The Swampland: from macro to micro

HARVARD UNIVERSITY
Graduate School of Arts and Sciences

DISSERTATION ACCEPTANCE CERTIFICATE

The undersigned, appointed by the

Department of Physics

have examined a dissertation entitled

The Swampland: from macro to micro

presented by Alek Bedroya

candidate for the degree of Doctor of Philosophy and hereby certify that it is worthy of acceptance.

Signature ______________________________

Typed name: Professor Cumrun Vafa, Chair

Signature ______________________________

Typed name: Professor Matthew Reece

Signature ______________________________

Typed name: Professor Cora Dvorkin

Date: April 24, 2023

The Swampland: from macro to micro

A DISSERTATION PRESENTED
BY
ALEK BEDROYA
TO
THE DEPARTMENT OF PHYSICS

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN THE SUBJECT OF
PHYSICS

HARVARD UNIVERSITY
CAMBRIDGE, MASSACHUSETTS
APRIL 2023

Thesis advisor: Professor Cumrun Vafa Alek Bedroya

The Swampland: from macro to micro

Abstract

UV completing a gravitational theory, unlike many non-gravitational effective field theories (EFTs), is expected to be highly restrictive. This insight is rooted in black hole physics, which imposes a universality in high-energy behavior. The Swampland program aims to identify anomaly-free EFTs that lack a gravitational UV completion.

In this dissertation, we review a number of Swampland conjectures, their motivations, and their implications for low-energy theories. Our analysis will be divided into two main categories: implications for macrophysics (cosmological data) and implications for microphysics (particle physics data). Macroscopic data refers to cosmological evolution, while microscopic data include gauge symmetry and mass spectrum. By exploring the Swampland program, we aim to gain a deeper understanding of the profound connections between UV physics and its ramifications for EFTs in the low-energy regime.

Contents

For my parents, Jacqueline and Johnny, and my sister Olinka.

Acknowledgments

I would like to express my heartfelt gratitude to my advisor, Cumrun Vafa, for being an exceptional mentor who not only taught me how to think critically about physics but also provided invaluable perspectives on life. I was privileged to observe and learn how Cumrun approaches difficult physics problems from different angles and understands how they fit with an interconnected web of ideas. In addition to his great academic mentorship, I am also grateful for Cumrun's patience and support in the ups and downs of graduate school. I would also like to extend my thanks to Cumrun Vafa and Afarin Sadr for their warm hospitality, which made Cambridge feel like home much sooner than I anticipated. I would like to express my sincere gratitude to my other committee members, Matthew Reece and Cora Dvorkin, for their invaluable support and guidance throughout my Ph.D. journey.

I am grateful to my collaborators, Hessamadin Arfaei, Robert Brandenberger, Yuta Hamada, Marlinea Laverde, Dieter Lüst, Miguel Montero, Georges Obied, Sanjay Raman, Houri Christina Tarazi, Mahdi Torabian, Irene Valenzuela, and Max Wiesner for the privilege of working with them and benefiting from their expertise and brilliant ideas. I am also thankful to Jamie Law-Smith, Rashmish K. Mishra, Paul Steinhardt, John Stout, and Tina Torkaman for many insightful discussions from which I have learned a lot.

I would like to acknowledge the members of my research group, including Kaan Baykara, Jacob Mcnamara, Georges Obied, Houri Tarazi, and David Wu, for their curiosity and friendly presence, which contributed to a stimulating and supportive research environment. I am especially thankful to Georges Obied and Houri Tarazi for their invaluable guidance on navigating graduate school and their patience in entertaining my questions, no matter how basic they are.

I am grateful to my class and office mates, Alex Atanasov, Dominic Chang, Nicholas L. Cuccia, Pavel Dolgirev, Nathan Leitao, Qianshu Lu, Noah Miller, Atınç Çağan Şengül, Yanting Teng, Maria Tikhanovskaya, and Arthur Tsangand, for their friendly presence and continuous support. In addition, I thank Alex Atanasov, Dominic Chang, and Noah Miller for their great friendship and countless physics discussions that reinforced my joy in doing physics. I especially thanks Alex Atanasov for his continued support, which made it possible to navigate through rough personal patches in my Ph.D.

I am deeply grateful to Hengameh Bagherian for the blessing of our great friendship, which has been a source of joy and support through the ups and downs in the past 5 years. My Ph.D. journey would indeed look very different without this delightful friendship. I would also like to express my gratitude to Hamidreza Mahmoudian, Nader Mostaan, Kamran Salehi Vaziri, and Shahrzad Zare for many having many fun chats about physics and their lasting friendships since my college days.

I am especially grateful to my undergraduate advisor, Hessamadin Arfaei, whose support and encouragement played a pivotal role in my pursuit of physics. I also extend my thanks to all of my teachers from grade school, olympiad, and college, for nurturing my curiosity and instilling in me the value of seeking truth, knowledge, and critical thinking.

I would like to acknowledge Harvard University for providing me with an exceptional opportunity to grow and pursue truth. Special thanks to Lisa Cacciabaudo and Jacob Barandes for creating a welcoming and accommodating environment in the Physics department. I am also grateful to the Simons Summer Workshop for fostering a conducive environment for fruitful discussions and learning.

I express my appreciation to David and Roselle Hechendorn for leading the Fellowship of International Students at Harvard (FISH), the rest of the FISH family, and the Trinity Cambridge Church for providing me with a true community and making Cambridge feel like a home away from home. Your expression of the love of Christ has left a lasting imprint on me. I am also grateful to many other fellow brothers and sisters in Christ for their prayers and words of encouragement over the past 5 years.

I am immensely grateful to my parents, Jacqueline and Johnny, for their unwavering love, sacrifice, and support, which has brought me to this point in my life. I also thank my sister, Olinka, for inspiring me with her curiosity in physics and paving the way for my academic journey. But, most importantly, I thank my family for their constant loving presence in my life. I dedicate this dissertation to them, which is negligible compared to how much I owe them for their immeasurable influence on my life.

Finally, I would like to express my deepest gratitude to God for His immeasurable love and for blessing me with the opportunity to study and enjoy the beauty of His creation.

0

Introduction

A great deal of success in modern particle physics is owed to quantum field theory (QFT). The formulation of quantum field theory is predicated upon a collection of approximately local physical observables (i.e. fields). The local formulation of QFT allows for arbitrarily short-scale physical configurations which naivly dominate the quantum amplitudes. In other words, the local formulation of QFT leads to divergences associated with short-distance physics in perturbative calculations. However, renormalization (it could be order by order, such as gravity) allows one to resolve this problem within an energy regime by recasting the calculations in terms of the low-energy observables. This overall approach can be dubbed as *effective field theory*. In short, effective field theory assumes that at low-enough energies, the theory is described by local observables (fields), and that the physics at low energies can be studied independently from that of high energies.

Effective field theory (EFT) comes with theoretical restrictions for consistency. These are the cancellation of gauge anomalies. However, apart from that, the theory is very flexible. This is because EFT is defined to only work in an energy window. One might expect that if we go to higher energies, there is a more complete EFT which becomes valid. This approach has been very successful in particle physics. However, it breaks down when gravity is added to the mix. The main problems are lack of locality as well as UV/IR connection in the presence of gravity, and black holes lie at the core of both of these issues.

Locality

Suppose we want to measure the local operators of an effective field theory using scattering amplitudes. A natural limitation arises due to the uncertainty principle. The wavepacket of an ingoing particle of momentum p has a spatial spread of at least $\sim \hbar/p$. Therefore, to increase the precision of local measurements, one has to resort to higher and higher energies. In effective field theory, there is no fundamental limitation to increasing the precision of local observables by going to higher and higher energies. However, in the presence of gravity, a fundamental limitation exists. We give three arguments for why locality can only be an emergent approximation in quantum gravity based on black holes, holography, and dualities.

Locality: black hole argument

Consider two ingoing neutral particles with a center of mass energy M and a very small impact parameter b. We will soon clarify what we mean by a very small impact parameter. Gravity has a universal coupling to mass/energy, and accordingly, the gravitational pull between the particles will create a curved background which only depends on M and b. In particular, if M is large enough and b is small compared to the Schwarzshild radius associated with M, we expect to have a black hole as an intermediate state. Note that the size of the corresponding black hole will have a size that increases with the energy. Therefore, beyond the energy that black holes form, the size that we can probe with scattering experiments increases instead of decreasing. The turnaround scale is the mass of the smallest black hole. The above argument shows that in gravitational theories, the effective field theory breaks down at length scales smaller than the size of the smallest black hole. The inverse of this length scale is called the quantum gravity cutoff Λ_{QG}. We can see the connection between Λ_{QG} and the size of the smallest black hole in a second way which is instructive.

We refer to some point-like states of the theory as particles. But every particle has a mass and hence curves the spacetime around it. This raises the question of what separates particles from black holes? The answer lies in wether we can trust the fields (including the metric) close enough to the center of mass of the particle. In other words, is the horizon far enough from the center that we can refer to the particle as a black hole? Suppose Λ_{QG}^{-1} is the smallest length scale for which an EFT description is valid. Then the heaviest particle is one which has a Schwarzschild radius smaller than Λ_{QG}^{-1}. Anything heavier than that would be a black hole. Therefore, the size of the smallest black hole is Λ_{QG}^{-1}, which is the same as the length scale at which the effective field theory breaks down.

Locality: holographic argument

Another argument for why locality is an emergent approximation in quantum gravity also follows from the holographic principle. What we mean by the holographic principle is the most conservative form of holography, which states that physical observables in a gravitational theory must live on the boundary of spacetime which implies that bulk, local operators only give rise to an emergent description. A simple motivation for the holographic principle is that in quantum gravity, amplitudes include summation over spacetimes of different topologies. Therefore, a unique spacetime manifold which is the underlying structure necessary to define local operators, does not exist. However, a classical approximation can arise when a particular configuration extremizes the Quantum Gravity path integral.

Locality: duality argument

The fact that any classical picture (including a field theory) in quantum gravity is emergent rather than fundamental is nicely captured by dualities in string theory. For example, consider two T-dual descriptions of a spacetime with one compact dimension of size R in one frame and l_s^2/R in the other frame. Here l_s is the string length. As we change R/l_s from very small values to very large values, the semi-classical spacetime that provides the sharpest approximation to the quantum theory transitions from one description to another. There is a mapping between the descriptions, but there is no direct mapping between the point in the two spacetimes. For example, a local wavepacket in the compact dimension in one picture maps to a winding string with no notion of position in the compact dimension. Generally, the notion of spacetime and any ”local” operator associated with spacetime are expected to be emergent in quantum gravity.

Naturalness

As we saw earlier, the presence of gravity strongly implies that the effective field theory must break down at some energy scale. Therefore, any notion of locality is only emergent and approximate. This is important because many intuitions that we might build from EFT are not reliable in quantum gravity. For example, in a truly local theory, every EFT is expected to have a more UV-complete EFT. One can use the UV theory to estimate a natural order of magnitude for the observables at lower energies based on the cutoff of the IR theory. This idea is called *naturalness*. The idea of naturalness led to a lot of success. However, it also has created a fair share of puzzles. These puzzles include the electroweak hierarchy problem and the cosmological constant problem. As we discussed above, the basic assumption that the cutoff of field theory can always be increased breaks down in quantum gravity.

UV/IR connection

In EFT, we expect low-energy physics to be described independently from UV physics. This idea sits at the core of renormalization. However, this idea fails in gravitational theories. To see why, we go back to black holes and Hawking's calculation of their entropy. The black hole entropy calculation uses quantum field theory in curved spacetime. The larger the black hole, the smaller the curvature of its near-horizon geometry would be. This would also lead to lower Hawking temperatures. Therefore, Hawking's calculation becomes more trustable for larger black holes as it becomes a more IR calculation. But let us take a step back and ask what does it actually calculate?

The entropy of a black hole at energy M can be thought of as $\ln(\Omega(E))$ where $\Omega(E)$ is the number of states with energy E. These are states that generalize the notion of particles and become black holes due to gravitational screening. Therefore, black hole entropy teaches us that due to the IR behavior of gravity, the density of states at high energies has a universal behavior. This suggests that, as opposed to field theory, in quantum gravity, IR physics and UV physics are deeply connected and our intuition of effective field theory is often misleading. This brings us to the Swampland program. Let us first define the terminology of landscape and Swampland.

Swampland and Landscape: the consistent EFTs (i.e. no gauge anomalies) that do not have a QG UV-completion are said to be in the "***Swampland***" while the ones that do are said to belong in the "***Landscape***" of quantum gravity.

The Swampland program
Delineating the landscape of quantum gravities by finding criteria that ensure a theory belongs to the Swampland. For finding such criteria, we use universal observations in string theory as well as arguments based on more basic physics (unitarity, black hole physics, etc.).

Note that, by definition, our universe is in the Landscape. So finding criteria that cut away corners of the theory of space from the Landscape could lead to direct predictions about our universe.

In this dissertation, we propose and motivate some principles for quantum gravity based on string theory, black hole physics, holography, and consistency with more established Swampland conjectures. Furthermore, we study the implications of these conjectural principles for macrophysics (e.g. cosmology) as well as microphysics (e.g. gauge theory). The non-trivial implication of quantum gravity at the micro-level might be less surprising.

However, as we explained above, due to gravity's UV/IR connection, it is natural to expect non-trivial constraints on macrophysics as well.

Macroscopic scales

The observed value of dark energy sets a natural IR scale given by the Hubble parameter. As we discussed in the introduction, IR physics and UV physics are expected to be connected in quantum gravity. In particular, the Swampland conjectures aim to capture the conditions that UV physics imposes in the IR.

If the value of dark energy is explained by a cosmological constant, it suggests that de Sitter space must be realizable in quantum gravity. However, there is no known construction of eternal de Sitter space. In particular, in string theory, there is no known de Sitter construction under parametric control. For example, in any infinite distance limit in which we expect to have parametric control over the perturbation, the potential is always exponential in terms of the canonically normalized scalar field expectation value.

Let us very briefly review the challenges of constructing de Sitter space in string theory. At the tree level, it is known that the potential does not have any local minimum at infinite distance limits [1,2,3,4,5,6]. For example, if one considers an M theory compactification on an arbitrary manifold with arbitrary fluxes, the volume modulus will always find a runaway direction.

One might expect that the exponential decay of the potential at infinity is due to the emergence of supersymmetry at the infinite distance limit. However, there are known non-tachyonic and non-supersymmetric examples in string theory which exhibit the same behavior, such as the $O(16) \times O(16)$ Heterotic string theory [7,8]. In this example, the non-zero potential comes from the non-vanishing of the one-loop amplitude, which is common in non-supersymmetric theories. Even though the one-loop vacuum energy is constant in the string frame, when going to the Einstein frame, it leads to an exponentially decaying potential in the dilaton. However, it is important to note that the tree-level contribution vanishes. In fact, this is always true in any string theory due to the existence of three conformal Killing vectors on the sphere[9].

There are various scenarios proposed for corrections (quantum and classical) to give rise to a metastable de Sitter space[10,11]. However, such constructions are currently under debate (see[12,13] for some examples). The observations from string theory prompted the authors in[14,15] to propose the Swampland de Sitter conjecture, which states that the scalar potential always satisfies

$$|\nabla V| > c_1 V \quad \text{or} \quad |\Delta^2 V| < -c_2 V, \tag{1}$$

for some universal positive constants c_1 and c_2 which only depend on the spacetime dimension. In particular, this conjecture rules out the existence of a local minimum for a positive potential which would give rise to a metastable de Sitter.

The main motivation for the de Sitter conjecture comes from the weak coupling regime of string theory in the asymptotics of the field space. However, the statement of the conjecture in the interior of the field space is not directly connected to a more physical principle. Here we propose the Trans-Planckian Censorship Conjecture, which makes the UV/IR connection concrete in the cosmological setup and provides a physical reasoning for the de Sitter conjecture in the asymptotics of the field space.

The principle we propose, the Trans-Planckian Censorship Conjecture (TCC), simply put, states that in an expanding universe that could realize in a consistent quantum gravity theory, the sub-Planckian quantum fluctuations should remain quantum and can never become larger than the Hubble horizon and classically freeze[1].

In this part, we will study the implications and motivations of Swampland conjectures that are relevant for cosmology. After formulating TCC in chapter 1, we study its implications for scalar potentials in 2. Then we study the implications of Swampland conjectures for early universe and late time cosmology in 3 and 4 respectively. Finally, we connect TCC to thermodynamic aspects of de Sitter space in 5 and provide a holographic motivation for it in 6.

[1] This notion is different from the similarly named phenomenon discussed in[16,17].

Microscopic implications

As discussed earlier, Effective Field Theory (EFT) is highly flexible in terms of the theoretical consistency of low-energy physics. For instance, the rank of the gauge symmetry in EFT can be arbitrarily large, allowing for a wide range of possibilities. However, it is known that UV physics in gravitational theories is influenced by the number of light degrees of freedom. For example, the entropy of the smallest black hole must be greater than the number of light species [18]. While this argument does not impose a universal bound on the number of species, as it could be possible that the size of the smallest black hole increases with the number of species, it does highlight the sensitivity of the UV theory to the number of light species. In fact, in string theory constructions with sufficient supersymmetry, the number of particles is strongly constrained. For instance, in theories with 16 supercharges in d dimensions, the rank of the gauge groups r is always found to be less than $26 - d$. Even in cases with fewer supersymmetries, though the exact bound on the rank may not be known, the finiteness of the rank is expected. For example, theories with 8 supercharges can be constructed by compactifying type II theories on Calabi-Yau threefolds. Since the number of Calabi-Yaus is anticipated to be finite, the number of theories in the Landscape is also expected to be finite.

Despite the observation of the finiteness in string theory, a more bottom-up approach is necessary to bolster confidence in the aforementioned intuition. In recent years, significant progress has been made in this direction through the Swampland program. For instance, utilizing Swampland principles and the unitarity of the worldvolume theory of the supergravity string, it was demonstrated that the inequality $r \leq 26 - d$ holds in all theories with 16 supercharges. Although such sharp statements are more challenging to establish with less supersymmetry, the ability to provide a bottom-up rationale in theories with higher supersymmetries serves as a compelling proof of principle for the finiteness of the landscape.

In this dissertation, we will present a bottom-up classification of all the possible gauge groups that can arise in supersymmetric gravitational theories in theories with dimension 7 and more. Our arguments are based on Swampland principles as well as black hole physics.

Additionally, we will utilize Swampland principles to argue for dualities between 10 and 11 dimensional theories. These arguments provide a bottom-up rationale for the existence of heavy states that are typically not part of the EFT, but become light and hence part of the EFT in certain corners of the moduli space.

Publications and collaborators

The material covered in this thesis are based on the following publications and preprints.

- Chapters 1 and 2: A. Bedroya and C. Vafa, "Trans-Planckian Censorship and the Swampland," JHEP **09**, 123 (2020) [arXiv:1909.11063 [hep-th]] [19].
- Chapter 3: A. Bedroya, R. Brandenberger, M. Loverde and C. Vafa, "Trans-Planckian Censorship and Inflationary Cosmology," Phys. Rev. D **101**, no.10, 103502 (2020) [arXiv:1909.11106 [hep-th]] [20].
- Chapter 4: A. Bedroya, M. Montero, C. Vafa and I. Valenzuela, "de Sitter Bubbles and the Swampland," Fortschritte der Physik, 2000084 (2020) [arXiv:2008.07555 [hep-th]] [21].
- Chapter 5: A. Bedroya, "de Sitter Complementarity, TCC, and the Swampland," LHEP **2021**, 187 [arXiv:2010.09760 [hep-th]] [22].
- Chapter 6: A. Bedroya, "Holographic origin of TCC and the Distance Conjecture," [arXiv:2211.09128 [hep-th]] [23].
- Chapters 7, and 8: A. Bedroya, Y. Hamada, M. Montero and C. Vafa, "Compactness of brane moduli and the String Lamppost Principle in $d > 6$," JHEP **02**, 082 (2022) [arXiv:2110.10157 [hep-th]] [24].
- Chapters 9: A. Bedroya, S. Raman and H. C. Tarazi, [arXiv:2303.13585 [hep-th]] [25].
- Chapter 10: A. Bedroya and Y. Hamada, [arXiv:2303.14203 [hep-th]] [26]. and A. Bedroya, S. Raman and H. C. Tarazi, [arXiv:2303.13585 [hep-th]] [25].

Part I

The Swampland: macro

1

The Trans-Planckian Censorship Conjecture (TCC)

In a quantum gravitational theory, we do not believe that the notion of spacetime as a continuum would make sense at distance scales smaller than Planck length. However, in such a theory we can nevertheless have expansions in the background, which raises the question of what happens to these scales becoming larger than Planck length. In a consistent QG theory, the quantum fluctuations of this kind should remain quantum, in a way not to be contradictory with a classical picture of spacetime at larger scales. However, as is known in the context of inflationary models, when sub-Planckian quantum fluctuations become larger than the Hubble horizon $1/H$, they can become classical and freeze. This would lead to the classical observation of a sub-Planckian quantum mode, which is a bit strange! This is known as the inflationary trans-Planckian problem [27,28,29,30,31]. The traditional view of this problem has been that either we need more information to figure out what happens to these modes or that the structure of the quantum gravitational theory would give the same answer as if the modes were smooth even in the trans-Planckian domain. Here we would like to propose an alternative viewpoint: That such questions should never arise in a consistent quantum gravitational theory! That no trajectory of a consistent quantum theory of gravity should lead to a classical blow-up of the sub-Planckian modes to become larger than the Hubble horizon $1/H$ and that all the QFT's that do lead to this scenario belong to the Swampland.

1.1 Statement of TCC

We conjecture that a field theory consistent with a quantum theory of gravity does not lead to a cosmological expansion where any perturbation with length scale greater than the Hubble radius trace back to trans-Planckian scales at an earlier time. This could be formulated in terms of initial and final scale factors, a_i and a_f, and final Hubble parameter H_f as

$$\frac{a_f}{a_i} \cdot l_{pl} < \frac{1}{H_f} \Rightarrow \int_{t_i}^{t_f} H dt < \ln \frac{M_{pl}}{H_f}. \tag{1.1}$$

Note that if we take $l_{pl} \to 0$ or equivalently $M_{pl} \to \infty$ this condition becomes trivial, as it should with any Swampland condition. In the following we set (the reduced Planck mass) $M_{pl} \to 1$.[1,2]

Since the fluctuations growing bigger than the Hubble radius freeze out, if the wavelength of sub-Planckian quantum fluctuations become larger than the Hubble-radius they turn into

[1] Perhaps, a more accurate statement would be to say $\frac{a_f}{a_i} < \frac{KM_{pl}}{H_f}$ for some $\mathcal{O}(1)$ constant K. However, unlike other Swampland conjectures which depend on some $\mathcal{O}(1)$ constants, the consequences of TCC are rather insensitive to the exact value of K as it usually appears as a logarithmic correction. Therefore, in this chapter, we set K equal to 1, but one can easily restore the K-dependence in all of the results.

[2] Under time-reversal, the statement (1.1) of TCC for expanding universes, transforms into the following statement for contracting universes. A field theory consistent with a quantum theory of gravity does not lead to a cosmological contraction where any perturbation with length scale larger than the Hubble scale $(-1/H)$ evolve into the sub-Planckian scales at a later time. This could be mathematically formulated in the form in reduced Planck units. $\frac{a_i}{a_f} < -\frac{1}{H_i}$.

classical non-dynamical fluctuations. This leads to the following equivalent statement of TCC in terms of the quantum fluctuations.

An equivalent statement of TCC:

Sub-Planckian quantum fluctuations should remain quantum.

1.2 Immediate Consequences

Upper bound on H

Perhaps, the most immediate consequence of the conjecture (1.1) is that for the field theory description to not break down, H must be smaller than 1 at all times. This is natural as the Hubble parameter is usually proportional to the energy density which must be smaller than Planck energy density for the field theory description to be valid.

Upper bound on lifetime

Suppose the equation of state $w = p/\rho$ is greater than -1, we can show that the lifetime of universe beginning from $t = t_i$ when it started expanding could be bounded from above by its current value of Hubble parameter, H_f. Note that for any combination of conventional matter and radiation, cosmological constant and all of the quintessence models the assumption $w \geq -1$ holds[3]. The rate of change of the Hubble parameter in terms of the energy density ρ and the equation of state w is given by,

$$\dot{H} = -(1+w)\frac{\rho}{d-2}. \tag{1.2}$$

For $w \geq -1$, the above equation would imply that H is monotonically decreasing. Therefore, for every co-moving time interval $[t_i, t_f]$, we have

$$H_f T \leq \int_{t_i}^{t_f} H dt = \ln\left(\frac{a_f}{a_i}\right), \tag{1.3}$$

[3] This may in principle be violated for phases involving extended objects.

where $T = t_f - t_i$ is the lifetime and $H_f = H(t_f)$. Using the above inequality to bound the LHS of (1.1) leads to

$$T \leq H_f^{-1} \ln\left(H_f^{-1}\right). \tag{1.4}$$

Note that this could also be viewed as an upper bound H in terms of lifetime T. The TCC through the inequality (1.4) provides a prediction for the current age of the universe. For $H \approx 70(km/s)/Mpc$ this upper bound is $\sim$ 2 trillion years which is consistent with the age of our universe.

In general, TCC implies that the Hubble parameter must significantly decrease in a time scale given by $H^{-1}\ln(H^{-1})$. This statement applies to meta-stable de Sitters as well as quintessence solutions.

Decelerating expansions are consistent with TCC

Following, we give a general argument why violating TCC requires accelerating expansion or trans-Planckian energy density $H \geq 1$. The inequality (1.1) could be written as

$$\dot{a}_f < a_i. \tag{1.5}$$

Therefore, violation of TCC requires initial and final points where,

$$\dot{a}_f \geq a_i. \tag{1.6}$$

Suppose H is smaller than the Planck scale, we know $\dot{a}_i/a_i = H < 1$. If we use this inequality in (1.6), we find

$$\dot{a}_f > \dot{a}_i. \tag{1.7}$$

Therefore, $\int_{t_i}^{t_f} \ddot{a} = \dot{a}_f - \dot{a}_i$ must be positive and there has been accelerating expansion somewhere along the way.

2

Consequences of TCC for Scalar Potentials

In this section, we find some of the consequences of TCC for scalar fields with a potential $V(\varphi)$. We assume V is positive and monotonic. As already noted non-monotonic potentials with critical points are forbidden classically but are allowed when we take into account quantum corrections as we will discuss in the next section. We divide our analysis in this section into three parts. First, we study the consequences of TCC for asymptotic behavior (long field ranges) of the single-field potentials. Next, we generalize some of these results to multi-field models. In the end, we study the short-range predictions of the conjecture for single-field potentials.

2.1 Long-Range Predictions

Using the definition of $H = \frac{\dot{a}}{a}$, we can rewrite the conjecture (1.1) in the form

$$\int_{\varphi_i}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi = \int_{t_i}^{t_f} H dt < -\ln\left(H_f\right). \tag{2.1}$$

In d spacetime dimensions, the Friedmann equation takes the form

$$\frac{(d-1)(d-2)}{2} H^2 = \frac{1}{2}\dot{\varphi}^2 + V, \tag{2.2}$$

and the equation of motion takes the form

$$\ddot{\varphi} + (d-1)H\dot{\varphi} + V' = 0, \tag{2.3}$$

where V' indicates the derivative of V with respect to φ. Note that we are working in the units where the reduced Planck mass ($M_{pl} = \frac{m_{pl}}{\sqrt{8\pi}}$) is equal to 1. Since V in the equation (2.2) is positive, we have

$$\frac{H}{|\dot{\varphi}|} > \frac{1}{\sqrt{(d-1)(d-2)}}. \tag{2.4}$$

If we use the above lower bound for the integrand in the equation (2.1), we find

$$\frac{|\varphi_f - \varphi_i|}{\sqrt{(d-1)(d-2)}} < -\ln\left(H_f\right), \tag{2.5}$$

which can be rearranged in the form

$$H_f < e^{-\frac{|\varphi_f - \varphi_i|}{\sqrt{(d-1)(d-2)}}}. \tag{2.6}$$

Due to the positivity of the kinetic term in the equation(2.2), V is bounded from above by $(d-1)(d-2)H^2/2$. If we combine this upper bound with the inequality (2.6), we find[1]

$$V(\varphi) < Ae^{-\frac{2}{\sqrt{(d-1)(d-2)}}|\varphi-\varphi_i|}, \tag{2.7}$$

where, $A = (d-1)(d-2)/2$ is a constant. For definiteness let us assume $V' < 0$. We can use the above inequality to find a lower bound for the average of $-V'/V$ over interval $[\varphi_i, \varphi_f]$ in the field space.

$$\left\langle \frac{-V'}{V} \right\rangle \Bigg|_{\varphi_i}^{\varphi_f} = \frac{1}{\Delta\varphi}\int_{\varphi_i}^{\varphi_f} \frac{-V'}{V} d\varphi = \frac{ln(V_i) - ln(V_f)}{\Delta\varphi}.$$

If we combine the upper bound (2.7) for V_f with the above identity, we find

$$\left\langle \frac{-V'}{V} \right\rangle \Bigg|_{\varphi_i}^{\varphi_f} > -\frac{B}{\Delta\varphi} + \frac{2}{\sqrt{(d-1)(d-2)}}, \tag{2.8}$$

where, $B = -\ln(V_i) + \ln(A)$ and $\langle \frac{-V'}{V} \rangle \Big|_{\varphi_i}^{\varphi_f}$ is the average of $\frac{-V'}{V}$ over $[\varphi_i, \varphi_f]$.

One may worry about the emergence of light states at large distances in field space expected from the Swampland distance conjecture[32]. In particular the interactions between φ and other fields cannot be ignored in this large field limit and the effective field theory of φ ignoring the other modes would be invalid in such a limit. However, these modifications do not affect the derivation of the inequalities (2.7) and (2.8) because all we needed to derive these was $(d-1)(d-2)H^2/2 > V$ which is true even if we have additional energy contributions to H. Therefore, even for values of φ where the effective field theory breaks down due to the emergence of a tower of light states, the inequalities (2.7) and (2.8) are still valid. By taking the limit φ_i and

[1] One may conclude that since we can take $\varphi_i \to -\infty$ this would imply that V has to vanish. As we shall discuss one cannot start from arbitrarily negative field value φ_i to reach arbirary φ_f, which is a necessity for this derivation. In other words there is a smallest value of φ_i one has in the above equation to reach a fixed value of φ_f including arbitrarily large values of φ.

$\varphi_f \to \infty$ in the eq(2.8), we find

$$(\frac{|V'|}{V})_\infty \geq \frac{2}{\sqrt{(d-1)(d-2)}}, \tag{2.9}$$

where

$$(\frac{|V'|}{V})_\infty := \liminf_{\varphi_i\to\infty} \liminf_{\varphi_f\to\infty} \left\langle \frac{-V'}{V} \right\rangle \Big|_{\varphi_i}^{\varphi_f}. \tag{2.10}$$

Thus the inequalities (2.7) and (2.9) are valid for every value of φ, even when the effective field theory breaks down due to the emergence of a tower of light particles. However, in[33] it was argued that this would never happen. To summarize, according to the emergent string conjecture, such light states are always either KK particles or a string tower. In either case, their mass scale must be greater than the Hubble parameter to make sure the effective equations of GR do not break down[34,35]. Therefore, we can assume that the tower of light states at the infinite distance limit are heavier than the Hubble scale and will not contribute to the dynamics. The inequality (2.9) shows that the potential decays exponentially fast in the asymptotics. In the following, we focus on the exponential potentials without the addition of light states. Let $V \propto e^{-\lambda\varphi}$.

$$\begin{aligned}
\frac{d}{d\varphi}(\frac{V}{\dot{\varphi}^2}) &= \frac{1}{\dot{\varphi}}\frac{d}{dt}(\frac{V}{\dot{\varphi}^2}) \\
&= \frac{V'}{\dot{\varphi}^2} - 2(\frac{\ddot{\varphi}}{\dot{\varphi}^2})(\frac{V}{\dot{\varphi}^2}) \\
&= \frac{V'}{\dot{\varphi}^2}(1+2(\frac{V}{\dot{\varphi}^2})) + \frac{2(d-1)H}{\dot{\varphi}}(\frac{V}{\dot{\varphi}^2}) \\
&= -(\frac{V}{\dot{\varphi}^2})\sqrt{1+2(\frac{V}{\dot{\varphi}^2})}(\lambda\sqrt{1+2(\frac{V}{\dot{\varphi}^2})} - 2\sqrt{\frac{d-1}{d-2}}),
\end{aligned} \tag{2.11}$$

where in the third line we used the equation of motion (2.3), and in the fourth line we used the Friedmann equation (2.2). We can rewrite the equation (2.11) in the form

$$x' = -x\sqrt{1+2x}(\lambda\sqrt{1+2x} - 2\sqrt{\frac{d-1}{d-2}}), \tag{2.12}$$

where $x := (V/\dot{\varphi}^2)$ and x' represents the derivative of x with respect to φ. The x is related to the equation of state parameter, w, as

$$w = \frac{2}{1+2x} - 1. \tag{2.13}$$

If $\lambda > 2\sqrt{(d-1)/(d-2)}$, the right hand side of the equation (2.12) is always negative and x decays exponentially to 0 as a function of φ. For $\lambda < 2\sqrt{(d-1)/(d-2)}$, the right hand side of the (2.12) has a positive root at $x_c = 2(d-1)\lambda^{-2}/(d-2) - 1/2$. By checking the signs one can see that $x = x_c$ is an attractor solution and x will converge to x_c. Plugging H from the equation(2.2) into (2.1), leads to the following form for the trans-Planckian censorship conjecture.

$$\sqrt{\frac{2x+1}{x(d-1)(d-2)}}V(\varphi_f)^{\frac{1}{2}} = H_f < e^{-\int_{\varphi_i}^{\varphi_f}\frac{H}{\dot{\varphi}}d\varphi} = e^{-\int_{\varphi_i}^{\varphi_f}\sqrt{\frac{1+2x}{(d-1)(d-2)}}d\varphi}. \tag{2.14}$$

If we look at the above inequality in the limit $\varphi \to \infty$ where x goes to $x_c = 2(d-1)\lambda^{-2}/(d-2) - 1/2$, we find

$$V(\varphi) \leq Ae^{-\frac{4}{(d-2)\lambda}(\varphi-\varphi_i)}, \tag{2.15}$$

where $A = x_c(d-1)(d-2)/(2x_c+1)$. Since $V \propto e^{-\lambda\varphi}$, also decays exponentially, we have

$$\lambda \geq \frac{4}{(d-1)\lambda} \to \lambda \geq \frac{2}{\sqrt{d-2}}. \tag{2.16}$$

This inequality could be expressed in terms of x_c and w as

$$\begin{aligned} x < x_{TC} &= \frac{d-2}{2} \\ w > w_{TC} &= \frac{2}{d-1} - 1. \end{aligned} \tag{2.17}$$

Note that for $\lambda > 2/\sqrt{d-2}$, in the attractor solution, aH/a_i goes to zero and because of the fast convergence of the solution to the attractor solution, it is always bounded from above by an $\mathcal{O}(1)$ number. Thus, TCC implies that $\lambda > 2/\sqrt{d-2}$.

Following we find the lower bounds for x_c and w in order to have inflation ($\ddot{a} > 0$) and we compare them to x_{TCC} and w_{TCC} in arbitrary dimensions,

$$q = \frac{(d-3)\rho + (d-1)p}{(d-1)(d-2)H^2}, \tag{2.18}$$

where $q = -\frac{\ddot{a}a}{\dot{a}^2}$ is the deceleration parameter, $p = \frac{1}{2}\dot{\varphi}^2 - V$ is the pressure and $\rho = \frac{1}{2}\dot{\varphi}^2 + V$ is the energy density. For $\ddot{a}$ to be positive, we must have

$$x_c > x_{inf} = \frac{d-2}{2}, \tag{2.19}$$

which can be expressed in terms of the equation of state as

$$w < w_{inf} = \frac{2}{d-1} - 1. \tag{2.20}$$

These are exactly the same values as (2.17). For exponential potentials, it seems that TCC is equivalent to not having long-field accelerating expansion. This relation is consistent with the general result that we proved in section 1 that violation of TCC necessitates accelerating expansion.

Note that in the above analysis we ignored the effects of the creation of light states which emerge as the field values roll to infinity. These effects would modify both the Friedmann equation (2.2) and the equation of motion (2.3). In this regard (2.9) is more robust because it allows for the emergence of a tower of light modes.

2.2 Generalization to Multi-Field Models

In this section we study the applicability of our results to multi-field models where the fields take value in an n-dimensional manifold $\mathcal{M}$. Let $\{\varphi^j\}_{j=1}^n$ be coordinates for a local patch and the metric induced by the kinetic term on $\mathcal{M}$ to take the form $ds^2 = G_{ij} d\varphi^i \varphi^j$ in this coordinate system. For a spatially constant field configuration, the Friedmann equation takes the form

$$\frac{(d-1)(d-2)}{2} H^2 = \frac{G_{ij}\partial_t \varphi^i \partial_t \varphi^j}{2} + V(\varphi).V_i \Delta\varphi \tag{2.21}$$

Let s be the Affine parametrization of the solution path such that

$$G_{ij}\partial_s \varphi^i(s)\partial_s \varphi^j(s) = 1. \tag{2.22}$$

We can rewrite (2.21) in terms of s as

$$\frac{(d-1)(d-2)}{2} H^2 = \frac{1}{2}(\frac{ds}{dt})^2 + V(\varphi(s)). \tag{2.23}$$

This is exactly the same as the Friedmann equation in the single field case which we used to derive (2.7) and then (2.17) with φ being replaced with s. Note that we did not need TCC to hold for all initial conditions to derive (2.17), we only needed TCC to hold for one initial condition. Therefore, the results (2.17) holds for the multi-field case as well,

$$V(s) < Ae^{-\frac{2}{\sqrt{d-2}} d^s(\varphi_i, \varphi_f)}, \tag{2.24}$$

where A is some constant and $d^s = \int_{\varphi_i}^{\varphi_f} ds$ is the canonical length of the solution path from φ_i to φ_f. Let $d(\varphi, \varphi_f)$ be the canonical length of the geodesic connecting the two points, then we have $d \leq d^s$. Therefore, we can replace d^s in (2.24) with d to get

$$V(s) < Ae^{-\frac{2}{\sqrt{(d-1)(d-2)}} d(\varphi_i, \varphi_f)}. \tag{2.25}$$

The above inequality holds for any two points φ_i and φ_f that can be connected through a solution to the equations of motion such that the potential remains positive along the path. The derivation of (2.7) and (2.17) extends without any modifications to the multi-field case and gives

$$(\frac{|V'|}{V})_\infty \geq \frac{2}{\sqrt{d-2}}, \tag{2.26}$$

where $(\frac{|V'|}{V})_\infty$ is defined as $\liminf_{s_i\to\infty} \liminf_{s_f\to\infty} \left\langle \frac{-V'(\varphi(s))}{V(\varphi(s))} \right\rangle_{[s_i,s_f]}$ where s is the canonical Affine parameter for an arbitrary path with infinite length in $\mathcal{M}$.

Note that the inequality (2.25) is only applicable to a pair of points (φ_i, φ_f) which are connected by a classical solution. Following, we further explore this relationship between the points in $\mathcal{M}$.

One can define a causal structure on the moduli space based on which initial conditions can evolve into other ones in an expanding universe. Suppose x and y are two points in the moduli space $\mathcal{M}$, we say x causally precedes y, if for some $\dot{\varphi}_i^2 < \mathcal{O}(1)$ the initial field configuration $\varphi = x$ can evolve into $\varphi = y$. We show this by $x \prec y$. The condition $\dot{\varphi}_i^2 < \mathcal{O}(1)$ makes sure that the field theory description does not break.

Due to the dissipative nature of the Friedmann equations, this causal structure is non-commutative. Generally, to go from a point with a lower potential to a point with a higher potential, we might need a trans-Planckian initial condition $\dot{\varphi}$ to overcome the potential difference in the presence of dissipation. In fact, by assuming our energy density must be sub-Planckian ($H < 1$), which is a much weaker assumption than the TCC, we can find an upper bound on the field range that the field φ can climb up a potential hill.

Suppose $\varphi(t)$ is climbing up a positive **monotonically increasing** potential V from φ_i to

φ_f, we find an upper bound on $\Delta\varphi = \varphi_f - \varphi_i$.

$$\begin{aligned}\ddot{\varphi} &= -(d-1)H\dot{\varphi} - V' \\ &< -(d-1)H\dot{\varphi} \\ &< -\sqrt{\frac{2(d-1)V}{d-2}}\dot{\varphi} \\ &< -\sqrt{\frac{2(d-1)V_i}{d-2}}\dot{\varphi}.\end{aligned} \tag{2.27}$$

Integrating the above inequality leads to

$$\Delta\dot{\varphi} + \sqrt{\frac{2(d-1)V_i}{d-2}}\Delta\varphi < 0. \tag{2.28}$$

Since $\dot{\varphi}_i < \sqrt{(d-1)(d-2)/2}$ (this results from $H < 1$), we find

$$\Delta\varphi < \frac{d-2}{2}\sqrt{\frac{1}{V_i}}. \tag{2.29}$$

Note that the above upper bound only depends on V_i the value of the potential at the initial point. We can use the full power of TCC to derive another upper bound which also depends on V_f the final value of the potential. From the equation (2.25), we know that an initial field value cannot be too far, because other wise the upper bound in (2.25) would be less than V_f. This gives

$$\Delta\varphi < \frac{\sqrt{(d-1)(d-2)}}{2}\ln\left(\frac{A}{V_f}\right). \tag{2.30}$$

In fact, this has the same nature as the inequality (2.29) since typically going back in the solution requires climbing up a potential hill. This obstruction for extending the solution in the field space only in the past direction happens because of the dissipation in our equations. If two points do not satisfy the inequality (2.30) for any order of them, they are causally unrelated. This could mean that there is a potential barrier between them that is high enough such that climbing it in the presence of dissipation would need trans-planckian initial conditions. Situations like this can naturally happen for two points in opposite asymptotic regions of the Moduli space, as the potential is highest in the interior and decays exponentially at infinity.

We can use this result to obtain a bound on the asymptotic gradient of the potential. We divide the moduli space into two parts, the interior $\mathcal{M}_I$ that contains all the local maxima of V and the asymptotic region $\mathcal{M}_\infty$ which is located far enough from $\mathcal{M}_I$ with respect to the canonical distance given by the metric defined on $\mathcal{M}$. Since $\mathcal{M}_I$ contains the critical points, the causal paths initiated from $\mathcal{M}_I$ can cover all of the moduli space including $\mathcal{M}_\infty$. Suppose $\mathcal{M}_\infty$ can be covered by causal paths $\{\gamma_\alpha\}_{\alpha\in\mathcal{I}}$ (with respect to the causal structure defined in 2.2) such that

- they all initiate in $\mathcal{M}_I$.
- the path γ_α is parametrized by the Affine parameter s_α.

