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DISCUSSION

BERMAN: I would like to ask with what accuracy must you measure the process $p\mu+p \rightarrow p\mu+p$ in order to see a difference between the $F=0$ and $F=1$ substates?

DZHELEPOV: The cross-section $\sigma_{p\mu+p} = (1.7^{+0.4}_{-0.5}) \times 10^{-19}$ cm² found by us is in a satisfactory agreement with the value 3×10^{-19} cm² which was theoretically calculated by Cohen *et al.* However, the value 3×10^{-19} cm² is calculated without considering the hyperfine structure of the mesonic atom $p\mu$. If the scattering lengths $a_u = +5$ and $a_g = -11$, determined by Cohen for symmetrical and antisymmetrical state of the $p\mu+p$ system, are used, the calculated cross-section in the $F=0$ state will be about 20 times less than the experimental one. The difficulty is that at present it is impossible to prove theoretically what is the right choice of values for the lengths mentioned. For example, if in the case of $a_u = 5$ we assume that $a_g = +3$ or -30, the theoretical cross-section for $F=0$ will be in a good agreement with the experimental value. Thus, to obtain more definite conclusions about the agreement of the experimental data with the theory, one must have more precise data on the scattering lengths. I think that a further increase of the precision of the experiment will be necessary.

ELECTRODYNAMICAL CROSS-SECTIONS AT HIGH ENERGY

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(presented by P. Budini)

In a preceding work [1] we have shown that the total cross-section for positron-electron annihilation into photons, taking into account terms up to e^6 order, can be written in the form

$$\sigma_T = \sigma_0(1+\delta_T)\,,$$

where σ_0 is the Born approximation. As the total energy E goes to infinity the leading term of δ_T is proportional to $(\alpha/\pi)\ln^2(2E/m)$, and it has been shown that this term comes from soft photon emission.

This result can be generalized. In fact it is known [2] that the contribution of soft photon emission to a process in which charged particles are emitted (absorbed) can be factorized in the matrix element, and this is true for every order of the S matrix expansion.

Those contributions can be summed up and, following a line of thought similar to that of Eriksson [3], we get for the probability density of the process described by the matrix element M, accompanied by the emission of any number of soft photons with total energy $\leqq \Delta E \ll m$ (in a particular reference system):

$$P(\Delta E) = \left(\frac{\Delta E}{E}\right)^c e^{B+\mathrm{Re}A} \frac{e^{-\gamma c}}{\Gamma(1+c)} |M|^2\,, \tag{1}$$

where

$$A = \int \frac{d^4k}{k^2+\Lambda^2} \cdot \frac{1}{2\pi i} S_\mu(k) S_\mu(-k)$$

and Λ is the usual fictitious mass of the photon. In the limit as Λ tends to zero A diverges logarithmically:

$$A = A_1(p_i, p_f) - c \ln \frac{\Lambda}{m}$$

with $$A_1 = \frac{\alpha}{\pi} \sum_{i,j} Q_i Q_j (p_i p_j) \int_0^1 \frac{dx}{2p_x^2} \ln \left(\frac{-p_x^2}{m^2} \right)$$

$$p_x = x(p_i \varepsilon_i - p_j \varepsilon_j) - \varepsilon_j p_j$$

The other symbols are those of Eriksson. $|M|^2$ contains all the radiative corrections due to virtual photons. The first three factors of Eq. (1) originate from soft photon emission. Of these factors exp (Re A_1) is the only one which may contain terms proportional to $\alpha \ln^2 (q^2/m^2)$. In particular for positron-electron annihilation one obtains

$$\text{Re} A_1 \approx \frac{2\alpha}{\pi} \ln^2 \frac{2E}{m}. \quad (2)$$

For elastic scattering in an external field we have:

$$\begin{cases} \text{Re} A_1 \approx \dfrac{\alpha}{2\pi} \ln^2 \dfrac{q^2}{m^2} & q^2 \gg m^2 \\ \text{Re} A_1 \approx -\dfrac{\alpha}{\pi} \left[\ln \dfrac{q^2}{m^2} + 6 \ln 2 + 2 \right] & q^2 \ll m^2 . \end{cases} \quad (3)$$

It is interesting to observe that Eq. (2) is independent of angles. This angular independence can be easily understood observing that in positron-electron annihilation soft photons are emitted only from the incoming particles.

The problem whether terms of the form $\alpha \ln^2 (q^2/m^2)$ show up in the expansion of $P(\Delta E)$ will depend on the behaviour of $|M|^2 e^{-c \ln(\Lambda/m)}$ in Eq. (1). First of all it can be easily shown that this term does not contain infra-red divergences as $\Lambda \to 0$. In fact they can be eliminated term by term in the expansion of $|M|^2$ and exp $[-c \ln (\Lambda/m)]$ in powers of α; we put then

$$|M|^2 e^{-c \ln (\Lambda/m)} = |M_0|^2 X(p_i, p_f)$$

where $|M_0|^2$ represents the first Born approximation. The discussion now is limited to the behaviour of X as a power series of α. In order to avoid for the moment the difficulty arising from the unsolved problem of the convergence of this series we will assume that all that is said is valid up to a certain approximation in α^n ; in other words, a finite number of terms of the expansion is taken into account everywhere.

