra of observables (either a local phase space with its functions, or an algebra of observables generated by creation-annihilation operators). It is important to note that in this model the isotony requirement for the quasilocal algebra is imposed in the tangent space, not the curved manifold, and thus the problem of algebras associated to "large" sets is circumvented. Instead there arises the problem of comparing algebras and their observables and states in the tangent spaces to different points of the manifold. We propose to use the connection associated to the metric for transporting observables and states. In particular, transport around a closed loop in the manifold induces an automorphism in the local algebra (which for obvious reasons is called a holonomy automorphism), and a transposed action on the states, such that expectation values are left invariant. In the simplest examples of homogeneous spaces the holonomy automorphism may turn out to be trivial, yielding no new restrictions on the model.

B. Locally Relativistic GRSM. This case is considerably more difficult, since it assumes that the local (tangent-space) theory is invariant under the Poincaré group. A model of this type could be obtained by considering quasilocal algebras based on "causally convex charts" of a four-dimensional pseudoriemannian manifold (roughly, a causally convex chart can be made to look like an ordinary double cone by means of a special choice of coordinates). The main problem here is how to characterize equilibrium states, i. e., whether one can impose reasonable conditions leading to something akin to a "local KMS property", and what the meaning of the appropriate β parameter is. It is worth recalling that in a relativistic Gibbs enesemble the reciprocal temperature becomes a four-vector by multiplication with the four-velocity, the Hamiltonian being replaced by the appropriate integral over the energy-momentum tensor, etc. In this model too, observables tangent to different points should be compared by means of the connection, one must investigate the action of holonomy automorphisms (at least for spacelike loops) and expectation values should be invariant. So far no definite results have been obtained, but the reward for constructing a meaningful model of this kind are great: it might be a prelude to an "algebraic" approach to quantum gravitation. Relations to the approaches of Arnowitt-Deser-Misner, De Witt and Faddeev-Popov will be explored.

4. RKT. As already mentioned, this area is well covered in the literature. There remain problems (e. g., deriving a BBGKY hierarchy and solving various kinetic equations) which are under investigation.

5. Conclusion. In this brief review I have managed only to point to a few of the difficult problems encountered in this subject. The models discussed are not very physical and the presentation given here, at least, would not qualify as highly mathematical. Therefore this talk could be classified in the category $\overline{M} \cup \overline{\Phi}$ (the bar means not) the dual of the proposition $M \cap \Phi$. But since the first statement is not the whole Universe, by duality we have another proof that $M \cap \Phi \neq \emptyset$!

In conclusion I would like to thank our Japanese hosts on behalf of all of us for a wonderful conference: ARIGATOH GOZAIMAS!

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Q: (R. Arens): Does the statement about non-interaction in relativistic mechanics remain meaningful in quantum theory?

A: In my opinion, no.