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2014 J. Phys.: Conf. Ser. 569 012084

(http://iopscience.iop.org/1742-6596/569/1/012084)

Journal of Physics: Conference Series **569** (2014) 012084 doi:10.1088/1742-6596/569/1/012084

Essential $\bar{K}$ cluster "K^-pp" studied with a coupled-channel Complex Scaling Method + Feshbach method

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Abstract. We have investigated an essential $\bar{K}$ cluster "K^-pp", which is the simplest system of kaonic nuclei, with a coupled-channel Complex Scaling Method (ccCSM). Combining the ccCSM with Feshbach method we can handle a coupled-channel problem effectively as a single-channel problem. As a result of a study of the K^-pp with the ccCSM+Feshbach method using an energy-dependent chiral-theory based potential, it is found that the K^-pp is shallowly bound with the binding energy of around 20-35 MeV. The mesonic decay width is rather dependent on the interaction parameters and ansatz; the half decay width is ranging from 20 to 65 MeV.

1. Introduction

In strange nuclear physics and hadron physics, kaonic nuclei (nuclear system with anti-kaons, $\bar{K} = K^-, \bar{K}^0$) have been a hot topic since the formation of dense state are interestingly expected due to strong $\bar{K}N$ attraction. To consider such an interesting system, let us start with an excited hyperon $\Lambda(1405)$ ($J^\pi = 1/2^-$, $I = 0$). The early work with a quark model succeeded the reproduction of most spectra of P-wave baryons, except for the $\Lambda(1405)$ whose mass was resulted to be larger than the observed value [1]. Since the $\Lambda(1405)$ exists below the $\bar{K}N$ threshold by just 30 MeV, it is expected to be a $\bar{K}N$ quasi-bound state. Actually, chiral unitary models, which are based on a meson-baryon picture, have succeeded to explain various properties of the $\Lambda(1405)$ [2]. Thus, we believe that the $\Lambda(1405)$ is a $\bar{K}N$ quasi-bound state, rather than a genuine three quark state. Based on this picture, Akaishi and Yamazaki proposed a phenomenological $\bar{K}N$ potential, which is nowadays called AY potential [3]. AY potential is quite attractive especially in the isospin-zero channel. As a result of a fully-microscopic calculation of kaonic nuclei with antisymmetrized molecular dynamics (AMD) method using AY potential [4], it is found that a single K^- meson can be deeply bound in various light nuclei with 100 MeV binding energy. The AMD study shows exotic properties of kaonic nuclei. Due to the strong $\bar{K}N$ attraction, nucleons are attracted by a K^- meson to form dense nuclear state; the average density amounts

Table 1. Summary of theoretical studies of K^-pp. "$B(K^-pp)$" and "Γ_M" are the binding energy and mesonic decay width of K^-pp, respectively. They are in unit of MeV. "Method" and "Potential" mean a method and a $\bar{K}N$ potential employed in each calculation, respectively. In variational calculations two kinds of basis function are used: Gaussian function "(Gauss)" and Hyperspherical Harmonics "(H.H.)". In these calculations, phenomenological potentials ("Pheno.") and chiral-theory based potentials ("Chiral") are examined. In the latter potentials, there are energy-dependent ("(E-dep.)") and energy-independent ("(E-indep.)") versions. "Non-rel." and "Semi-rel." indicate non-relativistic and semi-relativistic kinematics, respectively.

	Dote-Hyodo -Weise [6]	Akaishi -Yamazaki [7]	Barnea-Gal -Liverts [8]	Ikeda-Sato [9]	Shevchenko-Gal -Mares [10]
$B(K^-pp)$	20 ± 3	47	16	$60 \sim 95$	$50 \sim 70$
Γ_M	$40 \sim 70$	61	41	$45 \sim 80$	$90 \sim 110$
Method	Variational (Gauss)	Variational (Gauss)	Variational (H.H.)	Faddeev-AGS	Faddeev-AGS
Potential	Chiral (E-dep.)	Pheno.	Chiral (E-dep.)	Chiral (E-indep.)	Pheno.
Kinematics	Non-rel.	Non-rel.	Non-rel.	Semi-rel.	Non-rel.

to several times higher than the normal nuclear density. In addition even three protons can be bound by a single K^- meson with a funny structure. Thus, the kaonic nuclei are expected to be a exotic system which could be in particular a doorway to dense nuclear matter.