We call every $\alpha \in \mathcal{I}$ an asymptotic direction of the moduli space (fig 2.1). We define

$$(\frac{|\nabla_I V|}{V})_\alpha := \liminf_{s_{\alpha,i}\to\infty} \liminf_{s_{\alpha,f}\to\infty} \left\langle \frac{|\partial_{s_\alpha} V(\gamma(s_\alpha))|}{V(\gamma(s_\alpha))} \right\rangle_{[s_{\alpha,i},s_{\alpha,f}]}, \tag{2.31}$$

where on the right hand side $\langle \_ \rangle_{[s_{\alpha,i},s_{\alpha,f}]}$ is the average over $[s_{\alpha,i}, s_{\alpha,f}]$. This roughly represents the ratio $|V'|/V$ along the asymptotic direction $(\partial_{s_\alpha}\gamma(s_\alpha))$ going outward from the interior. We also define

$$(\frac{|\nabla V|}{V})_\alpha := \liminf_{s_{\alpha,i}\to\infty} \liminf_{s_{\alpha,f}\to\infty} \left\langle \frac{|\nabla V(\gamma(s_\alpha))|}{V(\gamma(s_\alpha))} \right\rangle_{[s_{\alpha,i},s_{\alpha,f}]}, \tag{2.32}$$

which roughly represents the limit of $|\nabla V|/V$ as we go to infinity in the asymptotic direction α. From the above definitions we know

$$(\frac{|\nabla V|}{V})_\alpha \geq (\frac{|\nabla_I V|}{V})_\alpha. \tag{2.33}$$

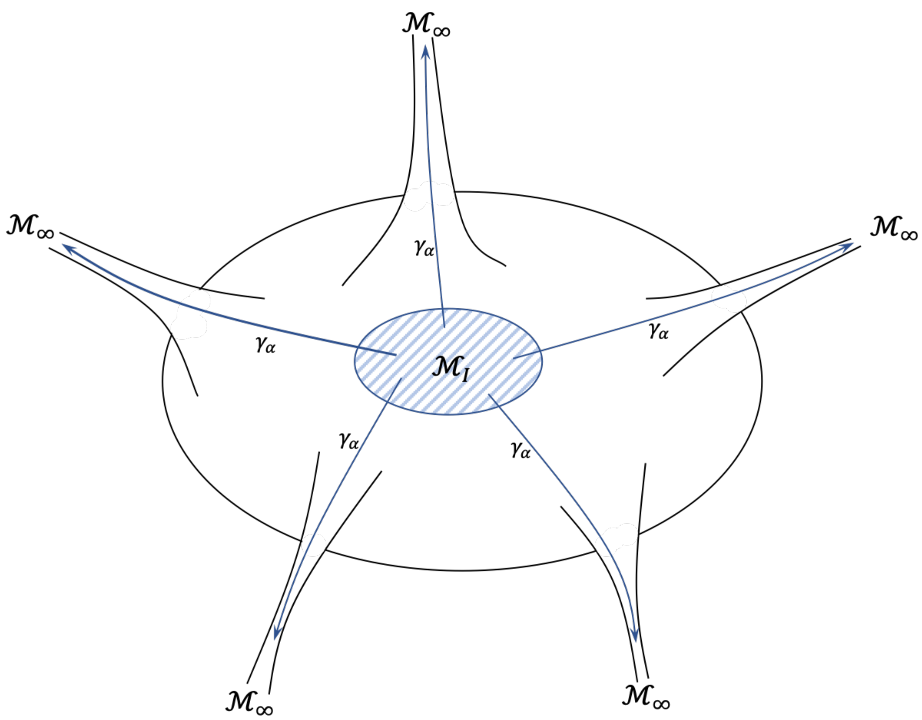

Figure 2.1: The curves γ_α are causal curves that initiate in the interior region $\mathcal{M}_I$ and collectively span the asymptotic region $\mathcal{M}_\infty$.

On the other hand, from the inequality (2.26), for every α we have

$$(\frac{|\nabla_I V|}{V})_\alpha \geq \frac{2}{\sqrt{d-2}}. \tag{2.34}$$

Combining (2.33) and (2.34) leads to

$$(\frac{|\nabla V|}{V})_\alpha \geq \frac{2}{\sqrt{d-2}}, \tag{2.35}$$

which has the same form as the dS Swampland conjecture[14] but is for the asymptotic region of the moduli space.

2.3 Short-Range Predictions

In this section, we prove several inequalities from TCC for the short-field-range behavior of monotonically decreasing positive potentials.

Obstruction of flatness

The trans-Planckian censorship conjecture clearly forbids a flat potential ($V' = 0$) as it can lead to accelerated expansion with a fixed Hubble parameter. In our first result in this subsection, we find an inequality which puts an upper bound on the length of the field range over which $|V'|$ is smaller than a constant. Suppose $|V'|_{\max}$ is the maximum of $|V'(\varphi)|$ over $\varphi \in [\varphi_i, \varphi_f]$, we have,

$$\frac{d\dot{\varphi}^2}{d\varphi} = 2\ddot{\varphi} \leq 2|V'| \leq 2|V'|_{\max}, \tag{2.36}$$

where we used the (2.3) for the first inequality. For the initial conditions $\dot{\varphi} = 0$ and $\varphi = \varphi_i$, integrating the above inequality gives

$$\dot{\varphi}(\varphi) = \sqrt{\int_{\varphi_i}^{\varphi} \frac{d\dot{\varphi}^2(\varphi')}{d\varphi'} d\varphi'} \leq \sqrt{2|V'|_{\max}\Delta\varphi}, \tag{2.37}$$

where $\Delta\varphi = \varphi - \varphi_i$. Using the above inequality in the TCC leads to

$$\begin{aligned} \ln\left(\sqrt{\frac{(d-1)(d-2)}{2V(\varphi)}}\right) &\geq -\ln(H) \\ &> \int_{\varphi_i}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi \\ &\geq \int_{\varphi_i}^{\varphi_f} \sqrt{\frac{1}{(d-1)(d-2)|V'|_{\max}}} \sqrt{\frac{V(\varphi_i)}{\varphi - \varphi_i}} d\varphi \\ &= \sqrt{\frac{V(\varphi_i)\Delta\varphi}{4(d-1)(d-2)|V'|_{\max}}}, \end{aligned} \tag{2.38}$$

where in the first and third lines we used $H^2(d-1)(d-2)/2 \geq V$, in the second line we used the TCC, and in the third line we used (2.37). We can rearrange the above inequality into the form

$$(\frac{|V'|_{\max}}{V_{\max}}) > \frac{(\varphi_f - \varphi)}{4(d-1)(d-2)} \ln\left(\sqrt{\frac{(d-1)(d-2)}{2V(\varphi_f)}}\right)^{-2}. \tag{2.39}$$

We used the monotonicity to replace $V(\varphi_i)$ with $V_{\max}$. Note that V' and V are not evaluated at the same point in (2.39). However, for regions where the potential is stable ($V'' > 0$), both V and V' attain their maximum at the same point $\varphi = \varphi_i$, and the LHS in (2.39) becomes a local quantity.

The integration in the statement of TCC makes it a global criterion in terms of the potential. In fact, it is very challenging to obtain a local statement about the potential from TCC, which is why the small field range inequalities are weaker than their long-field-range counterpart derived in the previous subsection. We now provide the results of some numerical analysis which supports this observation.

Let $C(\varphi_f) := H_f \frac{a_f}{a_i}$. For the conjecture to be true, C must be bounded from above by an $\mathcal{O}(1)$ constant for any physically allowed initial condition (one that V_i and $|\dot{\varphi}_i|$ are both less than 1). The maximum of C over a field range roughly measures the amount of violation of the conjecture.

Suppose λ is the decay rate of an exponential potential, we showed for $\lambda < \lambda_{TC} = \frac{2}{\sqrt{d-2}}$, the conjecture gets violated at infinity. Below, are the results of investigating the consistency of exponential potentials with the conjecture for all field ranges

1) For any value of $\lambda < \lambda_{TC}$, even though the conjecture is violated at infinity, it seems that the conjecture holds for any initial condition over field range $\Delta\varphi \sim \mathcal{O}(1)$, which by violation we mean $C > 1$. Surprisingly, this is true even for decay rates as small as $\lambda \sim 10^{-3}$ that are in contradiction with the conjecture at large field values.

2) For decay rates $\lambda > \lambda_{TC}$ which the conjecture holds at the limit $\varphi \to \infty$, it seems that it also holds for all field values. More specifically, in 4 dimensions, there are no physically allowed initial conditions that would result in a $C > 1$ for any $\lambda > \sqrt{2} + 0.01$.

Conclusions:

1) For exponential potentials, it seems that the conjecture is always satisfied for small field values ($\Delta\varphi < \mathcal{O}(1)$) and their consistency with the conjecture is determined based on their

large-φ behavior. In other words, the conjecture becomes more non-trivial at large field values.

2) The conjecture does not restrict the value of $|V'|/V$ over very small field ranges. We can have potentials with arbitrarily small λ that satisfy the conjecture for any physically permissible initial conditions over sufficiently small field ranges $\Delta\varphi << \mathcal{O}(1)$. Therefore, this conjecture does not rule out the quintessence models with small decay rates as long as they only last for $\Delta\varphi < \mathcal{O}(1)$. In particular, we have checked that the models discussed in [36,37] where $0 < \lambda \leq 0.6$ are compatible with TCC because the field ranges in those models are sufficiently smaller than Planck.

Accelerating roll

In this part, using a different assumption, we find an inequality very similar to (2.39) for small field regime behavior of the potential. Suppose we have a rolling scalar field with positive $\ddot{\varphi}$ over a field range $[\varphi_i, \varphi_f]$. The equation of motion (2.3) implies

$$(d-1)H\dot{\varphi} < |V'|. \tag{2.40}$$

This inequality leads to

$$\begin{aligned}
\int_{\varphi_i}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi &> \int_{\varphi_i}^{\varphi_f} \frac{(d-1)H^2}{|V'|} d\varphi \\
&\geq \int_{\varphi_i}^{\varphi_f} \frac{2}{d-2} \frac{V}{|V'|} d\varphi \\
&= \frac{2}{d-2} \left\langle \frac{V}{|V'|} \right\rangle \Delta\varphi,
\end{aligned} \tag{2.41}$$

where in the first line we used (2.40) and in the second line we used $H^2 \geq \frac{2V}{(d-1)(d-2)}$ from (2.2). Using the above result, in addition to $H \geq \sqrt{\frac{2V}{(d-1)(d-2)}}$ in (2.1), one can show

$$\frac{2}{d-2} \left\langle \frac{V}{|V'|} \right\rangle \Delta\varphi \leq \ln \sqrt{\frac{(d-1)(d-2)}{2V}}, \tag{2.42}$$

where $\left\langle \frac{V}{|V'|} \right\rangle$ is the average of $\frac{V}{|V'|}$ over $[\varphi_i, \varphi_f]$.

Strongest consequence of TCC for short-field-range behavior of V

We finish this subsection by discussing an inequality that is proved in the appendix A. For every pair of non-negative numbers c_1 and c_2 such that $c_2^2(2+c_1^2) < (d-2)/(d-1)$, we find

$$\min\left(\frac{V(\varphi)}{|V'(\varphi)|}c_1, c_2\right)A_1(c_1, c_2, \varphi) < \ln\left(\frac{A_2}{\sqrt{V(\varphi + A_3(c_1, c_2, \varphi))}}\right), \tag{2.43}$$

where the identities (A.27), (A.25), and (A.16), provide the definitions of functions A,B and C. In the derivation of (2.43), we have not weakened the inequalities for obtaining simpler looking result. That comes at the expense of complexity of our final result which makes it hard to physically interpret for an arbitrary potential. If one is interested in the consistency of a specific class of potentials with TCC, by restricting to that class, the inequality might take a much simpler form. In the appendix A, we discuss how this is the case for convex potentials. Moreover, unlike the original conjecture which must be checked for every initial conditions, (2.43) only depends on the potential and could be checked numerically more easily. The (2.43) is derived by estimating the initial condition that is in most tension with the conjecture and looking at the TCC for that initial condition.

2.4 Metastable dS

We show that the trans-Planckian censorship conjecture implies that the universe cannot get stuck in a local minimum for $V(\varphi)$ for an infinite amount of time. We find an upper bound on the lifetime τ by which every classical local minimum must decay into another state. Therefore, according to the trans-Planckian censorship conjecture, the potential cannot have a positive minimum, or in other words, $\inf V \leq 0$.

For meta-stable dS we have $\Lambda = (d-1)(d-2)H_\Lambda^2/2$. Using (1.4) we find

$$\tau < \frac{1}{H_\Lambda} \ln\left(\frac{1}{H_\Lambda}\right), \tag{2.44}$$

In a quantum theory of gravity, even though dS spaces seem to be impossible to attain as a vacuum, it is not implausible that sufficiently short-lived transient quasi-dS like phases could appear, and TCC allows this. The Hubble time of such a background provides a natural time scale and it is reasonable to expect that the lifetime of such an unstable state to be roughly proportional to this characteristic time scale. Indeed, if our universe is stuck in a metastable minimum with $V = \Lambda \approx 2.9 \times 10^{-122}$, the TCC predicts an upper bound of $\tau < 2.4$ trillion years on the lifetime of our universe. Thus also in such a case TCC gives an explanation of the coincidence problem: Not only the age of our universe is related to Hubble time, but its lifetime also cannot exceed the Hubble time, up to log corrections.[2]

Note that all the above analysis only applies to local minima with positive values of V. For example, for a Harmonic potential $V(\varphi) = \varphi^2$, from numerical analysis we found that the TCC is satisfied over a field range $[-0.9M_{pl}, 0.9M_{pl}]$. As the field oscillates about the local minimum within this range, the Hubble friction is strong enough that the field does not get stuck in high V for too long. In other words, thanks to the massless graviton, the energy of φ gets channeled to the gravity sector fast enough that it does not violate the conjecture.

2.5 Unstable dS

In this subsection, we show that for a potential with an unstable local maximum, $|V''|$ cannot be small over a large field interval. In other words, over any field interval around the local maximum,

[2] There is an interesting similarity between the upper bound on the dS lifetime predicted by TCC and the scrambling time associated to dS space where we use the scrambling time given by [38]

$$\tau_{scrambling} \propto \frac{\ln S}{T},$$

where T and S denote temperature and entropy. We see that the upper bound for the lifetime of dS space $\tau_{dS} \sim \tau_{scrambling}$ with the substitutions $T_{dS} = \frac{H}{2\pi}$ and $S_{dS} \sim 1/H^2$. We thank J. Maldacena for pointing out this connection.

there is a lower bound for $|V''|$ so that the quantum fluctuations could push the field away from the extremum point. Otherwise, the field could stay close to the local maximum for a long enough time that leads to a violation of TCC. First, we provide a more heuristic argument to demonstrate what would go wrong with a quadratic potential over a long field range. Afterward, we give a rigorous argument to prove a sharp inequality from TCC.

Suppose we have a quadratic potential given by

$$V(\varphi) = \frac{V''(\varphi_0)}{2}(\varphi - \varphi_0)^2 + V(\varphi_0), \tag{2.45}$$

where $V''(\varphi_0) < 0$. In[39], for the case of $d = 4$, it was shown that a gaussian probability distribution centered at $\varphi = \varphi_0$ solves the Fokker-Planck equation describing the evolution of quantum fluctuations. That result could be easily generalized to the following solution for any dimension $d > 2$.

$$Pr[\varphi = \varphi_c; t] \propto \frac{\exp\left[-\frac{\varphi_c^2}{2\sigma(t)^2}\right]}{\sigma(t)}, \tag{2.46}$$

where

$$\sigma(t) \propto \frac{H^2\left(e^{\frac{2|V''(\varphi_0)|t}{(d-1)H}} - 1\right)^{1/2}}{\sqrt{|V''(\varphi_0)|}}. \tag{2.47}$$

Note that the expectation value of H remains constant and equal to $\sqrt{2V(\varphi_0)/((d-1)(d-2))}$. If the field range over which (2.45) holds is large enough, the above equation would hold for large t. As t goes to infinity, $\sigma(t)$ would exponentially grow like $e^{|V''(\varphi_0)|t/[(d-1)H]}$. This leads to a lifetime of $(d-1)H/|V''(\varphi_0)|$. Comparing this with the upper bound (2.44) gives

$$\frac{|V''(\varphi_0)|}{V(\varphi_0)} \geq \frac{2}{d-2}\ln\left(\sqrt{\frac{(d-1)(d-2)}{2V}}\right)^{-1}. \tag{2.48}$$

This heuristic argument tells us that either the field range over which the potential is quadratic is bounded from above, or $|V''|/V$ is bounded from below. In the following, we present a similar but rigorous statement. Suppose $V(\varphi)$ is a positive potential such that $V'(\varphi_0) = 0$ and for every $\varphi \in [\varphi_0, \Delta\varphi]$, we have $V'(\varphi) < 0$ and $|V''| \leq |V''|_{max}$. If

$$\Delta\varphi \geq \frac{B_1(d)B_2(d)^{\frac{3}{4}}V_{max}^{\frac{d-1}{4}}V_{min}^{\frac{3}{4}}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{\frac{1}{2}}}{V_{min}B_2(d) - |V''|_{max}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^2}$$

, then

$$\frac{|V''|_{max}}{V_{min}} \geq B_2(d)\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{-2}, \tag{2.49}$$

where $V_{max} = V(\varphi_0)$ and $V_{min} = V(\varphi_0 + \Delta\varphi)$ are respectively the maximum and the minimum of V over $\varphi \in [\varphi_0, \Delta\varphi]$, and $B_1(d)$, $B_2(d)$, and $B_3(d)$ are $\mathcal{O}(1)$ numbers given by

$$\begin{aligned} B_1(d) =& \frac{\Gamma(\frac{d+1}{2})^{\frac{1}{2}}2^{1+\frac{d}{4}}}{\pi^{\frac{d-1}{4}}((d-1)(d-2))^{\frac{d-1}{4}}}, \\ B_2(d) =& \frac{4}{(d-1)(d-2)}, \\ B_3(d) =& \sqrt{\frac{(d-1)(d-2)}{2}}. \end{aligned} \tag{2.50}$$

These criteria tell us that if $|V''|$ is small enough over a long enough field range, then $|V''|/V$ is bounded from below by a logarithmic function in V. This result is very similar to the refined Swampland dS conjecture with a logarithmic correction. For details of the derivation of this result and its application to quadratic potentials see appendix B.

3

Early universe cosmology

In this section we study the cosmological implications of TCC in early universe. Cosmological observations provide detailed information about our universe on the largest observable scales. Cosmic microwave background (CMB) measurements[40,41,42], for instance, demonstrate that fluctuations in the matter and energy persist on cosmological scales. There is no causal explanation for the origin of these fluctuations in Standard Big Bang cosmology. Scenarios of early universe cosmology such as the Inflationary Universe[43,44,45,46,47] provide a causal mechanism to generate these fluctuations. A key aspect of both inflationary cosmology and of other scenarios that provide an explanation for the origin of structure in the universe (see e.g.[48] for a comparative review) is the existence of a phase in the early universe during which the

Hubble horizon $H^{-1}(t)$, where $H(t)$ is the Hubble expansion rate as a function of time t, shrinks in comoving coordinates. The Hubble horizon provides the limiting length above which causal physics that is local in time cannot create fluctuations. In both inflationary cosmology and in the proposed alternatives, comoving length scales which are probed in current cosmological experiments were inside the Hubble horizon at early times. It is postulated that fluctuations in both matter[49] and gravitational waves[50] originate as quantum vacuum perturbations that exit the Hubble radius during the early phase, are squeezed and classicalize, and then re-enter the Hubble radius at late times to produce the CMB anisotropies and matter density perturbations that we observe today.

In[51] (see also[52]) it was realized that if the inflationary phase lasts somewhat longer than the minimal period, then the length scales we observe today originate from modes that are smaller than the Planck length during inflation. This was called the *trans-Planckian problem* for cosmological fluctuations (see also[53,54,55]). This problem was viewed not so much as an issue with a particular model, but more as a question of how to treat trans-Planckian modes in such a situation. It has been conjectured[19], however, that this trans-Planckian problem can never arise in a consistent theory of quantum gravity and that all the models which would lead to such issues are inconsistent and belong to the Swampland. This is called the *Trans-Planckian Censorship Conjecture* (TCC).

According to the TCC no length scales which exit the Hubble horizon could ever have had a wavelength smaller than the Planck length. In Standard Big Bang cosmology no modes ever exit the Hubble horizon, and the TCC has no implications (indeed the TCC is automatically satisfied for all models with a $w \geq -1/3$). However, in all early universe scenarios which can provide an explanation for the origin of structure, modes exit the Hubble horizon in an early phase. If a_i is the value of the cosmological scale factor at the beginning of the new early universe phase, and a_f is the value at the time of the transition from the early phase to the phase of Standard Big Bang expansion, the TCC reads

$$\frac{a_f}{a_i} < \frac{M_{pl}}{H_f}, \tag{3.1}$$

where H_f is the radius of the Hubble horizon at the final time t_f and M_{pl} is the reduced Planck mass.

In [19], the relationship between the TCC and other *swampland conjectures* [32,14,15] which have recently attracted a lot of attention (see e.g. [56] for a recent review) was discussed. Here, we focus on the consequences of the TCC for inflationary cosmology.

It is clear from the form of (3.1) that the TCC will have strong implications for inflationary cosmology. In the case of de Sitter expansion, a_f is exponentially larger than a_i, and hence (3.1) strictly limits the time duration of any inflationary phase. The implications for some alternative early universe scenarios are weaker. For example, in *String Gas Cosmology* [57], the early phase is postulated to be quasi-static. Hence, the condition (3.1) is satisfied: no modes which were larger than the Hubble scale at the beginning of the Standard Cosmology phase ever had a wavelength smaller than the Planck length. The same is true for the various bouncing scenarios (e.g. the *matter bounce* [58], and the *Pre-Big-Bang* [59] and Ekpyrotic [60] scenarios)[1], where the initial phase is one of contraction. This is true as long as the energy scale at the bounce point is smaller than the Planck scale. In the following we will study the consequences of the TCC for inflationary cosmology.

The outline of this section is as follows: In the following section we discuss general constraints imposed by the TCC on the energy scale of inflation and the resulting consequences for the amplitude of gravitational waves. These conclusions do not depend on what drives inflation, only that it occurred. In subsection 3, we then specialize to slow-roll inflation models, and show that consistency with the TCC leads requires fine-tuning of the initial conditions.

We will work in the context of homogeneous and isotropic cosmologies with 4 space-time dimensions. For simplicity, we assume spatially flatness so that the metric can be written in the form

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \,, \tag{3.2}$$

where $\mathbf{x}$ are the spatial comoving coordinates and $a(t)$ is the scale factor (which can be

[1] Of course it is possible that these models have issues related to other Swampland conditions.

normalized such that $a(t_0) = 1$, where t_0 is the present time). The Hubble expansion rate is

$$H(t) \equiv \frac{\dot{a}}{a}, \tag{3.3}$$

and its inverse is the Hubble radius. As is well known (see[61] for an in-depth review of the theory of cosmological fluctuations and[62] for an overview), quantum-mechanical fluctuations oscillate on sub-Hubble scales, whereas they freeze out and become squeezed when the wavelength is larger than the Hubble radius. During a phase of accelerated expansion, the proper wavelengths of fluctuations initially smaller than the Hubble scale can be stretched to super-Hubble scales. This transition from sub-Hubble to super-Hubble is referred to as horizon-crossing. The TCC prohibits horizon-crossing for modes with initial wavelengths smaller than the Planck length.

We will be considering models of inflation in which a canonically normalized scalar field φ with potential energy $V(\varphi)$ constitutes the matter field driving the accelerated expansion of space. We will be using units in which the speed of sound, Boltzmann's constant and $\hbar$ are set to 1.

3.1 Implications of the TCC for the Energy Scale of Inflation

In this section we work in the approximation that the Hubble expansion rate during the period of inflation is constant. In order for inflation to provide a solution to the structure formation problem of Standard Big Bang cosmology, the current comoving Hubble radius must originate inside the Hubble radius at the beginning of the period of inflation (see Fig. 1). This condition reads

$$\frac{1}{H} \cdot e^{N_+} \cdot \frac{a_R}{a_{end}} \cdot \frac{T_R\, g_*(T_R)^{1/3}}{T_0\, g_*(T_0)^{1/3}} \simeq \frac{1}{H_0}, \tag{3.4}$$

where H is the Hubble scale during inflation, a_{end} and a_R are the values of the scale factor at the end of inflation and when reheating is completed, and g_* indicates the number of spin degrees of freedom in the thermal bath. Here N_+ is the number of e-foldings accrued during the inflation after the CMB-scale modes exit the horizon, T_0 is the temperature of the CMB at the present time, and T_R is the corresponding temperature after reheating. Equation (3.4) can be summarized as follows. We start with a Hubble horizon scale which at the time of the inflation is $1/H$, by the end of inflation it is magnified by e^{N_+}, by reheating it has grown again by $a_R/a_{\rm end}$, and between reheating and the present day it grows by the ratio of the T_R/T_0 (corrected by the number of degrees of freedom). To solve the horizon problem, this scale should then be larger than the Hubble scale of the current universe $1/H_0$. To obtain the order of magnitude of the constraints, we consider rapid reheating and take the reheating temperature to be given by the potential energy at the end of inflation, and hence set $a_R \sim a_{\rm end}$. For simplicity we also assume that the ratio of g_*'s is 1.

Under the assumption that the period of reheating lasts less than one Hubble time T_R is given by the potential energy V during inflation via $T_R \approx V^{1/4}$. Using the Friedmann equation, the Hubble scale $1/H_0$ is given by the current energy density ρ_0 via

$$\frac{1}{H_0} = \sqrt{3}\rho_0^{-1/2} M_{pl} \,. \tag{3.5}$$

In turn, ρ_0 can be re-expressed in terms of the temperature T_0:

$$\rho_0 \approx T_0^4 \frac{T_{eq}}{T_0} \frac{1}{\Omega_m} \tag{3.6}$$

where Ω_m is the fraction of energy density in matter and T_{eq} is the temperature at the time of equal matter and radiation, and we have used the fact that the matter energy density today is larger than the radiation energy density T_0^4 by the factor T_{eq}/T_0. Using $H = V^{1/2}/(\sqrt{3}M_{pl})$, the condition (3.4) then becomes

$$e^{N_+} \simeq \frac{V^{1/4}}{(T_0 T_{eq})^{1/2}} \sqrt{\Omega_m} \sim \frac{V^{1/4}}{(T_0 T_{eq})^{1/2}} \,. \tag{3.7}$$

In the approximation of constant value of H during inflation, the TCC condition (3.1) can be written in the form

$$e^{N_+} < \frac{M_{pl}}{H} \,. \tag{3.8}$$

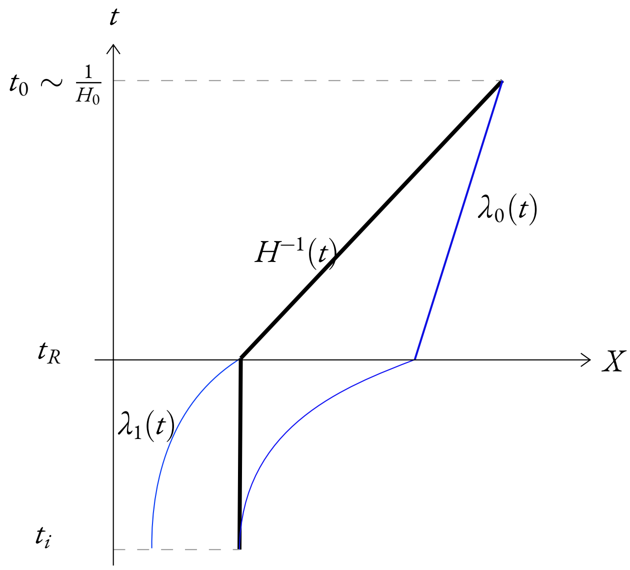

Figure 3.1: Space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis represents physical distance. The inflationary period lasts from t_i to t_R. Shown are the Hubble radius $H^{-1}(t)$ and two length scales $\lambda_0(t)$ and $\lambda_1(t)$ (fixed wavelength in comoving coordinates). For inflation to provide a possible explanation for the observed fluctuations on large scales, the scale $\lambda_0(t)$ corresponding to the current Hubble horizon must originate inside of the Hubble radius at the beginning of inflation. This leads to the condition (3.4). The TCC, on the other hand, demands that the length scale $\lambda_1(t)$which equals the Hubble radius at the end of inflation was never trans-Planckian. In the sketch, both conditions are marginally satisfied.

The equation (3.7) for N_+ and the upper bound (3.8) on N_+ coming from the TCC are compatible only provided that the condition

$$V^{3/4} < \sqrt{3} M_{pl}^2 (T_0 T_{eq})^{1/2} \tag{3.9}$$

is satisfied. Inserting the values of T_0, T_{eq} and M_{pl} we obtain

$$V^{1/4} < 6 \times 10^8 \text{GeV} \sim 3 \times 10^{-10} M_{pl} \,. \tag{3.10}$$

Note that this conclusion is independent of the assumption that quantum fluctuations during inflation are the seeds for primordial structure formation. While we have used a potential V to describe the energy density during inflation, our analysis holds for more general scenarios and Eq. (3.10) can be interpreted as a bound on the energy density during the inflationary epoch.

We now add the assumption that quantum fluctuations of the inflaton are responsible for the origin of structure. In this case, the power spectrum $\mathcal{P}$ of the curvature fluctuation $\mathcal{R}$ (see[61]) is given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{8\pi^2\varepsilon}\Big(\frac{H(k)}{M_{pl}}\Big)^2, \tag{3.11}$$

where k is the comoving wavenumber of the fluctuation mode and $H(k)$ is the value of H at the time when the mode k exits the Hubble radius. The parameter ε determines the deviation of the equation of state in the inflationary phase compared to pure de Sitter:

$$\varepsilon \equiv \frac{3}{2}\Big(\frac{p}{\rho} + 1\Big), \tag{3.12}$$

where p and ρ are pressure and energy densities, respectively. For inflation to provide the source of structure in the Universe, we need[63]

$$\mathcal{P}_{\mathcal{R}}(k) \sim 10^{-9}. \tag{3.13}$$

Combining (3.10) (3.11) and (3.13) leads to an upper bound on ε

$$\varepsilon \sim 10^9 \frac{1}{8\pi^2}\Big(\frac{H(k)}{M_{pl}}\Big)^2 \sim 10^9 \frac{V}{24\pi^2 M_{pl}^4} < 10^{-31}. \tag{3.14}$$

Since the power spectrum of gravitational waves is given by

$$\mathcal{P}_h(k) \sim \Big(\frac{H(k)}{M_{pl}}\Big)^2, \tag{3.15}$$

the tensor to scalar ratio r is given by

$$r = 16\varepsilon < 10^{-30}, \tag{3.16}$$

where the factor 16 comes from the different normalization conventions for the scalar and tensor spectra. While the discussion above assumed that the inflaton dominated the scalar perturbations it is important to note that the TCC constrains the absolute amplitude of the primordial gravitational waves. The bound on r therefore relies only on the TCC bound on the energy in Eq. (3.10) and the observed amplitude of $\mathcal{P}_{\mathcal{R}}$. Allowing scalar perturbations from additional fields or a modified sound speed for the inflaton, for example, will not relax Eq. (3.16).

From (3.16) we draw the conclusion that any detection of primordial gravitational waves on cosmological scales would provide evidence for a different origin of the primordial gravitational wave spectrum than any inflationary model consistent with the TCC. Note that a number of cosmological scenarios alternative to inflation do predict significant primordial tensor modes on cosmological scales. One example is String Gas Cosmology which predicts both a scale-invariant spectrum of cosmological perturbations with a slight red tilt[64] and a roughly scale-invariant spectrum of gravitational waves with a slight blue tilt[65].

Note that the above analysis applies not only to single field inflation, but also to multi-field inflation. The conclusions only depend on the fact that the parameter ε is $\ll 1$ which is self-consistent with what we found. The constraint also applies to *warm inflation*[66] models, models which can be consistent with the de Sitter swampland conjecture[67,68].

In this section, we have been general and have not assumed a slow-roll inflation. In the following section we will study the consequences of the TCC for slow-roll inflation.

3.2 Application to Slow-Roll Inflation

The equation of motion of a canonically normalized scalar field φ in a homogeneous and isotropic metric of the form (3.2) is

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0 \,, \tag{3.17}$$

where the prime indicates the derivative with respect to φ. Here, we have assumed that there is no important coupling of φ to other matter fields during inflation. Thus, the analysis in this section applies to cold inflation but not to warm inflation. The Friedmann equation takes the form

$$3H^2M_{pl}^2 = \frac{1}{2}\dot{\varphi}^2 + V, \tag{3.18}$$

In the case of single field slow-roll inflation, the slow-roll parameter ε is

$$\varepsilon \simeq \frac{M_{pl}^2}{2}\Big(\frac{V'}{V}\Big)^2 . \tag{3.19}$$

The slow-roll equation of motion is

$$3H\dot{\varphi} = -V' . \tag{3.20}$$

The field range $\Delta\varphi$ which the inflaton field φ moves during the period of inflation is given by

$$|\Delta\varphi| \simeq |\dot{\varphi}\Delta t| , \tag{3.21}$$

where Δt is the time period of inflation. We will show that $\Delta\varphi$ is very small compared to the Planck mass. In this case, it is self-consistent to assume that H and $\dot{\varphi}$ are constant. In this case using the TCC

$$\Delta t = H^{-1}N < H^{-1}\ln\Big(\frac{M_{pl}}{H}\Big) , \tag{3.22}$$

and, making use of (3.19), the field range becomes

$$\begin{aligned} |\Delta\varphi| &< \sqrt{2}\varepsilon^{1/2}\ln\Big(\frac{M_{pl}}{H}\Big)M_{pl} \\ &< \frac{10^{9/2}V^{1/2}}{M_{pl}}\ln\left(\frac{M_{pl}^2}{\sqrt{V}}\right) \\ &< 10^{-13}M_{pl} , \end{aligned} \tag{3.23}$$

where in the last inequality we used the monotonicity of $[x\ln(1/x)]$ for $x < e$ to substitute the upper bound on V from (3.10). As first studied in[69], in the case of *large field inflation*, i.e. $|\Delta\varphi| \gg M_{pl}$, the inflationary slow-roll trajectory is a local attractor in initial condition space, even taking into account metric fluctuations[70,71,72]. Small field inflation as is the case here, on the other hand, is not an attractor in initial condition space, as reviewed in[73]. If the field range for slow-roll inflation is constrained by the TCC conjecture to obey (3.23), then the inflationary scenario is faced with an initial condition problem. The expected initial field velocity is

$$\dot{\varphi}_i^2 \sim V \tag{3.24}$$

and hence

$$\frac{\dot{\varphi}_{SR}}{\dot{\varphi}_i} \sim \varepsilon^{1/2} < 10^{-15}\,, \tag{3.25}$$

and it takes fine tuning of the initial velocity in order to be sufficiently close to the slow-roll trajectory.

In the following we propose a model which can consistently explain observations, including the observational value of the tilt. We consider an inverted parabola potential $V(\varphi) = V_0 - |V''|\varphi^2/2$ over a small field range $[\varphi_i, \varphi_f]$ such that $\delta V/V \ll 1$ over the field range. Given the smallness of ε from (3.14), we have

$$M_{pl}^2\frac{V''}{V} \simeq \frac{n_s - 1}{2}, \tag{3.26}$$

where $n_s = 1 + 2\eta - 6\varepsilon$ is the tilt parameter and $\eta = M_{pl}^2 V''/V$ is the second slow-roll parameter. This fixes V'' from observation. From the equations (3.11) and (3.19) we find

$$\frac{V_0}{12\pi^2 M_{pl}^2 \varphi_{CMB}^2} \simeq \mathcal{P}(k)(\frac{n_s - 1}{2})^2. \tag{3.27}$$

where φ_{CMB} is the value of the field when the modes on CMB scales exited the Hubble horizon. Assuming H remains almost constant $H \simeq \sqrt{V_0/3M_{pl}^2}$ during the slow-roll inflation, one can show

$$\ln\left(\frac{\varphi_2}{\varphi_1}\right) \simeq |\eta| N(\varphi_1 \to \varphi_2), \tag{3.28}$$

where $N(\varphi_1 \to \varphi_2)$ is the number of e-folds accrued as φ goes from φ_1 to φ_2. If we plug $\varphi_1 = \varphi_{CMB}$ and $\varphi_2 = \varphi_f$ into the above identity and use the equation (3.7), we find

$$\frac{\varphi_f}{\varphi_{CMB}} = e^{|\eta| N_+} \simeq \frac{V_0^{\frac{|\eta|}{4}} \Omega_m^{\frac{|\eta|}{2}}}{(T_0 T_{eq})^{\frac{|\eta|}{2}}}. \tag{3.29}$$

Plugging φ_{CMB} from (3.27) into (3.29) leads to

$$\varphi_f \simeq \frac{V_0^{\frac{1}{2}+\frac{|\eta|}{4}} \Omega_m^{\frac{|\eta|}{2}}}{M_{pl}^2 (T_0 T_{eq})^{\frac{|\eta|}{2}} 12^{\frac{1}{2}} \mathcal{P}^{\frac{1}{2}} \pi |\eta|} \simeq 3.9 \times 10^5 \cdot \left(\frac{V_0}{M_{pl}} \right)^{0.505}, \tag{3.30}$$

where in the last step we substituted $|\eta| \simeq 0.02$, $\mathcal{P} \simeq 2 \cdot 10^{-9}$, $T_0 \simeq 3K$, $T_{eq} \sim 10^4 K$, and $\Omega_m \simeq 0.3$. This fixes the end of the field range φ_f in terms of the energy scale V_0. The only free parameters left are V_0 and φ_i.

Now we impost the TCC for the slow-roll trajectory to find a constraint in terms of φ_i and V_0. Plugging $\varphi_1 = \varphi_i$ and $\varphi_2 = \varphi_{CMB}$ in (3.28) gives

$$\varphi_{CMB} \simeq \varphi_i e^{\frac{1-n_s}{2} N_-}, \tag{3.31}$$

where N_- is the number of e-folds accrued before the modes on CMB scales exit the horizon. The total number of e-foldings is $N_{total} = N_- + N_+$. From (3.7) we find

$$e^N \simeq e^{N_-} \frac{V^{1/4}}{(T_0 T_{eq})^{1/2}}. \tag{3.32}$$

On the other hand, from the TCC, we know that the total number of e-folds is bounded by $e^N < M_{pl}/H$. Using (3.31), this can be expressed as

$$\left(\frac{\varphi_{CMB}}{\varphi_i}\right)^{\frac{2}{1-n_s}} < \frac{(3 M_{pl}^4 T_0 T_{eq})^{\frac{1}{2}}}{V_0^{\frac{3}{4}}}. \tag{3.33}$$

Plugging φ_{CMB} from (3.27) in (3.33) with $\mathcal{P} \simeq 2 \cdot 10^{-9}$, $n_s \simeq 0.96$, $T_0 \simeq 3K$, and $T_{eq} \sim 10^4 K$

leads to

$$(\frac{V_0}{M_{pl}^4})^{1.03} < 6.6 \times 10^{-12} \cdot (\frac{\varphi_i}{M_{pl}})^2. \tag{3.34}$$

The above inequality is necessary for the potential to be consistent with the TCC, but it is not sufficient. This is because a potential is consistent with the TCC if the inequality (3.1) is satisfied for every expansionary trajectory, not just one particular trajectory.

For energy scale $V_0^{1/4} = 10^{-10}M_{pl}$ the potential $V_0(1 - 0.02\varphi^2)$ defined over the field range $[\varphi_i, \varphi_f] = [9.7 \times 10^{-16}M_{pl}, 2.4 \times 10^{-15}M_{pl}]$ satisfies all the criteria (3.10), (3.30), and (3.34). These criteria were imposed by observation and consistency with (3.1) for the slow-roll trajectory. By numerical analysis, we further verified the consistency of this potential with the inequality (3.1) for every expansionary trajectory. This is an example of a simple potential that can explain the observation and be consistent with the TCC at the same time, however, due to its short field range, it suffers from the fine-tuning problem.

3.3 Conclusions and Discussion

We have studied the implications of the recently proposed the TCC for inflationary cosmology. Demanding that the TCC holds and that the largest scales that we currently probe in cosmology are sub-Hubble at the beginning of the inflationary phase (a necessary condition for the causal generation mechanism of fluctuations of inflationary cosmology to work) leads to an upper bound on the energy scale of inflation which is of the order of 10^9GeV. Demanding that the amplitude of the cosmological perturbations agree with observations then leads to an upper bound on the generalized slow-roll parameter ε of the order of $\varepsilon < 10^{-31}$. As a consequence, the tensor to scalar ratio is predicted to be smaller than 10^{-30}. A detection of primordial gravitational waves via B-mode polarization, pulsar timing measurements or direct detection, assuming the TCC, would then imply that the source of these gravitational waves is *not* due to quantum fluctuations during inflation.

The above conclusions are independent of any assumptions on the possible single-field nature of inflation. If we then specialize the discussion to the case of single field slow-roll inflation with a canonically normalized inflaton field, we find that the range $\Delta\varphi$ which the inflaton field traverses during the inflationary phase is of the order of $\varepsilon^{1/2}M_{pl}$. This raises an initial condition problem for most of the models since the expected field velocity is much larger than the field velocity along the slow-roll trajectory.

We proposed an inverted parabola potential as a simple example that is consistent with the TCC and can explain the observation at the same time.

4

Late time cosmology and dS bubbles

One of the major challenges facing present-day cosmology is understanding the nature of the observed dark energy. The simplest model is to assume that the dark energy is the energy of the minimum energy state of a theory. An example of this is represented by scalar fields with a potential. In such a scenario the minima of such scalar potential, if such points exist, would be (meta)-stable solutions to dark energy, leading to de Sitter spaces which seem to be a good approximation to the cosmological observations. Whether such a scenario would be absolutely stable or only metastable would depend on whether there are lower values of energy at other points in field space.