On the assumption, according to Eriksson and Petermann [3, 4], that for high momentum transfer the $P(\Delta E)$ expansion (in the c.m. system) does not contain $\alpha \ln^2 (q^2/m^2)$ terms, the behaviour of X will be such as to compensate the $\alpha \ln^2 (q^2/m^2)$ which comes from exp (Re A_1). This is confirmed by explicit calculations available for elastic scattering and also for inelastic processes at large angles in c.m.

It then remains to examine the situation at low momentum transfer for elastic scattering and for forward inelastic processes. In these cases X tends to a constant (at least the first terms of its expansion in powers of α). For elastic scattering Re A_1 does not contain $\alpha \ln^2 (q^2/m^2)$ terms, and so the cross-section also does not have contributions of this type. The problem of inelastic scattering is of particular interest for processes such as positron-electron annihilation into photons, where the forward differential cross-section is finite. In this case, Eq. (2) being independent of the kinematical situation, one can write for the forward differential cross-section at high energy:

$$d\sigma_f \to \left(\frac{\Delta E}{E} \right)^c e^{\frac{2\alpha}{\pi} \ln^2 \frac{2E}{m}} d\sigma_{0f} . \quad (5)$$

It is possible to generalize this result also to the case $m < \Delta E \ll E$. In particular if one assumes that the contribution from hard photon emission introduces powers less than $\alpha \ln^2 (E/m)$ (this can be verified up to the e^6 order), by integration of Eq. (5) over ΔE one obtains

$$d\sigma_f^i \to e^{\frac{2\alpha}{\pi} \ln^2 \frac{2E}{m}} d\sigma_{0f} , \quad (6)$$

where $d\sigma_f^i$ is the cross-section for inelastic forward annihilation (the forward photon has any energy).

Consider now that in Eq. (6) the correction to $d\sigma_{0f}$ is positive and that for non-forward annihilation the terms of the series will have only an $\alpha \ln (E/m)$ dependence. It is then of some interest to assume that the perturbation expansion of X is convergent and that in the high-energy limit only the higher powers of $\ln(E/m)$ contribute appreciably to the sum of the expansion. In this case one can give an upper limit to the asymptotic total cross-section for annihilation in photons:

$$\sigma_T \rightarrow \leqq e^{\frac{2\alpha}{\pi}\ln^2\frac{2E}{m}}\sigma_{0T} . \tag{7}$$

If the above assumption is not valid, then Eq. (7) refers to the expansion in powers of α, where only a finite number of terms is taken into account.

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DISCUSSION

KÄLLÉN: I always feel somewhat dizzy when I hear arguments of this kind and I would just like to ask you a few questions. I quite understand what happens when you limit yourself to the first two or three or four terms in the expansion. This is what is most interesting from the experimental point of view. However, you don't stop there, but you go further and sum the series over all n's. Now, it is believed by several people, including myself, that this series really is divergent. You pick out here what is supposed to be the most important contribution of each term, and that seems to form a convergent series. Would you say that your argument here " proves " the convergence of the series?

BUDINI: Oh well, no... I do *suppose* that it is convergent.

KÄLLÉN: I could possibly write one or two formulae, which I hope clarify the situation a little. Of course, I emphasize from the beginning that no one really knows what is going on here in the high E high n limit, but, just for the sake of argument, let us suppose that the real series looks the following way:

$$\sum_{n=0}^{\infty} \alpha^n \left[n\,! + \frac{(\ln E)^n}{n\,!} \right]$$

Just suppose that is how the series looks. Of course, if you take a given term (that is a fixed value of n) and ask what is going to happen in the high E limit you will pick out the $(\ln E)^n/n!$ term and you will get a convergent series. Apart from a couple of 2's and π's it is just the series that you have given before. However, the series is really divergent, and the asymptotic sum you get has nothing to do with the real sum. You can make another example which is slightly less drastic, namely if you consider the following series:

$$\sum_{n=0}^{\infty} \alpha^n \left[1 + \frac{(\ln E)^n}{n\,!} \right]$$

Again, considering this for fixed values of n and large E's you will pick out the same series as before. In reality the convergence of this series is determined by the parameter a and for the high n fixed E limit the important thing is the first term. You would rather expect the whole sum to be proportional to $1/(1-\alpha)$ instead of what you found, which is roughly $E^{\alpha \ln E}$. Please, do not misunderstand me. I am not saying that electrodynamics looks like this or looks like that, but I think this example, especially the last one, where you are really operating on a convergent series, shows that these arguments are extremely tricky. One must be careful. Even if it is clear what happens in the first few terms with fixed n and high E, it is very dangerous to go further and sum the series in this way. It may be right, but it's dangerous.

BUDINI: I agree with Prof. Källén's argument. But there is one consideration that one has to bear in mind, namely, it looks as if the contribution which can be summed comes from soft photons, which can be treated quite generally with not very difficult mathematical methods, so one can easily take into account the general term in that case.

SALECKER: I would propose to transfer these considerations to the Compton effect. For example, in the forward direction the double Compton effect vanishes exactly. So in that case we are free from the energy-resolution limit and free from the complications coming from the infra-red terms.

BUDINI: The Compton effect is very similar to the case of electron scattering in an external field. Again you have an angular dependence of $\mathrm{Re}\,A_1$, and again you have the $\ln^2 E$ terms cancelled, so in the Compton effect you have the " regular behaviour ".