To clarify exotic properties, which are expected, involved with kaonic nuclei, we have investigated the most simplest kaonic nucleus "K^-pp" before considering complicated many-body systems. The K^-pp seems to be an *essential cluster* of kaonic nuclei, while the $\Lambda(1405)$ (= a quasi-bound state of K^-p) can be regarded as a building block of kaonic nuclei. Since the K^-pp is only a three-body system composed of two protons and a single K^- meson, it has been investigated with various ways as summarized in Table 1. The binding energy and decay width are rather dependent on approaches and employed potentials. However, all theoretical studies show that the K^-pp can be bound with less than 100 MeV binding energy. Since there is $\pi\Sigma N$ threshold at 103 MeV below $\bar{K}NN$ threshold, these calculations indicate that the K^-pp should be a resonant state between the two thresholds. In addition, we know the fact that the $\bar{K}N$ is strongly coupled with the $\pi\Sigma$ through the study of the $\Lambda(1405)$ [2]. Thus, we consider that *1. Resonance* and *2. Coupled-channel problem* are key ingredients in the theoretical study of the K^-pp. We, here, employ a coupled-channel Complex Scaling Method (ccCSM) since this approach can simultaneously treat these two ingredients. It should be noted that the complex scaling method has greatly succeeded in the study of resonances of unstable nuclei [11]. In our previous study [12], we applied the ccCSM to the two-body system of $\bar{K}N$-πY. (Y means Λ and Σ hyperons.) Since that study shows that the ccCSM is a useful tool also for the study of hadronic system, we now tackle the three-body system of K^-pp.

2. Methodology

In this section, we explain our method to investigate the K^-pp resonance shortly. At first, we remark that the K^-pp is considered as a coupled-channel system of $\bar{K}NN$-$\pi\Sigma N$-$\pi\Lambda N$ with $J^\pi = 0^-$ and $I = 1/2$ in theoretical studies.

Basically, we follow the usual prescription of the complex scaling method [11] to calculate

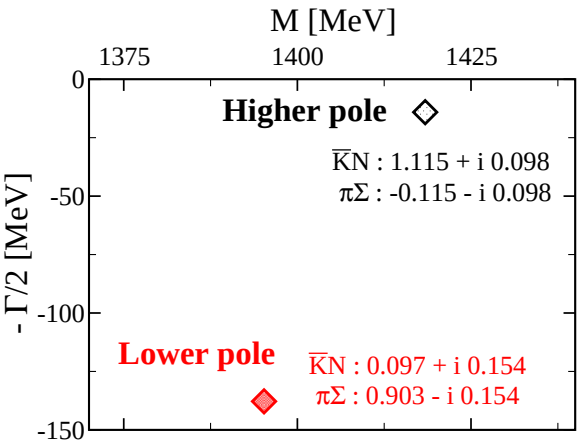

Figure 1. Double pole of $\Lambda(1405)$ obtained with the ccCSM. NRv2 potential with $f_\pi = 110$ MeV is used. The complex values of "$\bar{K}N$" and "$\pi\Sigma$" are norms of $\bar{K}N$ and $\pi\Sigma$ components in the pole states, respectively.

complex eigenvalues of the three-body system of K^-pp. The Hamiltonian for the K^-pp is complex-scaled, with a complex-scaling operator $U(\theta)$ the coordinate and the conjugate momentum is transformed as $\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}$ and $\boldsymbol{p} \to \boldsymbol{p}e^{-i\theta}$, respectively. Diagonalizing the complex-scaled Hamiltonian with a basis function, complex eigenvalues are obtained. Among those eigenvalues, the eigenvalues which are independent of the scaling angle θ are those of resonant states. In our study we employ the correlated Gaussian function [13] as a basis function. We give detailed explanation on important points of our study as below.

2.1. A chiral-theory based $\bar{K}N$ potential and treatment of its energy dependence

In our previous work [12], we proposed a chiral-theory based $\bar{K}N$-πY potential which is a local potential with a single-range Gaussian form factor in the coordinate space so that we can easily handle it with Gaussian basis function:

$$V_{ij}(r) = -\frac{C_{ij}}{8f_\pi^2}(\omega_i + \omega_j) \times [\text{flux factor}] \times (\text{Gaussian form factor}), \tag{1}$$

where ω_i indicates the meson energy in the channel i and C_{ij} is the Clebsch-Gordan coefficient in the SU(3) algebra. In the present study we use one of non-relativistic versions of our potential, which is denoted as NRv2 potential. Detail of the NRv2 potential is given in Eq. (8) in Ref. [12]. Note that the pion decay constant f_π in Eq. (1) is treated as a parameter in our potential.