Trans-Planckian Censorship Conjecture (TCC) broadly leads to the dS swampland conjecture which forbids metastable de Sitter, however it is less restrictive. In particular does allow for the existence of metastable dS spaces, as long as their lifetime is short. The short-lived dS spaces decay by transitioning to a state with lower energy. In this chapter, we study the consequences of such short-lived dS spaces. In particular, we consider a sequence of transitions from one metastable dS space to the next, nucleated by membranes, and capture this in terms of a dual effective theory of a scalar whose rolling in discrete steps captures these transitions. This scenario is reminiscent of the inflationary models in[74,75], which also involve a cascade of metastable dS vacua.

The transitions between nearby dS vacua are severely restricted by swampland conditions. In particular, TCC puts a strong upper bound on the lifetime of such a transition. Additionally, we can ask how do other swampland conjectures such as the Weak Gravity Conjecture (WGC) restrict the possibilities. Indeed WGC leads to the statement that the tension of the membranes which nucleate the decay cannot be too large. Surprisingly, we find that the fast decay implied by TCC already implies this as a consequence. Moreover, TCC leads to light enough membranes which in some limits can be viewed as localized excitations. For sufficiently small cosmological constant the generalized distance conjecture leads to predictions of the mass of the tower of such light states. We find that the TCC is again compatible with this prediction. This interwoven relationship between different Swampland conjectures which is also seen in many other contexts is indeed reassuring.

One could ask whether the resulting dual effective potentials that emerge are of the generic type allowed by TCC or the fact that they are generated by dS transitions makes them more restrictive. Indeed we find that they are more restrictive. In particular eternal inflation which naively is compatible with TCC is marginally ruled out as being dual to such transitions. This points to the possibility that eternal inflation is never allowed and is in the swampland as has been suggested in[39].

The organization of this section is as follows: In subsection 2 we review the membrane dynamics which lead to decays of the dS space. We also derive effective dual potentials capturing such transitions. In section 3 we apply WGC and TCC to the membrane dynamics. In subsection 4 we study the emergent potential and study its properties, and in particular, observe that eternal inflation is not compatible in this dual formulation. In subsection 5 we discuss the cosmological implications of our observations. In subsection 6 we end with some conclusions. Some of the technical aspects are presented in the appendices.

4.1 Membrane nucleation in metastable de Sitter

The basic point of this chapter is to study Swampland constraints in a de Sitter space whose cosmological constant changes via non-perturbative membrane nucleation processes. So we first need to understand how this process takes place, and how it translates to an "effective potential". We do both things in this section, relegating most details to the appendices.

Review: Thin-wall membrane nucleation Let us assume the existence of some metastable de Sitter vacuum. This vacuum should eventually decay to some lower energy configuration. The most standard decay channel is via Coleman-de-Luccia bubble nucleation[76,77], in which a bubble of true vacuum nucleates inside the false vacuum and starts expanding in an accelerated fashion, almost at the speed of light. This is a non-perturbative semiclassical instability whose transition rate can be estimated in terms of a Euclidean instanton solution,

$$\Gamma = P e^{-S} \tag{4.1}$$

where S is the euclidean classical instanton action and P is some prefactor involving the quantum fluctuations. For the bounce solution to exist, the bubble needs to nucleate with a critical radius R such that the cost of energy of expanding the bubble (the surface tension) is smaller than the energy gain associated with the difference of energies outside and inside the bubble. The result for S and P can be computed in the thin wall approximation, which neglects the physical width of the domain wall in comparison to its critical radius. This is done in appendix D, while here we will only present the results when gravitational corrections are negligible[1].

The critical radius R of the bubble in de Sitter is given by

$$(RH)^2 \simeq \frac{1}{1 + (R_0 H)^{-2}}\,, \quad R_0 = \frac{T}{\Delta\Lambda} \tag{4.2}$$

where $H = \Lambda^{1/2}$ is the Hubble scale and throughout the chapter, we will be working in Planck units. Here T is the tension of the domain wall and $\Delta\Lambda$ is the difference of vacuum energies on the two sides of the bubble. Note that R is smaller than the Hubble length, $R \leq H^{-1}$, and that in the flat space limit where $H \to 0$ we get $R = R_0$. The instanton action in (4.1) is given by

$$S \simeq \frac{T}{H^3} w(R_0 H)\,, \quad \frac{w(q)}{2\pi^2} = \frac{1 + 2/q^2}{\sqrt{1 + 1/q^2}} - \frac{2}{q} \tag{4.3}$$

while the instanton prefactor, up to order one factors, reads

$$P \simeq T^2 R^2 \simeq \frac{T^2 R_0^2}{1 + (R_0 H)^2} \tag{4.4}$$

More details of the computation of the prefactor can be found in[78]. Due to the gravitational effects, it is also possible to have up-tunneling in de Sitter space, but it is much more suppressed if $\Delta\Lambda < \Lambda$ (see appendix D).

In the flat space limit, i.e. when the critical radius of the bubble is much smaller than the Hubble scale, the instanton action and prefactor can be approximated by

$$S \simeq \frac{2\pi^2 T^4}{\Delta\Lambda^3}\,, \quad P \simeq \frac{T^4}{\Delta\Lambda^2} \tag{4.5}$$

while $R \simeq R_0$.

Our analysis will be mostly in the thin wall approximation, which we just described. In the opposite limit, when the membrane becomes very thick, there is a decay channel known as the

[1] Gravitational corrections are negligible when the tension os the bubble T is much smaller than the Hubble scale $\sqrt{\Lambda}$ in Planck units. This approximation will be sufficient for this chapter, as we will see that larger values of T are not consistent with the swampland constraints.

Hawking-Moss transition[79], which dominates over the thin wall Coleman-de-Luccia bubble nucleation. While we focus on the thin wall approximation, we can also put some constraints on the Hawking-Moss scenario, which we describe in appendix F.

We finish the review with a couple of comments. In subsequent sections, we study a sequence of successive mild tunnelings that could be effectively described by a smoothly evolving scalar field with a potential. This can be a good approximation only if we assume that the physical observables do not drastically change from one vacuum to the next. Because of that, in the following, we focus on cases where the de Sitter minimum decays to a less energetic nearby local de Sitter minimum with positive energy. This, in particular, implies that

$$\Delta\Lambda < \Lambda. \tag{4.6}$$

Depending on the model, this process could be repeated multiple times, going through different metastable dS vacua until reaching either an AdS supersymmetric vacuum or decaying to nothing[2].

In both cases, we expect a drastic change of the physical observables, either by suffering a Big Crunch or because the vacuum annihilates to nothing. In fact, a drastic change when $\Delta\Lambda$ becomes of order Λ is also motivated by a generalization of the swampland distance conjecture applied to the space of metric configurations[83] since the flat space limit $\Lambda \to 0$ is at infinite distance in this field space. Therefore, we will not discuss these final transitions here but focus on the chain of CdL transitions that will discharge the positive vacuum energy little by little, but staying on a quasi-de Sitter phase and assuming that the physics does not significantly change in the process.

Let us finally remark that in the following we will use the above Coleman De Luccia formulae even if the action (4.3) is of order one and there is no exponential suppression. This

[2] It has been shown in certain setups of AdS flux vacua[80,81,82] that there is an alternate decay channel where all of the flux is eaten up all at once and spacetime just ends at a "bubble of nothing". It would be interesting to study if the bubble of nothing in dS can also be understood as a limiting process of the thin-wall transitions we are describing here, and whether this can be used to put an interesting upper bound on the decay rate of a de Sitter vacuum.

is justified because the coupling of the domain wall is small, as argued in detail in appendix D.

4.1.1 The effective potential

We have just discussed the dynamics of a universe in which bubbles nucleate and expand in an accelerated fashion. But what precisely does a single observer see, on average? Sitting at the center of her very own static patch, things will not change much and will look approximately de Sitter, until she is hit by a bubble, which nucleated somewhere else.

After the bubble hits, the vacuum energy has changed by a little bit. Averaging over many transitions, we can replace these discrete jumps in the value of the cosmological constant by an effective scalar φ with a potential $V(\varphi)$. The characteristics of this potential are in turn determined solely by the fundamental parameters of the membrane picture, T and $\Delta\Lambda$.

This allows us to connect directly with the usual quintessence/slow-roll inflation literature, and indeed, over large distances and times the two descriptions are interchangeable[3].

A detailed derivation of the potential can be found in appendix E. The basic idea is that to compute the vacuum energy one only needs to compute how many bubbles reach the observer per unit time. At first, it would seem one needs to integrate the bubble production rate over the past lightcone of the observer. However, a bubble will not reach the observer if it hits another bubble and annihilates with it first. As a result, we only need to integrate the bubble production rate over some spherical effective volume $\mathcal{V}_{\text{eff}}$. Thus, the number of bubbles per unit of proper time is

$$\frac{dN}{dt} = \Gamma\,\mathcal{V}_{\text{eff}}. \tag{4.7}$$

Equation (4.7) is all we needed to compute the potential explicitly since

$$\frac{dV}{dt} = \Delta\Lambda\frac{dN}{dt} = \Delta\Lambda\Gamma\,\mathcal{V}_{\text{eff}} \sim \frac{(V')^2}{\sqrt{V}}, \tag{4.8}$$

[3] There are two main differences with the standard picture: at short enough times or length scales, the changes in the vacuum energy are discrete as we just discussed; and as we will see later on, we cannot get just any $V(\varphi)$ from the membrane perspective; the potential gets additional constraints.

where the last equality is a slow-roll expression. That is, we assumed that the vacuum energy can be described by a slow-rolling scalar with potential $V(\varphi)$, and then equated the slow-roll expression for dV/dt from what we get from membranes. Rearranging, one gets

$$\left(\frac{V'}{V}\right)^2 \sim \frac{\Delta\Lambda}{\Lambda^{3/2}}\Gamma\,\mathcal{V}_{\text{eff}}, \tag{4.9}$$

which determines the potential completely once we know $\mathcal{V}_{\text{eff}}$. A detailed derivation of this effective volume can be found in appendix E (see (E.15) for the general result for $\mathcal{V}_{\text{eff}}$.). Here, we will only note the two limiting cases that are relevant for our constraints:

- It is intuitively obvious that $\mathcal{V}_{\text{eff}}$ cannot grow larger than the Hubble horizon. In case that $\mathcal{V}_{\text{eff}}$ is this large, we find

  $$\frac{V'}{V} = \frac{\Delta\Lambda^{1/2}}{\Lambda^{3/2}}\,\Gamma^{1/2}. \tag{4.10}$$

  This corresponds to the case where collisions are rare ($\Gamma \ll H^4$) and basically all membranes which are produced in the past lightcone reach the observer.

- On the other hand, when the critical radius is much smaller than the Hubble scale and collisions are common ($\Gamma \gg H^4$), the effective volume is determined by the distance to the closest nucleating event, which is of order $\Gamma^{-1/4}$. So in this case

  $$\frac{V'}{V} = \frac{\Delta\Lambda^{1/2}}{\Lambda^{3/4}}\,\Gamma^{1/8}. \tag{4.11}$$

So to sum up, we have membranes that discharge the background cosmological constant, and a convenient description in terms of an effective potential. Without any further assumptions, this could take a very long time, the effective potential would be extremely flat, and the de Sitter could be extremely long-lived. In this chapter, we will see that Swampland conditions such as TCC and WGC place constraints on just how fast these decays can happen.

4.2 Swampland constraints on bubble nucleation

The goal of this section is to investigate the swampland constraints on the decay rate of bubble nucleation in metastable de Sitter vacua. We will see that, in particular, the Weak Gravity Conjecture and the Transplanckian Censorship Conjecture imply non-trivial constraints on the properties of the bubbles/membranes. It will be very convenient from now on to parametrize the scaling of the tension of the bubble and the difference in the vacuum energy in terms of Λ as follows,

$$T \sim \Lambda^{\alpha}\,, \quad \Delta\Lambda \sim \Lambda^{\beta}. \tag{4.12}$$

For the time being, we can think of α, β as constants although we will later allow them to depend on Λ as well.

4.2.1 The Weak Gravity Conjecture

The Weak Gravity Conjecture[84] states that, given a theory with a p-form gauge field weakly coupled to Einstein gravity, there must exist an electrically charged state satisfying

$$\gamma T \leq Q \tag{4.13}$$

where $Q = g_p q$ is the physical charge (including the gauge coupling g_p), T is the tension and γ is the charge to tension ratio of an extremal black brane in that theory. We will be applying this to a codimension-1 object, a membrane coupled to a 3-form gauge field with gauge coupling g_3. Since our primary interest is weakly curved de Sitter space, we will be ignoring potential corrections to the WGC bound from the positive cosmological constant[4].

The interpretation of the WGC for codimension-one objects is a bit subtle as the backreaction of these objects is very strong and destroys the asymptotic structure of the vacuum. Hence, they should not be understood as normalizable states around a given vacuum but rather

[4] See [85,86,87] for work on the (particle) version of the WGC in dS.

as defects sourcing localized EFT operators[88]—a perspective that has been recently analyzed in[89] in relation to the Swampland conjectures—. The 4d backreaction away from the object translates then into a classical RG flow to low energies that makes the tension scale-dependent. In this chapter, we will assume that the WGC applies to any energy scale and will impose the WGC to the domain wall solutions in the IR (an approach already taken in[90] when using the WGC to argue for a bubble instability for any non-supersymmetric vacuum).

In order to apply the WGC to the domain walls, we are assuming that the CdL bubble nucleation corresponds to a Brown-Teitelboim transition[91] in the sense that the domain wall contains a localised membrane on its core charged under a 3-form gauge field. This is characteristic for example of vacua arising from compactifications with internal fluxes. When crossing one WGC domain wall of quantized charge q, the quantized background 4-form field strength F_4 changes by q units. Since this field strength parametrises the vacuum energy, the charge of the domain wall roughly corresponds to the difference of vacuum energies in the tunneling transition. Consider for instance a single 3-form, with a (possibly field-dependent) gauge coupling g_3. The vacuum energy is such that the potential reads

$$\Lambda = \frac{1}{2} g_3^2 n^2. \tag{4.14}$$

We allow g_3 to depend on n polynomially, and assume that any other contribution to the vacuum energy is subleading with respect to (4.14). Then, one has that

$$Q = g_3 \, \Delta n \simeq \Delta(\sqrt{\Lambda}). \tag{4.15}$$

By plugging (4.15) into (4.13) we get that the WGC for domain walls implies

$$T \lesssim \frac{\Delta\Lambda}{\Lambda^{1/2}} \tag{4.16}$$

where we have assumed that the variation of vacuum energy is small. We have also neglected an order one factor coming from the extremality factor γ in (4.13) as we will only be interested in the scaling of the tension with the vacuum energy. Upon using (4.12), the above inequality translates into the following constraint on α, β,

$$\alpha - \beta + \frac{1}{2} \gtrsim 0\,. \tag{4.17}$$

It is interesting to note that (4.16) is equivalent to imposing $R_0 \lesssim H^{-1}$ where R_0 is the flat space radius defined in (4.2). Recall from section 4.1 that the decay rate can be written as a function of two variables, T and R_0 in Hubble units. The WGC bound $R_0 \lesssim H^{-1}$ makes the instanton action small, so ameliorates the exponential suppression, but also decreases the prefactor, as can be checked using (4.3) and (4.4). Hence, for a given value of the tension, the WGC implies an upper bound on the decay rate of bubble nucleation. This is reminiscent of the situation in [86], where WGC-like considerations led to an upper bound on how fast black holes should decay.

4.2.2 Transplanckian censorship conjecture

dS space seems to be difficult to realize in controllable regimes of String Theory. An example of this tension is a class of no-go theorems that forbid a metastable dS in the asymptotic of the field space which motivated the dS Swampland conjecture [14] (for related Swampland ideas see [92,93,15,94,95,96,97,98,99,100,83,86,89]). This key observation has led to multiple Swampland conditions that aim to find a more general principle that could explain the tension between the dS space and consistent quantum theories of gravity. One of such Swampland conditions, the Trans-Planckian Censorship Conjecture (TCC), states that the expansion of the universe must slow down before all Planckian modes are stretched beyond the Hubble radius [19]. If TCC gets violated, the Planckian quantum fluctuations exit the Hubble horizon, freeze out and classicalize which is, at the very least, strange. A variety of non-trivial consequences of TCC for scalar field potentials were studied in [19] and shown to be consistent with all known controllable string theory constructions. In this chapter, we will not enter into motivating the TCC, but simply study its implications for the case of metastable de Sitter vacua in more detail. A survey of the motivations for TCC can be found in [22].

For metastable de Sitter spaces where the Hubble parameter stays constant, the TCC imposes an upper bound on the lifetime as follows[19],

$$\tau \lesssim \frac{1}{H}\frac{1}{\log(1/H)}. \tag{4.18}$$

We will be referring to this upper bound as the TCC time τ_{TCC}. In the rest of this chapter, we focus on the leading terms in our computations and will ignore the logarithmic correction factor above. We will come back and discuss the effect of the subleading corrections in subsection 4.3.4.

Let us now study the consistency of the TCC with the CdL decay mechanism reviewed in section 4.1. In particular, we will momentarily see that thin-wall tunneling could be consistent depending on the characteristics of the domain wall. In appendix F we also discuss the Hawking-Moss transition to show that when it is the dominant decay channel it is (marginally) inconsistent with the TCC.

In section 4.1 we provided $\Gamma = Pe^{-S}$ in terms of T and $\Delta\Lambda$ for thin-walls. By plugging (4.12) into (4.3) and (4.4), we find the TCC takes the following form in terms of α and β,

$$\Gamma > H^4 \;\rightarrow\; \frac{\Lambda^{4\alpha-2\beta-2}}{1+\Lambda^{2\alpha-2\beta+1}}\exp\left(-\Lambda^{\alpha-3/2}w(\Lambda^{\alpha-\beta+1/2})\right) \gtrsim 1. \tag{4.19}$$

When Λ is very small, the above inequality can be approximated by

$$\Lambda^{4\alpha-2\beta-2}\exp\left(-\Lambda^{4\alpha-3\beta}\right) \gtrsim 1, \tag{4.20}$$

which can also be derived by using the flat space approximations for P and S in (4.5).

4.2.3 Constraints on domain walls

In the previous two subsections, we discussed how the Swampland conditions could be applied to the domain walls. In this subsection, we combine those results and perform a systematic study of what domain walls belong to the Swampland.

Figure 4.1 shows how the WGC (eq. (4.17)) and the TCC (eq. (4.19)) constrain the values of α and β which characterize thin-walls to lie in a confined blue region. We only present the results $\beta > 1$ as this is implied by (4.6), taking into account that in Planck units $\Lambda < 1$. An interesting feature is that TCC imposes a stronger constraint than WGC; in other words, in most of the parameter space, TCC implies WGC for domain walls. There is a region near $(2, 3/2)$ where the two curves intersect. Which one imposes the stronger constraint is sensitive to $\mathcal{O}(1)$ factors; we will comment on these in section 4.3.4.

The boundary of the blue region, which represents the TCC condition (4.19), does not have a simple analytic form in α and β for a general Λ. However, for exponentially small values of Λ, such as would be in our universe, the blue region given by $\Gamma = Pe^{-S} \gtrsim H^4$ can be well approximated by a triangle whose boundaries can be easily determined by looking at (4.20), from which we can extract $P \simeq \Lambda^{4\alpha-2\beta}$ and $S \simeq \Lambda^{4\alpha-3\beta}$. The triangle is delimited by two lines, one corresponding to $P \gtrsim H^4$, and another to $S \lesssim \mathcal{O}(1)$ to eliminate the exponential suppression, as follows

$$S \leq 1 \ \rightarrow \ 4\alpha \geq 3\beta, \tag{4.21}$$

$$\mathcal{P} \geq H^4 \ \rightarrow \ 4\alpha - 2\beta \leq 2 \tag{4.22}$$

These two lines provide a fairly accurate envelope of the numerical blue region if Λ is very small, except for the region at the tip of the triangle. The point where the triangle almost touches the WGC line corresponds to eternal inflation potentials, as we will discuss in more detail in section 4.3.3.

We also note in passing that for the whole approach to be valid, we should impose that the radius of the bubbles is above the cutoff of the EFT. Choosing the cutoff to be at the Planck scale, this just removes the point $(\alpha, \beta) = (1, 1)$, as all the bubbles inside the blue region have a subplanckian radius. Lowering the cutoff from the Planck scale to e.g. GUT scale would remove a very small region around this point, but this does not affect our constraints very much and the qualitative features of the plot remain unaltered.

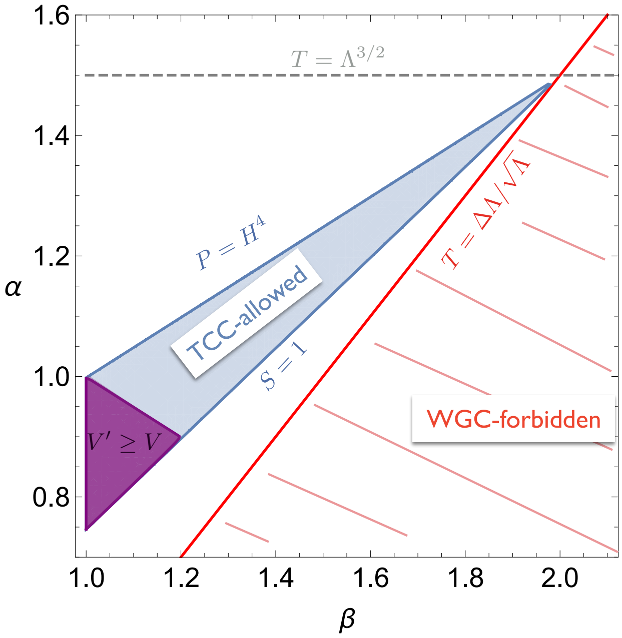

Figure 4.1: Allowed regions in the (α, β) plane for $\Lambda = 10^{-120}$. The left hand side of the red line is allowed by the WGC for membranes (4.17), while the light blue region corresponds to the TCC allowed region (4.19). The purple region corresponds to $V'/V > 1$ and points above the horizontal dotted line have $T \leq \Lambda^{3/2}$, which are disfavoured by the Higuchi bound and Distance Conjecture.

We have also added a few more lines in figure 4.1. First, the horizontal dotted grey line represents membranes with $T = \Lambda^{3/2}$. In the next section, we will provide some arguments that motivate us to only allow membranes below this line. Finally, we have highlighted in purple the region corresponding to $V'/V > 1$ by using the full derivation of the effective potential in terms of the decay rate in (4.9). This region is excluded by observational constraints in our universe. The line bounding this region and the rest of the blue triangle can be simply derived from (4.11), which is a good approximation since $\Gamma > H^4$ inside the blue region. Hence, by plugging $\Gamma \simeq P \simeq \Lambda^{4\alpha - 2\beta}$ into (4.11) we get

$$\frac{V'}{V} = 1 \rightarrow 2\alpha + \frac{\beta}{4} - \frac{3}{4} = 0\,. \tag{4.23}$$

4.2.4 Comments on Higuchi bound and Distance Conjecture

In figure 4.1 we have drawn a horizontal dotted line at the value associated with $T = \Lambda^{3/2}$. Here we will provide three different arguments in favor of imposing $\alpha \leq 3/2$ which, even if not completely conclusive, motivates this upper bound. These arguments involve

(1) the breakdown of effective field theory (2) applying Higuchi bound to the membrane and (3) the application of membrane excitations as leading to light states predicted by the generalized distance conjecture[83]. Note that the combination of this upper bound with the WGC constraints implies a finite region on the (α,β)-plane, which implies by itself an upper bound on the decay rate (independently of the TCC). Interestingly, this upper bound is a bit less restrictive but still consistent with the TCC.

Our domain wall solutions contain fundamental membranes on their core, which mediate transitions between different flux vacua. Often in string compactifications, the domain wall solutions involve additional scalar fields which get a nontrivial profile in the membrane background. If we go high up in energies, the membranes can be seen as localized free objects with a tension T_{mem} which can differ from the tension of the domain wall due to the contribution from the scalar flow driven by the membrane backreaction, so that $T \geq T_{mem}$. For the semiclassical description of these membranes not to break down, we need the tension to be above the cut-off of the EFT, $T_{\text{mem}} \geq \Lambda_{\text{cutoff}}^{3/2}$[89]. Otherwise, it would not be possible to describe the membrane within a local EFT. Since this cut-off is associated with the membrane sector, it can be disconnected from the SM of particle physics and could, in principle, take any value. However, it seems reasonable to impose that it is above the Hubble scale in an expanding universe, $\Lambda_{\text{cutoff}} > H$.

Hence, we get that $T_{\text{mem}} \geq \Lambda^{3/2}$ so there is a lower bound for the membrane tension in terms of the cosmological constant, which in turn implies a lower bound for the domain wall tension in the IR as $T \geq T_{mem} \geq \Lambda^{3/2}$, implying

$$\alpha \leq 3/2\,. \tag{4.24}$$

This is also consistent with the fact that in dS space, any mass scale less than Hubble is physically not measurable in the current phase of the universe. In other words, having the mass scale associated with the membrane $T^{1/3} \leq \Lambda^{1/2}$ will be unobservable. So we may as well restrict to $\alpha \leq 3/2$. To sum up, as long as the domain wall has a fundamental membrane on its core which can be described semiclassically with a local EFT, and the EFT cut-off is bigger than the Hubble scale, then one needs to impose (4.24). Of course, not every CdL transition needs to have the interpretation of a Brown-Teitelboim flux transition with a fundamental semiclassical membrane on its core, but otherwise, the justification of applying the WGC to the domain walls is less clear. If the membrane cannot be described semiclassically, then the EFT is non-local and it is not clear how to even start defining a charge under a gauge field and how to apply the WGC then.

The second argument comes from applying the Higuchi bound to the membranes. First of all, notice that it is not possible to apply the Higuchi bound directly to the IR domain walls, as the membranes are confined and the mass scale of the excitation modes is not associated with $T^{1/3}$. In fact, if the bubble is expanding, the volume contribution in $E \sim TR^2 - \Delta\Lambda R^3$ dominates over the tension surface, and the modes are tachyonic as they are describing a vacuum instability. However, it is possible to apply the Higuchi bound to the localized membranes at the core of the domain walls as long as the relevant energy scale is above the confinement scale and they behave as free objects. Indeed, the condition $T_{\rm mem} \geq \Lambda^{3/2}_{\rm cut-off}$ guarantees that there is a regime in energies in which very small spherical membranes behave as semiclassical unstable particles with a lengthscale at least of the order of their Compton length $l_c \sim T_{\rm mem}^{-1/3}$.

In other words, there are small unstable pockets that contract as soon as they are formed, with an energy that it is well approximated by $E \sim T_{\rm mem} l_c^2 \sim T_{\rm mem}^{1/3}$ since $T_{\rm mem} l_c^2 \gg \Delta\Lambda\, l_c^3$ if $T_{\rm mem} \geq \Lambda^3_{\rm cutoff}$ and $\Delta\Lambda \ll \Lambda_{\rm cutoff}$[5]. By applying the Higuchi bound to these small spherical membranes, we get that $T_{\rm mem}^{1/3} \geq \Lambda^{1/2}$ implying again (4.24).

The last argument is more a proposal for an interpretation of the role of these membranes in case they satisfy (4.24). Interestingly, these membranes are candidates to fulfill the AdS Distance Conjecture in de Sitter space[83]. The conjecture states that there should an infinite tower of states

[5]Notice that $T_{\rm mem} l_c^3 \gg \Delta\Lambda\, l_c^3$ is equivalent to require $S \sim T_{\rm mem}^4/\Delta\Lambda^3 \gg 1$. Thanks precisely to the scalar contribution due to the backreaction induced by the membrane, we can satisfy this condition for the membranes but still violate it for the domain wall in the IR (so that the domain wall will be consistent with the TCC later on). For this to happen one needs to have $T/Q|_{DW} < (T/Q)_{\rm mem}$, which is expected by the WGC if we have a vacuum which breaks spontaneously supersymmetry but the membranes were originally BPS.

with a mass of order

$$m \sim \Lambda^{\partial} \tag{4.25}$$

since the flat space limit $\Lambda \to 0$ is at infinite distance in the space of metric deformations. In de Sitter space, the Higuchi bound forces ∂ to be $\partial \leq 1/2$, which is equivalent to having $T_{\text{mem}} \geq \Lambda^{3/2}$. The conjecture does not specify what is the origin of the tower of states. In AdS space, they usually correspond to particles, KK towers for concreteness, underlying the absence of scale separation typically observed in AdS vacua. An interesting possibility is that, in de Sitter space, the tower of states comes from membranes and it is, therefore, eventually underlying the instability of these vacua. We could turn the argument around and say that, if the membranes provide the states satisfying the AdS Distance Conjecture, then they need to satisfy (4.24).

4.3 Emergent Potential and the Swampland

In the previous section, we studied how the Swampland conditions constrain the domain wall parameter space. As we saw in subsection 4.1.1, successive tunnelings between neighboring vacua can be effectively described by a smooth rolling of an emergent scalar field in a potential. We can either apply TCC in the membrane perspective or directly to the emergent effective potential without taking its microscopic origin into account. We will find that the membrane perspective leads to a restrictive class of emergent potentials that could not be obtained otherwise. This seems to extend the meaning of TCC and in particular, leads to essentially forbidding eternal inflation.

4.3.1 Flat potentials and TCC

The TCC implies the general statement that a quasi-deSitter phase cannot last more than $\frac{1}{H}\ln(1/H)$. We will now tailor this statement to the particular case of very flat ($|V'| \ll \frac{V}{|\ln(V)|}$) monotonic potentials. As we will see in subsection 4.2, these are the kind of potentials we get from the membrane picture on a range of parameter spaces. We aim to find the strongest condition that TCC alone imposes on this kind of potential.

First, we show that the TCC implies the field range needs to be sub-Planckian. We prove this by contradiction. Suppose the field range is trans-Planckian. Consider a slow-roll trajectory over an $\mathcal{O}(1)$ sub-interval of the field range. For the slow-roll trajectory we have

$$d\varphi \simeq \frac{|V'|}{\sqrt{3V}}dt. \tag{4.26}$$

Since $|V'| \ll V$, the change in V over this field range is negligible and V can be taken to be constant. From the TCC, we find $\Delta t < \frac{1}{H}\ln\left(\frac{1}{H}\right) \sim \frac{|\ln(V)|}{\sqrt{V}}$. Plugging this and $|V'| \ll \frac{V}{|\ln(V)|}$ in equation (4.26) gives

$$\Delta\varphi \sim \frac{|V' \ln(V)|}{V} \ll 1, \tag{4.27}$$

which is in contradiction with our assumption. Thus, the field range must be sub-Planckian. This fact combined with the fact that the potential is very flat, allows us to take V and H to be nearly constant in our computations. Given that H is almost constant, the consistency of the slow-roll trajectory with TCC would imply that any other trajectory is consistent with TCC as well. This is because it only takes one Hubble time for a trajectory to become slow-roll and TCC upper bound for the duration of the inflation is $\frac{1}{H}\ln\left(\frac{1}{H}\right)$ which for small values of the cosmological constant is much greater than the Hubble time. So the first part of the trajectory before the slow-roll is negligible. In any case, note that the derivation of the effective potential (4.9) in appendix E assumes slow-roll.

This can also be expressed in terms of the potential alone, without referring to the slow-roll trajectory. By rearranging (4.26) and imposing TCC, we get

$$\sqrt{3V}\int \frac{d\varphi}{|V'|} = \int dt < \sqrt{\frac{3}{V}}\ln\left(\sqrt{\frac{3}{V}}\right) \quad \rightarrow \quad 2V\int \frac{d\varphi}{|V'|} \lesssim |\ln(V)|. \tag{4.28}$$

In short, TCC only imposes that the potential must get steep ($|V'| \gtrsim V$) after the time

$\tau_{TCC} \sim \frac{1}{H} \ln\left(\frac{1}{H}\right)$. When the potential is induced by successive tunnelings as in section 4.1.1, this constraint could be interpreted as a statement about the time evolution in the $\alpha - \beta$ plane in figure 4.1. TCC is equivalent to requiring the trajectory in $\alpha - \beta$ plane to reach the purple region ($|V'| \gtrsim V$) in less than τ_{TCC} time, and nothing else. In particular, it does not lead to any pointwise constraints on the potential. As we will now see, combining with the membrane picture, it is possible to do better.

4.3.2 Swampland constraints on the membrane effective potential

We start by finding the characteristics of the effective potentials that can arise from membrane tunneling. Suppose we have a nearly flat ($|V'| \ll V$) monotonic potential. We investigate the possibility of dividing up the field range into small enough intervals $(\Delta\varphi)_i$ such that each piece can be approximated by a linear function, and each discrete jump can be realized by an allowed membrane nucleation. We define parameters θ_n and γ_n for the n-th piece as follows.

$$V_n^{\theta_n} = \frac{|V_n'|}{V_n},$$
$$V_n^{\gamma_n} = (\Delta\varphi)_n, \tag{4.29}$$

where V_n and V_n' are the potential and its slope at the n-th interval. Supposing $\Delta\varphi$ is small enough we find

$$V_n^{\beta_n} = (\Delta V)_n \simeq |V_n'|(\Delta\varphi)_n \tag{4.30}$$

which gives the following relation for β,

$$\beta_n = \theta_n + \gamma_n + 1. \tag{4.31}$$

Applying the slow-roll condition gives

$$(\Delta t)_n \simeq \frac{(\Delta V)_n}{|V'_n|\dot{\varphi}_n} \sim \frac{(\Delta V)_n\sqrt{V_n}}{|V'_n|^2} = V_n^{\beta_n - 2\theta_n - \frac{3}{2}}. \tag{4.32}$$

Plugging β in terms of θ and γ leads to

$$(\Delta t)_n \sim V_n^{\gamma_n - \theta_n}\frac{1}{H} \tag{4.33}$$

Note that the derivation of the effective potential in section 4.1.1 allows us to compute θ as a function of α and β defined in (4.12). Using this as well as (4.31), we can translate the swampland constraints on the (α, β)-plane of figure 4.1 to the (θ, γ)-plane instead, as shown in figure 4.2. It is very important that not every point in the (θ, γ) is the image of a point in the (α, β) plane; points that are not in the image (green region in the figure) are not physically meaningful from the point of view of the membranes. In addition, the map is 2-to-1; two different points in the (α, β) plane map to the same point on (θ, γ)[6]. The blue region in figure 4.1 "folds over itself" along $S \sim 1$ to be mapped to the blue region in figure 4.2. Every point in the blue region in figure 4.2 has two preimages; one with $S > 1$ and the other with $S < 1$. More generally, the entire (α, β) plane folds over itself along the curve $\partial_\alpha(\Gamma) = 0$, which is very close to, but not exactly at, the lower boundary of the TCC region in figure 4.1, and after the vertex of the TCC triangle it goes on to a line of almost constant α. The folding line gets mapped to the boundary of the green region in figure 4.2, which is approximately described by the following function:

$$\gamma = \begin{cases} \frac{1}{5}\left(1 + 3\,\theta\right) & \theta \lesssim 1/2 \\ \theta - 0.013 & \theta \gtrsim 1/2 \end{cases}. \tag{4.34}$$

This provides, for each value of θ, the maximum value of γ consistent with a membrane origin of the effective potential. Notice that $\gamma = \frac{1}{5}(1 + 3\,\theta)$ is equivalent to the condition for the instanton action to be $S \sim 1$ in the flat space limit.

[6] This is related to the fact that the lifetime of the dS can be unchanged if while the membrane action increases the prefactor increases in a way to compensate this.

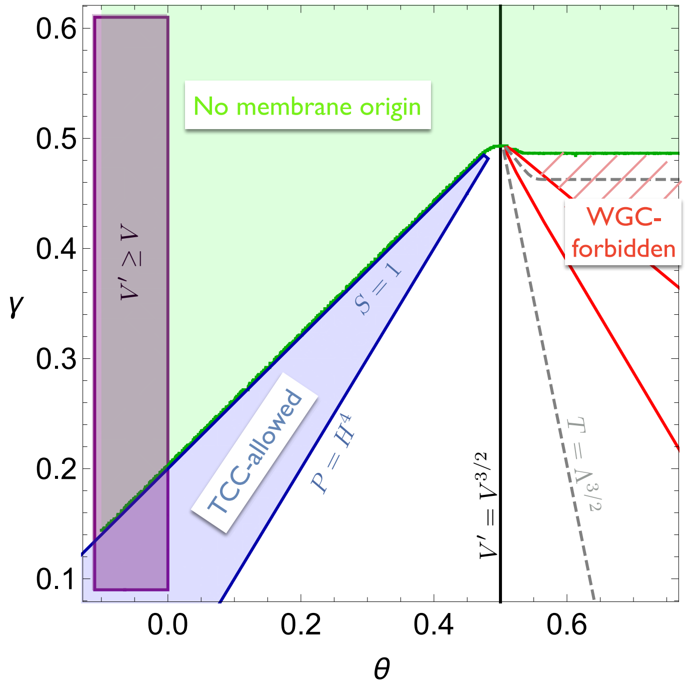

Figure 4.2: Allowed regions in the (γ, θ) plane for $\Lambda = 10^{-120}$. Not all of the (θ, γ) plane corresponds to a valid membrane picture (i.e. the map from the (α, β) plane to (θ, γ) is not onto); we have shaded in light green the region which is not part of the image. The purple region corresponds to $V'/V > 1$, while the light blue region corresponds to the TCC allowed region. The red line saturates the WGC for membranes and the region above the upper red branch is forbidden by the WGC. The region to the right of the grey dotted line one has $T < \Lambda^{3/2}$ and is disfavoured by the Higuchi bound and Distance Conjecture. The black line represents the "eternal inflation" locus, defined by $|V'| = V^{3/2}$.

There is a potential ambiguity in the definition of the decay time that needs to be addressed. The time Δt in (4.33) is the time it takes for the transition to occur everywhere in the Hubble patch which is different from the lifetime associated to an individual bubble[7] used when applying the TCC in section 4.2.3. Applying the TCC to the former time, i.e. $\Delta t \leq H^{-1}$ simply implies the condition $\gamma \geq \theta$, which a priori might seem different (even weaker) than the constraint coming from applying the TCC to the membrane picture ($\Gamma > H^4$) represented as the blue region in figure 4.1. However, as we see from the figure, in practice applying the TCC to the effective potential provides the same constraints as applying the TCC to the individual membranes as long as we restrict ourselves only to those potentials that can be interpreted

[7]The lifetime associated to an individual membrane is the time scale for only one bubble to form somewhere in the universe and shift the value of the potential by ΔV within that bubble. Imposing that this time scale is smaller than Hubble time is equivalent to the TCC constraint for membranes imposed in section 4.2.2, i.e. $\Gamma > H^4$. The homogeneous time scale Δt in (4.33) is when the average of V over the whole Hubble patch decreases by ΔV and is given by $\Delta t = \Gamma^{-1}\mathcal{V}_{\text{eff}}^{-1}$, where the effective volume $\mathcal{V}_{\text{eff}}$ is derived in (E.14).

as originating from averaging over a cascade of membrane nucleation transitions[8]. This is because the top boundary of the blue region coincides with the limit of the region admitting a membrane origin.

In figure 4.2, there is a maximum value of θ allowed by TCC[9]. In subsection 4.3.1 we saw that TCC by itself does not bound θ in general, so the constraint comes from the assumption that the potential has a fundamental description in terms of membranes. The membrane picture strengthens TCC, turning it into a constraint on the potential.

Finally, it is important to note that while γ is a physical observable since it quantifies inhomogeneities in the bubble nucleation process, this is a piece of information that gets lost in the effective potential description, which only tracks the Hubble scale evolution. In other words, the only constraint that determines whether a potential can be chopped into pieces generated by membranes is the upper bound on θ.

4.3.3 No Eternal Inflation

So what general lessons can we learn from the membrane picture? We will now argue that the eternal inflation point is marginally excluded by our constraints.

As could be seen in figure 4.2, the maximum allowed values for γ and θ are realized when the TCC and/or the WGC get saturated and hit the boundary of the no-membrane origin region, so that the lower boundary of the blue region and/or the red line intersect (4.34). The intersection of these three curves nearly happens at the same point which, by using (4.33), satisfies

$$\gamma_{max} = \theta_{max} + \dots \tag{4.35}$$

where the "…" denote subleading corrections that go away in the limit $\Lambda \to 0$. From equation

[8] Using (E.14) one could analytically check the equivalence between $\Delta t = \Gamma^{-1}\mathcal{V}_{\text{eff}}^{-1} < H^{-1}$ and $\Gamma > H^4$ by noting that $\mathcal{V}_{\text{eff}}$ takes values in between H^{-3} (when the decay rate is small) and $\Gamma^{-3/4} < H^{-3}$ (when the decay rate is large).

[9] This feature is sensitive to $\mathcal{O}(1)$ factors, but we will show in the next subsections that the TCC in combination with the WGC always implies an upper bound on θ.

(4.31), we find that β is maximized at this point as well,

$$\theta_{\max} \simeq \frac{\beta_{\max} - 1}{2} + \dots \tag{4.36}$$

As discussed in subsection 4.2.1, applying the WGC to the $\beta_{\max}$ point implies $\alpha \geq \beta - \frac{1}{2}$, while the saturation of TCC implies $P \sim H^4$, where P is the prefactor of the decay rate defined in (4.4) as discussed in section 4.19. Using (4.4) we find

$$P \sim H^4 \to \frac{\Lambda^{4\alpha-2\beta}}{1+\Lambda^{2\alpha-2\beta+1}} \sim \Lambda^2. \tag{4.37}$$

For Λ small, the denominator becomes an order one factor $1+\Lambda^{2\alpha-2\beta+1} \sim \mathcal{O}(1)$. Plugging in $\alpha \geq \beta - \frac{1}{2}$ gives

$$\Lambda^{2\beta_{\max}-2} \gtrsim \mathcal{O}(1) \times \Lambda^2 \ \to \beta_{\max} \lesssim 2 + \dots \tag{4.38}$$

The sign of the next to leading term above depends on the value of order one factors coming from the prefactor as well as corrections to the TCC and the WGC. We will comment on the effect of these corrections in section 4.3.4.

Plugging the above inequality in (4.36) leads to the following inequality for the potential

$$|V'| > CV^{\frac{3}{2}}, \tag{4.39}$$

for some constant C. Interestingly, the constraint $|V'| > CV^{\frac{3}{2}}$ is also the standard condition for no-eternal inflation [39] (see [101] for an alternate scenario which is able to provide eternal inflation even if this condition is not satisfied). This is consistent with the results of figure 4.2, where we can see that the TCC-allowed region excludes the eternal inflation locus represented as a black vertical line.