Here, we mention the recent progress of our study. According to many studies based on the chiral SU(3) theory, there appear two poles in s-wave and $I = 0$ channel, which corresponds to the $\Lambda(1405)$, when we use the energy-dependent chiral potential [2]. However, such a *double-pole structure* of the $\Lambda(1405)$ was not clearly observed in our previous work [12], though the potential was constructed based on the chiral SU(3) theory. Recently, we have successfully confirmed the double-pole structure in our potential by using the improved Gaussian basis function [14], as shown in Fig. 1. Analyzing the norm of $\bar{K}N$ and $\pi\Sigma$ components in each pole state, we have found that the higher-pole state is dominated by the $\bar{K}N$ component since the $\bar{K}N$ norm is a large magnitude. On the other hand, in the lower pole the $\pi\Sigma$ component is found to be a major component.

By the way, as mentioned above our $\bar{K}N$ potential has an energy dependence. We need to take into account the self-consistency for $\bar{K}N$ energy when we treat bound and resonant states. But, the definition of the $\bar{K}N$ energy in the K^-pp is non-trivial because the two-body $\bar{K}N$ is a subsystem of the three-body $\bar{K}NN$ system. We define the $\bar{K}N$ energy (E_{KN}) in such a way as proposed in an earlier study [6]. In that study the E_{KN} is defined in two ways by considering extreme two pictures; The antikaon is regarded as a field (Ansatz 1) or it is considered as a

particle (Ansatz 2). Details of calculation of the E_{KN} is given in Eqs. (20) and (21) in Ref. [6]. Also in the present study, we examine these two ansatz. We remark that this self-consistency is realized for the *complex* $\bar{K}N$ energy since the complex energy of the resonance pole is directly treated in the current work, while the real $\bar{K}N$ energy is considered in in the earlier work with a variational approach [6].

2.2. Essence of the ccCSM+Feshbach method

The K^-pp system is a coupled-channel system of $\bar{K}NN$, $\pi\Sigma N$ and $\pi\Lambda N$. We treat such a multi-channel problem as a single-channel problem with help of Feshbach method [15]. In Feshbach method, we set a model space (P space) and outer space of the model space (Q space). Then, the Schrödiner equation is given as a coupled-channel equation of wave functions for P and Q spaces. Eliminating the Q-space wave function, we can construct a Schrödiner equation only for the P-space wave function Φ_P as $[T_P + U_P^{Eff}(E)]\Phi_P = E\Phi_P$. ($T_P$ is a kinetic-energy operator for the P space.) Here, the effective potential for P space, $U_P^{Eff}(E)$, is formally given as

$$U_P^{eff}(E) \;=\; V_{PP} \,+\, V_{PQ}\, G_Q(E)\, V_{QP} \quad \text{with} \quad G_Q(E) \;=\; \frac{1}{E - H_{QQ}}, \tag{2}$$

where $G_Q(E)$ is the Green function for the Q space and $\{V_{XY}\}$ indicates the coupled-channel potential for the P and Q spaces with (X, Y)=P or Q.

Certainly, we can eliminate the Q-space component with the Feshbach method. However, the problem is how to express the $G_Q(E)$ in actual calculations. We overcome this matter with an interesting nature of the complex scaling method. The closure relation is proven to hold also in the CSM, including resonant states explicitly in addition to bound and non-resonant continuum states. (*Extended Closure Relation*, ECR [11]). The ECR is known to be well described approximately with a set of finite number of the eigenstates $\{|\chi_n^\theta\rangle\}$ which are obtained by the diagonalization of a complex-scaled Hamiltonian H^θ with a Gaussian basis function; $\sum_n |\chi_n^\theta\rangle\langle\tilde{\chi}_n^\theta| \simeq 1$ [16].