It is worth noting that the setup is generally sensitive to $\mathcal{O}(1)$ factors which get hidden on the value of the constant C. For example, the actual curve of $\theta = 1/2$ in figure 4.1 gets a logarithmic correction $\theta = 1/2 + \frac{\log(C)}{\log \Lambda}$ if we keep track of C in calculating θ, where C can even get a mild dependence on Λ. Hence, eternal inflation is only marginally ruled out, and some models with a large enough constant C might still be allowed. We will discuss this in more detail in subsection 4.3.4.

Reference[39] also proposed that eternal inflation might be in the Swampland; here, we have derived this condition on the effective potential from applying TCC to metastable de Sitter vacua. As explained in[39], a metastable dS scenario is only compatible with eternal inflation if $\Gamma/H^4 \leq \mathcal{O}(1)$; this is the exact opposite of what TCC requires. We have also seen that this condition maps exactly to the usual $V' \gtrsim V^{3/2}$ for the effective potential. This is evidence that the dual description we have constructed correctly captures the physics and it is a non-trivial consistency check for our computations.

This relation between TCC and no-eternal inflation is actually intriguing from the perspective of the effective potential, since as shown in subsection 4.3.1 there is no obvious a priori reason why the TCC should imply (4.39). This result comes about only when we include membranes in the picture. One might have tried to show that TCC forbids eternal inflation by arguing that if inflation is eternal, there will be some patch where Planckian modes will be stretched beyond the Hubble horizon, naively leading to a violation of TCC, and thus, to the conclusion that TCC forbids eternal inflation. There are two problems with this naive argument:

- To violate TCC, an inflationary patch with a homogenous Hubble parameter must contain a mode as it goes from Planckian to Hubble size. Such a patch does not typically exist in eternal inflation since bubbles of true vacuum are constantly appearing.
- Since inflation lasts forever, one could argue that all sorts of unlikely things will happen somewhere eventually, including a TCC-violating Hubble patch. This illustrates that the current formulation of TCC is a semiclassical statement in terms of expectation values of quantum operators that only deals with what happens "on average", and it might be violated statistically, like the second law of thermodynamics, and point towards a more

fundamental quantum mechanical version of TCC that is absolute.

To sum up, TCC is a statement about the overall shape of the potential, but assuming the potential effectively describes tunneling between nearby vacua, we can get an additional point-wise result which implies eternal inflation is marginally ruled out.

4.3.4 Subleading corrections

Throughout most of this chapter, we have been cavalier regarding $\mathcal{O}(1)$ factors and other subleading corrections. For instance, we have neglected the $\log(1/H)$ logarithmic term in the TCC bound, or the WGC extremality factor in (4.13). The reason for this is that we cannot compute some of these in complete generality, such as $\mathcal{O}(1)$ corrections to the prefactor in (4.4). Although the qualitative results and conclusions we present in this chapter are insensitive to such subleading corrections, they become important when determining the fate of effective potentials satisfying

$$|V'| \sim C\, V^{3/2}\,. \tag{4.40}$$

The region near the tip of the TCC-allowed blue triangle in figure 4.2 is sensitive to these numerical factors, and might get extended to cross the vertical line at $\theta = 1/2$ marginally allowing potentials satisfying (4.40) for a certain value[10] of C.

For concreteness, dS gravitational corrections to the prefactor and instanton action push the TCC-allowed region to the left, moving it away from the eternal inflation locus by introducing a negative correction to β_{max} in (4.38) of order $\mathcal{O}(1/|\log\Lambda|)$ that increases the value of C in (4.39). Contrarily, the logarithmic term in the TCC bound, $\Delta t < H^{-1}/\log H$ implies a positive correction to β_{max} of order $\mathcal{O}(\log(\log\Lambda)/|\log\Lambda|)$ that pushes the TCC+WGC-allowed region to the right. Depending on the exact value of this correction, the TCC-allowed region might get extended to parametrically large values of β, as illustrated in figure 4.3. However, the WGC will always cut this region providing, even in this case, a maximum value of θ. Hence, in

[10] The proposed values for C in the condition for eternal inflation in the literature varies, e.g. $C = \frac{1}{\sqrt{2}\pi}$ in [39] and $\frac{1}{2\pi\sqrt{3}}$ in [102].

this case, the bound (4.39) is still valid but the value of C will be smaller than one and depend logarithmically on Λ. Other corrections coming from the prefactor or the WGC could also in principle push the bound in one direction or another.

In any case, we can conclude that potentials satisfying $|V'| \leq CV^{3/2}$ are forbidden by the swampland constraints for a certain factor C that is sensitive to all these corrections and could have a logarithmic Λ dependence. Therefore, any attempt to rule out a concrete model of eternal inflation would require a better knowledge of all possible subleading corrections. This is certainly an interesting avenue to further study in the future. At the moment, we can only conclude that eternal inflation is *marginally* forbidden by the swampland constraints.

4.3.5 Higher dimensions

We showed that the TCC marginally implies WGC and the no-eternal inflation condition in 4 dimensions. One can generalize all the calculations to show the same holds in higher dimensions as well. Following is a naive computation to demonstrate how this plays out int higher dimensions. In d-dimensions, the equations (4.21) and (4.22) change to

$$\begin{aligned} S &\sim \frac{T^d}{(\Delta\Lambda)^{d-1}} \lesssim 1 \to \alpha \geq \frac{d-1}{d}\beta, \\ P &\sim \frac{T^d}{(\Delta\Lambda)^{d-2}} \gtrsim H^d \to \alpha \leq \frac{d-2}{d}\beta + \frac{1}{2}. \end{aligned} \tag{4.41}$$

These two lines constraints together imply $\alpha \geq \beta - \frac{1}{2}$ which is the WGC. Moreover, the above inequalities set an upper bound $\frac{d}{2}$ on β. Plugging that upper bound into (4.9) leads to

$$\left(\frac{|V'|}{V}\right)^2 \sim \Lambda^{\beta-\frac{3}{2}}\Gamma\mathcal{V}_{\text{eff}} \gtrsim \Lambda^{\beta-1} \gtrsim \Lambda^{\frac{d}{2}-1}, \tag{4.42}$$

where we used TCC in the second equation. We can write the above inequality as $|V'| \gtrsim V^{\frac{d+2}{4}}$ which is the no-eternal inflation condition in d dimensions[11]. Therefore, we find TCC marginally implies WGC and no-eternal inflation condition in all higher dimensions as well. This points to a deeper relationship between TCC and WGC as this result holds in all dimensions and not just 4.

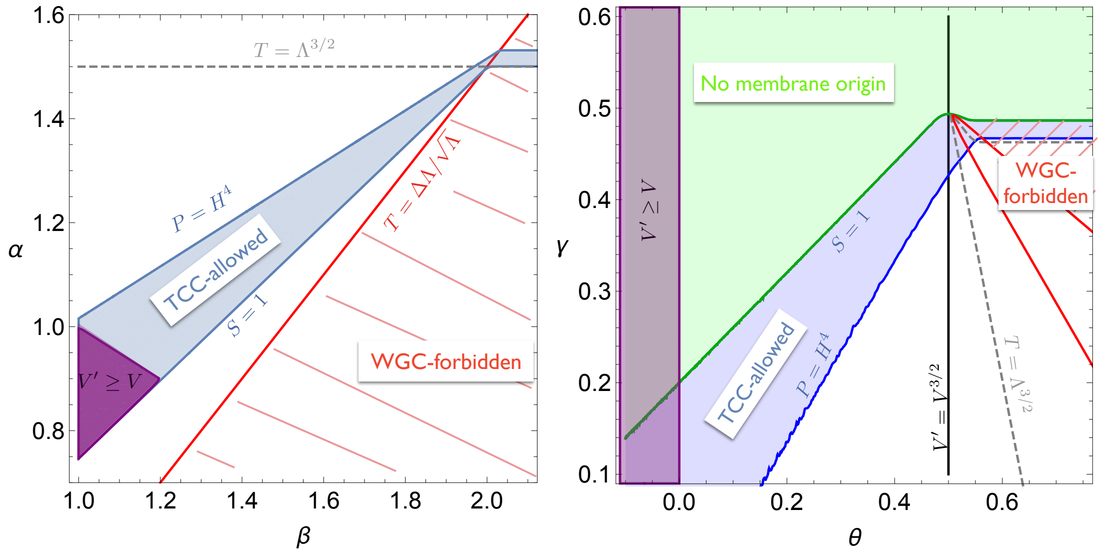

Figure 4.3: Allowed regions in the (α, β) plane (left panel) and (γ, θ) (right panel) for $\Lambda = 10^{-120}$ taking into account the logarithmic correction on the TCC. The light blue TCC region now grows an extra "tube" that makes it consistent with any value of θ. The red curve correspond to WGC, purple region corresponds to $V'/V > 1$. Eternal inflation is marginally ruled out by TCC and WGC, but not by either on its own.

4.4 Cosmological implications

In this section, we study the cosmological implications of our results assuming that the relevant potentials are dual to a fast decaying dS. In particular, we are interested in the consequences of our results for the emergent inflationary models and the dark energy.

[11]The no eternal inflation condition derived in[39] could be generalized to higher dimensions as follows. In higher dimensions, the Fokker Planck equation (2.11) in[39] takes the form $\dot{P}[\varphi, t] = A\partial_i\partial^i P[\varphi, t] + B\partial_i((\partial^i V(\varphi))P[\varphi, t])$ where $A \sim H^{d-1}$ and $B \sim H^{-1}$. This modifies the Gaussian solution (3.7) in[39] to $Pr[\varphi > \varphi_c, t] \sim \exp[-\frac{t}{\sigma^2}]$ where $\sigma \sim H^{\frac{d+1}{2}}/|V'|$. In order to have eternal inflation, the Hubble expansion must beat this exponential decay i.e. $H \gtrsim \frac{|V'|^2}{H^{d+1}}$. This results in the no eternal inflation condition $|V'| > KV^{\frac{d+2}{4}}$ for some constant K which depends on $\mathcal{O}(1)$ factors in computation of A and B.

4.4.1 Inflation

In[20] it was shown that the simplest TCC-compatible potential that could fit the observations such as the CMB power spectrum is an inverted parabola. A similar hilltop model was discussed in[103]. A concrete example of this can be taken to be $V = V_0(1 - 0.02\varphi^2)$ defined over $[\varphi_i, \varphi_f]$ where φ_f is fixed by observation to be

$$\varphi_f \simeq 3.9 \times 10^5 \cdot \left(\frac{V_0}{M_{pl}}\right)^{0.505}. \tag{4.43}$$

Plugging this into the potential gives

$$|V'|(\varphi_f) \simeq 8 \times 10^3 \, V_0^{1.505}. \tag{4.44}$$

For small enough V_0 and large enough φ_i the above potential is consistent with the no-eternal inflation condition as well as the TCC. Even though the potential is consistent with the TCC, it still suffers from a severe fine-tuning problem due to its short-field-range. This is because the field range is not long enough that a generic trajectory converges the slow-roll attractor. This initial condition problem seems to be an unavoidable consequence of the TCC for inflationary models[20]. As discussed in[22] there is an additional fine-tuning problem that goes back to the freedom in choosing the dS vacuum among the α-vacua. The only α-vacuum that can produce the scale-invariant CMB fluctuations is the Bunch-Davis (BD) vacuum. It was argued that if the dS space lives long enough the fine-tuning problem goes away because any α-vacuum will thermalize into the BD vacuum[30]. This argument does not apply to TCC-compatible dS spaces due to their short lifetime.

4.4.2 Dark Energy

Suppose the evolution of the cosmological constant is given by a scalar field whose potential comes from the successive short inter-vacua tunneling as discussed in this chapter. As the scalar field rolls down, the characteristics of the domain wall corresponding to the potential evolve. We can view the rolling of the scalar field as a trajectory in the $\alpha - \beta$ plane in the membrane picture. TCC tells us that in the asymptotic of the field space $\frac{|V'|}{V} \gtrsim \mathcal{O}(1)$. Thus, the trajectory in the $\alpha - \beta$ plane (figure 4.1) eventually reaches the purple curve. In fact, this must happen within a TCC time. This is because before we hit the purple curve the potential is very flat ($|V'| \ll V$) and the Hubble parameter is almost constant. From observations we know that the equation of state parameter w is close to -1 which means the quintessence potential is not steep, i.e. $|V'| \lesssim V$. This leaves two possibilities for the current state of our universe: we are very close to the purple curve where the potential begins to steep down, or we are still wandering in the blue region with a plateau potential while moving toward the purple curve.

Case 1: Near the $|V'| \sim V$ curve

In that case, the universe while remaining in the blue region must be close to the purple curve. That means we are close to the line that connects $(\alpha, \beta) \simeq (1, 1)$ to $(\alpha, \beta) \simeq (0.9, 1.2)$. All these points correspond to the same slope $\frac{|V'|}{V} \sim \mathcal{O}(1)$, however they differ in the scale of the bounce radius. From the equation (D.7) we find $R \simeq R_0 \sim \Lambda^{\alpha - \beta}$. This gives a range $1 \lesssim R \lesssim \Lambda^{-0.3}$ for the bounce radius in Planck units. After restoring the Planck length, it implies $l_{pl} \lesssim R \lesssim 10^{35} l_{pl} \sim 1\,\mathrm{m}$.

This scenario is also phenomenologically appealing as it provides a cosmological relaxation mechanism to generate a small cosmological constant, consistent with the current expansion of our universe. As long as we are close the to the purple boundary with $V'/V \sim \mathcal{O}(1)$, it is just a matter of time to lower the cosmological constant by bubble nucleation to a very small value, even if the initial value at the beginning of the cosmological evolution was very big. This is very similar to the dynamical neutralization of the cosmological constant by Brown-Teitelboim and Bousso-Polchinski, whenever we have a landscape of flux vacua. If we are close to $\beta = 1$, the variation of vacuum energy $\Delta\Lambda$ becomes very close to Λ, so after a few transitions, one would end up with a very small value for the cosmological constant. The drawback is that the effective description in which we can average over the discrete jumps breaks down and one would need

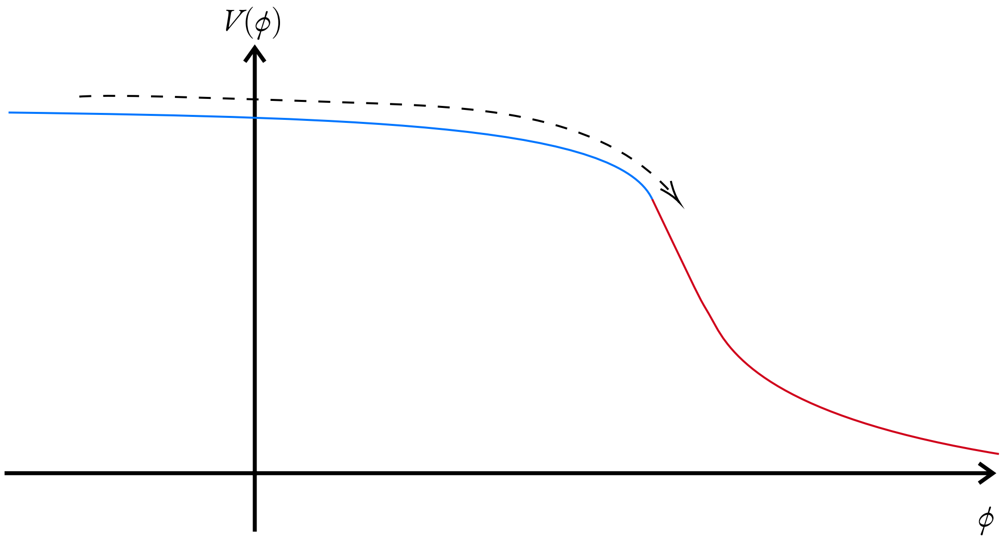

Figure 4.4: The graph shows a generic effective potential that would emerge from successive inter-vacua tunnelings. In the plateau part of the graph which is drawn in green, we have $|V'| \ll V$. This corresponds to part of the cosmic evolution spent in the blue region of the figure 4.1. The steep part of the potential drawn in red has $|V'| \gtrsim V$. This corresponds to the part of the cosmic evolution spent inside or near the purple region in the figure 4.1.

a very finely grained landscape in order not to miss our current tiny value of Λ. In any case, regardless of where we are in the (α, β)-plane, as long as it is close to the purple region, it is always possible to find some effective potential that reaches a tiny value of the cosmological constant in less than the Hubble time since the whole blue region is consistent with the TCC. This scenario could also explain the cosmological coincidence problem if something drastic happens when we reach a small value of Λ. A tantalizing possibility is that the effective field theory drastically breaks down precisely when getting a small value of Λ and entering into the purple region, because we could get then access to transplanckian field ranges and infinite towers of states should become light according to the SDC.

Case 2: Far from the $|V'| \sim V$ curve

Suppose our universe in the blue region the $\alpha - \beta$ plane in figure 4.1 sufficiently far from the purple curve. In that case, we have $|V'| \ll V$ which means the potential is so flat that effectively behaves like a cosmological constant. This is consistent with observations. Moreover, the TCC is also satisfied because as discussed in subsection 4.3.1, the only constraint that TCC imposes on nearly flat potentials is that the age of the universe must be less than $\frac{1}{H}\ln\left(\frac{1}{H}\right)$ which is true in our universe. The drawback is the usual naturalness problem of the cosmological constant since one would need to start originally with a very tiny cosmological constant as the membrane nucleation transitions will not modify its value in a significant way.

4.5 Conclusions

In this chapter, we applied some of the Swampland conjectures to short-lived de Sitter spaces and described the resulting decay in terms of an effective theory. We saw that WGC and the generalized distance conjecture lead to a restricted region in parameter space which surprisingly includes the full region implied by the TCC. In other words, TCC in this context seems to "know" about WGC and the generalized Distance Conjecture. This relationship between different Swampland conjectures which is frequently encountered, reinforces the belief in their validity. In studying the resulting effective theory, we found that they lead to a scalar field with a restricted type of potential. In particular, even though a potential allowing eternal inflation is consistent with TCC, the resulting potentials we obtain from dS membrane picture are marginally inconsistent with eternal inflation. However eternal inflation is not completely ruled out by our considerations and there is a small region in parameter space that may in principle allow it depending on subleading corrections we have neglected in our analysis. This is an interesting question that should be explored in the future.

Our results apply to situations that can be described as a cascade of non-perturbative nucleation of bubbles in de Sitter space. We have constructed an effective potential that provides a dual description of the low-energy physics of the cascading membranes, connecting the membrane and cascade pictures. Our methods can be extended to a variety of interesting situations, e.g. when the membranes are charged under multiple 3-form gauge fields or the membranes are unstable and can grow holes in their surface corresponding to strings magnetically charged under axions coupled to the 3-form gauge fields.

So how general is the membrane picture? On a more speculative note, we could take the radical position that *any* quasi-dS potential consistent with string theory always admits such a membrane origin. If so, the conclusion that eternal inflation is in the Swampland, which we got here from the membrane picture, would be general. In particular, near the infinite distance boundaries of the field space in string theory compactifications, the scalar potential is known to exhibit a runaway behavior towards the asymptotic limit that typically satisfies the de Sitter conjecture. Such a runaway could be explained if it is actually the effective description of a cascade of membrane nucleation with a tension approaching the region $V' \geq V$ in figure 4.1. To determine whether this is a sensible scenario, we would need a better understanding of possible constraints on the dynamics on the (α, β)-plane in addition to the ones studied in this chapter.

It would be interesting to see if other Swampland conjectures can also be brought to play in this context. For example, the cobordism conjecture [104] predicts that there is always a bubble of nothing in a quantum theory of gravity. What is the relation of our "minimal bubble" to the bubble of nothing? Can this place an upper bound on the lifetime of dS which is even stronger? Given the importance of a deeper understanding of dark energy for the future evolution of our universe, it is worthwhile pursuing aspects of short-lived dS from all possible perspectives.

5

TCC and scrambling

It is remarkably more challenging to construct a de Sitter vacuum in string theory than a flat or an Anti-de Sitter vacuum. TCC is one of several Swampland conjectures that have been proposed to pinpoint a mutual property among theories in the Landscape that could explain this hurdle in string theory[14,15,19]. However, string theory is not the only place where de Sitter space sets itself apart from flat and Anti-de Sitter spaces. Another notable example is its finite-sized Hilbert space and thermal properties, which are absent from other backgrounds. It is natural to think that all the unique features of de Sitter space should be fundamentally interconnected. If true, there should be some relation between the Swampland program and the thermodynamic features of de Sitter space.

Interestingly, TCC provides a strange coincidence which is strongly suggestive of such a connection. The maximum allowed de Sitter space lifetime by TCC matches the scrambling time of de Sitter space! However, we need to make a comment on what we mean by scrambling time here. Recent works [105] have suggested that the scrambling time in de Sitter space is given by $1/H$ rather than $1/H \ln(1/H)$. A key ingredient in this argument is that the majority of the degrees of freedom of de Sitter space in the static patch must be localized near the Hubble horizon. Therefore, a generic perturbation does not need any time to get to the Hubble horizon. However, for generic field theory perturbations inside the static patch, this is not the case. Therefore, by scrambling time in this section, we refer to the scrambling time of degrees of freedom that can penetrate inside the static patch and admit an EFT description at low energies.

At first, the coincedence between the scrambling time and the TCC time scale might seem strange as one time-scale is motivated by string theory, and the other comes from de Sitter complementarity. However, as we discussed earlier, this connection is natural since both contexts study a feature of de Sitter space, which sets it apart from flat and Anti-de Sitter backgrounds.

We investigate this non-trivial coincidence to find a thermodynamic interpretation for TCC. The goal of this section is to take a small step in bridging the gap between the Swampland program and the extensive literature about thermal aspects of de Sitter space.

The organization of the section is as follows. In subsection 5.1, we study the consequences of the Swampland conditions for the de Sitter space with a particular focus on TCC. However, before that, we mention some of the motivations for TCC that lend support to those results. In section 5.2, we study de Sitter space from the lens of de Sitter complementarity and other perspectives that view de Sitter space as a thermal background. We show that all those ideas point towards the same result that if de Sitter space lives long enough, it would be a thermal background with a thermalization time of $\sim \frac{1}{H}\ln\left(\frac{1}{H}\right)$. In subsection 5.3, we put the Swampland picture next to the other pictures to arrive at a thermodynamic interpretation of TCC. We argue that TCC, in its essence, tells us that de Sitter space is not stable enough to be viewed as a thermal background. We elaborate on the physical meaning of this interpretation and support it by other Swampland conditions. Finally, we show in subsection 5.4 that TCC imposes a severe initial condition problem on inflation.

5.1 Swampland picture

This section explores the picture that Swampland conditions, especially TCC, offer of de Sitter space. We begin by reviewing TCC and some motivations for it that lend support to its implications. We then study the implications of Swampland conditions for different possible realizations of de Sitter space.

TCC and its motivations

Trans-Planckian censorship conjecture (TCC) postulates that in a consistent quantum theory of gravity, an expansionary universe in which Planckian modes exit the Hubble horizon cannot be realized[19]. What is special about the Hubble radius is that when a mode exits the Hubble horizon, it becomes non-dynamical and freezes out[51,106,29]. Moreover, super-Hubble modes undergo decoherence which makes them equivalent to stochastic classical perturbations[107] and the modes will remain classical even if/when they re-enter the Hubble horizon. As shown in figure 5.1, a violation of TCC would lead to the classicalization of all dynamical quantum fluctuations $H < k < l_P^{-1}$.

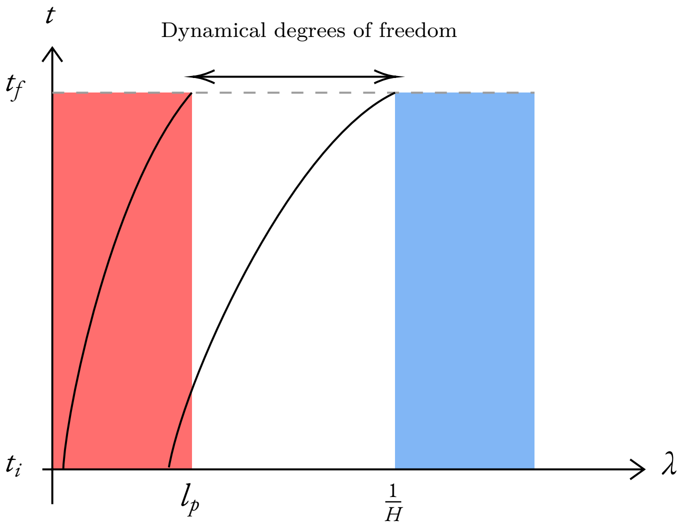

Figure 5.1: The curved lines denote the expansion of wave-lengths of two comoving modes. These two modes correspond to the greatest, and smallest dynamical modes at time t_f. As shown in the figure, if TCC is violated over some time interval $[t_i, t_f]$, no comoving mode would stay dynamical throughout that time window.

Following we review some of the arguments in favor of TCC.

5.1.1 Central node in the Swampland web

The TCC implies some versions of several Swampland conditions. These versions have shown to be true in known controllable string theory constructions. These non-trivial consistencies strongly support TCC. The connections between TCC and other Swampland conditions suggest that TCC is a central node in the web of Swampland conditions. One that brings many of them together and provides a simple physical explanation for them. Following, we review some of the non-trivial implications of TCC that resemble other Swampland conditions and their consistency with string theory.

The de Sitter Conjecture: One of the most notable implication of TCC is that in d-non-compact dimensions, $\frac{|\nabla V|}{V} \geq \frac{2}{\sqrt{(d-1)(d-2)}}$ in the asymptotics of the field space. This is similar to the de Sitter conjecture [14], but it provides a definite lower bound which is remarkably satisfied in multitudes of string theory constructions [19,96].

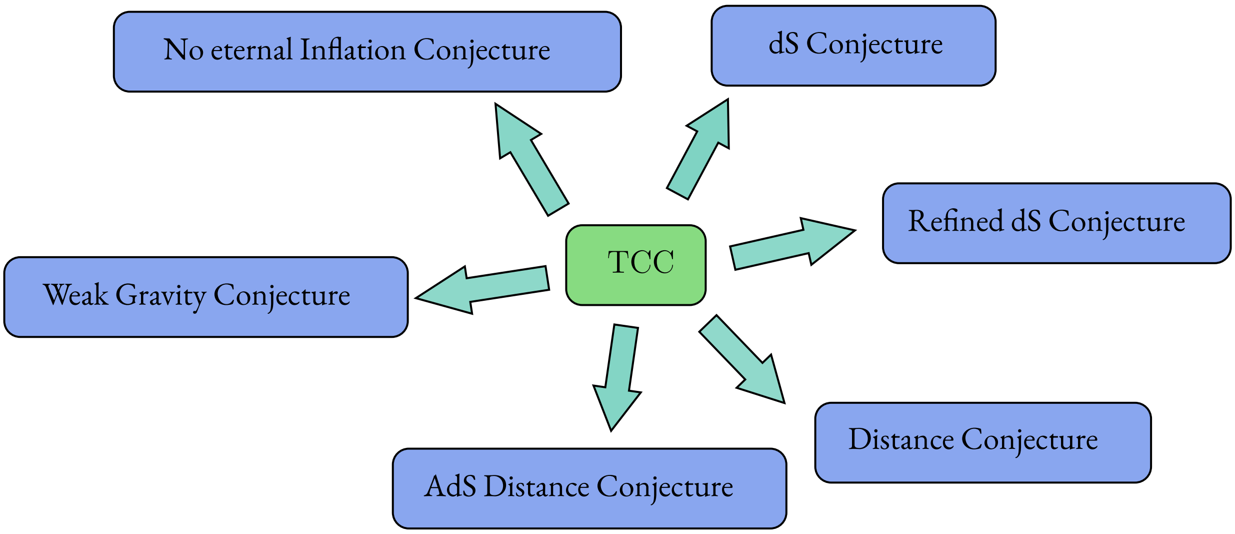

Figure 5.2: TCC non-trivially implies some version of a number of Swampland conditions. This is remarkable given how simple the physical idea behind TCC is.

Distance conjecture : As proposed in[96], TCC suggests a definite lower bound $\frac{1}{\sqrt{6}}$ for the order one constant λ in the distance conjecture in 4d[32]. This bound could also be motivated by the following heuristic argument. Suppose φ_1 and φ_2 are canonically normalized fields where φ_2 is a scalar field in a tower of light states emerging as $\varphi_1 \to \infty$. As φ_1 goes to infinity, mass of φ_2 exponentially decays. After some point, φ_2 becomes light enough to be added to the spectrum of the low energy field theory. In this regime, the potential depends on both φ_1 and φ_2 and the mass of φ_2 is roughly given by $m \sim \sqrt{\partial^2_{\varphi_2} V}$. In conventional string theory constructions, the potential decays exponentially in asymptotic directions. Suppose the potential behaves as $V \sim f(\varphi_2)\exp(-g(\varphi_2)\varphi_1)$ for large φ_1, the mass scale m would decay like $\sim \exp\left(-\frac{g(\varphi_2)}{2}\varphi_1\right)$ as φ_1 goes to infinity. From TCC, we know that $g(\varphi_2) \geq \frac{2}{\sqrt{(d-1)(d-2)}}$ [19] which leads to

$$\lambda \geq \lambda_{TCC} = \frac{1}{\sqrt{(d-1)(d-2)}}. \tag{5.1}$$

In[108], authors found a lower bound for λ

$$\begin{aligned} \lambda &\geq \sqrt{\frac{1}{10-d}} \quad \frac{d}{2} = even \\ \lambda &\geq \sqrt{\frac{2}{10-d}} \quad \frac{d}{2} = odd, \end{aligned} \tag{5.2}$$

for single modulus limit in various Calabi-Yau compactifications. Remarkably, this bound is stronger than the TCC-motivated bound (5.1) for every $d \geq 4$. The TCC-motivated bound has been checked more generally in a variety of 4d string theory constructions in [109,96].

Refined de Sitter conjecture: The refined de Sitter conjecture states that there is a universal $\mathcal{O}(1)$ lower bound for $|\Delta V|/V$ at local maxima [15,93]. Interestingly, TCC implies a logarithmically corrected version of this condition for local maxima [19]. The modified condition roughly is $\frac{|\Delta V|}{V} > \frac{16}{(d-1)(d-2)\ln(V)^2}$.

Weak gravity conjecture (WGC) and generalized distance conjecture: Consider a flux generated potential with a charged co-dimension 1 brane. The brane serves as the domain wall for tunneling between neighbouring vacua. TCC implies both the WGC and the generalized distance conjecture for the brane in all dimensions [21].

No eternal inflation: Suppose tunnellings between neighbouring vacua are non-drastic enough that a monotonic quintessence potential could effectively describe the universe's evolution. In that case, one can show TCC marginally forbids eternal inflation in any dimension [21].

Figure 5.2 shows all the Swampland conjectures that, in some form, are implied by TCC.

5.1.2 Coincidence problem

An immediate consequence of *TCC* is that the age of the dark energy dominated epoch T_Λ must be less than $\frac{1}{H}\ln\left(\frac{1}{H}\right)$ which is true in our universe. This consistency already provides a simple, yet non-trivial, experimental test for TCC. Perhaps the more interesting fact is that our universe only marginally satisfies this inequality thanks to the logarithmic term $\ln\left(\frac{1}{H}\right)$. This "accident" is precisely the coincidence problem in cosmology. If the cosmological constant is a constant, i.e. the universe is stuck in a local minimum of the potential, there is no apriori reason for $T_\Lambda \sim \frac{1}{H}$.

Statistically speaking, for most of the lifetime of a metastable universe, its age is of the order of its lifetime, and TCC relates that lifetime to the Hubble time. In fact, according to TCC, no matter the universe is in metastable equilibrium, or it is rolling, the coincidence of $T \sim \frac{1}{H}$ is anticipated, which is another non-trivial consistency with observation.

5.1.3 TCC as gravitational renormalizability

We argue that TCC could be viewed as a natural modification of the renormalizability condition for gravitational theories where the conventional notion of renormalizability does not apply. To see this, let us review what renormalizability means in field theories.

Quantum field theories typically come with UV divergences that prevent them from being effective descriptions at all energy scales. We usually resolve this issue by restricting to the low-energy modes. To obtain a low-energy field theory, we integrate out the high-energy modes. This procedure effectively sets the high-energy modes in their ground states and gives us a unitary theory for the remaining low-energy modes. Renomalizability is the condition that makes it possible to have a closed theory for the low-energy modes. The implication of such scale separation for the classical theory is that UV perturbations must not significantly impact or be impacted by the time evolution of low-energy classical modes. Therefore, in flat space, renormalizability naively implies that there exists a momentum cutoff Λ such that classical high momentum perturbations with $k > \Lambda$ do not influence the dynamics of low energy modes with $K < \Lambda$. However, this naive notion of scale separation does not apply to GR since any expansion of the universe stretches some modes with $k > \Lambda$ into modes with $k < \Lambda$. The above naive argument is a simple way of understanding why GR is non-renormalizable.

Similar to how quantum field theory's consistency imposes renormalizability, it is natural to expect a UV-complete quantum theory of gravity must satisfy some renormalizability-like condition. As we saw earlier, the naive scale separation (renormalizability) does not hold in gravitational theories. The simplest relaxation to a scale separation between classical modes with $k > \Lambda$ and $k < \Lambda$, is to postulate that there are **two energy scales** $\Lambda_{UV} \gg \Lambda_{IR}$ such that the deep UV modes $k > \Lambda_{UV}$ do not stretch into deep IR modes $k < \Lambda_{IR}$. There are natural candidates for these energy scales in any de Sitter background; the Planck scale and the Hubble scale. Using these scales, our candidate for gravitational renormalizability takes the following form.

Modes with $k > \frac{1}{l_P}$ or equivalently $\lambda < l_P$ cannot evolve into modes with $k < H$ or equivalently $\lambda > \frac{1}{H}$.

In other words, sub-Planckian modes cannot exit the Hubble horizon, which is precisely the statement of TCC. In a sense, TCC is a natural gravitational analogue of the renormalizability condition in field theory.

5.1.4 Initial condition problem for inflation

We want to take this opportunity to clarify a possible confusion that a particular initial condition problem for inflation, has been a motivation for TCC. A violation of TCC poses two apparent initial condition problems for inflation. We briefly review each of these problems, the resolutions proposed in the literature, and how they relate to TCC.

First initial condition problem:

If some fluctuations, e.g. Hubble sized CMB fluctuations, trace back to trans-Planckian fluctuations, it seems part of the needed initial condition is inaccessible to the field theory. This raises a practical question. What initial state should we consider for those modes as they become sub-Planckian and enter the range of field theory?

Resolution:

Requiring the vacuum to be like the Minkowski vacuum at short distances fixes the de Sitter vacuum at Planckian momenta[30]. Essentially, the equivalence principle naturally screens the trans-Planckian physics from sub-Planckian observers. This argument resolves the problem for as long as de Sitter space is stable as a semi-classical background. This "problem" has not been a motivation for TCC, and the mentioned resolution is not affected by TCC.

Second initial condition problem:

In contrary to the Minkowski space, the de Sitter space does not have a unique vacuum. There is a family of vacua called α-vacua that all are invariant under symmetries of de Sitter space. However, the only α-vacuum that would give scale-invariant CMB fluctuations is the Bunch-Davies (BD) vacuum. Why did the universe choose this particular vacuum state in the inflationary era?

Resolution:

The authors in[30] argued that any deviation from the BD vacuum eventually exits the Hubble horizon and disappears. Therefore, inflation automatically sets the universe in the BD vacuum. We revisit the de Sitter vacua and this argument in section 5.2, and we contrast it with TCC in section 5.3. We show that the argument in[30] makes an assumption that is fundamentally inconsistent with TCC. In other words, a violation of TCC is baked in the argument. Assuming TCC is correct, this argument no longer works. We will come back to this initial condition problem later in section 5.4.

de Sitter space and the Swampland

Swampland conditions suggest that the de Sitter space is in tension with a UV-complete theory of gravity. The de Sitter conjecture forbids the de Sitter space all-together, but TCC allows it as long as it is sufficiently short-lived. TCC requires the de Sitter space to undergo some significant transformation by $\tau_{TCC} = \frac{1}{H}\ln\left(\frac{1}{H}\right)$ so that the Planckain fluctuations do not exit the Hubble horizon. This transformation could be a significant drop in cosmological constant, tunnelling to a different vacuum, quantum breaking of spacetime, or the effective field theory's breakdown. Following, we study each of these possibilities separately.

Suppose the cosmological constant continuously discharges (quintessence), TCC tells us that a major part of Λ have to be discharged by τ_{TCC}. This implies that the quintessence potential could not be too flat. For monotonic potentials, this leads to $\frac{|V'|}{V} \gtrsim \frac{2}{\sqrt{(d-1)(d-2)}}$ in asymptotics of the field space and $\frac{|V'|}{V} \gtrsim \mathcal{O}(\ln(V)^{-2})$ in the interior of the field space[19]. For local maxima on the other hand, TCC roughly implies $\frac{|V''|}{V} \gtrsim \frac{16}{(d-1)(d-2)\ln(V)^2}$ [19].

Suppose the universe undergoes tunnelling, i.e. a bubble of a more stable vacuum forms and expands until it takes over the Hubble patch. TCC implies the metastable vacuum's lifetime must be less than τ_{TCC}. This has important implications for the domain wall of the bubble. In particular, the domain wall must satisfy both WGC and the generalized distance conjecture[21].

Another way to avoid violating TCC is that the QFT in curved background description breaks down. This could happen in two ways; 1) Break down of the classical background, often referred to as quantum breaking, 2) Break down of the EFT due to the emergence of new light states in the theory. Whichever happens first, TCC tells us that it must take place by τ_{TCC}. Authors in [110,111,112] argued that the quantum breaking time for de Sitter space is $\sim H^{-3}$, which is greater than τ_{TCC}. However, the de Sitter and the distance conjectures show that the EFT breaks down by τ_{TCC} in the asymptotics of the field space[113].

All in all, we expect the de Sitter space to undergo some significant physical transformation before τ_{TCC}. No matter which scenario happens, we come across the same timescale τ_{TCC}, after which the initial de Sitter description should no longer work.

5.2 Complementarity picture

In this section, we study the de Sitter space from the complementarity perspective. First, we review black hole complementarity from which most of the ideas for de Sitter complementarity have originated. As we will see, black holes do not share some of the strange features of de Sitter space, which makes black hole complementarity simpler and easier to understand than its de Sitter counterpart.

Review of black hole complementarity

In nutshell, the black hole complementarity is a partial resolution to a tension between well-established physical principles. We briefly review the argument in [114] that leads to the idea of complementarity.

We assume the following postulates[1]:

1) *Unitary semi-classical QFT in curved spacetime*: We treat gravity classically and other fields quantum mechanically. We assume every spacelike Cauchy surface has an associated Hilbert space for the quantum fields living on it. The time evolution between any two such Cauchy surfaces is given by a time-dependent unitary operator dependent on the gravitational background.

2) *Equivalence principle*: Every free-falling observer must be unable to distinguish the spacetime from Minkowski space through performing local experiments.

3) *No remnant*: We assume the black hole will completely evaporate at a finite time.

Consider a mater distribution that collapses into a black hole and evoporates in a finite time. Suppose Σ_- and Σ_+ are two Cauchy surfaces in the far past and the far future with respect to the black hole as shown in figure 5.3. Cauchy surface Σ_{BH} passes through the formal intersection of the horizon and the singularity in the Penrose diagram 5.3. Σ_{in} and Σ_{out} denote the parts of Σ_{BH} that are respectively inside and outside of the black hole. Suppose $|\psi_-\rangle$, $|\psi_+\rangle$, and $|\psi_{BH}\rangle$ are the state of the quantum fields respectively on Σ_-, Σ_+, and Σ_{BH}, and ρ_{in} and ρ_{out} are the density matrices associated to the quantum fields inside and outside the black hole given by

$$\begin{aligned} \rho_{in} &= Tr_{\mathcal{H}_{out}} |\psi_{BH}\rangle \langle\psi_{BH}| \, , \\ \rho_{out} &= Tr_{\mathcal{H}_{in}} |\psi_{BH}\rangle \langle\psi_{BH}| \, . \end{aligned} \tag{5.3}$$

From postulate 1, we know that some unitary transformation maps $|\psi_-\rangle$ to $|\psi_+\rangle$.

$$|\psi_+\rangle = U_1 |\psi_-\rangle \, . \tag{5.4}$$

[1] Our list of postulates is slightly different from that of[114], but the following argument is almost identical.

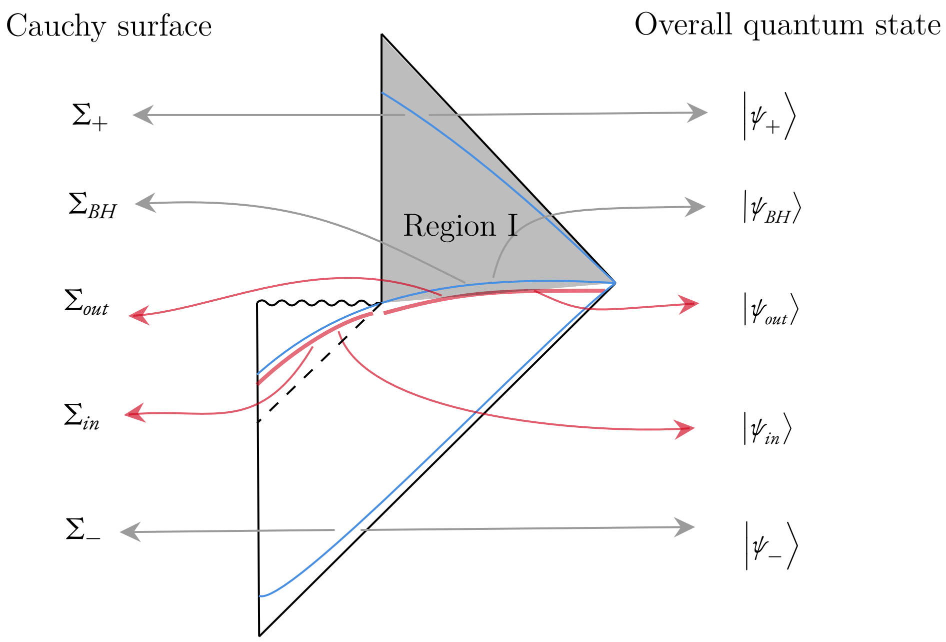

Figure 5.3: Penrose diagram of evaporating black hole.

Similarly, since Σ_+ and Σ_{out} are both Cauchy surfaces of region I in figure 5.3, ρ_{out} is related to $|\psi_+\rangle\langle\psi_+|$ by a unitary transformation. Therefore, there must be a pure state $|\psi_{out}\rangle$ which satisfies $\rho_{out} = |\psi_{out}\rangle\langle\psi_{out}|$ and is related to $|\psi_+\rangle$ via some unitary transformation U_2.

$$|\psi_{out}\rangle = U_2 |\psi_+\rangle . \tag{5.5}$$

Combining (5.4) and (5.5) gives

$$|\psi_{out}\rangle = U_3 |\psi_-\rangle , \tag{5.6}$$

where $U_3 = U_2 U_1$. Postulate 1 tells us that $|\psi_-\rangle$ is unitarily mapped to $|\psi_{BH}\rangle$ as well. The only way $|\psi_-\rangle$ could be unitarily mapped to both $|\psi_{BH}\rangle$ and $|\psi_{out}\rangle$ is that $|\psi_{BH}\rangle = |\psi_{in}\rangle \otimes |\psi_{out}\rangle$ for some constant state $|\psi_{in}\rangle \in \mathcal{H}_{in}$. This however would mean that if a free falling observer falls into the black hole and hit Σ_{in}, they would see a fixed state $|\psi_{in}\rangle$ independent from the initial state $|\psi_-\rangle$. This is different from what an inertial observer would see in flat spacetime and therefore is a violation of the equivalence principle.

The black hole complementarity is a principle to get around this paradox. The principle is

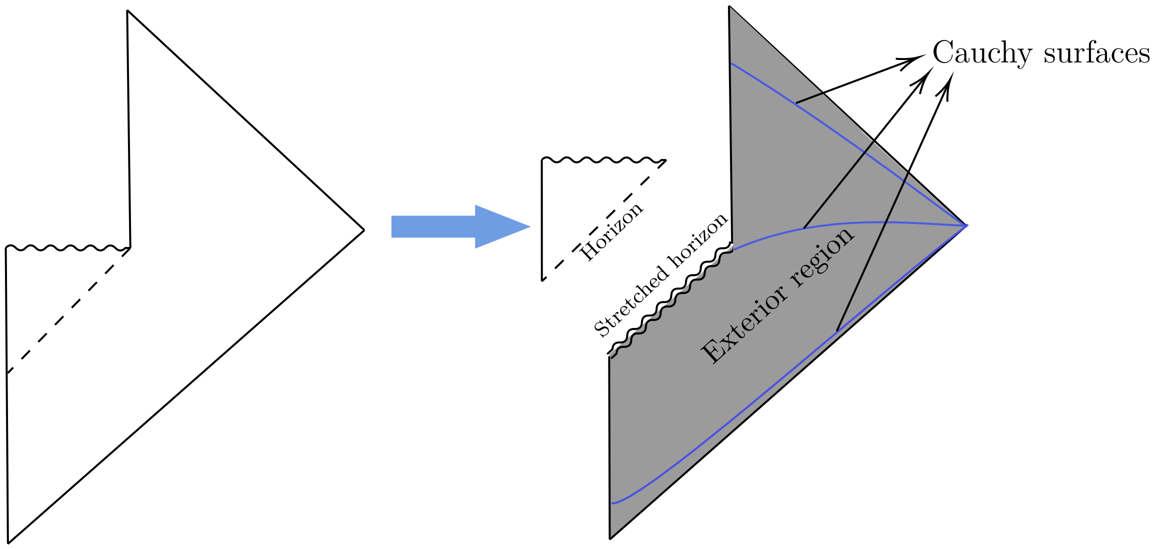

Figure 5.4: The black hole complementarity allows us to isolate the black hole's exterior and study the evolution in it independently.

that there are two different but complementary descriptions for the physics depending on the observer's trajectory. For outside observers at fixed distance from the black hole, the evolution could be studied independently from the interior of the black hole as shown in figure 5.4. All the black hole interactions with the exterior are explained by a real physical membrane at Planckian distance from the horizon. This membrane is called the *stretched horizon*. The falling of matter inside the black hole could be viewed as the stretched horizon absorbing its energy. For outside observers, the Hawking radiation is the thermal radiation of the stretched horizon.

In contrast to the accelerating observers at a fixed distance from the black hole, a free-falling observer falling into the black hole will not see the stretched horizon. This leads to two different descriptions of the physical events on a given Cauchy surface that extends to the inside of the black hole. This might seem paradoxical at first; however, since the two observers are causally disconnected, they cannot communicate their different narratives to each other. In other words, no observer can experience both physics [2].

As we mentioned above, the black hole's Hawking radiation could be interpreted as the stretched horizon's thermal radiation. When an object falls in the black hole, it perturbs the stretched horizon and causes a small deviation from the equilibrium. After some time, the

[2] Such an observer is often called a *superobserver* in the literature.

system's information thermalizes and can be radiated away in the form of thermal radiation. The time it takes for external perturbations to thermalize is called the *scrambling time*. Following, we give a more rigorous definition of it and study it in more detail.

The scrambling time of a system is the time that it takes for the information of a generic pure state to disperse among all microscopic degrees of freedom. More precisely, it is the time by which the density matrix $\rho = Tr_{\mathcal{H}_s} |\psi\rangle \langle\psi|$ becomes thermal for almost every subsystem $\mathcal{H}_s$ with half the degrees of freedom. In [115], it was conjectured that for any quantum system at inverse temperature β, the scrambling time τ_S is bounded from below by

$$\tau_s > f(\beta) \ln(S), \tag{5.7}$$

where $f(\beta)$ captures the interaction strength and S is the total entropy. The systems that saturate the above bound for some function $f(\beta)$ are called *fast scramblers*. Based on the complementarity principle, it was conjectured that both black holes and de Sitter space are fast scramblers with the scrambling time given by [115,38]

$$\tau_s \sim \frac{1}{T} \ln(S). \tag{5.8}$$

It is worth taking a while to review the argument that bounds the black hole scrambling time from below. We review a thought experiment presented in [115] that shows if the scrambling time is too short, the no-cloning theorem could be violated.

Suppose we have two observers Alice and Bob each carrying a q-bit that are fully entangled with each other. Bob jumps in the black hole, and right after he passes the horizon by a Planck length[3], he measures the q-bit. Then, he sends a null signal in the outward radial direction that carries the measurement's outcome information. On the other side, Alice waits at a short distance outside the horizon until the black hole radiates away Bob's q-bit information. She

[3] smaller distances are not meaningful to a semi-classical observer.

then collects the Hawking radiation that contains Bob's q-bit's information[4].

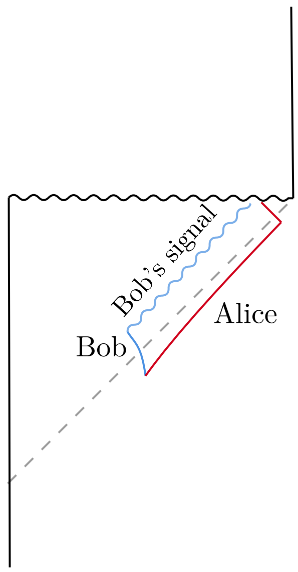

Figure 5.5: If Alice's worldline (the red curve) could meet Bob's signal (blue wavy curve), no-cloning theorem would be violated.

The time Alice needs to wait is precisely the scrambling time. As soon as Alice receives a copy of Bob's q-bit's information through Hawking radiation, she jumps in to catch Bob's signal (figure 5.5). If she succeeds, she will have two copies of Bob's information, which violates the no-cloning theorem. Moreover, both copies are fully entangled with Alice's q-bit, which violates the monogamy theorem. The scrambling time must be long enough so that Alice's future light cone and Bob's signal do not intersect to avoid these contradictions. This gives a lower bound of $\sim \frac{1}{M}\log(M)$ for the scrambling time[115].

In the next subsection, we apply the ideas we covered in this subsection to de Sitter space to develop the de Sitter complementarity.

de Sitter complementarity

Several authors[118,119,120,121,122,123,124,125,126] have proposed a complementarity principle for de Sitter space similar to the black hole version that we discussed in the previous subsection. The proposal is that the physics inside the Hubble horizon could be described independently

[4] The experiment is done after the Page time so that Alice can retrieve Bob's q-bit's information with $\mathcal{O}(1)$ bits of Hawking radiation[116,117].

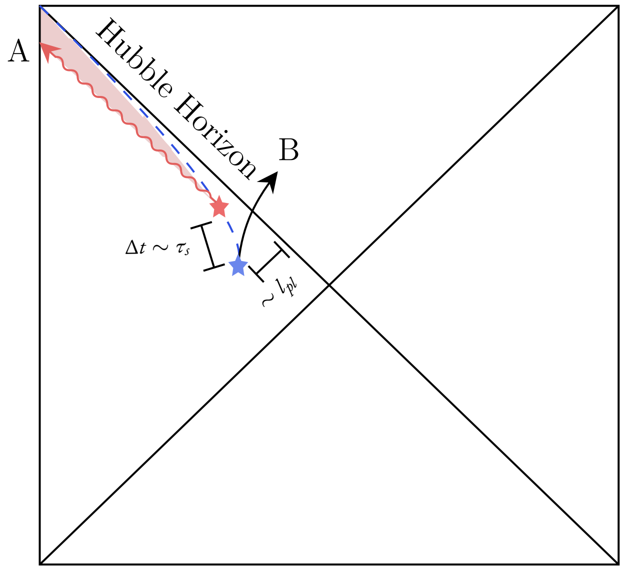

Figure 5.6: Penrose diagram of de Sitter space. The left side is the world line of a comoving observer A and the dashed blue curve denotes its corresponding stretched horizon. B is a system that exits A's Hubble horizon. The information of B thermalizes over the stretched horizon after the scrambling time τ_s and is partially radiated by the Hawking radiation in the red region.

from the outside of the Hubble patch. Similar to the black hole version, there is a stretched horizon located a Planck distance from the Hubble horizon. The stretched horizon disappears from free-falling observers crossing the Hubble horizon. When a system crosses the Hubble horizon of a comoving observer, the information of that system thermalizes over the stretched horizon and gets radiated toward the observer after a scrambling time (see figure 5.6).

A key difference between black hole complementarity and its de Sitter counterpart is in information recovery. In order to recover the information of a system that has crossed the black hole's horizon, one needs to wait out the Hawking-Page transition[116]. This is to ensure that half of the black hole's entropy is radiated away so that Hayden-Perskill protocol[117] could be implemented. Another way to think about the Page time is when the radiated matter is maximally entangled with the remaining of the black hole. The significant role of the maximal entanglement is especially evident in ER=EPR duality[127]. Suppose one collapses the maximally entangled radiated matter into a second black hole. It was conjectured that this would create a wormhole geometry, which makes the information recovery possible[128,129].

Information recovery is a bit trickier in de Sitter space. For starters, there is no Hawking-Page transition in de Sitter space. This is due to the fact that the maximum entropy that can be stored in the Hubble patch is a third of de Sitter entropy[130]. At first, this might seem to suggest that information in de Sitter space is irretrievable. However, collecting half the entropy is unnecessary as long as we have access to a maximally entangled state with the stretched horizon. In that case, the information can be recovered as soon as it is scrambled. As we will discuss in more detail in subsection 5.2, after the scrambling time, the vacuum evolves into the Bunch-Davies vacuum, which is maximally entangled across the horizon. Therefore, after the scrambling time, all the necessary ingredients to recover information are in place. Authors in[131] present an elegant method to recover information after scrambling time with the use of shockwaves[5].

For de Sitter space, the conjectured value (5.8) for the scrambling time takes the form

$$\tau_s \sim \frac{1}{H} \ln\left(\frac{1}{H}\right), \tag{5.9}$$

where $H \propto \sqrt{\Lambda}$ is the Hubble parameter. As pointed out earlier, the scrambling time matches TCC time τ_{TCC}. In this section we focus on de Sitter scrambling time and try to understand (5.9) better. We try to find an argument similar to the thought experiment we mentioned for black holes that justifies (5.9). In Appendix G, we present a thought experiment analogous to the one we discussed for black holes. We show that τ_s must be greater than $\sim \frac{1}{H}\ln\left(\frac{1}{H}\right)$ to avoid a violation of the no-cloning principle. Here, we propose a different thought experiment that offers a clear insight into the relation between the scrambling time and the maximum lifetime from TCC.

Thought experiment: Observational consistency

In the framework of complementarity, different observers experience different physics. The comoving observers see a stretched horizon while the free-falling observers crossing the Hubble horizon do not. This is consistent as long as observers who experience different physics cannot communicate their different narratives to each other. This is trivially satisfied for black holes,

[5] By computing out-of-time-order correlators, they show that de Sitter space is a fast scrambler ($\tau_s \sim \frac{1}{H}\ln\left(\frac{1}{H}\right)$). A similar argument for fast scrambling in de Sitter space was discussed in[132].

but in de Sitter space, the situation is more tricky. We consider a thought experiment to check if observers who experience different physics can communicate their experience to each other. We show that the consistency of complementarity imposes a lower bound $\frac{1}{H}\ln\left(\frac{1}{H}\right)$ on the scrambling time.

Consider two comoving observers Alice and Bob with initial positions of $r_A = 0$ and $r_B = l$ at $t = 0$. Consider a q-bit crosses Alice's horizon at $t = 0$, but it accelerates afterwards such that it never crosses Bob's horizon. Suppose Alice and Bob continue to stay on their comoving paths until $t = \tau_s$. From Alice's perspective, Hawking radiation carrying part of (even if very small) the q-bit's information radiates inward. However, from Bob's point of view, that would not happen since the q-bit has never exited his horizon. If Bob is still within Alice's Hubble horizon at this time, Alice can catch the radiation with the q-bit's information and communicate her different narrative to Bob (see figure 5.7). Alice and Bob must have exited each other's Hubble horizon by the scrambling time to prevent these inconsistent narratives from ever meeting each other. Therefore, scrambling time should be longer than the time it takes for a comoving length l to stretch beyond the Hubble radius.

$$\tau_s > \frac{1}{H}\ln\left(\frac{1}{Hl}\right). \tag{5.10}$$

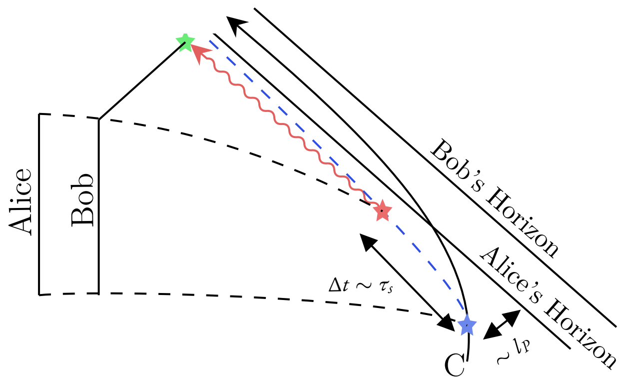

Figure 5.7: Setup of the thought experiment. System C exits Alice's Hubble horizon but stays inside Bob's Hubble patch. The dashed blue line is Alice's stretched horizon and the red squiggly line is the Hawking radiation which carries part of C's information.

We set l to the smallest possible meaningful distance l_{min}.

$$\tau_s > \frac{1}{H} \ln\left(\frac{1}{Hl_{min}}\right). \tag{5.11}$$

If $l_{min} = l_P$, the above lower bound matches TCC time. This thought experiment elucidates the relation between scrambling time and TCC time. They are both the time it takes for Planckian lengths to stretch beyond the Hubble radius. What is nice about this argument is that it tells us that the two times would match even if the smallest physical length scale l_{min} were different from Planck length[6]. It is reasonable to believe that in such a background, TCC time scale as the maximum lifetime of de Sitter space gets replaced with $\frac{1}{H} \ln\left(\frac{1}{Hl_{min}}\right)$. This is because the fundamental idea behind TCC time scale is to ensure that the smallest physical quantum fluctuations do not exit the Hubble horizon and classicalize[7].

Thermalization in de Sitter space

In the previous subsection, we studied thermalization in the framework of complementarity and showed that it takes $t \sim \frac{1}{H} \ln\left(\frac{1}{H}\right)$ to happen. In this subsection, we support that result by studying the de Sitter vacua. We review quantum backgrounds with approximate de Sitter symmetries (de Sitter vacua) and an argument that shows in a time of order τ_{TCC} all of them evolve into a particular one called the *Bunch-Davies* (BD) vacuum. The BD vacuum is a thermal background, which is why we call this process thermalization.

If we have a de Sitter vacuum $|\Omega\rangle$ the Wightman two-point functions $\langle\Omega|\,\varphi(x)\varphi(y)\,|\Omega\rangle$ must respect the isometries of the de Sitter space. Additionally, they are Green's functions of the free scalar field equation of motion. All such functions can be parametrized by a complex number α with a negative real part (see [133] for the analytic expression). The states $|\alpha\rangle$ for which

$$G_\alpha(x,y) = \langle\alpha|\,\varphi(x)\varphi(y)\,|\alpha\rangle\,, \tag{5.12}$$

are called α-vacua. For $\alpha = -\infty$, the Green's function matches the thermal Green's

[6] For example, the size of the compact dimensions could set a greater lower bound for physical lengths.
[7] We thank Matthew Reece for sharing this insight.

function[133,134]. This vacuum is called the Bunch-Davis vacuum. A review of the mode expansions that lead to these vacua can be found in[135]. The modes $\varphi^{\pm}(k)$ corresponding to the BD vacuum have a special property that

$$as\ k \to \infty \ \ \Rightarrow \ \ \varphi^{\pm}(k) \sim \frac{H\eta^{\frac{1}{2}}}{k} e^{i\vec{k}\cdot\vec{x} \mp ik\eta}, \tag{5.13}$$

where $(\vec{x}, \eta)$ are comoving coordinates. Equation (5.13) tells us that at large momenta (short distances) the BD modes exhibit the same behavior as their Minkowski counterparts. Therefore, the BD vacuum looks like the Minkowski vacuum at short distances which is consistent with the equivalence principle. The creation and annihilation operators for the rest of α vacuua do not share this property. The annihlation operators for α vacua are related to the ones for the BD through Boguliobov transformations[136],

$$a_k^{\alpha} = \frac{1}{\sqrt{1 - e^{\alpha+\alpha^*}}}(a_k^{BD} - a_k^{BD\,\dagger} e^{\alpha^*}). \tag{5.14}$$

The stability of de Sitter space is built in the symmetry group of it. So the instability of de Sitter space must manifest in the form of the impossibility of defining states that respect all de Sitter symmetries, i.e. de Sitter vacua. Following, we mention several issues with α-vacua that are closely related to the instability of the de Sitter space.

- We use adiabatic approximation as we assume that for a comoving mode k, the state satisfying $a_k |\psi\rangle = 0$ will continue satisfying it at later times. However, in[137], it was shown that this assumption does not hold. In other words, quantum effects make it impossible to find a true de Sitter vacuum that respects this symmetry.

- Due to the previous point, the time at which the condition $a_k^{\alpha} |\psi\rangle = 0$ is imposed for a given comoving mode matters. In[97], it was argued that the BD vacuum is the state we get by imposing these conditions at $t = -\infty$. This makes BD unphysical since, at that time, all the comoving modes are trans-Planckian. Physically, we can impose the annihilation conditions only after the modes enter the sub-Planckian regime. In[138], this issue was resolved by introducing a more physical alternative to the BD vacuum by imposing the annihilation condition of each mode at the time it enters the sub-Planckian regime. This state is called the *Instantaneous Minkowski Vacuum* (IMV). It was showed that this modification leads to decay of Λ with a lifetime of $\sim H^{-1}$ [139,140].

- Equation (5.14) tells us that all α-vacua (except BD) are UV divergent. This is because any α-vacuum other than the BD vacuum has arbitrarily high momentum excitations with respect to the BD vacuum, which leads to a divergent energy-momentum tensor. The fact that the short distance behavior of α-vacua are different from Minkowski violates the equivalence principle.

The α-vacua must be regulated at short distances to avoid the last problem. This could be done by introducing a cutoff Λ and imposing $a_k^{BD} |\alpha\rangle_{reg} = 0$ for $k > \Lambda$ and $a_k^{\alpha} |\alpha\rangle_{reg} = 0$ for $k < \Lambda$. Note that the two-point function at scale $\sim 1/\Lambda$ is no longer invariant under de Sitter space isometries. This is another example that preserving the symmetries of de Sitter space at all scales is impossible.

The regulated α-vacua $|\alpha\rangle_{reg}$ differ from BD only for modes with $k < \Lambda$. All of these modes exit the Hubble horizon in $\frac{1}{H}\ln\left(\frac{\Lambda}{H}\right)$. In other words, after $\frac{1}{H}\ln\left(\frac{\Lambda}{H}\right)$ any de Sitter vacuum evolves into the BD vacuum which is a thermal background. This result is identical to what we found in the previous subsection from de Sitter complementarity.

Complementarity in de Sitter space

Both of the perspectives that we discussed point towards the same expression $\frac{1}{H}\ln\left(\frac{1}{H}\right)$ for de Sitter thermalization/scrambling time which matches the conjectured value in [38]. The thought experiment in 5.2 gives us a unique insight into complementarity. It tells us that after the scrambling time, all the Hubble patch information exits the Hubble horizon and gets radiated back in the form of Hawking radiation. Let us say we have put observers on a comoving lattice with the initial spacing of Planck length. After a scrambling time, all observers exit each other's respective Hubble horizons. In a sense, each observer gets their own universe! Each observer will see all the other ones exit their horizon and receive their information in Hawking radiation after the scrambling time. There will be many isolated universes, each having a copy of the initial information in the form of Hawking radiation of everything else that crossed their Hubble horizon. This is the complementarity picture of de Sitter space.

5.3 Complementarity and the Swampland

In the last section, we provided arguments from different standpoints that the scrambling time in a de Sitter background is of the order of TCC time. The Swampland conditions suggest that the de Sitter space cannot be viewed as an equilibrium thermal background. This, however, does not mean that the de Sitter space does not have any statistical interpretation. For instance, the de Sitter entropy is still a meaningful quantity that counts the number of quasi-stable ds microstates. However, this should be viewed as a fine-grained entropy not to be confused with the thermodynamic entropy, which satisfies the second law of thermodynamics (see [141] for a review of fine and coarse-grained entropies). For the thermodynamic entropy to make sense, the system must be able to reach equilibrium. In a sense, the complementarity picture breaks down according to TCC.

It is worth mentioning that de Sitter space, if viewed as a thermal background, has some strange features. For example, the number of particles in the thermal radiation is $\mathcal{O}(1)$ [142]. So in a sense, the de Sitter space would be the minimal thermodynamical system that quantum mechanically could make sense.

There are some key differences between black hole complementarity and de Sitter complementarity. For example, Hawking-Page phase transition has no analogue in de Sitter space [130]. Another fundamental difference between the two is that the horizon is real and observer-independent in the black hole version. In contrast, in de Sitter space, the horizon is apparent and observer dependent that could significantly differ from the real horizon. The difference between real and apparent horizons is significant especially for fastly decaying de Sitter spaces such as those predicted by TCC. It is intriguing to see if there is a modified version of the complementarity principle that applies to all real horizons and is consistent with the Swampland picture.

A nice demonstration of the tension between Swampland conditions other than TCC and thermal aspects of de Sitter space can be found in[113]. The number of light degrees of freedom in a 4d de Sitter space is given by the de Sitter entropy $N_\Lambda \sim \frac{1}{\Lambda}$. Applying the de Sitter conjecture and the distance conjecture to a rolling quintessence potential shows that the number of accessible degrees of freedom increases by more than N_Λ over a scrambling time[113]. Therefore, the low energy EFT breaks before the de Sitter space can thermalize any non-thermal perturbation.

5.4 Cosmological implications

The fluctuations of CMB are scale-invariant. The only α-vacuum that generates scale-invariant fluctuations is the BD vacuum. This poses a fine-tuning problem for inflation's initial condition unless there is a natural mechanism that sets the vacuum to BD. As discussed in the last section, if the de Sitter space lasts more than the scrambling time, any initial vacuum would eventually evolve to BD. However, we saw that the Swampland conditions forbid this. Following, assuming a lifetime τ for the de Sitter space, we find the range of the α vacua that get turn into BD vacuum within the lifetime of de Sitter space. The smallness of this range determines the severity of the initial condition problem for inflation.

Suppose $|\psi\rangle_k$ is the projection of $|\alpha\rangle$ over the Fock space of particles with momentum k. Let $|\psi\rangle_k = \sum_n c_n |n\rangle$, where $|n\rangle$ is the n-particle state with respect to BD modes. Since $a^\alpha |\psi\rangle_k = 0$, from (5.14) we find

$$c_{2i+1} = 0 \;\; and \;\; c_{2i} = \frac{(2i-1)!!}{\sqrt{(2i)!}} e^{i\alpha^*}. \tag{5.15}$$

Therefore, the average excitation number of the momentum-k mode is

$$n_k \approx \frac{\sum_{i\geq 0} 2i\frac{(2i-1)!!}{\sqrt{(2i)!}}e^{2i\,\mathrm{Re}(\alpha)}}{\sum_{i\geq 0}\frac{(2i-1)!!}{\sqrt{(2i)!}}e^{2i\,\mathrm{Re}(\alpha)}}. \tag{5.16}$$

For large n we have

$$\begin{aligned}
\ln\frac{(2n-1)!!}{\sqrt{(2n)!}} &= \frac{1}{2}\sum_{i=1}^{n}\ln\left(\frac{2i-1}{2i}\right) \\
&= \frac{1}{2}\sum_{i=1}^{n}\ln\left(1-\frac{1}{2i}\right) \\
&\approx \frac{1}{2}\sum_{i=1}^{n}-\frac{1}{2i} \\
&= -\frac{1}{4}H(n) \\
&\approx -\frac{1}{4}\ln(n),
\end{aligned} \tag{5.17}$$

where $H(n)$ is the harmonic series. Therefore, from exponentiating the above equation we find,

$$\frac{(2n-1)!!}{\sqrt{(2n)!}} \approx n^{-1/4}. \tag{5.18}$$

Plugging this into (5.16) leads to

$$n_k \approx \frac{\sum_{i\geq 0} 2i^{3/4}e^{2i\,\mathrm{Re}(\alpha)}}{\sum_{i\geq 0} i^{-1/4}e^{2i\,\mathrm{Re}(\alpha)}}. \tag{5.19}$$

This can be expressed in terms of the polylogarithm functions as

$$n_k \approx f(\xi), \tag{5.20}$$

where $\xi = \exp(2\,\mathrm{Re}(\alpha))$ measures the deviation from the Bunch-Davis vacuum and

$$f(x) := 2\frac{Li_{-\frac{3}{4}}(x)}{Li_{\frac{1}{4}}(x)}. \tag{5.21}$$

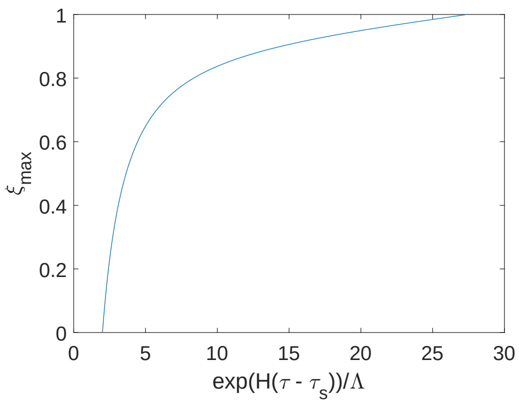

Figure 5.8: Plot of $\xi_{\max}$ versus $e^{H(\tau-\tau_s)}/\Lambda$.

The average momentum is

$$\langle k\rangle \sim \frac{\int_{|k|<\Lambda} d^{D-1}k \;\, n_k k}{\int_{|k|<\Lambda} d^{D-1}k} \\ \sim \Lambda f(\xi). \tag{5.22}$$

where Λ is the field theory cut-off and D is the dimension of space-time. The time that it takes for these excitations to freeze out (thermalize) is

$$\tau_\xi = \frac{1}{H}\ln\left(\frac{\Lambda f(\xi)}{H}\right). \tag{5.23}$$

For the fluctuations to be scale invariant at the end of inflation, the duration inflation τ must be longer than this time.

$$\tau > \frac{1}{H}\ln\left(\frac{\Lambda f(\xi)}{H}\right), \tag{5.24}$$

which can be rearranged into the following form.

$$\xi < \xi_{\max} = f^{-1}(e^{H(\tau-\tau_s)}/\Lambda), \tag{5.25}$$

where $\tau_s = \frac{1}{H}\ln\left(\frac{1}{H}\right)$ is the de Sitter scrambling time and also the maximum de Sitter life time according to TCC. Figure 5.8 shows the graph of $\xi_{\max} = f^{-1}(e^{H(\tau-\tau_s)}/\Lambda)$.

To explain the scale-invariance of CMB fluctuations without finetuning of initial vaccum state, generic initial states, including those with $1-\xi \ll 1$, must have enough time to thermalize into BD by the end of inflation. Therefore, we need $1-\xi_{\max} \ll 1$. The graph 5.8 tells us that this condition implies $e^{H(\tau-\tau_s)}/\Lambda \gg 1$. If $\Lambda = O(M_{pl})$, we would need $\tau - \tau_s \gg H^{-1}$ which is inconsistent with TCC. Thus, TCC implies that inflation cannot last long enough for a generic initial vacuum to create the observed almost scale invariant CMB fluctuations. This imposes a severe fine-tuning problem on inflation. This is in addition to another fine-tuning problem that TCC imposes on inflationary models due to the very short field range of the inflaton[29]. In conclusion, any conventional form of inflation seems to be in severe tension with TCC. A TCC-compatible potential alternative for inflation was recently proposed by Prateek Agrawalet. al. for the early phase of our universe[143].

5.5 Summary

We saw from two different points of view that suppose de Sitter space lives long enough, the time $\frac{1}{H}\ln\left(\frac{1}{H}\right)$ could be viewed as the thermalization time. From the complementarity standpoint, this is when out-of-equilibrium perturbations thermalize over the stretched horizon before getting radiated back into the Hubble patch. From another point of view, this is when deviations from the thermal BD vacuum exit the Hubble horizon. These are two different, yet compatible, ways of viewing the thermalization process in de Sitter space.

The TCC states that the lifetime of de Sitter space is less than the de Sitter thermalization time. In other words, the universe will quickly access more light degrees of freedom than the ones available to it in a give de Sitter background and will not stay in the de Sitter Hilbert space long enough to reach thermal equilibrium. Because of this, TCC poses a severe initial condition problem for any conventional inflationary scenario producing the scale-invariant CMB fluctuations.

It would be interesting to study the possibility of a more general principle that quantum gravity forbids finite-dimensional thermal systems in the sense that the thermal distribution in any finite-dimensional subspace cannot be confined to that subspace for more than its thermalization time.

6

Holography in scalar field cosmologies

In this section, we provide a holographic derivation of TCC in the asymptotics of the field space. We have learned a lot about the nature of quantum gravity from string theory and our understanding of string theory largely falls into two categories: 1) The non-perturbative data from BPS objects and dualities, and 2) very detailed perturbative understanding of weak coupling limits which lie at the infinite distance limits of the moduli space. The understanding that string theory provides for the weak coupling limits is an important component of what makes string theory what it is. After all, string theory owes its name to these weakly coupled descriptions and finding a fundamental understanding of general features of the infinite distance limits is as deep as understanding why string theory works.

All infinite distance limits of the moduli space in string theory exhibit some universal features which have led to various Swampland conjectures that postulate that these features are fundamental to quantum gravity. Here we focus on two conjectures. 1) The distance conjecture[32] which postulates the existence of a tower of light weakly coupled states at every infinite distance limit of the field space with masses that depend exponentially on the distance in the field space. 2) The Trans-Planckian Censorship Conjecture (TCC)[19] which prohibits the classicalization of Planckian quantum fluctuations and provides a concrete exponentially small upper bound for the scalar potential in the infinite distance limit of the field space.

Some of the Swampland conjectures have been connected to more fundamental physical principles such as unitarity, causality, or holography[144,145,146,147]. However, for the distance conjecture and de Sitter conjectures such as TCC, considerable part of the evidence comes from observations in string theory[14,19,148,96,149,150,108,151,95] and compatibility with more established Swampland conjectures[21,22,15]. In this work, we present an explanation for some aspects of these conjectures based on the holographic principle. What we mean by holographic principle is the most conservative form of holography, which states that physical observables in a gravitational theory must live on the boundary of spacetime. This statement is equivalent to the statement that there are no local physical observables in quantum gravity. One way to see this is that in quantum gravity, we sum over spacetimes of different topologies, and therefore, a unique spacetime manifold does not exist. However, a classical approximation can arise when a particular configuration extremizes the Quantum Gravity path integral. The fact that the classical picture is emergent rather than fundamental is nicely captured by dualities in string theory. For example, consider two T-dual descriptions of a spacetime with one compact dimension of size R in one frame and l_s^2/R in the other frame. As we change R/l_s from very small values to very large values, the semi-classical spacetime that provides the sharpest approximation to the quantum theory transitions from one description to another. There is a mapping between the descriptions, but there is no direct mapping between the spacetimes. For examples, a local wavepacket in the compact dimension in one picture maps to a winding string with no notion of position in the compact dimension. Generally, the notion of spacetime, and any "local" operator associated with the spacetime are expected to be emergent in quantum gravity.

Assuming that the potential always decays exponentially in the asymptotics of field space, we show that there must be universal lower bounds on 1) the decay rate of the potential and 2) the masses of the weakly coupled particles.

This section is organized as follows. Before going into details about the effective potential, we first examine its meaning in subsection 6.1. We review different definitions for effective action, both based on local observables (e.g. CFT correlators) and boundary observables (e.g. scattering amplitudes). In subsection 6.2, we show that a boundary definition for the effective potential is especially needed when the potential is positive. This is mainly due to the fact that any local extremum of the potential will be a de Sitter space with finite spatial volume. Therefore, the convexity theorem applies to any effective potential obtained from the QFT path integral. But as we will discuss, a positive effective potential cannot be everywhere convex. Therefore, a boundary definition of the effective action is required in theories with gravity.

In subsection 6.3 we consider a pure quintessence cosmology driven by a scalar field that rolls to infinity in the field space. We study the boundary data that weakly coupled fields can produce in such a cosmology. We show that in some cases, there is no non-trivial boundary data at all. In other words, the field theory would be a uniquely bulk phenomenon, and therefore, it cannot be holographically reproduced. The conditions that we find are

- The expansion cannot be accelerated $a \sim t^p,\ \ p \leq 1$.
- For $p > 1/2$, the masses of the weakly coupled particles must decay polynomially fast in time $m \lesssim t^{1-2p}$.

Note that the results of this section do not apply to cases where the thermal energy-momentum of other fields dominates the evolution (e.g. matter or radiation dominated cosmologies).

In subsection 6.4, we discuss the relation between these results and various Swampland Conjectures. The first result is simply equivalent to the TCC at future infinity, which could also be expressed in the language of the de Sitter conjecture. As explained in[19], TCC implies that the potential must decay as $\exp(-\lambda\varphi)$ where $\lambda \geq 2/\sqrt{d-2}$, unless the contribution of an emerging light tower of states to the energy density becomes significant in future infinity. In that case, TCC implies the more conservative bound of $\lambda \geq 2/\sqrt{(d-1)(d-2)}$. In [33], it was argued that emergent string conjecture [152] prevents the emergence of such light states in future infinity. In that case, TCC always implies $\lambda \geq 2/\sqrt{d-2}$.

As for the second condition, when expressed in terms of the scalar field that drives the expansion, it becomes identical to the statement of the distance conjecture $m \lesssim \exp(-c\varphi)$. The c that we find depends on λ. However, the greatest $c(\lambda)$ that we find is $c = 1/\sqrt{d-2}$, which remarkably matches with the proposal in [153], which is proposed on different grounds.

Note that our arguments only apply to universes that have polynomial expansion. Thus, we cannot rule out eternal de Sitter. In fact, an EFT in de Sitter can famously produce boundary data according to dS/CFT [154]. It is known that the asymptotic information in universes with an accelerated expansion is spread across different Hubble patches and might not be measurable by a bulk observer [155,156], which might be a reason to think such universes are not physical [157]. However, our argument points out a specific and fundamental problem with a subclass of universes with accelerated expansion. We argue that boundary observables do not exist in universes with accelerated polynomial expansions, whether we allow meta-observers or not.

6.1 Effective action

In this section we review two approaches for writing down an effective action for a physical system based on the data used to produce it. Generally, an effective action is the approximation of a quantum physical system by a classical field theory. The physical data that is used to construct the effective action is either the algebra of some local observables, or some boundary observables (e.g. S-matrix). CFTs are examples of the former class that do not have asymptotic states [158,159], and quantum gravities are believed to fall in the second category since they do not have gauge-invariant local observables [160]. Note that it is possible to have two dual descriptions of the same theory that fall into different categories. AdS/CFT correspondence [161] is an examples of this where one theory is more naturally defined using boundary observables, while the other theory is a CFT that has local observables.

In the following, we briefly review both of these approaches to the effective action, and study their requirements as well as their physical implications for the underlying physical theory.

Effective action for local observables

The most common definition of a field theory is a theory with local observables and the effective action captures the local interactions in such a theory. We view a quantum field theory as a theory of local observables defined via path-integral rather than a prescription to calculate scattering amplitudes.

Suppose we have a field theory with a bare action S_0 and a scalar field $\varphi(x)$. The effective action $\Gamma(\varphi)$ is defined to give us the quantum corrected equations of motion for the expectation values of the operators. We can evaluate the vacuum expectation value of the operator $\hat{\varphi}(x)$ by inserting it into the path integral. If the theory has a Minkowski vacuum, this vev would not depend on x. However, we can consider non-vacuum backgrounds for which $\langle\varphi(x)\rangle$ evolves in time. We are particularly interested in classical (or coherent) backgrounds. We can construct a coherent background by acting on the vacuum with $e^{\alpha\hat{a}-\alpha^*\hat{a}^\dagger}$, where $\hat{a}^\dagger$ and $\hat{a}$ are creation and annihilation operators. In order to act with an operator on vacuum, we can insert it into the path integral. Therefore, to construct a background which looks classical, we can insert an operator $e^{\alpha_p\hat{a}_p-\alpha_p^*\hat{a}_p^\dagger}$ for every harmonic oscillator corresponding to a momentum p:

$$\int \mathcal{D}\varphi e^{iS} e^{\int d^{d-1}p \;\; \alpha_p\hat{a}_p-\alpha_p^*\hat{a}_p^\dagger}. \tag{6.1}$$

Given that a_p is the Fourier transform of $\varphi(x)$, any linear combinations of $\varphi(x)$ takes the form above. Therefore, we can express a coherent state as follows

$$\int \mathcal{D}\varphi e^{iS} e^{i\int d^dx J(x)\varphi(x)}, \tag{6.2}$$

The insertion of $e^{i\int d^dx J(x)\varphi(x)}$ creates a classical background and is referred to as the source term.

Now that we have created a non-vacuum background, we can re-evaluate the expectation value of our local operator $\hat{\varphi}(x)$ and see how it evolves in time. Let the expectation value of $\varphi(x)$ in this background be $\varphi_J(x)$. If we take the limit of $\hbar \to 0$, the quantum fluctuations disappear and the path integral is replaced by its integrand. Therefore, we find that

$$\int \mathcal{D}\varphi e^{iS} e^{i\int d^d x J(x)\varphi(x)} \to e^{iS(\varphi_J)+i\int d^d x J(x)\varphi(x)}. \tag{6.3}$$

In this limit the equations of motion are given exactly by the action S. However, when $\hbar$ is non-zero, the above equation breaks. The effective action Γ_{full} is defined to capture these quantum effects and maintain the above equation.

$$\int \mathcal{D}\varphi e^{iS+i\int d^d x J(x)\varphi(x)} = \varphi e^{i\Gamma_{full}(\varphi_J)+i\int d^d x J(x)\varphi(x)}. \tag{6.4}$$

This definition turns out to capture the fully quantum corrected equations of motion, meaning that the expectation value of local operators follow the equations of motion given by minimizing Γ_{full}.

$$\frac{\delta\Gamma_{full}(\langle\varphi\rangle)}{\delta\langle\varphi(x)\rangle} = 0. \tag{6.5}$$

If we calculate Γ_{full} according to above prescription, it would have all sorts of non-local terms. An important manifestation of the non-local effects is the running of the coupling constants. What we usually mean by an effective action, is an approximation of Γ_{full} with a local action Γ with finite number of terms. This effective action can then be used to calculate long-range correlation functions. In other words, Γ captures the full quantum effect as far as long-range observables are concerned.

Approximating Γ_{full} with a local action Γ is not always possible. Even if we start with a local bare action S, the quantum effects generate variety of non-local terms in Γ_{full}. If the field theory is strongly interacting, the non-local terms can be too strong to neglect. Typically, only weakly interacting QFTs can be described an effective action at long ranges.

Suppose the effective action can be written as a sum of a canonical kinetic term and an effective potential term $V(\varphi)$ that only depends on $\langle\varphi\rangle$. In that case the equation (6.5) implies that the vacuum expectation value of φ must be at a local minimum of $V(\varphi)$.

Effective action for boundary observables

In quantum gravity, it is often the case that the theory is formulated in terms of some asymptotic observables defined on the conformal boundary of spacetime. The asymptotic data is string theory is the string worldsheet amplitudes, however, the interpretation and the structure governing these amplitudes vary depending on the background. In Minkowski spacetime, the string amplitudes define the S-matrix, while in the AdS spacetime, they have a much richer structure that represent the correlation functions in the dual CFT. The formulation of quantum gravity in terms of boundary data is the general basis for holography. In both Minkowski and AdS backgrounds, we can reproduce the low-energy boundary data of the quantum gravity using a QFT in curved spacetime with an appropriate effective action. This is how one calculates effective action in quantum gravity.

In this section, we review the boundary data in different backgrounds and how they can be calculated using the effective action. If a theory of quantum gravity is formulated based on the corresponding boundary data, one can use the results of this section to reverse engineer an effective action that approximates that theory.

Minkowski space ($V = 0$)

The boundary observables in a weakly coupled quantum field theory in Minkowski spacetime is the usual S-matrix elements. In Minkowski spacetime, the LSZ reduction formula allows us to define S-matrix elements in terms of the field theory correlation functions. The resulting S-matrix elements are asymptotic data, as they only depend on the initial condition in the asymptotic past/future[1]. For example, for massive scalar particles, the LSZ formula states [163]

[1] There are some subtleties with massless particles in 4d due to IR divergences which we will not discuss here. For a discussion of this problem and a proposed resolution see [162].

$$\int d^D p_1 e^{ip_1 x_1} \dots \int d^D p_m e^{ip_m x_m} \int d^D q_1 e^{-iq_1 y_1} \dots \int d^D q_n e^{-iq_n y_n} \langle \mathcal{T}\{\varphi(x_1)\dots\varphi(x_m)\varphi(y_1)\dots\varphi(y_n)\}\rangle$$
$$\sim \ (\Pi_{i=1}^m \frac{i}{p_i^2 - m^2 + i\varepsilon})(\Pi_{j=1}^n \frac{i}{q_j^2 - m^2 + i\varepsilon}) \langle \mathbf{p_1} \dots \mathbf{p_m} | S | \mathbf{q_1} \dots \mathbf{q_n} \rangle \,, \qquad (6.6)$$

where $\mathcal{T}$ is the time ordering operator and $\sim$ is equality up to analytic terms. Note that we have assumed that φ is canonically normalized which means the kinetic terms of φ in the effective action takes the form $-\frac{1}{2}(\partial_\mu \varphi)^2$. Since the correlation functions can be calculated from the effective action using the Schwinger–Dyson equation, we can also calculate the S-matrix elements from the effective action using tree-level diagrams.