With help of the ECR, the complex-scaled Green function $G_Q^\theta(E) = (E - H_{QQ}^\theta)^{-1}$ is represented as

$$G_Q^\theta(E) \;=\; \sum_n \frac{|\chi_{Q,n}^\theta\rangle\langle\tilde{\chi}_{Q,n}^\theta|}{E - \epsilon_n^\theta} \quad \text{with} \quad H_{QQ}^\theta\,|\chi_{Q,n}^\theta\rangle \;=\; \epsilon_n^\theta\,|\chi_{Q,n}^\theta\rangle, \tag{3}$$

where the energy eigenvalue ϵ_n^θ and eigenstate $|\chi_{Q,n}^\theta\rangle$ are obtained by the diagonalization of the complex-scaled Hamiltonian H_{QQ}^θ with Gaussian basis functions. Since the original Green function for the Q space is obtained from $G_Q^\theta(E)$ as $G_Q(E) \;=\; U^{-1}(\theta)\, G_Q^\theta(E)\, U(\theta)$, the effective potential for the P space is

$$U_P^{eff}(E) \;=\; V_{PP} \,+\, V_{PQ}\, G_Q(E)\, U^{-1}(\theta)\, G_Q^\theta(E)\, U(\theta)\, V_{QP}. \tag{4}$$

Here, the $G_Q^\theta(E)$ is given as Eq. (3) and is represented with Gaussian functions. Therefore, since the effective potential $U_P^{eff}(E)$ is also composed of Gaussian functions, it can be used as usual in the complex scaling method with Gaussian basis function.

We apply this technique to the calculation of the K^-pp system. Setting the $\bar{K}N$ channel as P space and the $\pi\Sigma$ and $\pi\Lambda$ channels as Q space, we construct an effective $\bar{K}N$ potential $U_{\bar{K}N}^{eff}(E)$. In other words, the πY channels are eliminated at the step of two-body calculation. The $U_{\bar{K}N}^{eff}(E)$ plugged in the three-body Hamiltonian of the $\bar{K}NN$, we solve the single-channel problem of the $\bar{K}NN$ system with the complex scaling method.

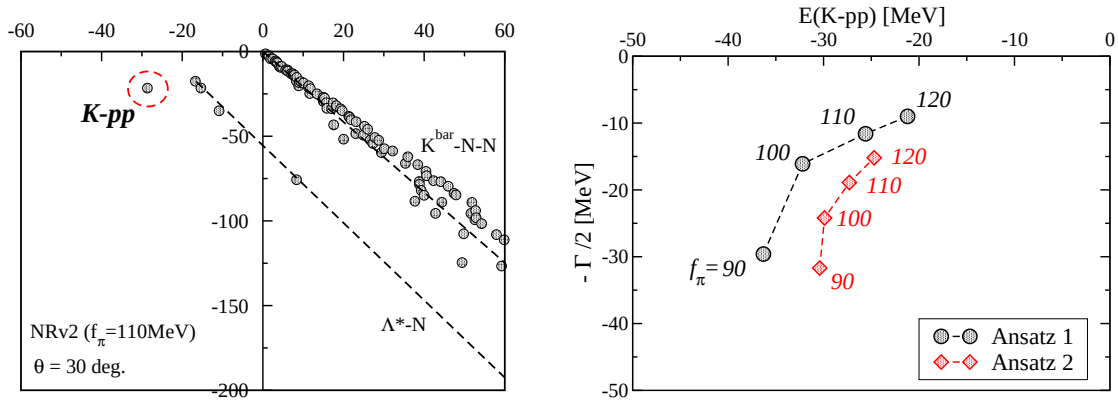

Figure 2. (Left) Complex-energy eigenvalue distribution when $\bar{K}N$ energy for the effective potential is fixed to the Λ^* energy. (Right) Pole position of K^-pp obtained by self-consistent calculation, where f_π is varied from 90 MeV to 120 MeV and two ansatz for $\bar{K}N$ energy are examined.

3. Result

We show our result of the K^-pp calculated with the ccCSM+Feshbach method. Here, we employ a version of our potential, NRv2, in which the f_π value is set to be 110 MeV as a typical case.