It is helpful to rewrite the scattering amplitude in the following form for reasons that will become clear shortly.

$$< \tilde{\varphi}_b(P_1)\tilde{\varphi}_b(P_2)...\tilde{\varphi}_b(P_n) >= \lim_{p_i \to P_i} \Pi_i \tilde{G}(p_i)^{-1} < \tilde{\varphi}(p_1)\tilde{\varphi}(p_2)...\tilde{\varphi}(p_n) >, \qquad (6.7)$$

where P_i and p_i are respectively on-shell and off-shell momenta, $G(p)$ is the propagator in the momentum space. The P_i's with positive energy correspond to in-going states while the ones with negative energy correspond to out going states.

One of the significant properties of QFT in Minkowski space is that the Hilbert space of asymptotic past/future admits a Fock space representation. As we will see shortly, this is not generically true in an arbitrary spacetime.

Anti-de Sitter space ($V < 0$)

In AdS space, there is a very rich asymptotic structure given by the rescaled correlation functions at the asymptotic boundary. One can intuitively understand the difference between Minkowski space and AdS space as follows. In AdS space an observer can send a null ray to the boundary and receive its reflection in a finite time. This means, the boundary condition in AdS space is relevant to the experiments inside the AdS. Therefore, there must be a meaningful way of measuring the correlation functions in AdS, even when some of those points approach the conformal boundary. The boundary data for QFT in AdS space is the basic ingredient that makes AdS/CFT correspondence possible. Anti de Sitter space in the global coordinate has the following metric.

$$ds^2 = -(1+\frac{r^2}{l_{AdS}^2})dt^2 + (1+\frac{r^2}{l_{AdS}^2})^{-1}dr^2 + r^2 d\Omega_{d-1}^2. \tag{6.8}$$

Suppose, $\{\mathbf{X_i}\}_{1\leq i\leq n}$ are n sets of coordinates on the $(D-1)$-sphere. Then, the boundary data is

$$\lim_{r\to\infty} \langle \mathcal{T}\{(r^{\Delta_1}\varphi_1(t_1,r,\mathbf{X}_1)r^{\Delta_2}\varphi_2(t_2,r,\mathbf{X}_2)\ldots r^{\Delta_n}\varphi_n(t_n,r,\mathbf{X_n})\}\rangle\,, \tag{6.9}$$

where φ_i are fields and Δ_i are appropriate weights that make the above expression convergent. For a free field with mass m, Δ is given by $\Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4m^2}$[164].

The normalized time-ordered correlation functions become singular in the limit where two of the points coincide and the corresponding singularity behaves like an OPE of primary fields in a CFT[164]. The ADS/CFT correspondence postulates that for any theory of quantum gravity, the normalized time-ordered correlations on the boundary must in fact correspond to the correlation functions of a CFT. Therefore, the asymptotic data of an EFT in AdS is correlation functions. Let us take a moment to explain how these boundary correlation functions are related to string worldsheet amplitudes.

In flat space, the string worldsheet amplitudes have an insertion of the form $\exp(ik.X)$ for any external leg with momentum k. Therefore, a generic S-matrix element looks schematically like

$$A_{\{q_i\}} = \left\langle \int \Pi_i dz d\bar{z} : e^{iq_i\cdot x_i} : \mathcal{V}(z_i,\bar{z}_i;X,\ldots))\right\rangle_{worldsheet}, \tag{6.10}$$

where : _ : is the proper ordering corresponding to the topology of the worldsheet. If we plug this into the LSZ equation and take a Fourier transform of both sides we find

$$\int \Pi_j [d^D q_j e^{-iq_j x_j} (\frac{i}{q_j^2 - m^2 + i\varepsilon})^{-1}] \langle \mathcal{T}\{\varphi(x_1) \ldots \varphi(x_n)\}\rangle \simeq A_{\{q_i\}}, \tag{6.11}$$

where x_i's are taken to the asymptotic boundary. Let us look at all the ingredients of the above equation and $(\frac{i}{q_j^2-m^2+i\varepsilon})^{-1}$ is the inverse of the Green's function to cancel the evolution of $\varphi_j(x_j)$ as we take x_j to the asymptotic boundary and ensure that the expression converges to something. In fact, the AdS boundary correlators are also defined using the same prescription. We multiply bulk correlators by the inverse of the Green's function to make sure the evolution of the two cancel each other out as we take the points to the asymptotic boundary. The result of this calculation is parametrized by boundary coordinates. In flat space these are the momenta q_i which pick out a direction, and in AdS they are coordinates x_i. This final final expression is also the string worldsheet amplitude. In other words, sting theory directly calculates the well-defined boundary observables and effective action is just model that approximates those boundary observables.

So in general, we have the following prescription to find the asymptotic observables produced by a field theory. If the particles freeze in position space we use

$$\lim_{x_i \to X_i \in \text{boundary}} \Pi_j G(X_j, x_j)^{-1} \langle \mathcal{T}\{\varphi(x_1) \ldots \varphi(x_n)\}\rangle \,. \tag{6.12}$$

and if they freeze in the momentum space, we use

$$\lim_{p_i \to P_i:\text{on-shell}} \Pi_i \tilde{G}(p_i)^{-1} < \tilde{\varphi}(p_1)\tilde{\varphi}(p_2)...\tilde{\varphi}(p_n) > . \tag{6.13}$$

6.2 Why gravity needs boundary observables when $V > 0$?

In this section we want to point out a limitation to the observable-based approach when the scalar potential is positive. Let us first see what happens when the spatial volume is finite. The effective action captures the equations of motion of local quantum observables. So one might think that the lack of need of asymptotic states will put finite and infinite spacetimes on equal footings. However, it turns out that there is a major distinction between the two cases. In finite spacetimes, one can go from any field configuration $\varphi(x) = \varphi_1$ to any other field configuration $\varphi(x) = \varphi_2$ with finite action. In other words, the amplitude of tunneling between the two is finite. This is in sharp contrast with infinite spacetimes with dimensions $d > 2$ where it takes infinite energy to change the boundary conditions. For this reason, infinite spacetimes can have different boundary conditions parameterized by moduli, that set the theory in different vacua. But in finite spacetimes, the moduli cannot be frozen, as quantum fluctuations can take the system from any field configuration to any other configuration with finite amplitude.

In the presence of different vacua, the effective potential can have multiple minima each corresponding to a different vacuum. Note that all the minima must have the same energy, otherwise, we can tunnel from one to another using a Coleman instanton in finite time[76,165]. Given that the effective action represents the fully quantum corrected equations, being in the local minima of the effective action must mean that we are in the true vacuum. Therefore, the local minima of the effective action must be impervious to Coleman instantons. Thus, the local minima of the effective potential, must be the global minima of the effective potential.

As we discussed above, infinite spacetimes with $d > 2$ dimensions can have multiple vacua. Therefore, the effective potential in infinite spacetimes can have multiple global minima. This is because, the energy cost to transition between the two is infinite and therefore, the amplitude is zero.

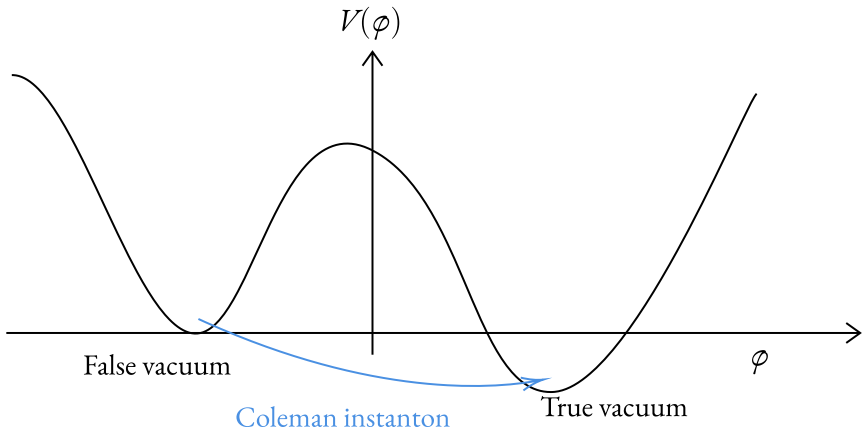

Figure 6.1: If the potential has another local minimum with a smaller energy, due to Coleman instantons, the universe will nucleate expanding bubbles of the true vacuum which collectively fill the universe in finite time.

It is often said that the effective action is convex, however, the convexity theorem only applies when one has not specified the boundary conditions of path integral at infinity. When one does not specify that boundary condition, they are effectively averaging over all different vacua. But, when fixing the boundary condition at infinity, one can derive a non-convex effective action that captures the fully quantum equations of motion. Higgs potential is an example of this.

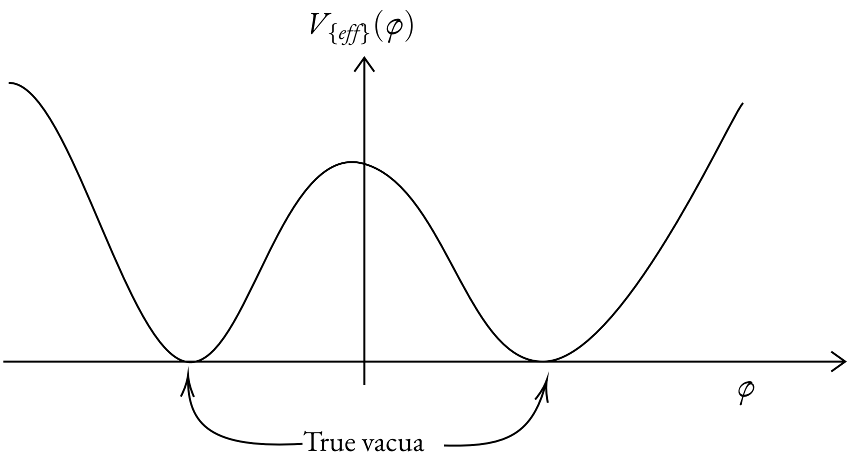

Figure 6.2: In infinite spacetimes with dimension $d > 2$, there could be different vacua. All of these vacua must have the same energy to ensure their stable against Coleman instantons. Moreover, a true vacuum must be the minimum of the effective potential. Therefore, an effective potential that is calculated via fixing the boundary condition at infinity, can have multiple minima, and is not necessarily convex.

Going back to case of finite volume, no two minima are separated vacua since we can always transition between states with finite amplitude. Therefore, there must be a unique vacuum in the theory. This implies that the effective potential must have a unique global minimum. This in fact follows from the convexity theorem as well[163]. In finite volume there are no boundary conditions to fix, and therefore the convexity theorem applies. A convex function has a unique minimum, therefore, the vacuum is unique. de Sitter space has a finite volume, therefore, if it is realized as a minimum of an effective potential which is defined based on the local observable approach, that potential must be convex. However, in all known examples in string theory the potential dies off exponentially in all directions of the field space[14] and the following three criteria are mutually inconsistent:

- V is convex.
- V decays exponentially at infinities.
- V has an extremum with $V > 0$.

In fact, as long as $V > 0$ and decays exponentially at infinities, it will always have an extremum. Thus, we do not even need to assume the existence of an unstable or metastable de Sitter as long as $V > 0$.

The above argument shows that the effective action based on local observables especially does not make sense when $V > 0$ and at best, it can only explain a finite window of field space. Therefore, to have a meaningful definition of effective potential, gravity necessitates existence of boundary observables for $V > 0$.

6.3 Boundary observables in rolling backgrounds

In this section we study the boundary observables that can be defined in FRW backgrounds with polynomial expansions $a \sim t^p$. The reason we are interested in these backgrounds is that as we show in the Appendix K any expansion driven by an exponentially decaying potential is polynomial[2]. Exponential potentials are ubiquitous in string theory and in all the known

[2]Most of the calculations in the Appendix K are standard calculations that we include for the sake of completeness (see for example[157,166,22]).

examples, the scalar potentials decays exponentially at the infinity of the filed space. This also follows from the Swampland de Sitter conjecture[14].

6.3.1 Asymptotic cosmology for exponential potentials

Let us summarize the results of the Appendix K. Assuming that the evolution of spacetime is driven by an exponentially decaying scalar potential, we find

- Expansionary/contracting solutions expand/contract polynomially in the future/past infinity.
- Unless there is a bounce, every spacetime has a spacelike singularity at finite time (i.e. big bang or big crunch).
- Suppose the scale factor goes like $a \sim t^p$ at the asymptotic of the field space and scalar potential goes like $\exp(-\lambda\varphi)$. We have

$$\begin{aligned} \lambda < 2\sqrt{\frac{d-1}{d-2}} &: p = \frac{4}{(d-2)\lambda^2} \\ \lambda > 2\sqrt{\frac{d-1}{d-2}} &: p = \frac{1}{d-1}. \end{aligned} \tag{6.14}$$

- $p > 1$ iff $\lambda < \frac{2}{\sqrt{d-2}}$.

The above results show the connection between exponential potentials and polynomial evolutions. In the rest of the section, we focus on the cosmological evolution and study the boundary observables for a given background that expands polynomially as

$$a(t) \sim t^p, \tag{6.15}$$

as $t \to \infty$ where $a(t)$ is the scale factor in the FRW solution with flat spatial curvature

$$ds^2 = -dt^2 + a(t)^2 (\sum_{i=1}^{d-1} dx^{i\,2}). \tag{6.16}$$

6.3.2 Heuristic calculations

Before doing any precise calculations, let us do some heuristic calculations to see what we should expect about the boundary data. Consider a particle with mass m and comoving momentum k. The proper momentum of the particle is k/a. Therefore, at late times, we expect the particle to have energy $\omega \simeq m + \frac{k^2}{2ma^2}$ and comoving velocity $v \simeq \frac{k}{ma^2}$. The maximum distance that this particle can travel in comoving coordinate is

$$\int dt \frac{k}{ma^2}, \tag{6.17}$$

which is finite if $p > 1/2$. Even if m depends on the running scalar and changes with time as t^{-q}, as long as $q < 2p - 1$, the particle cannot make it to spatial infinity and freezes out at some comoving coordinates. Therefore, we cannot define an S-matrix and our boundary data must be frozen correlators.

Now let us consider the massless case. For massless particles, the particle horizon is given by

$$\int \frac{dt}{a}. \tag{6.18}$$

Therefore, if $p > 1$, the particle massless particle freezes out at some comoving coordinate and our boundary data must be frozen correlators rather than scattering amplitudes.

In the following subsections we see how the boundary correlation functions freeze when $q < 2p - 1$ for massive particles or $p > 1$ for massless particles. Not only that, but we will also see that the frozen correlation functions are trivial and they vanish when the boundary points are not coincident.

6.3.3 Boundary correlation functions

The Ricci curvature in this background is

$$\mathcal{R} = 2(d-1)\frac{\ddot{a}}{a^2} + (d-1)(d-2)\frac{\dot{a}^2}{a^2}. \tag{6.19}$$

In a general curved background, the equation of motion of a scalar field is given by

$$[-\frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu) + m^2 + \xi\mathcal{R}]\varphi = 0, \tag{6.20}$$

where m is the mass and ξ is the coefficient of the $\varphi\mathcal{R}$ coupling. We assume $\xi = 0$ as its value does not affect our conclusions.

$$\frac{1}{a^2}[-\Delta^2 + (d-1)\dot{a}a\partial_t + a^2\partial_t^2 + m^2a^2]\varphi = 0. \tag{6.21}$$

Since we have translational symmetry in space, we can consider the following solution ansatz.

$$\tilde{\varphi}_k = e^{-i\omega(t)\cdot t + i\vec{k}\cdot\vec{x}}. \tag{6.22}$$

Note that ω depends on t. As $t \to \infty$, the last two terms dominate and we find

$$\lim_{t\to\infty}\omega(t) \to \pm m. \tag{6.23}$$

Let us focus on the positive frequency modes for now. We define $\omega = m + \delta\omega$, where $\delta\omega$ vanishes at $t = \infty$.

$$\frac{1}{a^2}[k^2 - i(d-1)\dot{a}a(\omega + t\dot{\omega}) - (\omega + t\dot{\omega})^2a^2 + m^2a^2]\tilde{\varphi}_k = 0. \tag{6.24}$$

For ω to converge we need $t\dot{\omega} \to 0$. Therefore, at late times, we can replace $\dot{a}a(\omega + t\dot{\omega})$ with $\dot{a}am$. However, we cannot do the same with $(\omega + t\dot{\omega})^2a^2$. This is because $t\dot{\omega}\omega a^2$, $2\omega\delta\omega a^2$ could be relevant. However, the rest of the terms will be subleading. So we find

$$\frac{1}{a^2}[k^2 - i(d-1)\dot{a}am - 2m\delta\omega a^2 - 2t\delta\dot{\omega}ma^2] = 0, \tag{6.25}$$

which leads to

$$t(\dot{\delta\omega}) + \delta\omega + i\frac{d-1}{2}\cdot\frac{\dot{a}}{a} - \frac{k^2}{2ma^2} = 0. \tag{6.26}$$

The general solution to the above differential equation is

$$\delta\omega = \frac{c}{t} - i\frac{(d-1)p}{2}\frac{\ln(t)}{t} - \frac{k^2}{2ma^2(2p-1)}, \tag{6.27}$$

where c is a constant. The first term amounts to an overall normalization of $\tilde{\varphi}$. We choose c such that

$$\delta\omega = -i\frac{1}{t}\ln\left((\frac{a}{a_0})^{(d-1)/2}\right) - \frac{k^2}{2ma^2(2p-1)}. \tag{6.28}$$

For $\omega = m + \delta\omega$ we find

$$\omega = m - i\frac{1}{t}\ln\left((\frac{a}{a_0})^{(d-1)/2}\right) - \frac{k^2}{2ma^2(2p-1)}. \tag{6.29}$$

As we states above, there is also a negative frequency mode which we can obtain by fliping the sign of m in the above equation. So we find

$$\omega_\pm(k,t) = \pm m - i\frac{1}{t}\ln\left((\frac{a}{a_0})^{(d-1)/2}\right) \mp \frac{k^2}{2ma^2(2p-1)}. \tag{6.30}$$

We could relate the two solutions to each other by

$$i\omega_\pm = (i\omega_\mp)^*. \tag{6.31}$$

This follows from the fact that if $\tilde{\varphi}$ is a solution so must be $\tilde{\varphi}^*$ because the equations are real.

This implies that if ω gives a solution, so does $-\omega^*$. Therefore, either ω is purely imaginary or the two frequencies are related by the above relation. As we will see later, the first case happens for the massless fields.

We consider $p > 1/2$ where the friction term in (6.30) dominates and the modes freeze at time infinity.

Now let us quantize our scalar field. The canonical quantization relations lead to

$$[\varphi(x), \pi(y)] = i\delta^{d-1}(x-y)a^{-(d-1)}, \tag{6.32}$$

where $\pi = \partial_t \varphi$. The factor of $a^{-(d-1)}$ shows up because the distance $x-y$ in the delta function is not the proper distance. We could equivalently write

$$[\varphi(x), \pi(y)] = i\delta^{d-1}(a(x-y)). \tag{6.33}$$

Note that the above equation uses the fact that metric is diagonal and $g_{tt} = -1$. Otherwise, we would have $\pi = -\partial^t \varphi$ which is not necessarily the same as $\partial_t \varphi$.

Now let us expand the operators φ and π in terms of the solutions $\tilde{\varphi}_k$.

$$\varphi(x,t) = \int d^{d-1}k\, a_k e^{-i\omega_+(k,t)t + i\vec{k}\cdot\vec{x}} + a_k^\dagger e^{-i\omega_-(k,t)t - i\vec{k}\cdot\vec{x}}. \tag{6.34}$$

The reality of the above expression follows from (6.31). Then we have

$$\pi(y,t) = \int d^{d-1}k'\, a_{k'}(-i)(\omega_+ + \dot{\omega}_+ t)e^{-i\omega_+(k',t) + i\vec{k}'\cdot\vec{y}} + a_{k'}^\dagger(-i)(\omega_- + \dot{\omega}_- t)e^{-i\omega_-(k',t) - i\vec{k}'\cdot\vec{y}}. \tag{6.35}$$

From the above equations we find

$$[\varphi(x,t),\pi(y,t)] = \int d^{d-1}k\, d^{d-1}k' [a_k, a_{k'}](-i)(\omega_+ + \dot{\omega}_+ t)e^{-i\omega_+(k,t)t - i\omega_+(k',t)t + i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{x}}$$
$$+[a_k^\dagger, a_{k'}^\dagger](-i)(\omega_- + \dot{\omega}_- t)e^{-i\omega_-(k,t)t - i\omega_-(k',t)t - i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{x}}$$
$$+[a_k, a_{k'}^\dagger](-i)(\omega_- + \dot{\omega}_- t)e^{-i\omega_+(k,t)t - i\omega_-(k',t)t + i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{x}}$$
$$+[a_k^\dagger, a_{k'}](-i)(\omega_+ + \dot{\omega}_+ t)e^{-i\omega_-(k,t)t - i\omega_+(k',t)t - i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{x}}. \tag{6.36}$$

From (6.32) we know that the right hand side must depend on time like $a^{-(d-1)}$. If we plug in the asymptotic behavior of $\omega_\pm$ from (6.30), we will see that the only terms that reproduce that asymptotic behavior are coefficients of $[a_k, a_{k'}^\dagger]$ and $[a_k^\dagger, a_{k'}]$. Therefore, we find

$$[a_k, a_{k'}] = 0,$$
$$[a_k^\dagger, a_{k'}^\dagger] = 0. \tag{6.37}$$

We can rewrite the rest of the expression (6.36) follows.

$$[\varphi(x,t),\pi(y,t)] = \int d^{d-1}k\, d^{d-1}k' [a_k, a_{k'}^\dagger](-2i)(\omega_- + \dot{\omega}_- t)e^{-i\omega_-(k,t)t - i\omega_-(k',t) - i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{x}}. \tag{6.38}$$

Using (6.30), at $t \to \infty$, we find

$$[\varphi(x,t),\pi(y,t)] \simeq \int d^{d-1}k\, d^{d-1}k' (2im)(\frac{a_0}{a})^{d-1}[a_k, a_{k'}^\dagger]. \tag{6.39}$$

For (6.36) to be true, we need

$$[a_k, a_{k'}^\dagger] = \frac{\delta^{d-1}(k-k')}{2ma_0^{d-1}}. \tag{6.40}$$

Now let us define the vacuum $|\Omega\rangle$ to be the state that is annihilated by all the operators a_k. This is a natural choice to ensure that at small scales, the vacuum looks like Minkowski (i.e.

equivalence principle). Because, any comoving mode is trans-Planckian at some time and if it is not in the vacuum, we will have a state that has highly UV excitations.

Let us calculate the equal-time correlation function for this state. When we expand the fields in terms of the creation and annihilation operators, there is only one combination that does not vanish.

$$\langle\Omega|\,\varphi(x,t)\varphi(y,t)\,|\Omega\rangle = \int d^{d-1}k\, d^{d-1}k'\,\langle\Omega|\,a_k a^\dagger_{k'}\,|\Omega\rangle\, e^{-i\omega_+(k,t)t - i\omega_-(k',t)+i\vec{k}\cdot\vec{x}-i\vec{k}'\cdot\vec{y}}. \quad (6.41)$$

Using $\langle\Omega|\,a_k a^\dagger_{k'}\,|\Omega\rangle = \langle\Omega|\,[a_k, a^\dagger_{k'}]\,|\Omega\rangle$ and the commutation relation (6.40), we find

$$\varphi(x,t)\varphi(y,t) = \int d^{d-1}k\, \frac{1}{2ma_0^{d-1}} e^{-2i\mathrm{Im}(\omega(k,t)t)+i\vec{k}\cdot(\vec{x}-\vec{y})}. \quad (6.42)$$

We have dropped the state $|\Omega\rangle$ for simplicity. Since the leading k-dependence of ω in (6.30), the above expression gives a simple delta function at $t\to\infty$.

$$\varphi(x,t)\varphi(y,t) \simeq \frac{1}{2ma^{d-1}}\delta^{d-1}(x-y). \quad (6.43)$$

We can use factor out the asymptotic time-dependence of ω by defining boundary fields as $\varphi_b(x) \equiv \lim_{t\to\infty}\varphi(x,t)a(t)^{(d-1)/2}$. We find that the boundary correlators are given by

$$\varphi_b(x)\varphi_b(y) = \frac{1}{2m}\delta^{d-1}(x-y), \quad (6.44)$$

and so are trivial.

We can use adiabatic approximation to see what happens if m depends on the running scalar and therefore changes with time. Assuming m depends polynomially on time, as long as $1/ma^2$ decays faster than $1/t$ the k-dependent term in (6.30) is sub-leading and the correct asymptotic data is frozen correlators rather than scattering amplitudes. Therefore, as long as $ma^2H \gtrsim 1$, there are no non-trivial boundary data associated with the field.

Now let us study the massless fields. Since the asymptotic behavior of the solution is different for massless fields, we choose a different Ansatz.

$$\tilde{\varphi}_k = f_k(t)e^{i\vec{k}\cdot\vec{x}}. \tag{6.45}$$

Looking at equation (6.21) for $m = 0$, we find

$$\frac{k^2}{a^2} + (d-1)\frac{p}{t}\partial_t f_k + \partial_t^2 f_k = 0 \tag{6.46}$$

At $t \to \infty$, the first term vanishes, and without that term, $f =$ constant is a solution. This makes up expect that f converges at $t \to \infty$ in which case, we can view it as a function of $z = 1/t$ and study its behavior around $z = 0$. We take this as an assumption and confirm it a posteriori. The above differential equation takes the following form in z.

$$z^4\partial_z^2 f + z^3(2-(d-1)p)\partial_z f + \frac{k^2}{a_0^2}z^{2p}f = 0, \tag{6.47}$$

where $a_0 = a/t^p$. We can simplify the equation by writing it in terms of $g(z) = \ln(f(z))$ as follows.

$$z^4\partial_z^2 g + 2z^4(\partial_z g)^2 + z^3(2-(d-1)p)\partial_z g + \frac{k^2}{a_0^2}z^{2p}. \tag{6.48}$$

If we neglect the last term, the general solution takes the following form.

$$g_0(z) = \ln\left(c_0 + c_1 z^{p(d-1)-1}\right), \tag{6.49}$$

where c_0 and c_1 are constants. To take the effect of the last term in (6.47) into account we define $g = g_0 + \delta g$ such that $\delta g(0) = 0$. After pluging this term in the equation (6.47) and keeeping the leading terms for small z we find

$$z^4\partial_z^2\delta g + z^3(2-(d-1)p)\partial_z\delta g + \frac{k^2}{a_0^2}z^{2p} = 0. \tag{6.50}$$

The solution to the above equation with the initial condition $\delta g(0) = 0$ is

$$\delta g(z) = -Az^{2p-2}k^2, \tag{6.51}$$

where

$$A = [(2p-2)(1+(d-3)p)a_0^2]^{-1}. \tag{6.52}$$

Therefore, for the solution (6.45) at large t we find

$$f_k(t) \simeq (c_0 + \frac{c_1}{t^{p(d-1)-1}}) \exp\left(-A\frac{k^2}{t^{2p-2}}\right). \tag{6.53}$$

We can choose $c_0 = 1$ as a normalization convention to define the following two modes.

$$\tilde{\varphi}_{k\,\pm} = (c_0 \pm \frac{c_1}{t^{p(d-1)-1}}) \exp\left(-A\frac{k^2}{t^{2p-2}}\right). \tag{6.54}$$

Second quantization in terms of these modes is more complicated than the massive case, however, it is actually not necessary. There is an easy way of seeing why the boundary data would still be trivial. Note that the solution converges at $t \to \infty$ and completely freezes. Therefore, the propagator converges to a constant as we take one operator to the boundary. In other words, the boundary operators, are simply the bulk operators up to a constant.

$$\left\langle \varphi_b(X)\varphi_b(Y)\right\rangle \propto \lim_{t\to\infty} \langle \varphi(X,t)\varphi(Y,t)\rangle\,. \tag{6.55}$$

The proper distance between (X,t) and (Y,t) goes to infinity and we expect the bulk propagator on the right hand side to go to zero due to the cluster decomposition theorem. Therefore, in the absence of any scaling factor that could compensate this decay, the boundary correlators at non-coincident points vanish again.

$$X \neq Y: \;\; \varphi_b(X)\varphi_b(Y) = 0. \tag{6.56}$$

Let us summarize the results of this section.

Compactness of brane moduli space

Suppose we have an FRW background that expands as $a \sim t^p$ in the future infinity. If $p > 1$, no field (massless or massive) yields any non-trivial boundary observable. If $p < 1$, a weakly coupled massive particle can produce a non-trivial boundary observable if and only if $m \lesssim t^{1-2p}$ as $t \to \infty$.

6.3.4 Some important remarks

- **Higher spins:** The calculation of the previous subsection is qualitatively similar for particles of higher spins and the same results apply.
- **Weak coupling:** Our results only apply to particles hat become weakly coupled in the future infinity. There could be more massive states that cannot be understood as sharp resonances and therefore, do not lead to a boundary field. For example in a scattering theory, such states cannot be in/out going states because they are not stable. However, they leave an imprint on the scattering amplitude.
- **Meaning of trivial boundary correlators:** Let us point out that the boundary correlators could be different if they are evaluated for a non-vacuum state in the bulk. In dS/CFT this corresponds to having some operator insertions at the past boundary, which can be expressed as operator insertions at the antipodal point in the future boundary. On the other hand, in AdS/CFT, this corresponds to evaluating the boundary correlators at a state other than the AdS vacuum. In that case, we can create the other state by inserting some other operator on the boundary and the said correlator is equivalent to a more complicated vacuum correlation function. This is a consequence of crossing symmetry or CPT and the same thing can be done here. We can study the two-point function in a non-vacuum background by inserting other operators that change the state from vacuum. For example, $\int d^{d-1}z f(z)\varphi_b(z)$ corresponds to a one-particle state and sandwiching the two-point function between this operator corresponds to the two-point function in presence

of a particle.

$$\left\langle \varphi_b(X)\varphi_b(Y)\right\rangle_{\text{1-particle state}} = \left\langle \left(\int d^{d-1}z' f(z')^* \varphi_b(z')\right)\varphi_b(X)\varphi_b(Y)\left(\int d^{d-1}z f(z)\varphi_b(z)\right)\right\rangle_{|\Omega\rangle} . \tag{6.57}$$

As one can see from the above equation, such two-point functions can be reduced to higher-order correlation functions. However, when the two-point functions are delta functions, the correlation functions factorize and can be calculated using Wick's theorem. For example, for the above correlation function we have

$$\left\langle \varphi_b(X)\varphi_b(Y)\right\rangle_{\text{1-particle state}} \propto f(X)^* f(Y) + f(Y)^* f(X). \tag{6.58}$$

Therefore, there is no additional data to the boundary observables and the boundary data is trivial. In other words, the prefactors of the delta functions (e.g. the factor of $1/2m$ in (6.43)) are not sufficient to capture the interactions in the bulk and reconstruct the EFT.

Intrestingly, the factorization phenomenon seems to be a universal feature of field theories at infinite distance limits where the theory becomes weakly coupled[167]. Therefore, it should be expected that frozen correlation functions will factorize and not be able to capture the interactions away from the infinite distance limit of the field space.

- **Uniqueness of boundary observables:** One might ask why did we take the boundary limit as $t \to \infty$ while keeping the spatial co-moving coordinates fixed? For example, if we had taken the proper distance to be fixed rather than the comoving distance, then we would have gotten a different result. However, we argue that there is a unique good choice which is forced on us by the background. Suppose we had chosen to define the boundary propagators as $\varphi_b(X) = \lim_{t\to\infty} f(t)\varphi(Xt^\alpha)$ for some alpha, where $f(t)$ is chosen such that the limit of boundary correlators exist. If $\alpha < 0$, then we are measuring correlators at distances that are shrinking in comoving coordinates. Therefore, such a correlator is set by the modes with decreasing comoving wavelength. However, since these modes were

Trans-Planckian at some point, they must be set in the vacuum. In other words, even though the correlation functions are non-trivial, the information of an initial condition is completely lost. On the other hand if $\alpha > 0$, the boundary correlator will be even more trivial as it will be identically zero and we will lose the singularity. In general, we must choose the boundary coordinates such that we have a well-defined quasi-stationary mode expansion with constant wavenumbers.

- **Significance of polynomial expansion:** Note that the polynomial dependence on time played an important role in the analysis of this section. The future boundary of universes with exponential expansion or polynomial accelerated expansion are both spacelike. However, in de Sitter space where the expansion is exponential, one can define non-trivial boundary correlators. Since the points on the future boundary are not causually related, the boundary correlators correspond to non-vanishing superhorizon correlations. These correlators are the basis for dS/CFT[154]. Heuristically, the difference is that when the expansion is exponential, the correlators freeze out so fast, that their x-dependence does not fizzle out. However, in polynomial accelerated expansion, the expansion is fast enough to freeze out the correlators at infinite time, but not fast enough to preserve a non-trivial x-dependence. Now let us give a more precise explanation and point out how the calculations are different between the exponential and polynomial expansions. In exponential expansion, the friction term in the equation of motion (6.21) is of the same order as the mass term and does not die off with time. The friction term gives an imaginary component to $\omega(t \to \infty)$ which then leads to imaginary dependence on k in (6.42). Therefore, the boundary propagator has a non-vanishing k-dependence as $t \to \infty$.

In [168], some exotic string theory backgrounds using negative branes where studied. If such theories exist, the worldvolume theory of the negative branes would be a non-unitary supergroup gauge theory and their dual near horizon geometry are dS and AdS spaces of exotic signatures. There are still some open questions about these theories, in particular, an independent path integral definition of the worldvolume theory of the branes is missing. However, there is some evidence from dualities and string worldsheet for their existence[168]. What is interesting is that all of the exotic string theory backgrounds that where find in[168] have an exponential expansion of the induced volume in the stationary coordinate, and therefore, cannot be ruled out by our argument.

6.4 Swampland conjectures from holography

In section 6.2 we showed that in quantum gravity, a global positive potential requires a boundary observables to be well-defined which is the most conservative form of holography. In the previous section, we showed that depending on the masses of the particles and the rate of expansion, such boundary condition might not exist. In this section we show that these conditions match with some of the Swampland constraints and can be viewed as a holographic derivation for them.

6.4.1 Trans-Planckian Censorship Conjecture

Trans-Planckian Censorship Conjecture (TCC) states that in a theory of quantum gravity, we cannot have an expansion that is fast and long enough to stretch Planckian modes beyond the Hubble scale[19]. In other words, the scale factor and Hubble parameter must satisfy the following condition for any given times t_i and t_f.

$$\frac{a(t_f)}{a(t_i)} < \frac{1}{H(t_f)}. \tag{6.59}$$

The initial motivation of TCC was that such expansions erase the quantum information as they classicalize the initial quantum fluctuations. The motivations for this conjecture were expanded in a subsequent work[22]. For backgrounds with polynomial expansion $a \sim t^p$, TCC is equivalent to $p \leq 1$. Therefore, the bound we found for the exponent of the expansion, is equivalent to the statement of TCC in the asymptotic regions of the field space. This is very much in the spirit of the initial motivation of TCC that information somehow gets lost if TCC is violated. As we can see now that the information (whether quantum or classical) completely gets lost at future infinity if TCC is violated.

6.4.2 de Sitter conjecture

We can express the statement of TCC in the asymptotic region of field space in terms of the scalar potential. Using the results of Appendix K, we can see that $p \leq 1$ is equivalent to $\lambda \geq 2/\sqrt{d-2}$ where $V \sim \exp(-\lambda\varphi)$. However, as was initially pointed out in [19], this is only true if we assume that the scalar fields are the main driver of the expansion. One could imagine a scenario where the tower of light states that emerge in the infinite distance limit are excited and contribute to the expansion. In that case the constraint on the potential would be milder and it would be $\lambda \geq 2/\sqrt{(d-1)(d-2)}$ [19].

In [33], using the emergent string conjecture, it was argued that the tower of light states are always heavier than the Hubble scale in which case they would not significantly contribute to the expansion. In that case, TCC gives $\lambda \geq 2/\sqrt{d-2}$.

6.4.3 Distance conjecture

In the previous section, we saw that if the mass of the states in the bulk do not go to zero fast enough, they will not lead to any boundary observables. Assuming that the expansion is driven by a potential $V \sim \exp(-\lambda\varphi)$, we can express the conditions imposed on their masses as $m \lesssim \exp(-c(\lambda)\varphi)$ where c would depend on λ. For $\lambda < \frac{2}{\sqrt{d-2}}$ no state yields boundary observables and for $\lambda > \frac{8}{\sqrt{(d-2)}}$, any mass yields scattering amplitude. However, for a λ in the above range, using the results of Appendix K, our conditions lead to the following non-trivial bound.

$$m \lesssim \exp(-c(\lambda)\varphi), \quad c(\lambda) = \frac{4}{(d-2)\lambda} - \frac{\lambda}{2}. \tag{6.60}$$

Note that $c(\lambda)$ is greatest at $\lambda = 2/\sqrt{d-2}$ for which $c = 1/\sqrt{d-2}$. This is a derivation of distance conjecture in rolling backgrounds. What is even more interesting, is that the coefficient

$c = 1/\sqrt{d-2}$ matched with the proposal in [153] to sharpen the distance conjecture. However, we arrive at these decay rates from a completely different perspective based on the most basic form of holography.

6.4.4 Generalized distance conjecture

The bound $m \lesssim t^{1-2p}$ can be expressed in terms of the Hubble parameter as $m \lesssim H^{2p-1}$. We can understand this bound in the following way: as we change the geometry of the spacetime in the future infinity, there must be a tower of light states whose mass depends polynomially on the energy density. This statement is similar to the distance conjecture when we view the background metric as a modulus and we think about moving in the space of geometries as going to the asymptotic direction in the moduli space[3]. This formulation was made more precise for AdS spaces as generalized distance conjecture which states that as Λ goes to zero, there must be a tower of light states whose masses go to zero polynomialy fast in Λ [169]. If we replace Λ with $\sim H^2$ and extend it to time-dependent backgrounds, we recover $m \lesssim H^c$ for some positive constant c.

Let us make the connection between our statement and the distance conjecture more precise. Consider a homogeneous background of an auxiliary canonically normalized massless field Φ that would reproduce the same action as the background we consider.

$$S = \int d^d x \sqrt{g} \frac{1}{2} (\partial_t \Phi)^2. \tag{6.61}$$

By matching the above action with the actual action $S = \int d^d s \sqrt{g} \mathcal{L}$ where $\mathcal{L} = \frac{1}{2\kappa^2} \mathcal{R} - V(\varphi) - \frac{1}{2}(\partial_\mu \varphi)^2$, we can define a formal notion of distance $\Delta\Phi$ for our rolling geometry that is given by $\int dt \sqrt{2\mathcal{L}}$. On the other hand, we have $\sqrt{\mathcal{L}} \propto H \propto t^{-1}$ as $t \to \infty$. Therefore, we find that this formal notion of distance between times t_i and t_f goes like $\propto \ln\left(t_f/t_i\right)$. If we apply the distance conjecture to this notion of distance, it would imply that there must be a tower of light states whose masses are exponential in $\ln(t/t_0)$, which would be polynomial in t. This

[3] We are thankful to Cumrun Vafa for pointing this connection out to us.

is exactly what we find! Therefore, we find a derivation for generalized distance conjecture in rolling backgrounds.

6.5 Conclusions

We showed that the most basic form of holography, that observables of quantum gravity live on the boundary of spacetime, has non-trivial implications for scalar field cosmologies. In particular, we showed that the expansion of the universe, if polynomial, must be decelerated. Moreover, if the expansion goes like t^p, the masses of the weakly coupled particles must satisfy $m \lesssim t^{1-2p}$. We also showed that these conditions are deeply connected to the Swampland conjectures. In particular, the condition $p \leq 1$ is equivalent to Trans-Planckian Censorship Conjecture in the future infinity, and the condition $m \lesssim t^{1-2p}$ matches with the generalized distance conjecture. Moreover, it provides an explicit bound for the coefficient of the distance conjecture that depends on the potential. However, the strongest constraint that we find is $m \lesssim \exp\left(-\varphi/\sqrt{d-2}\right)$ which is identical to the proposal of[153].

Our work presents a more fundamental explanation for various Swampland conjectures by connecting them to a more fundamental principle of quantum gravity, which is the holographic principle.

Part II

The Swampland: micro

7

Small instantons

A consistent quantum theory of gravity often includes long-range gauge forces as part of its low-energy description. This is particularly true of supersymmetric theories, where these are often required by supersymmetry. For instance, in theories with sixteen supercharges, the gravity multiplet includes a massless 2-form field. Completeness of the spectrum [170] then requires that all possible values of the charges are populated by physical objects in the theory. Studying the worldvolume theory of these branes, which map in the context of the String Landscape to "probe branes" has been the source of much recent progress [171,172,18,173,174,175] and can be used to banish to the Swampland some naively consistent theories of quantum gravity that do not appear in the String Landscape.

When the branes are supersymmetric, they often have exact moduli spaces – exactly massless directions of their worldvolume field theory. Some of these moduli have a direct spacetime interpretation, like the scalars parametrizing the "center of mass" degrees of freedom of the object in spacetime. But others characterize purely internal degrees of freedom. For instance, in theories of quantum gravity arising via compactification from a higher-dimensional theory, we will often have additional scalars parametrizing the position of the branes in the compact space. Of course, scalars can also have other interpretations, such as Wilson lines of higher-dimensional gauge fields or more exotic origins. In any case, the low-energy effective field theory controlling the dynamics of the moduli is simply a sigma model from the brane worldvolume to the moduli space $\mathcal{M}$:

$$\mathcal{L} \supset \int_{\text{brane}} \sqrt{-g}\frac{G_{IJ}}{2}\partial_\mu\varphi^I\partial_\nu\varphi^J, \tag{7.1}$$

where greek indices run over brane's worldvolume directions, φ are the moduli, and Latin uppercase indices live in the tangent bundle of $\mathcal{M}$. To understand the dynamics of (7.1), it is often useful to compactify all spatial worldvolume directions of the p-brane on the torus T^p to obtain an effective quantum mechanics with target space $\mathcal{M}$ (times any additional degrees of freedom that may arise due to compactification, such as Wilson lines, etc.). Here we assume $p < d - 2$, so the resulting 0-brane has codimension more than 2 in the uncompactified spacetime. Canonical quantization then produces a spectrum whose energies are equal to the eigenvalues of the Laplacian on $\mathcal{M}$. We reach the conclusion that the sigma model (7.1) has a spectrum dictated by the Laplacian on $\mathcal{M}$ (times any additional space). In particular, if $\mathcal{M}$ is non-compact and the Laplacian has a continuous spectrum, the set of asymptotic states of the theory described in (7.1) has infinitely many modes of any given finite energy range. In particular, this means that the entropy density is infinite. Such behavior is fine in quantum field theory, but it is unacceptable in a quantum theory of gravity, where the entropy is upper bounded by that of the corresponding Schwarzschild's black hole or black brane, as specified by Bekenstein's bound. Thus, the consistency of quantum gravity leads us to the basic claim [174] which is the main tool we use in this chapter:

Compactness of brane moduli space

The moduli space of any p-brane with $p < d-2$ is compact (or more precisely has a discrete spectrum of the Laplacian) in a consistent quantum theory of gravity.

Throughout this chapter, we will apply this principle to the magnetic $(d-5)$-brane associated with the magnetic dual of the B field in the gravity multiplet of d-dimensional theory with 16 supercharges, extending the results in [174]. These branes preserve half the supercharges in their worldvolume, and when the d-dimensional moduli are tuned such that the d-dimensional theory has non-Abelian gauge fields, the $(d-5)$-branes correspond to the zero-size limit of gauge theory instantons. An example is the heterotic NS5 brane in 10 dimensions [176,177,178].

When the instanton is of finite size, larger than the cutoff of the low-energy supergravity description, the low-energy dynamics can be read off from the set of zero modes of supergravity fields in the instanton background. This can be efficiently analyzed using supersymmetry and the index theorem to obtain the number of fermion zero modes. The corresponding theory, including the modulus ρ that parametrizes the instanton size, is known as the "Higgs branch" of the brane worldvolume theory. It connects to another branch of the moduli space, the "Coulomb branch", at $\rho = 0$ (Figure 7.1). The Higgs branch receives its name because the low-energy effective field theory description there does not contain worldvolume gauge fields, while at low energies on the Coulomb branch, the low-energy effective description is a supersymmetric gauge theory. The phase transition between the two at $\rho = 0$ is described by a (possibly free) SCFT. While the Higgs branch can be described via supergravity, the small instanton SCFT and the Coulomb branch cannot.

The basic question about the Coulomb branch of the theory is its dimension, known as the rank of the theory (see [179] for a recent review). Even though this cannot be addressed from bulk supergravity, the dimension of the Coulomb branch for a small instanton can be accessed by supersymmetry due to constructions like ADHM for classical groups or Minahan-Nemeschansky theories [180,181] for exceptional groups. Specifically, the ADHM construction for classical groups allows one to parametrize the low-energy dynamics in terms of linear degrees of

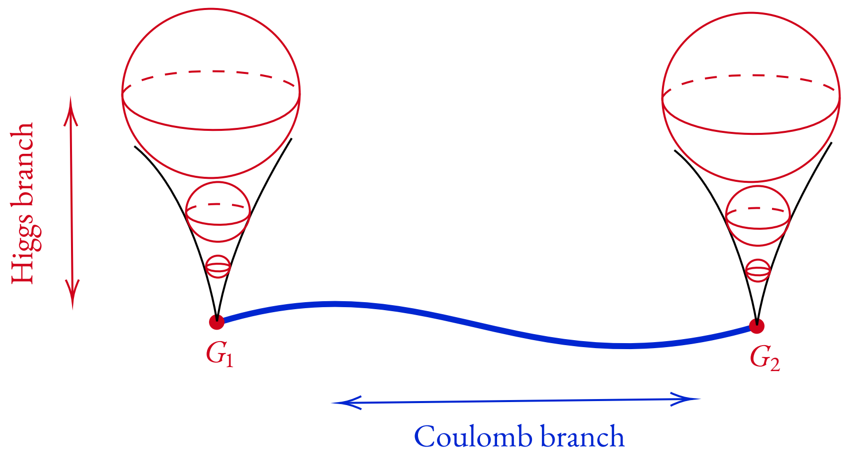

Figure 7.1: The above figure shows a path between two instantons in the instanton moduli space. The path connects a G_1 instanton to a G_2 instanton where G_1 and G_2 are independent non-Abelian components of the spacetime gauge group G. We first shrink the G_1 instanton down to zero-size, which corresponds to moving along its Higgs branch to the Coulomb branch. Then we move in the Coulomb branch to deform a G_1 zero-size instanton to a G_2 zero-size instanton. Finally, we can move in the G_2 instanton Higgs branch by increasing the size of the instanton.

freedom (which gives a natural description of the low-energy dynamics of zero-size instantons, see [182]) and includes one scalar parametrizing the Coulomb branch. As a result, for branes that can arise as a small instanton limit, the Coulomb branch will be rank one[1].

Although it never happens in known string constructions, a priori, we may also consider the case where a given brane never arises as a small instanton of a non-Abelian group. In this case, the non-commutative geometry version of the ADHM construction works for $U(1)$ instantons in non-commutative space [183], and again yields a one-dimensional Coulomb branch.

We emphasize that the argument outlined there essentially is that there is a unique field theory description of the moduli space of instantons, including zero-size configurations, that predicts a one-dimensional Coulomb branch. This and its exceptional versions are *pure field theory phenomena*, even though some were first discovered in the context of string backgrounds, that we use as building blocks in the construction of quantum theories of gravity below.

The moduli space theory is not only rank one: it is also connected. In [174], this was argued via a strengthened version of the Cobordism Conjecture [104] which was argued there to hold for

[1] A priori one may think that this does not exclude the possibility that, e.g., an exotic small E_8 instanton can be described by a yet to be discovered SCFT which has a rank higher than 1. However, since there is a unique moduli space of E_8 instantons, studying the small E_8 moduli space, using string theory is perfectly allowed to deduce that its Coulomb branch is one dimensional. This is a local statement that does not rely on the existence of quantum gravity in particular. We will provide another supporting argument in Section 8.2.

theories with 8 supercharges because no superpotential is allowed for scalar fields[2].

Connectedness of the moduli space refers to instantons of a fixed charge – components of different charge are obviously disconnected –. In some string compactifications, like the rank 10 theory in eight dimensions, where one can have symplectic groups where the small instanton has a zero-dimensional Coulomb branch. This corresponds to fractional $D3$ branes stuck at $O7^{+}$ planes. However, crucially, there is never more than one non-Abelian factor with a zero-dimensional Coulomb branch[3]. In the stringy description, this is due to the fact that we have a single $O7^{+}$ plane. The corresponding moduli space is therefore a single point, which is connected. For these theories, we consider the more interesting instanton moduli space with an instanton number of two, where the Coulomb branch is again one-dimensional.

Finally, we also note that the connectedness of the moduli space for the small instanton limit of every possible non-Abelian gauge group with eight supercharges is deeply connected with the fact that they all carry the *same* physical brane charge. As discussed in [185], this is related to demanding absence of Chern-Weil global symmetries.

To sum up, we end up with the conclusion that the space of the $(d-5)$-branes in theories with 16 supercharges is a connected moduli space, corresponding to a rank one Coulomb branch. The basic consistency principle that will allow us to fully classify these Coulomb branches is just the simple fact that brane worldvolume couplings should be well defined on the moduli space, up to duality transformations.

This seemingly mild principle will turn out to have far-reaching consequences, to the extent that we can determine the full moduli space of theories with sixteen supercharges in seven and higher dimensions!

[2] The only way scalar fields pick up mass in theories with eight supercharge is via coupling to vector multiplets, as in going to their Coulomb branch, and not through self-interaction of the scalar multiplets. See also the discussion in [184].

[3] A redundant Swampland prediction coming out of this picture is that there can never be a point in moduli space with more than one $Sp(n)$ factor in the rank 10 8d theory. As was shown in [174], this prediction is correct.

8

Swampland constraints in various dimensions

In this Section, we consider rank one worldvolume theories with 8 supercharges that describe codimension-4 small instantons in various spacetime dimensions. By imposing consistency conditions on worldvolume theories, we derive new swampland constraints and are even able to reconstruct, purely from the brane perspective, the internal geometries that are familiar from string theory.

8.1 9D

In this Section, we show that the SLP holds for 9-dimensional supergravity theories.

The gauge instantons are 4-branes, which are described by 5d $\mathcal{N} = 1$ theory. The consistency conditions on the brane theory impose strong restrictions on the gauge algebras. In particular, we argue that theories with $\mathfrak{sp}(n)$ gauge symmetry are in the swampland for $n > 1$ with dynamical quantum gravity, but fine without gravity. We also reconstruct the internal space $S^1/\mathbb{Z}_2$ of type I' string theory [186] from the viewpoint of the 4-brane.

Consistency condition from 4-brane

We consider a 5d $\mathcal{N} = 1$ rank one theory as a worldvolume theory of 4-brane. This theory has a Coulomb branch of real dimension one. The Coulomb branch is parametrized by the expectation value of the real scalar field φ belonging to the vector multiplet $\mathcal{A}$. The gauge symmetry is $U(1)$ on a generic point of the Coulomb branch.

The 9d theory with sixteen supercharges has gauge symmetry, with propagating gauge bosons. This spacetime gauge symmetry is seen as a global symmetry of the worldvolume theory of the brane. If there are non-Abelian factors in the spacetime gauge group, there is a point of symmetry enhancement at the Coulomb branch in the worldvolume theory. At this special point, the Coulomb branch connects to a Higgs branch, in which the brane "fattens up" as an instanton of the spacetime gauge group.

As discussed in the previous chapter, the Coulomb branch moduli space must be compact and connected. The compactness of the moduli space is required by the finiteness of the black hole entropy, and the connectedness is required by the stronger cobordism conjecture. Therefore, the Coulomb branch moduli space is connected, one dimensional, and compact, and so it is either S^1 or $S^1/\mathbb{Z}_2$.

We will now argue that the case of 16 supercharges we are interested in corresponds to $S^1/\mathbb{Z}_2$, while the S^1 moduli space corresponds to nine-dimensional theories with 32 supercharges. Take the 9d theory and compactify on S^1, to obtain an $\mathcal{N} = 1$ eight-dimensional theory. The instanton moduli space is now complex one-dimensional, with the additional scalar coming from the Wilson line of the photon. In compactifications coming from theories where the instanton Coulomb branch in 9d is $S^1/\mathbb{Z}_2$, the 8d geometry is an elliptic K3, and in particular, it has curvature. By contrast, if the 9d instanton Coulomb branch is S^1, the 8d instanton Coulomb branch has geometry T^2, which is flat.

String theory makes a prediction for the possible Coulomb branches of the probe brane worldvolume theory, and these can be studied by analyzing noncompact configurations of branes in string theory. The stringy prediction for the moduli space geometry from this noncompact analysis is always that the moduli space geometry is not flat and has singularities at a finite distance (consider, for instance, a $D3$ probing a $D7$). We will take this prediction that string theory makes for field theory as an assumption; from this, it follows that the moduli space geometry can only be K3, which uplifts to $S^1/\mathbb{Z}_2$ in nine dimensions.

We can provide another heuristic argument for the same conclusion, which may have wider applicability than the current context. From the brane point of view, the basic difference between S^1 and an interval is that the former has an isometry[1]. The Coulomb branch parameter φ is then actually an axion with a continuous shift symmetry. This means that the currents

$$J_1 = d\varphi, \quad J_4 = *d\varphi \tag{8.1}$$

are exactly conserved, $d * J_1 = d * J_4 = 0$, and generate a 0-form and a 4-form global symmetry on the brane worldvolume. The object charged under the 4-form current is simply solitonic membranes of φ, i.e. field configurations $\varphi(x_5)$ that depend nontrivially on one spatial coordinate transverse to the membrane and that wind around the target space circle once.

Because we have these symmetries, we can include topological couplings in the worldvolume

[1] As one can see by the arguments later in the Section, there cannot be any points on the moduli space where the 4-brane theory corresponds to a gauge instanton when the moduli space is S^1. So indeed, the Coulomb branch has no special points, and it will automatically have continuous shift symmetry, at least in the IR.

brane action,

$$\int *J_1 \wedge A_1, \quad \int *J_4 \wedge A_4, \tag{8.2}$$

which introduce a coupling to the corresponding background connections. This is a standard procedure in field theory. We will now give an argument that, whenever this happens in the worldvolume theory of a brane coupled to quantum gravity, the background connections *must* correspond to dynamical fields of the bulk theory. Otherwise, the worldvolume symmetry becomes an exact global symmetry, which is forbidden in quantum gravity. We can argue for this in a manner similar to Sen's construction of branes within branes via tachyon condensation [187]. Consider a brane- antibrane pair, with a worldvolume charged state on the brane. The two branes condense, but the condensation cannot be complete since otherwise, the global charge on the worldvolume theory would be violated. More concretely, the winding of the worldvolume scalar forces the tachyon condensation to remain incomplete in an appropriate locus. As a result, one is left with a remnant solitonic object in the bulk spacetime. Whichever process can make this object break or decay would uplift to the original field theory, contradicting the assumption that the theory had a global symmetry.

In the case under consideration, we would therefore conclude that the bulk theory has a 4-form field. Such a field (or rather, its dual 3-form) is part of the 9d $\mathcal{N} = 2$ supergravity multiplet, but not part of the 9d $\mathcal{N} = 1$ multiplet. As a consequence, S^1 is only compatible with 32 supercharges, as advertised.

An important caveat is that this argument only applies to *exact* symmetries of the worldvolume brane theory. One could have worldvolume accidental IR symmetries, which will not be coupled to a dynamical bulk field. An example is the BPS string in the rank 1 component of the 9d moduli space obtained as M theory on the Möbius strip [188], where there is accidental supersymmetry enhancement from $(8, 0)$ to $(8, 8)$ at low energies. While we believe the above is morally correct, we cannot argue that these symmetries must be exact in the worldvolume theory; this is why the previous argument using the geometry of the $U(1)$ instanton moduli space is required. Furthermore, only in the $S^1/\mathbb{Z}_2$ case non-Abelian symmetries arise [188]. In the following, we will consider only this case.

The low energy $U(1)$ theory at the general point of the Coulomb branch is specified by the prepotential $\mathcal{F}(\varphi)$, which is at most cubic. The prepotential of 5d $\mathcal{N}=1$ rank 1 theory is

$$\mathcal{F}=\frac{1}{2g^2}\varphi^2+\sum_i\frac{c_i}{6}\left(|\varphi-\varphi_i|^3+|\varphi+\varphi_i|^3\right), \tag{8.3}$$

where we take $\varphi=0$ and φ_e are the endpoints of the interval $S^1/\mathbb{Z}_2$, and g, c_i are parameters. If the endpoint theory is a SCFT, then the gauge coupling is infinite. The cubic term c_i is only generated by a one-loop computation[189], where the field which becomes light at $\varphi=\pm\varphi_i$ contributes. In principle, there could be a tree-level contribution to the cubic term, but it is absent due to the $\mathbb{Z}_2$ quotient. Note that the effective prepotential on the Coulomb branch is valid even if SCFTs do not admit a gauge theory description. The effective gauge coupling is given by the second-order derivative of prepotential:

$$\frac{1}{g^2(\varphi)}=\frac{\partial^2\mathcal{F}}{\partial\varphi^2}=\frac{1}{g^2}+\sum_i c_i\left(|\varphi-\varphi_i|+|\varphi+\varphi_i|\right). \tag{8.4}$$

The Coulomb branch moduli space metric is

$$ds^2=\frac{1}{g^2(\varphi)}d\varphi^2. \tag{8.5}$$

The Chern-Simons term is given by the third-order derivative of a prepotential:

$$\frac{\partial^3\mathcal{F}}{\partial\varphi^3}\frac{1}{24\pi^2}A\wedge F\wedge F=\sum_i c_i\left(\mathrm{sign}\left(\varphi-\varphi_i\right)+\mathrm{sign}\left(\varphi+\varphi_i\right)\right)\frac{1}{24\pi^2}A\wedge F\wedge F. \tag{8.6}$$

Thus, the coefficient c_i represents the "jump" Δk_i in the Chern-Simons term.

The consistency condition can be obtained as follows: consider a double cover S^1 with the interval $S^1/\mathbb{Z}_2$. Let us move the scalar field φ around S^1 once. At this time, the level of the Chern-Simons term (or the gauge coupling) must come back to its original value. Since the coefficient c of the cubic term corresponds to the jump in level, the well-definedness of the level, when we come back to the same point, requires that

$$\sum_i c_i = \sum_i \Delta k_i = 0. \tag{8.7}$$

This equation, which comes from the compactness of the brane moduli, which in turn comes from the finiteness of black hole entropy, can be used to constrain the bulk gauge symmetry[2]. At any point in the Coulomb branch, the worldvolume theory flows to SCFT or IR free theory at low energy. Suppose we have a complete classification of 5d rank-1 theories and know the coefficients c_i and the extended global symmetry. Then from (8.7), we can restrict the possible global symmetries. This translates into a restriction to the bulk gauge symmetry. Interestingly, as we will soon review, the classification of 5d theory has developed significantly in recent years. This helps us to obtain new swampland constraints.

In the following, we first review the classification of 5d SCFTs. Then, we list the IR-free theories whose Higgs branch is isomorphic to the instanton moduli space.

Classification of 5d SCFTs

A 5d gauge theory is not renormalizable, because the gauge coupling has a negative mass dimension. However, if the theory has nontrivial UV fixed points, it can become UV complete as a field theory. Originally, 5d SCFTs were discovered as theories on $D4$-branes[190]. A large class of 5d SCFTs is obtained by M-theory on local CY threefold with shrinking 4-cycle[191], and (p, q) 5-brane webs in type IIB string theory[192].

In recent years, there has been significant progress in the classification of 5d SCFTs[193,194,195,196,197,198,199,200,201,202,203,204,205,206,207]. There are several classification methods. For example, there is a classification based on geometry[195,201], a classification based on gauge theory description[194,206], and a classification based on the S^1 compactification of the 6d SCFT (which may involve a twist)[195,196,197,204,207]. In particular, all 5d SCFTs are conjectured to be

[2]This condition can also be used to exclude the existence of non-Abelian symmetries when the Coulomb branch geometry is S^1.

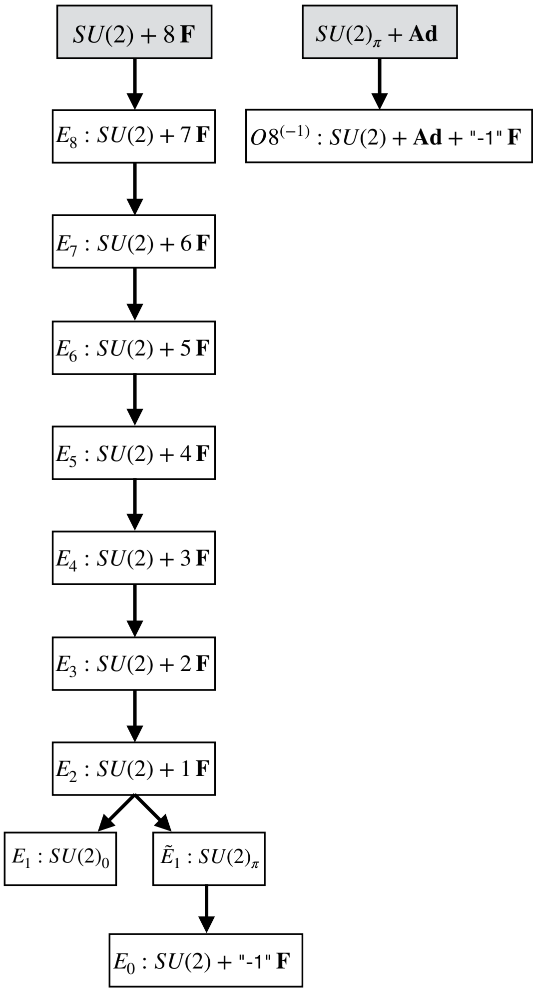

Figure 8.1: The RG flow among 5d rank 1 SCFTs obtained by mass deformations. The shaded boxes correspond to 5d KK theories which are UV completed by 6d SCFTs. Each box represents a gauge theory description. Since it is rank 1, the gauge group is $SU(2)$ in all cases, where $\mathbf{F}$ represents matter in the fundamental representation and $\mathbf{Ad}$ represents matter in the adjoint representation. In the case of pure gauge theory and adjoint matter only, discrete theta angles are possible, which are denoted by subscripts. There are no gauge theory descriptions for E_0 and $O8^{(-1)}$ theories, but we write "-1"$\mathbf{F}$ because RG flow corresponds to formally removing the fundamental representation matter.

obtained as RG flows of the 5d KK theory (which is a compactification of 6d SCFT)[195].

Here we sketch the classification based on 5d gauge theories obtained from supersymmetry preserving relevant deformations. The necessary conditions for obtaining nontrivial SCFTs are proposed in[194], which is an improved version of[208]. The conditions are that there exists a physical Coulomb branch where the monopole string has positive tension and the instanton particle has positive mass squared and that the gauge coupling there is positive. The possible gauge groups of rank 1 are $U(1)$, $O(2)$, and $SU(2)$, of which only $SU(2)$ satisfies the necessary condition. The $SU(2)$ gauge theories with $N_f = 0, 1, \cdots, 7$ fundamental matters correspond to the gauge coupling deformation of E_{N_f+1} theory, as listed in Fig. 8.1. For $N_f = 0$, there exists a freedom to add discrete theta angle [191]. The SCFT that corresponds to the theory with discrete theta angle π is $\tilde{E}_1$ theory. The $\tilde{E}_1$ theory is amenable to further relevant deformation, which yields the E_0 theory. Note that the $SU(2)$ gauge theory with $N_f = 8$ fundamental matters is UV completed by a 6d SCFT. All E_n and $\tilde{E}_1$ theories are obtained from the RG flow of the 6d SCFT, and the Coulomb branch geometries are locally $\mathbb{R}/\mathbb{Z}_2$. The SCFTs are realized at the fixed point.

Similarly, the $SU(2)$ gauge theories with an adjoint matter and discrete theta angle π is UV completed by 6d SCFT. The relevant deformation of this theory flows to another 5d SCFT in the IR (right side of Fig. 8.1). This theory was found in [201]. We call this $O8^{(-1)}$ theory because, as we will see below, it is natural to regard this as the worldvolume theory of the $D4$-brane probing the $O8^{(-1)}$-plane. Here the $O8^{(-1)}$-plane is the orientifold plane whose $D8$-brane charge is -1 [188]. The Coulomb branch geometry of the $O8^{(-1)}$ theory is $\mathbb{R}/\mathbb{Z}_2$.

If we include the matter in the $SU(2)$ gauge theory with representations that do not appear in Fig. 8.1, the value of c_i becomes negative. This makes the gauge coupling negative and does not satisfy the necessary condition in [194].

In this chapter, we derive a consistency condition of the worldvolume theory of probe branes, assuming that the classification above is complete. In principle, it is possible that there exist unknown SCFTs which do not admit a gauge theory description and are associated with unknown geometry, although we believe that this is unlikely.

The values of c_i in the SCFTs described above are as follows: [190,209,201]

$$\frac{c}{c_{A_0}} = \begin{cases} 9 - n \quad \text{for } E_n \text{ theory } (n = 0, 1, \cdots, 8) \\ 8 \quad \text{for } \tilde{E}_1 \text{ theory} \\ 1 \quad \text{for } O8^{(-1)} \text{ theory} \end{cases}, \tag{8.8}$$

where c_{A_0} is the value of c for A_0 theory ($U(1)$ gauge theory with one electron).

IR free theories

The theory of the symmetry enhanced point on the 4-brane can be an IR-free theory as well as a SCFT. Here we list the rank-1 free theories in which the Higgs branch is isomorphic to an instanton moduli space.

There are two theories in which the Higgs branch is a one-instanton moduli space: A_n theory and D_n theory. The A_n theory is a $U(1)$ gauge theory containing $(n+1)$ "electrons" as matter. This theory has an $\mathfrak{su}(n+1)$ global symmetry. A D_n theory is a $SU(2)$ gauge theory containing n "quarks" as matter. This theory has an $\mathfrak{spin}(2n)$ global symmetry. The Coulomb branch geometry of A_n theory is $\mathbb{R}$ and that of D_n theory is $\mathbb{R}/\mathbb{Z}_2$ considering the Weyl group. Compactifying the A_n theory to S^1 results in a 4d $\mathcal{N}=2$ theory corresponding to the $\mathfrak{su}(n+1)$ small instanton in 8d spacetime. This corresponds to the I_n singularity of the 4d Coulomb branch. Similarly, the S^1 compactification of the D_n theory is the I_n^* singularity of the 4d Coulomb branch.

For the C_n case, the one-instanton Higgs branch is simply given by a free half-hypermultiplet, and has a zero-dimensional Coulomb branch. This means that the corresponding probe brane is "stuck", and indeed this theory is realized by the worldvolume of a stuck $D4$ branes. By contrast, the theory with instanton number of two has a one-dimensional Coulomb branch, and as argued in [174] it is the one describing mobile probe branes that connect to small instantons of other non-Abelian factors. Therefore, we will focus on this theory, which is given by an $O(2)=U(1)\rtimes\mathbb{Z}_2$ gauge theory containing two hypermultiplets of charge 2 and $\mathfrak{sp}(n)$ hypermultiplets of fundamental representation as matter [210]. The global symmetry of this theory is $\mathfrak{sp}(n)$. The Coulomb branch geometry of C_n theory is $\mathbb{R}/\mathbb{Z}_2$ because of a discrete gauging. The S^1 compactification of this theory is the frozen I_{n+8}^* singularity in the 4d Coulomb branch. The lack of mass deformation due to $\mathbb{Z}_2$ gauging in the $O(2)$ gauge symmetry corresponds to the singularity being frozen [210].

The jump in the level of the Chern-Simons term (which is the same as the change in the slope of the gauge coupling) can be obtained exactly by a 1-loop calculation[189]. The result is

$$\frac{c}{c_{A_0}} = \begin{cases} -(n+1) & \text{for } A_n \text{ theory} \\ 8-n & \text{for } D_n \text{ theory} \\ -(8+n) & \text{for } C_n \text{ theory} \end{cases} . \tag{8.9}$$

One could also ask why these are all the possibilities for the global symmetry of IR free theories. Naively, these are easy to construct: For any G, just consider $U(1)$ gauge theory with matter in a representation of G. At low energies, the scalars in the matter sector are described just by a kinetic term

$$\mathcal{L} \supset \int \sqrt{-g} \left[\frac{\kappa_{ab}}{2} \partial_\mu \varphi^a \partial^\mu \varphi^b \right], \tag{8.10}$$

where the indices a, b take values in some representation of G, and κ_{ab} is the corresponding quadratic form. At low energies, this theory has a G global symmetry. In particular, there seems to be no obstacle to things like $G = G_2$ in eight dimensions, which we know does not arise in the landscape of known 8d $\mathcal{N} = 1$ theories[174].

However, for any theory with lagrangian (8.10), the global symmetry in the IR enhances to $Spin(n)$, $Sp(n)$, or $U(n)$, according to whether the representation under consideration is real, pseudoreal, or complex, respectively. The point is that any finite-dimensional representation of any group comes with the quadratic form κ_{ab} that one uses to construct the non-degenerate kinetic term, and that the symmetry of the lagrangian (8.10) is that of the quadratic form. When the representation is real, κ_{ab} is a real symmetric matrix, and the corresponding symmetry group is the orthogonal group. When the representation is pseudoreal, it preserves a symplectic form; the symmetry is the symplectic group. And when the representation is complex, κ_{ab} preserves a Hermitian form, whose symmetry group is unitary.

So, assuming there is no accidental symmetry enhancement in the deep IR, the only possibilities are the ones we have listed. We will now use these to classify the possible consistent

Name	free or CFT	Symmetry	Geometry	Brane	c/c_{A_0}
$A_n(n=0,\cdots)$	free	$\mathfrak{su}(n+1)$	$\mathbb{R}$	$(n+1)D8$	$-(n+1)$
$C_n(n=0,\cdots)$	free	$\mathfrak{sp}(n)$	$\mathbb{R}/\mathbb{Z}_2$	$O8^+ + nD8$	$-(8+n)$
$D_n(n=0,\cdots)$	free	$\mathfrak{spin}(2n)$	$\mathbb{R}/\mathbb{Z}_2$	$O8^- + nD8$	$8-n$
$E_n(n=1,\cdots,8)$	CFT	caption	$\mathbb{R}/\mathbb{Z}_2$	$O8^- + (n-1)D8$	$9-n$
$\tilde{E}_1$	CFT	$\mathfrak{u}(1)$	$\mathbb{R}/\mathbb{Z}_2$	$O8^-$	8
E_0	CFT	$\varnothing$	$\mathbb{R}/\mathbb{Z}_2$	$O8^{(-9)}$	9
$O8^{(-1)}$	CFT	$\varnothing$	$\mathbb{R}/\mathbb{Z}_2$	$O8^{(-1)}$	1

Table 8.1: List of 4-brane worldvolume theories with one-dimensional Coulomb branch. Global symmetries of E_n and $\tilde{E}_1$ theories are $E_8 = \mathfrak{e}_8, E_7 = \mathfrak{e}_7, E_6 = \mathfrak{e}_6, E_5 = \mathfrak{spin}(10), E_4 = \mathfrak{su}(5), E_3 = \mathfrak{su}(3) + \mathfrak{su}(2), E_2 = \mathfrak{su}(2) + \mathfrak{u}(1), E_1 = \mathfrak{su}(2), \tilde{E}_1 = \mathfrak{u}(1), E_0 = \varnothing$. The Higgs branch is given by one-instanton moduli space (two-instantons moduli space for C_n). The geometry column refers to the local geometry around the symmetry enhanced point. See Eqs. (8.8) and (8.9) for the details of the c/c_{A_0} column, and the text after (8.13) for details behind the brane column.

quantum gravity vacua that one can have in nine dimensions.

Comparison of (8.7) and string vacua

So far, we have listed the possible brane worldvolume theories that can arise at the endpoints or singular points of the $S^1/\mathbb{Z}_2$ Coulomb branch moduli space. These are summarized in Table 8.1. Since the overall geometry is $S^1/\mathbb{Z}_2$, there are worldvolume theories with a geometry of $\mathbb{R}/\mathbb{Z}_2$ at the two endpoints. It is important to emphasize that there should be exactly two worldvolume theories with $\mathbb{R}/\mathbb{Z}_2$ geometries.

Only the A_n theory can appear as a singularity inside a line segment. The fact that the cubic coefficient c_i in A_n theory is negative is important in obtaining the bound.

The different choices of theories on the two endpoints correspond to different classes of vacua. There are 10 ways to choose two endpoints from C_n, D_n, E_n (including $\tilde{E}_1$), and $O8^{(-1)}$ theories. However, in order to satisfy (8.7), given that c_{A_n} is negative, the sum of c_i of the theory on the endpoints must be non-negative. This excludes the case where both endpoint theories are C_n and the case where C_n and $O8^{(-1)}$ are chosen. The remaining eight patterns are as follows:

$$\begin{aligned}
&2\,D, D+E, 2\,E : \text{Rank 17 theories,} \\
&D+O8^{(-1)}, E+O8^{(-1)} : \text{Rank 9 theories,} \\
&2\,O8^{(-1)} : \text{Rank 1 theories A,} \\
&C+D, C+E : \text{Rank 1 theories B.}
\end{aligned} \tag{8.11}$$

Here, on the right side, we have written the rank of the symmetry group, which is consistent with the result in[211].

The non-Abelian symmetry can be read from the global symmetry groups at the singularity, but it is not a priori clear how to count the number of $\mathfrak{u}(1)$ symmetries. Here we make two important comments about the counting of $\mathfrak{u}(1)$ factors. Later in the Subsection, we will provide a more complete and systematic study of the $\mathfrak{u}(1)$ factors.

First, there are $\mathfrak{u}(1)$'s associated with the relative positions of A_n singularities. This is understood by starting from the A_1 singularity and deforming it. The 5d theory at the A_1 singularity has an $\mathfrak{su}(2)$ global symmetry that rotates the two electrons, which corresponds to the bulk gauge symmetry. Then, let us consider breaking the $\mathfrak{su}(2)$ symmetry. This is achieved, in the bulk, by giving the vacuum expectation value to the Cartan component of the scalar field in the vector multiplet. On the brane, on the other hand, it is achieved by giving a mass difference to the two electrons (corresponding to an A_1 singularity splits into two A_0 singularities). Therefore, we see that the $\mathfrak{u}(1)$ vector multiplet in the bulk is coupled to the mass difference operator on the brane. Similar arguments show that the relative position of A_n and D_m singularities are also associated with $\mathfrak{u}(1)$'s.

Next, there may be an additional $\mathfrak{u}(1)$ corresponding to the 5d instanton number. If a 5d theory at the fixed point is IR free, then there is a conserved current[190]

$$j = *\,\mathrm{Tr}\,(F \wedge F)\,. \tag{8.12}$$

This generates an $\mathfrak{u}(1)$ symmetry under which the BPS instanton particle is charged. As before, this must couple to the 9d bulk vector multiplet in a supersymmetric way. This means that there is a $\mathfrak{u}(1)$ gauge symmetry in the bulk and that the 5d gauge coupling (and the mass of instanton particle) is controlled by the corresponding 9d scalar field.

We will now explain how the gauge group (including its Abelian factor) can be completely determined from the data of the Coulomb branch of the brane theory. The data that we are interested in is the collection of global symmetries of the brane worldvolume theory and the points where those global symmetries are realized on the Coulomb branch. But for now, we consider theories where the relative positions of all points of symmetry enhancement on the Coulomb branch are frozen. The positions can freeze due to the continuity of the brane coupling constant g across the brane moduli space. For example, consider the case where there are two E_8 theories at the endpoints and an A_0 theory somewhere in the middle. In such a theory, the location of the A_0 is forced to be exactly at the center of the interval. This is because $1/g^2$ vanishes at the E_8 endpoints and symmetrically increases in the middle. Therefore, the tipping point of $1/g^2$, where A_0 is located, must be at the center.

In theories where the relative positions of points of symmetry enhancement for a fixed gauge group are completely frozen, enhancing the symmetry algebra is impossible[3]. Later we will show that these theories are the only maximally enhanced theories. In other words, we show that the gauge group can always be enhanced to one of these symmetry groups.

In most cases, the maximally enhanced theory has a semisimple symmetry algebra of rank of 17, 9, or 1. However, in a few cases, the rank of the semisimple algebra is off by one. Therefore, for such theories to be realized, there must be an extra $\mathfrak{u}(1)$ in the gauge group. All the frozen geometries in the sense discussed above and their corresponding gauge algebras are listed in the following tables. The theories with ranks 17, 9, and 1, are respectively listed in Tables 8.2, 8.3, and 8.4.

[3]Note that our definition for maximally enhanced theory is one where the gauge group cannot be further enhanced to a larger group. This is different from the definition in[212] where maximal enhancement refers to the absence of $\mathfrak{u}(1)$ factors.

#	Placement of enhanced theories on the Coulumb branch	Gauge algebra	Root lattice
1	E_8——A_1——E_8	$\mathfrak{e}_8 + \mathfrak{e}_8 + \mathfrak{su}(2)$	$2E_8 + A_1$
2	E_8——A_2——E_7	$\mathfrak{e}_8 + \mathfrak{e}_7 + \mathfrak{su}(3)$	$E_8 + E_7 + A_2$
3	E_8——A_3——E_6	$\mathfrak{e}_8 + \mathfrak{e}_6 + \mathfrak{su}(4)$	$E_8 + E_6 + A_3$
4	E_8——A_4——E_5	$\mathfrak{e}_8 + \mathfrak{spin}(10) + \mathfrak{su}(5)$	$E_8 + D_5 + A_4$
5	E_8——A_5——E_4	$\mathfrak{e}_8 + \mathfrak{su}(6) + \mathfrak{su}(5)$	$E_8 + A_5 + A_4$
6	E_8——A_6——E_3	$\mathfrak{e}_8 + \mathfrak{su}(7) + \mathfrak{su}(3) + \mathfrak{su}(2)$	$E_8 + A_6 + A_2 + A_1$
7	E_8——A_8——E_1	$\mathfrak{e}_8 + \mathfrak{su}(9) + \mathfrak{su}(2)$	$E_8 + A_8 + A_1$
8	E_8——A_9——E_0	$\mathfrak{e}_8 + \mathfrak{su}(10)$	$E_8 + A_9$
9	E_7——A_3——E_7	$\mathfrak{e}_7 + \mathfrak{e}_7 + \mathfrak{su}(4)$	$2E_7 + A_3$
10	E_7——A_4——E_6	$\mathfrak{e}_7 + \mathfrak{e}_6 + \mathfrak{su}(5)$	$E_7 + E_6 + A_4$
11	E_7——A_5——E_5	$\mathfrak{e}_7 + \mathfrak{spin}(10) + \mathfrak{su}(6)$	$E_7 + D_5 + A_5$
12	E_7——A_6——E_4	$\mathfrak{e}_7 + \mathfrak{su}(7) + \mathfrak{su}(5)$	$E_7 + A_6 + A_4$
13	E_7——A_7——E_3	$\mathfrak{e}_7 + \mathfrak{su}(8) + \mathfrak{su}(3) + \mathfrak{su}(2)$	$E_7 + A_7 + A_2 + A_1$
14	E_7——A_9——E_1	$\mathfrak{e}_7 + \mathfrak{su}(10) + \mathfrak{su}(2)$	$E_7 + A_9 + A_1$
15	E_7——A_{10}——E_0	$\mathfrak{e}_7 + \mathfrak{su}(11)$	$E_7 + A_{10}$
16	E_6——A_5——E_6	$\mathfrak{e}_6 + \mathfrak{e}_6 + \mathfrak{su}(6)$	$2E_6 + A_5$
17	E_6——A_6——E_5	$\mathfrak{e}_6 + \mathfrak{spin}(10) + \mathfrak{su}(7)$	$E_6 + D_5 + A_6$

18	E_6——A_7——E_4	$\mathfrak{e}_6 + \mathfrak{su}(8) + \mathfrak{su}(5)$	$E_6 + A_7 + A_4$
19	E_6——A_8——E_3	$\mathfrak{e}_6 + \mathfrak{su}(9) + \mathfrak{su}(3) + \mathfrak{su}(2)$	$E_6 + A_8 + A_2 + A_1$
20	E_6——A_{10}——E_1	$\mathfrak{e}_6 + \mathfrak{su}(11) + \mathfrak{su}(2)$	$E_6 + A_{10} + A_1$
21	E_6——A_{11}——E_0	$\mathfrak{e}_6 + \mathfrak{su}(12)$	$E_6 + A_{11}$
22	E_5——A_7——E_5	$\mathfrak{spin}(10) + \mathfrak{spin}(10) + \mathfrak{su}(8)$	$2D_5 + A_7$
23	E_5——A_8——E_4	$\mathfrak{spin}(10) + \mathfrak{su}(9) + \mathfrak{su}(5)$	$D_5 + A_8 + A_4$
24	E_5——A_9——E_3	$\mathfrak{spin}(10) + \mathfrak{su}(10)$ $+\mathfrak{su}(3) + \mathfrak{su}(2)$	$D_5 + A_9 + A_2 + A_1$
25	E_5——A_{11}——E_1	$\mathfrak{spin}(10) + \mathfrak{su}(12) + \mathfrak{su}(2)$	$D_5 + A_{11} + A_1$
26	E_5——A_{12}——E_0	$\mathfrak{spin}(10) + \mathfrak{su}(13)$	$D_5 + A_{12}$
27	E_4——A_9——E_4	$\mathfrak{su}(10) + \mathfrak{su}(5) + \mathfrak{su}(5)$	$A_9 + 2A_4$
28	E_4——A_{10}——E_3	$\mathfrak{su}(11) + \mathfrak{su}(5)$ $+\mathfrak{su}(3) + \mathfrak{su}(2)$	$A_{10} + A_4 + A_2 + A_1$
29	E_4——A_{12}——E_1	$\mathfrak{su}(13) + \mathfrak{su}(5) + \mathfrak{su}(2)$	$A_{12} + A_4 + A_1$
30	E_4——A_{13}——E_0	$\mathfrak{su}(14) + \mathfrak{su}(5)$	$A_{13} + A_4$
31	E_3——A_{11}——E_3	$\mathfrak{su}(12) + \mathfrak{su}(3) + \mathfrak{su}(3)$ $+\mathfrak{su}(2) + \mathfrak{su}(2)$	$A_{11} + 2A_2 + 2A_1$
32	E_3——A_{13}——E_1	$\mathfrak{su}(14) + \mathfrak{su}(3)$ $+\mathfrak{su}(2) + \mathfrak{su}(2)$	$A_{13} + A_2 + 2A_1$
33	E_3——A_{14}——E_0	$\mathfrak{su}(15) + \mathfrak{su}(3) + \mathfrak{su}(2)$	$A_{14} + A_2 + A_1$
34	E_1——A_{15}——E_1	$\mathfrak{su}(16) + \mathfrak{su}(2) + \mathfrak{su}(2)$	$A_{15} + 2A_1$
35	E_1——A_{16}——E_0	$\mathfrak{su}(17) + \mathfrak{su}(2)$	$A_{16} + A_1$

36	E_0——A_{17}——E_0	$\mathfrak{su}(18)$	A_{17}
37	E_8————D_9	$\mathfrak{e}_8 + \mathfrak{spin}(18)$	$E_8 + D_9$
38	E_7————D_{10}	$\mathfrak{e}_7 + \mathfrak{spin}(20)$	$E_7 + D_{10}$
39	E_6————D_{11}	$\mathfrak{e}_6 + \mathfrak{spin}(22)$	$E_6 + D_{11}$
40	E_5————D_{12}	$\mathfrak{spin}(24) + \mathfrak{spin}(10)$	$D_{12} + D_5$
41	E_4————D_{13}	$\mathfrak{su}(5) + \mathfrak{spin}(26)$	$D_{13} + A_4$
42	E_3————D_{14}	$\mathfrak{su}(3) + \mathfrak{su}(2) + \mathfrak{spin}(28)$	$D_{14} + A_2 + A_1$
43	E_1————D_{16}	$\mathfrak{su}(2) + \mathfrak{spin}(32)$	$D_{16} + A_1$
44	E_0————D_{17}	$\mathfrak{spin}(34)$	D_{17}
45 -53	$0 \leq n \leq 8 : D_n$————D_{16-n}	$\mathfrak{spin}(2n) + \mathfrak{spin}(32-2n)$ $+\mathfrak{u}(1)$	$D_n + D_{16-n}$

Table 8.2: List of possible maximally enhanced rank 17 theories in nine dimensions which is obtained by Swampland considerations. The first 44 lines where the algebra is semisimple match with Table 3 in[212] which have string theory realizations.