At first, we consider the case where the $\bar{K}N$ energy in the energy-dependent $\bar{K}N$ potential is fixed to that for the $\Lambda(1405)$. In other words, the $\bar{K}N$ energy is not self-consistent in the K^-pp, but it is self-consistent for the isolated two-body system of $\bar{K}N$-$\pi\Sigma$ which forms the $\Lambda(1405)$ resonance. The obtained complex-energy eigenvalue distribution is shown in the left panel of Fig. 2. In the figure the origin of the real energy axis corresponds to the $\bar{K}NN$ three-body threshold. It is known that in the complex scaling method the continuum states appear on so-called 2θ line ($\tan^{-1}(\mathrm{Im}\,E/\mathrm{Re}\,E) = -2\theta$) when the scaling angle is θ [11]. Therefore, the eigenvalues along the 2θ line running from the origin indicate the continuum states of $\bar{K}NN$ three-body system. There are eigenvalues on the other line. The starting point of this line is $(-16.7,\ -17.5)$ MeV which is almost identical to the complex energy of the Λ^* [12]. (Λ^* denotes the higher pole of the $\Lambda(1405)$.) So, the eigenvalues on the second line indicate the Λ^*N two-body continuum states. We can see a point which is isolated from the two lines mentioned above, as marked with a red circle in the figure. This point means the K^-pp resonance. In this case where the $\bar{K}N$ energy is fixed at that of the Λ^*, the pole energy of the K^-pp resonance is found to be $(-B(K^-pp),\ -\Gamma_M/2) = (-28.6,\ -21.6)$ MeV.

Next, we take into account the self-consistency for the $\bar{K}N$ energy in the three-body system K^-pp, following the two ansatz as explained in the section 2.1. Searching a self-consistent solution, we have succeeded to find such a solution of the K^-pp resonance. The resonance energy is obtained to be $(-B(K^-pp),\ -\Gamma_M/2) = (-25.6,\ -11.6)$ MeV for Ansatz 1 and $(-27.3,\ -18.9)$ MeV for Ansatz 2. The binding energy $B(K^-pp)$ is not so dependent on the ansatz, whereas the decay width Γ_M strongly depends on it. Compared with the earlier work of the variational calculation with a chiral potential [6], the present calculation gives slightly deeper binding and narrower width.

We examine several f_π values, since the f_π value is a parameter of our $\bar{K}N$ potential. We have found the pole of the K^-pp resonance when the f_π value is varied from 90 MeV to 120 MeV, as depicted in the right panel of Fig. 2. As a result, the binding energy is found to be small, 20-35 MeV. The decay width is obtained to spread widely, 20-65 MeV. We note on the dependence of the ansatz. Both ansatz give similar binding energy. However, they give rather different decay width; Ansatz 1 tends to give small decay width, compared to Ansatz 2.

4. Summary and future plans

We have investigated the essential $\bar{K}$ cluster, K^-pp, with a coupled-channel Complex Scaling Method combined with Feshbach method. In the method, we reduce a coupled-channel problem to a single-channel problem in order to handle the system more easily, by utilizing the extended closure relation in the complex scaling method. Applying this method to the K^-pp, we have confirmed that the method works well to find the resonant pole clearly on the complex energy plane. Using a chiral SU(3)-based $\bar{K}N$ potential with a local Gaussian form, we have found that the binding energy and the mesonic decay width of K^-pp are roughly 20-35 MeV and 20-65 MeV, respectively. We consider that the K^-pp is shallowly bound as suggested by the early studies with variational methods using an energy-dependent chiral potential.

On the experimental side, there are several reports related to the K^-pp. According to the reports of Refs. [17, 18], if the observed state is a K^-pp state, the binding energy is more than 100 MeV. However, the situation is still controversial [19]. Recently, two experimental results on the K^-pp have been reported from J-PARC [20, 21]. Further analysis of these results will help to reveal the detailed nature of the K^-pp, compared with theoretical studies.

In the present study, we have incorporated the $\pi\Sigma$ and $\pi\Lambda$ components into the effective $\bar{K}N$ potential. In our future, we will consider the $\bar{K}NN$-$\pi\Sigma N$-$\pi\Lambda N$ coupled-channel problem without any channel elimination. By the explicit treatment of $\pi\Sigma$ and $\pi\Lambda$ channels we will obtain more accurate result on the K^-pp. In addition, the role of the πYN three-body dynamics is expected to be clarified as well as that of the $\bar{K}NN$ dynamics.

Acknowledgments

This work was developed through several activities (meetings and workshops) at the J-PARC Branch of the KEK Theory Center. This work is supported by JSPS KAKENHI Grant Number 25400286 and partially by Grant Number 24105008.

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