#	Placement of enhanced theories on the Coulomb branch	Gauge algebra	Root lattice
1	E_8——A_1——$O8^{(-1)}$	$\mathfrak{e}_8 + \mathfrak{su}(2)$	$E_8 + A_1$
2	E_7——A_2——$O8^{(-1)}$	$\mathfrak{e}_7 + \mathfrak{su}(3)$	$E_7 + A_2$
3	E_6——A_3——$O8^{(-1)}$	$\mathfrak{e}_6 + \mathfrak{su}(4)$	$E_6 + A_3$
4	E_5——A_4——$O8^{(-1)}$	$\mathfrak{spin}(10) + \mathfrak{su}(5)$	$D_5 + A_4$

5	E_4——A_5——$O8^{(-1)}$	$\mathfrak{su}(6)+\mathfrak{su}(5)$	A_5+A_4
6	E_3——A_6——$O8^{(-1)}$	$\mathfrak{su}(7)+\mathfrak{su}(3)+\mathfrak{su}(2)$	$A_6+A_2+A_1$
7	E_1——A_8——$O8^{(-1)}$	$\mathfrak{su}(9)+\mathfrak{su}(2)$	A_8+A_1
8	E_0——A_9——$O8^{(-1)}$	$\mathfrak{su}(10)$	A_9
9	D_9————$O8^{(-1)}$	$\mathfrak{spin}(18)$	D_9

Table 8.3: List of possible maximally enhanced rank 9 theories in nine dimensions which is obtained by Swampland considerations. The above table matches with Table 3 in[213] which have string theory realizations.

#	Placement of enhanced theories on the Coulumb branch	Gauge algebra	Root lattice
1	$O8^{(-1)}$——A_1——$O8^{(-1)}$	$\mathfrak{su}(2)$	A_1
2	E_1————C_0	$\mathfrak{su}(2)$	A_1
3	E_0————C_1	$\mathfrak{su}(2)$	A_1
4	D_0————C_0	$\mathfrak{u}(1)$	$\varnothing$

Table 8.4: List of possible maximally enhanced rank 1 theories in nine dimensions which is obtained by Swampland considerations.

Let us compare the above with string compactifications. In string theory, there are four classes of vacua[214,215,188,18,212,213]. To make the comparison with (8.11) easier to understand, we write each class as follows.

$$
\begin{aligned}
&2\,O8^- + 16D8 \,:\, \text{Rank 17 theories,}\\
&O8^- + O8^0 + 8D8 \,:\, \text{Rank 9 theories,}\\
&2\,O8^0 \,:\, \text{Rank 1 theories A,}\\
&O8^- + O8^+ \,:\, \text{Rank 1 theories B,}
\end{aligned}
\tag{8.13}
$$

where $O8^0$ is the shift orientifold[215,188], and $O8^\pm$ are the orientifold planes with D8-charge ± 8.

Theories with rank 17 are obtained by circle compactification of heterotic/type I strings, as well as type I' string where there are two $O8^-$-planes and 16 $D8$-branes[186]. In this context, the 4-brane obtained as a small instanton is a $D4$-brane. The theory on the $D4$-brane that probes the $(n+1)$ $D8$-branes is the A_n theory, and the theory on the $D4$-brane that probes the $O8^- + nD8$ is the D_n theory. When the dilaton diverges at the position of $O8^- + (n-1)\,D8$, then the E_n theory is realized. It is also possible for $O8^-$ to emit $D8$ non-perturbatively ($O8^- \to O8^{(-9)} + D8$)[4]. The worldvolume theory of a $D4$-brane that probes the $O8^{(-9)}$ becomes the E_0 theory. This is not captured by the perturbative type I' description, but it can be understood from the language of geometry in real K3[216]. Table 11 of[212] lists all the patterns of the maximally enhanced gauge groups in the 9-dimensional heterotic string vacuum, which matches the ones in Table 8.2.

Theories with rank 9 are obtained from the CHL string[217,218], M-theory on the Möbius strip[219,220], and IIA string with $O8^- + O8^0 + 8D8$[188]. Again, when the O-plane emits $D8$ non-perturbatively ($O8^- \to O8^{(-9)} + D8$ and $O8^0 \to O8^{(-1)} + D8$), the maximally enhanced gauge symmetries are realized. By comparing with (8.11), we can see that the $O8^{(-1)}$ theory in Table 8.1 is naturally interpreted as a worldvolume theory for $D4$-brane probing the $O8^{(-1)}$ plane. Table 3 of[213] lists all the patterns of the maximally enhanced gauge groups in the 9-dimensional CHL string vacuum, which matches the ones in Table 8.3.

Finally, there are two inequivalent theories that have rank-1. One is M-theory on the Klein

[4] The superscript corresponds to $D8$-brane charge.

Bottle [220], the Asymmetric Orbifold of IIA, and is IIA with 2 $O8^0$ [188]. The other is IIB on the Dabholkar-Park background [220], the Asymmetric Orbifold of IIB, and IIA with $O8^- + O8^+$ [188]. It is known that in both classes of theories, the symmetry can be enhanced to $SU(2)$, which matches the list in Table 8.4.

Spacetime gauge theory and instanton moduli space

For brane moduli spaces listed in Tables 8.2, 8.3, and 8.4, we found the corresponding spacetime gauge group and showed that the gauge group is maximally enhanced. In the following, we complete our analysis by determining the spacetime gauge group for the ones not listed in the tables. The semisimple part of the gauge algebra is easy to find as it is given by the global symmetries of the brane theory. However, counting the number of additional $\mathfrak{u}(1)$ components turns out to be non-trivial. Moreover, we will show that the theories listed in the tables are the only maximally enhanced theories. In other words, the gauge group of any theory with a different brane moduli space could be enhanced to one of the entries of Tables 8.2, 8.3, or 8.4.

To show any other theory can be a=enhanced, we look at the deformations of the Coulomb branch of the brane resulting from moving around in the Coulomb branch of the bulk theory. In addition to the continuous change in the position of A_n points in the interior of the interval, multiple groups can fuse or break up. These can be most easily read off from the string theory realization of these theories. Note that this is a field theory statement, even though we use the string theory realizations of these theories to verify it. We find that the following transitions are allowed:

1. E_0 and $A_0 \leftrightarrow \tilde{E}_1$ corresponding to $O8^{(-9)} + D8 \leftrightarrow O8^-$.
2. E_1 and $A_0 \leftrightarrow E_2$ corresponding to moving away a $D8$ from the E_2 point.
3. A_m and $A_n \leftrightarrow A_{m+n+1}$ corresponding to joining/separating two stack of $m+1$ and $n+1$ $D8$ branes .

4. A_m and $D_n \leftrightarrow D_{m+n+1}$ corresponding to moving to/away a stack of $m + 1$ $D8$ branes to/from a stack of $O8^-$ and n $D8$ branes.

5. C_0 and $A_0 \leftrightarrow C_1$ corresponding to moving joining/separating a $D8$ brane to/from the $O8^+$ brane.

We did not include transitions involving $C_{n>1}$ since, as we will see later, including such theories makes it impossible to satisfy the condition (8.7).

Note that the first two transitions do not change the rank of the semisimple Lie algebra of the gauge group. However, the last three transitions change the rank of the semisimple Lie algebra by one. Since the the total rank of the group is invariant, these transitions must also involve the appearance of additional $\mathfrak{u}(1)$. In other words, the rank change comes from (un)Higgsing mechanism that absorbs/breaks up a $\mathfrak{u}(1)$ to/from the Lie algebra[5].

Following, we implement an algorithmic series of these transitions that will maximally enhance the gauge group.

One can first use transitions 1 and 2 to convert the enhanced theories on both ends into one of $\{C_{0 \text{ or } 1}, E_{n\neq2}, D_n\}$. Then, one can use transition 3 to fuse all the A-type points of symmetry enhancement into one. If one of the endpoints is a D theory, one can use transition 4 to absorb the remaining A-singularity into the D. If one of the endpoints is C_0, either there is nothing in the middle, or there is just an A_0. In the latter case, one can use transition 5 to absorb the A_0 fiber into C_0 and change it into C_1. At the end of this series of transitions, the condition 8.7 is still satisfied, and none of the transitions 1-5 can be done anymore. The only configurations that have these properties are the ones listed in Tables 8.2, 8.3, and 8.4. Therefore, any gauge group can be enhanced to a semisimple group listed in the third column of Tables 8.2, 8.3, and 8.4. An example of this algorithm is illustrated in Figure 8.2.

Note that for transitions 3-5, the brane theory encodes the data of the $U(1)$ gauge coupling through the relative position of the enhanced global symmetry points on its Coulomb branch.

[5] This is consistent with the argument below (8.11).

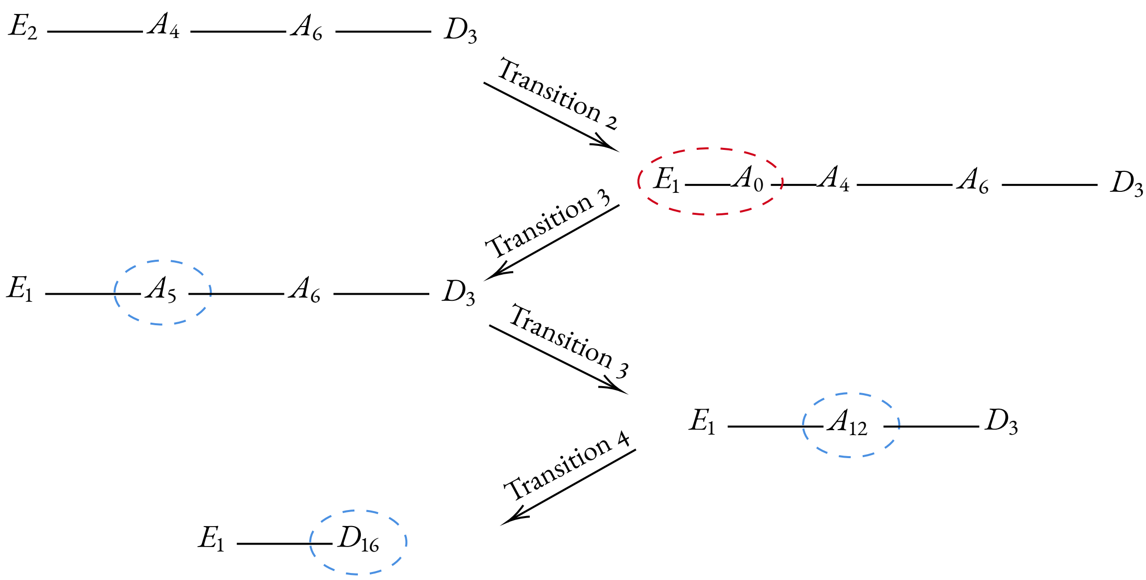

Figure 8.2: The above graph demonstrates the algorithm to deform a theory consistent with the condition (8.7) to a maximally enhanced theory in Table 8.2.

When the distance between these points is shrunk to zero, the appropriate adjoint vector bosons become massless, and the symmetry is enhanced. In the string theory language, this corresponds to the string states connecting the D-branes becoming massless.

The above algorithm offers an easy way to count the number of $\mathfrak{u}(1)$'s. The number of $\mathfrak{u}(1)$'s is the number of transitions 3-5 used plus the number of $\mathfrak{u}(1)$ at the end of the algorithm! This allows us to read off the full gauge group by looking at the brane's Coulomb branch.

There is a small loophole in the above argument that we address below. In the above algorithm we assumed that we can arbitrarily move the position of the points of symmetry enhancement to perform the transitions 8.1, as long as they are consistent with equations (8.4) and (8.7). However, we might not have such a control over the brane moduli space through variations of the spacetime moduli. In the following we show that variations of bulk moduli indeed allow for such arbitrary deformations of the brane moduli space. Our argument has two steps. First we show that as long as the points do not cross, we can arbitrarily move them around subject to the equations (8.4) and (8.7). Then we show that after moving two points corresponding to one of the transitions arbitrarily close to each other, they can be fused by changing bulk moduli.

First step: In Subsection 8.3, we will show that by varying the vev of bulk $\mathfrak{u}(1)$ scalars, any sufficiently small movement of the points of symmetry enhancement that satisfies (8.4) and (8.7) is possible. Now we argue that any large movement must also be possible. Suppose we have a canonically normalized bulk modulus φ_{9d} that controls the distance between two points of symmetry enhancement. We assume that increasing φ_{9d} corresponds to bringing the two points closer to each other. The unwanted scenario happens when even by taking φ_{9d} to infinity, the points do not get arbitrarily close to each other and stop at some finite distance. As we will explain next, this cannot happen.

The 9d supersymmetry fixes the moduli space of the spacetime theory to be the moduli of the $\Gamma^{17,1}$ Narain lattice, for which all infinite distance limits are decompactification limits. Fortunately, we understand the decompactification limits for the brane theory. When two points of symmetry enhancement that cannot be fused are brought closer to one another, we get a theory that does not have any 5d UV completion and decompactifies into a 6d theory. For example, this happens when we try to fuse A_0 point to an E_8 endpoint. Therefore, the decompactification limit corresponds to the situation when two points of symmetry enhancement converge but cannot be fused. Therefore, the worrisome situation mentioned before where the points stop at some finite distance from each other never happens. This completes the first step of the argument. Now we prove the second step.

Second step (fusibility of points): We want to show that suppose a pair of points corresponding to one of the transitions are brought sufficiently close to each other, they can be fused. To see why, note that we can always perform the transitions in the direction of splitting a point of symmetry enhancement into two. This can be done by Higgsing the spacetime gauge symmetry. Thus, at sufficiently small distances between two points of symmetry enhancement, the relation between the canonical distance of two points of symmetry enhancement on the brane moduli and the spacetime moduli is such that the points can be fused at a finite distance of the 9d moduli space. Doing it in reverse must also be possible at finite distance of bulk moduli space.

This completes the argument that closes the loophole. We showed that any movement of the points of symmetry enhancement that satisfies equations (8.4) and (8.7) in addition to all of the five transitions (in both directions) are always possible through variations of the spacetime moduli. In particular, this shows that all the rank 17 gauge groups are connected by the spacetime moduli space.

Excluding 9d supergravity theories with $\mathfrak{sp}(n)$ symmetry

Here we show that theories with $\mathfrak{sp}(n \geq 2)$ symmetry can be excluded using (8.7) and Table 8.1. In order to achieve $\mathfrak{sp}(n)$ symmetry, we must take the C_n theory as one of the endpoints of $S^1/\mathbb{Z}_2$. At this time, in order to satisfy (8.7)

$$-(8+n)+\frac{c_{S^1/\mathbb{Z}_2}}{c_{A_0}} \geq 0, \qquad \Rightarrow \qquad n \leq \frac{c_{S^1/\mathbb{Z}_2}}{c_{A_0}} - 8, \tag{8.14}$$

is required, where $c_{S^1/\mathbb{Z}_2}$ is the value of c_i at the other endpoint. From here, we can see that the upper bound of n for $\mathfrak{sp}(n)$ symmetry is determined by the maximum value of $c_{S^1/\mathbb{Z}_2}/c_{A_0}$ that can be taken. Table 8.1 shows that $c_{S^1/\mathbb{Z}_2}/c_{A_0}$ is maximized in the E_0 theory, where the value is 9. This means that

$$n \leq \frac{c_{S^1/\mathbb{Z}_2}}{c_{A_0}} - 8 \leq 1. \tag{8.15}$$

Thus, theories with $\mathfrak{sp}(1) = \mathfrak{su}(2)$ symmetry are feasible, but theories with $\mathfrak{sp}(n \geq 2)$ symmetry are not. Note that there is nothing problematic with a supersymmetric $\mathfrak{sp}(n)$ symmetry without coupling to gravity. Indeed the $O8^+$ orientifold in string theory realizes it on a non-compact space, where 9d gravity is decoupled, except for the running of the dilaton which leads to infinitely strong coupling at finite distance. In the string construction, the point at strong coupling can only be probed by very long strings stretching out of $O8^+$, which have a high energy. Therefore, this singularity can be interpreted as a stringy version of the Landau pole.

Also, note that supersymmetry is essential for the conclusions reached here. It is known that in a non-supersymmetric gravity theory, one can obtain a theory with $\mathfrak{sp}(n)$ gauge symmetry in 10 dimensions[221], and by compactifying it to S^1 one can obtain the same symmetry in 9 dimensions.

8.2 8d

In this Section, we review the argument for the 8d case discussed in[174], and extend it to derive the SLP in 8d supergravity theories.

The gauge instantons in 8d are 3-branes, which are described by a 4d $\mathcal{N} = 2$ rank-1 theory. The coupling constant is represented by an elliptic curve, and the total space is an hyperkähler geometry. The only known compact and connected hyperkähler manifolds with real dimension 4 are the torus T^4 or K3, and only the latter produces non-Abelian symmetry. A direct application of the arguments in Section 8.1 shows that only K3 is relevant for theories with 16 supercharges. In this way, the elliptic K3 geometry of the F-theory compactification[222,223,224] is reconstructed from the 3-brane[174]. Moreover, by studying the structure of the Coulomb branch for gauge instantons from the bottom-up perspective[225], one recovers the dictionary between K3 singularities and enhanced gauge symmetries. See[226,211,227,228] for swampland constraints on the global structure of the gauge group in 8d.

In Section 7, we have provided the argument that the rank of the brane theory is one. As remarked in the footnote there, we used the extra input from the string theory constructions to exclude the possibility of the yet-to-be-discovered SCFT. In 8d, we can exclude such a possibility in yet another way based on the central charge[6]. It is known that other gauge algebras cannot be realized[229,174], so in the following, we will only consider the case of the simply-laced gauge algebra and the $\mathfrak{sp}(n)$ gauge algebra. Assuming that the Higgs branch is given by that of the one-instanton moduli space, the central charges a and c of SCFT are determined[230,231]. This is because the Higgs branch corresponds to a non-Abelian instanton of nonzero size, and the

[6] This statement applies to 9d as well, since it is related by a simple S^1 compactification.

low-energy dynamics is uniquely determined by supergravity and the index theorem. Moreover, an upper bound to the rank can be obtained by using the relationship[232] between the central charge and the scaling dimension of the Coulomb branch coordinates:[174]

$$(\text{Rank}) \leq 4(2a - c). \tag{8.16}$$

This can be used to rule out the existence of nontrivial SCFTs with ranks greater than one that we do not yet know about. For $\mathfrak{sp}(n)$, the central charges are those of the trivial SCFTs, and there are no nontrivial SCFTs. For simply-laced gauge algebras, by giving the bulk scalar a vacuum expectation value, we can always break the symmetry to $SU(2)$. The central charges of the interacting SCFT on the corresponding small instanton 3-brane are $a = 11/24$ and $c = 1/2$. By substituting these values into (8.16), this leads to (Rank) $\leq 5/3$, which means that ranks higher than one are excluded.

We will now briefly comment on the reduced rank cases, which are related to frozen singularities in F-theory. In eight dimensions, there are three theories of rank 18, 10, and 2[211]. The gauge group of the theory with rank 18 is completely reproduced from the 3-brane. A theory of smaller rank corresponds to the case with D_{8+n} frozen singularities, and the theory on the 3-brane that probes this singularity is a S^1 compactification of the C_n theory of Table 8.1. A theory with rank 10 corresponds to geometry with a single frozen D_{8+n} singularity, while a theory with rank 2 corresponds to geometry with two. This mapping allows us to reproduce the gauge symmetry in theories with reduced rank too, which completes the SLP in 8d supergravity theories as well.

8.3 7D

In this Section, we apply the methodology of previous Sections to 7d theories with 16 supercharges. In 7d, the brane is magnetically charged under the 3-form field in the gravity multiplet. Therefore, the brane is a 2-brane instanton. Assuming the brane is BPS, the worldvolume theory of the 2-brane is a 3d $\mathcal{N} = 4$ theory, which becomes $U(1)$ in the Coulomb branch and at low energies.

Following the black hole argument and the strong version of the cobordism conjecture reviewed in Section 7, we can respectively argue that the moduli space is compact and connected. Moreover, from the $\mathcal{N} = 4$ supersymmetry, we know that the moduli space is hyperkähler[233]. To sum up, we are led to conclude that the moduli space is a four-dimensional compact, connected hyperkähler manifold. As mentioned above, only two such examples are known: T^4 and K3. The case of T^4 has no symmetry enhancement at any point in the moduli space and corresponds to seven-dimensional theories with 32 supercharges. We will therefore focus on the remaining case of 16 supercharges described by K3.

In principle, we can use this knowledge of moduli space to constrain the landscape of gauge theories, similar to what we did in the 9d case in Section 7. To systematically classify the possible 7d theories of maximal rank, we will need to construct all possible SCFT's that arise at singular points in the Coulomb branch, just as we did in the 9d case and was done for the 7d case in[234]. Unlike in the 9d case in Section 7, we do not have a systematic classification of 3d $\mathcal{N} = 4$ SCFT's, so we cannot use it to produce a list of allowed singularities (including the frozen ones); this means that strictly speaking our results here are weaker than the 9d and 8d cases. What we will do instead is list the known 3d rank $\mathcal{N} = 4$ SCFT's that arise at singular points in known string theory constructions,[7] and use these singularities to reconstruct all 7d $\mathcal{N} = 1$ maximal enhancement points. We can then reverse the field and string theory roles: Rather than using a field theory construction to construct all 7d $\mathcal{N} = 1$ theories, we will employ the known existing constructions to *predict* a classification of those rank-1 3d $\mathcal{N} = 4$ SCFT's which have the instanton moduli space as the Higgs branch!

Before embarking on the classification, we must discuss a subtlety that is absent in the 8d and 9d cases. The local singularities that can arise in K3 are codimension 4. The geometry, in a neighborhood of the singularity, looks like $\mathbb{R}^4/\Gamma_g$, where Γ_g is an ADE group. The four scalars

[7] The $M2$ brane probing M-theory on K3 was discussed in[235,236].

in the worldvolume of the brane are the three scalars that live in the 3d $\mathcal{N} = 4$ vector multiplet, and the dual photon. Since the low-energy theory is a sigma model into K3, there is a possibility of including a topological coupling in the brane worldvolume,

$$\int C_{IJK}\epsilon^{\alpha\beta\gamma}\partial_\alpha X^I \partial_\beta X^J \partial_\gamma X^K = \int \pi^*(C), \tag{8.17}$$

where $\{\alpha, \beta, \gamma\}$ are brane worldvolume indices, Latin uppercase indices correspond to the K3 tangent space, and the notation $\pi^*(C)$ just denotes the pullback, to the 2-brane worldvolume, of the K3 3-form C. A smooth K3 has no nontrivial 3-cycles, and so in such a case $C = 0$. But in a singular K3, one can excise the local singularity, and the resulting space has a nontrivial linking 3-cycle of topology S^3/Γ_g. It follows that, if in a given quantum theory of gravity, we find a brane with $C \neq 0$ around some singularity, it will not be possible to deform the K3 to be smooth: the corresponding singularity must be frozen.

What we have just given is a Swampland derivation of the existence of frozen singularities, which are very familiar from F and M theory constructions[214]. In particular, we have recovered their M-theory description as geometric singularities frozen by 3-form flux. In a sense, the brane perspective is telling us that any consistent 7d $\mathcal{N} = 1$ theory can arise from K3 with frozen singularities, and so, it gets us tantalizingly close to the statement that M-theory is the unique quantum theory of gravity in seven dimensions.

From the definition (8.17), it is pretty clear that the 3-form C is only defined modulo an integer, since the coupling remains the same upon shifting C by a 3-form that integrates to 1 on the relevant 3-cycle. Furthermore, 3d $\mathcal{N} = 4$ supersymmetry requires that the coupling is topological, which amounts to the statement that the 3-form C is closed, $dC = 0$. This local condition must be true globally in the compact K3 (with singularities removed), so if there are $p = 1, \ldots$ frozen singularities in K3, the holonomies $\int_{S^3/\Gamma_g^{(p)}} C$ around each of the frozen singularities must satisfy

$$\sum_p \int_{S^3/\Gamma_g^{(p)}} C \equiv 0 \bmod 1. \tag{8.18}$$

The constraint (8.18) can also be recovered in known compactifications to seven dimensions: it just becomes the condition that there cannot be G_4 flux on M-theory on K3 [214].

Armed with the above, we can reproduce the list of maximal enhancements in known theories in seven dimensions [214,234]. The following table lists all known (possibly frozen) local singularities that can arise in K3, the global symmetry on the brane theory at that point (corresponding to the non-Abelian enhanced symmetry), the local geometry of the Coulomb branch near the singularity and taken from [214,237], and the corresponding discrete flux threading the singularity.

Unfrozen Algebra	Flux $\int C_3$	Frozen Algebra
$\mathfrak{so}(2n+8)$	$1/2$	$\mathfrak{sp}(n)$
$\mathfrak{e}_6$	$1/2$	$\mathfrak{su}(3)$
$\mathfrak{e}_6$	$1/3, 2/3$	$\varnothing$
$\mathfrak{e}_7$	$1/2$	$\mathfrak{so}(7)$
$\mathfrak{e}_7$	$1/3, 2/3$	$\mathfrak{su}(2)$
$\mathfrak{e}_7$	$1/4, 3/4$	$\varnothing$
$\mathfrak{e}_8$	$1/2$	$\mathfrak{f}_4$
$\mathfrak{e}_8$	$1/3, 2/3$	$\mathfrak{g}_2$
$\mathfrak{e}_8$	$1/4, 3/4$	$\mathfrak{su}(2)$
$\mathfrak{e}_8$	$1/5, 2/5, 3/5, 4/5$	$\varnothing$
$\mathfrak{e}_8$	$1/6, 5/6$	$\varnothing$

Table 8.5: The frozen singularities in the context of the compactification of M-theory on K3 [214,237].

These have to be combined in all possible ways to form a K3 with singular fluxes. We list all the possibilities in the table below, matching the known list of 7d theories with reduced rank [214,18,234]. So assuming this table can be derived independently from the classification of 3d SCFT's with $\mathcal{N}=4$, this would complete the SLP program for supergravity theories in 7d as well.

As we have seen so far, much information about the spacetime theory is encoded in the small instanton moduli space. For example, the gauge group of the spacetime theory is related

Rank	Flux $\int C_3$	Freezing rule	Dual description
11	$\frac{1}{2}+\frac{1}{2}$	$\mathfrak{so}(2n+8)\oplus\mathfrak{so}(2m+8)\to\mathfrak{sp}(n)\oplus\mathfrak{sp}(m)$ $2\,\mathfrak{e}_6\to 2\,\mathfrak{su}(3),\ 2\,\mathfrak{e}_7\to 2\,\mathfrak{so}(7),\ 2\,\mathfrak{e}_8\to 2\,\mathfrak{f}_4$ $\mathfrak{so}(2n+8)\oplus(e_6,e_7,e_8)$ $\to\mathfrak{sp}(n)\oplus(\mathfrak{su}(3),\mathfrak{su}(7),\mathfrak{f}_4)$ $\mathfrak{e}_6\oplus\mathfrak{e}_7\to\mathfrak{su}(3)\oplus\mathfrak{so}(7),$ $\mathfrak{e}_6\oplus\mathfrak{e}_8\to\mathfrak{su}(3)\oplus\mathfrak{f}_4,\ \mathfrak{e}_7\oplus\mathfrak{e}_8\to\mathfrak{so}(7)\oplus\mathfrak{f}_4$	Hetero $\mathbb{Z}_2$ triple CHL string IIA $6O6^-+2O6^+$ no vector structure F on K3$\times S^1/\mathbb{Z}_2$
7	$\frac{1}{3}+\frac{2}{3}$	$2\,\mathfrak{e}_6\to\varnothing,\ 2\,\mathfrak{e}_7\to 2\,\mathfrak{su}(2),\ 2\,\mathfrak{e}_8\to 2\,\mathfrak{g}_2$ $\mathfrak{e}_6\oplus\mathfrak{e}_7\to\mathfrak{su}(2),\ \mathfrak{e}_6\oplus\mathfrak{e}_8\to\mathfrak{g}_2,$ $\mathfrak{e}_7\oplus\mathfrak{e}_8\to\mathfrak{su}(2)\oplus\mathfrak{g}_2$	Hetero $\mathbb{Z}_3$ triple F on K3$\times S^1/\mathbb{Z}_3$
5	$\frac{1}{4}+\frac{3}{4}$	$2\,\mathfrak{e}_7\to\varnothing,\ 2\,\mathfrak{e}_8\to 2\,\mathfrak{su}(2)$ $\mathfrak{e}_7\oplus\mathfrak{e}_8\to\mathfrak{su}(2)$	Hetero $\mathbb{Z}_4$ triple F on K3$\times S^1/\mathbb{Z}_4$
3	$\frac{1}{5}+\frac{4}{5}$	$2\,\mathfrak{e}_8\to\varnothing$	Hetero $\mathbb{Z}_5$ triple F on K3$\times S^1/\mathbb{Z}_5$
3	$\frac{1}{6}+\frac{5}{6}$	$2\,\mathfrak{e}_8\to\varnothing$	Hetero $\mathbb{Z}_6$ triple F on K3$\times S^1/\mathbb{Z}_6$
3	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	$\mathfrak{so}(8)\oplus\mathfrak{so}(2n+8)\oplus\mathfrak{so}(2m+8)\oplus\mathfrak{so}(2\ell+8)$ $\to\mathfrak{sp}(n)\oplus\mathfrak{sp}(m)\oplus\mathfrak{sp}(\ell)$	IIA $4O6^-+4O6^+$
3	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$	$\mathfrak{so}(8)\oplus\mathfrak{so}(2n+8)\oplus\mathfrak{so}(2m+8)\oplus\mathfrak{so}(2\ell+8)$ $\to\mathfrak{sp}(n)\oplus\mathfrak{sp}(m)\oplus\mathfrak{sp}(\ell)$	IIA $4O6^-+4O6^+$ F on $T^4\times S^1/\mathbb{Z}_2$
1	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$	$3\,\mathfrak{e}_6\to\varnothing,\ 2\,\mathfrak{e}_6\oplus\mathfrak{e}_7\to\mathfrak{su}(2)$	F on $T^4\times S^1/\mathbb{Z}_3$
1	$\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$	$\mathfrak{so}(8+2n)\oplus\mathfrak{e}_7\oplus(\mathfrak{e}_7,\mathfrak{e}_8)$ $\to\mathfrak{sp}(n)\oplus(\varnothing,\mathfrak{su}(2))$	F on $T^4\times S^1/\mathbb{Z}_4$
1	$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$	$\mathfrak{so}(2n+8)\oplus(\mathfrak{e}_6,\mathfrak{e}_7)\oplus\mathfrak{e}_8\to\mathfrak{sp}(n)\oplus(\varnothing,\mathfrak{su}(2))$	F on $T^4\times S^1/\mathbb{Z}_6$

Table 8.6: List of 7d theories with reduced rank[214,18,234]. The reduced rank theories are obtained by putting the flux in Table 8.5, where the total flux must vanish mod 1 in the compact manifold. The gauge algebra is obtained by the replacement of the maximal rank theories (the list of maximally enhanced gauge algebra is given in[234]). The number of inequivalent rank-3 theories is not entirely certain. It may be four rather than three (see footnote 22 in[214]). Applying the freezing rule to the list provided in[234], we see that the gauge algebra of all rank-1 theories can be enhanced to $\mathfrak{su}(2)$. The only maximal enhanced gauge algebra of rank-3 theories corresponding to $4O6^-+4O6^+$ is $3\,\mathfrak{su}(2)$. The maximal gauge algebra of the other cases is given in[234].

to the singularities of the brane moduli space. An ambitious improvement of this relationship would be to understand how the Coulomb branch of the brane moduli deforms by changing the spacetime moduli. In fact, in 7d, we can make an elegant connection between the geometry of the brane moduli space and the spacetime moduli.

Take a 2-cycle in the small instanton moduli space and consider a skyrmion where the brane moduli wrap around the 2-cycle on the spatial slices of the brane. Suppose we can localize the

2+1 dimensional skyrmion so that it pinches off from the brane worldvolume. The pinched-off skyrmion is a 0+1 spacetime particle. Note that the mass of the scalar field corresponding to this particle controls the size of the 2-cycle. Therefore, we can locally control the complex structure of the K3 moduli space subject to the frozen singularities by changing the periods of K3 through varying the spacetime moduli. This establishes a direct relationship between the brane moduli space and spacetime moduli using only field theory.

The above result holds for higher dimensions as well. For example, take the 9d theory and compactify it on a T^2 down to 7d. The points of symmetry enhancements on the brane Coulomb branch map to singularities of K3. Therefore, we can locally move the location of the singularities subject to the K3 geometry by changing the spacetime moduli. If we decompactify one S^1, the path in the 7d moduli space lifts to a path in the 8d moduli space, which moves the location of the singular fibers on the brane moduli space subject to the geometry of the elliptic K3. Suppose we further decompactify the extra S^1. In that case, the path lifts to a local movement of the points of symmetry enhancement on the brane Coulomb branch subject to the single valuedness of gauge coupling g in (8.4) and $\sum_i c_i = 0$ from (8.7).

If the spacetime gauge group of the 9d theory differs from the ones listed in Tables 8.2, 8.3, and 8.4, two things happen simultaneously:

1. Given that the entries of the tables have maximal semisimple algebras, the gauge algebra must be a subalgebra of one of the entries with an additional $\mathfrak{u}(1)$ factor.

2. Since the entries of the tables are the only configurations where the relative positions of the points of symmetry enhancement are completely frozen, the relative position between at least two of the points of symmetry enhancement must be tunable.

Therefore, we conclude the relative position between two points of symmetry enhancements is tunable if and only if the gauge algebra has an additional $\mathfrak{u}(1)$ factor. In other words, the Coulomb branch of the brane moduli space is sensitive to the scalars of the vector multiplets corresponding to the Abelian $\mathfrak{u}(1)$'s.

In 6d, there are two types of theories with 16 supersymmetries: chiral ($\mathcal{N} = (2,0)$) and non-chiral ($\mathcal{N} = (1,1)$). Of these, the chiral $\mathcal{N} = (2,0)$ theory is known to have such a strong restriction that the massless spectrum is determined by symmetry alone[238], so we will consider the non-chiral $\mathcal{N} = (1,1)$ theory.

The initial analysis of the 6d theories parallels that of the 7d. The brane is $(1+1)$ dimensional and has $\mathcal{N} = (4,4)$ supersymmetry by studying the gauge instanton solution in the bulk 6d theory. From a combination of the strong cobordism conjecture and the ADHM construction, we find that the moduli space of the 1-brane is a connected two-dimensional complex manifold. Moreover, from the black hole argument reviewed in 7, we know that the moduli space must be compact. Contrary to the 7d case, the $\mathcal{N} = (4,4)$ supersymmetry does not lead to an hyperkähler manifold as the target space of the sigma model in general[239,240,241]. Only when there is an extra $U(1)$ isometry does the target space becomes hyperkähler.

If we assume an additional $U(1)$ isometry, these facts collectively narrow down the possibilities to either K3 or T^4. Like the 7d case, T^4 corresponds to theories with 32 supercharges. Thus, we conclude that the moduli space of the 1-brane is K3. As in the cases of the other dimensions, by classifying 2d $\mathcal{N} = (4,4)$ theories, the possible gauge algebras of 6d theories are obtained. It would be interesting to complete the classification.

In fact, without assuming $U(1)$ isometry, we can see the appearance of K3 geometry from another argument. Since the 1-brane we are considering has rank-1 and $\mathcal{N} = (4,4)$, the worldsheet theory has four scalars and four fermions, and the total central charge is $c = 6$. In[242], by calculating the elliptic genus of the $\mathcal{N} = (4,4)$ theory with $c = 6$, it is shown that this theory is interpreted as a string propagating on K3[8]. In this sense, we can reconstruct the K3 geometry as the target space of the sigma model. Note that this argument can only be applied to smooth K3 and not to singular K3 with frozen singularities.

[8] As a technical assumption, it is required that the massless representations with isospin $l = 0$ and $l = 1/2$ do not mix[242]. This rules out torus compactifications.

The arguments above indicate that the geometry is morally K3, but we can not rule out the possibility that there is a $\mathcal{N} = (4, 4)$ SCFT which is different from the SCFT we get from K3 (despite that fact that at least in the smooth case it must have the same elliptic genus as K3)[9]. Modulo the assumption that all the $\mathcal{N} = (4, 4)$ SCFT's with $c = 6$ and compact target spaces are somehow equivalent (e.g., T-duality) to the SCFT with K3 target space (possibly with singularities), the SLP is valid.

[9] See [243] for the study on the patterns of frozen singularities from this point of view.

9

Infinite distance limits

9.1 Refinements of the distance conjecture

In this section, we review some variations of the distance conjecture that are very powerful for bottom-up arguments. In particular, we focus on the emergent string conjecture and the sharpened distance conjecture

Emergent string conjecture
In any infinite distance limit of the field space, the lightest tower of states is either a KK tower coming from the higher dimensional field theory or excitations of a fundamental string[152,34].

Note the word fundamental in describing the string. The criterion that the string is fundamental makes the conjecture very strong. What we mean by a fundamental string is a string that has graviton as a string state, and moreover, the scattering amplitudes of the string states are dominated by processes involving string worldsheets (see Fig. 9.1). In other words, the gravitational amplitudes in the weak-coupling limit are given by a tree-level string amplitude.

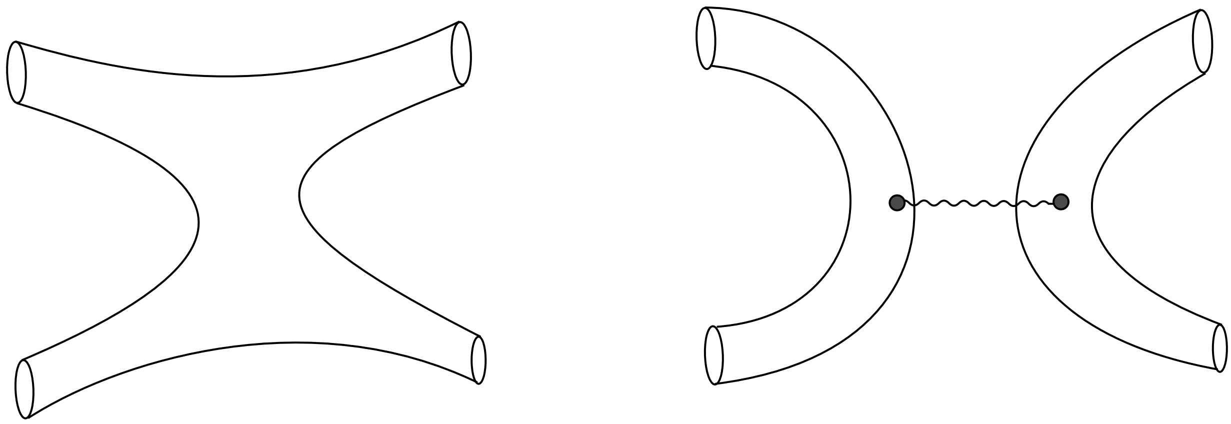

Figure 9.1: (a) The worldsheet diagrams that contribute to the amplitude of string states. (b) For an ordinary defect that couples to spacetime fields, such diagrams need to be summed. However, for fundamental strings, including such diagrams would lead to overcounting of the amplitude since the spacetime fields are supposed to emerge from string perturbation.

The emergent string conjecture is a very strong refinement of the distance conjecture. However, we will be using a weaker refinement of the distance conjecture in our studies which is the sharpened distance conjecture [153]. The sharpened distance conjecture states that the value of the numerical coefficient λ in the distance conjecture must be lower bounded by $1/\sqrt{d-2}$. Moreover, the authors in Ref. [153] also made the observation that the inequality is only saturated when one of the string towers is the lightest tower. Our arguments will be based on the following conjecture.

Sharpened Distance Conjecture [153]

The numerical coefficient λ in the distance conjecture satisfies

$$\lambda \geq \frac{1}{\sqrt{d-2}} =: \lambda_{\min} \, . \tag{9.1}$$

The inequality is saturated if and only if the lightest tower is the string states.

Note that in the above conjecture, it is not assumed that the corresponding string is necessarily fundamental. In fact, for this precise reason, the sharpened distance conjecture is a weaker conjecture than the emergent string conjecture. A derivation of the sharpened distance conjecture from the emergent string conjecture was given in Ref.[56]. One could formulate a weaker version of the emergent string conjecture without the assumption that the light string would be fundamental. The sharpened distance conjecture is stronger than the weak version of the emergent string conjecture and weaker than the emergent string conjecture.

Emergent string conjecture

$\Downarrow$

Sharpened distance conjecture

$\Downarrow$

Weak emergent string conjecture (without fundamentality of the string)

In the following sections, we show that the sharpened distance conjecture combined with the finiteness of the black hole entropy shows that the number of string limits is always countable. This result implies that with the exception of a measure zero subset of infinite distance limits, any other limit decompactifies. This is a powerful result that we will use for bottom-up arguments of various string dualities.

9.2 Countability of strings

We first argue that the number of inequivalent strings in the moduli space must be countable. We prove this statement by contradiction. Suppose we have uncountable inequivalent strings. For each string labeled by α, we pick a point P_α such that the string wolrdsheet description is valid in the ε-neighborhood of that point in the moduli space. ε is a positive number which can be arbitrarily small, and the radius of the neighborhood is measured using the canonical metric on the moduli space. Let us call these neighbourhoods string domains. Consider a covering of the moduli space with a countable number of $\varepsilon/2$-neighborhoods[1]. One of these open sets, let us call it U, will include uncountably many points P_α. Since the diameter of U is ε, the worldsheet description of every string α where $P_\alpha \in U$ is valid across all of U. Therefore, U has uncountably many inequivalent strings with valid descriptions at the same points in the moduli space. However, that would mean that uncountably many of those strings have a mass scale below some finite mass M, which would violate the Bekenstein-Hawking entropy formula. Therefore, the number of inequivalent strings across the moduli space is countable.

9.3 Countability of string limits

Now that we have shown that the number of inequivalent strings is countable, we will show that only countably many directions in the moduli space are string limits. For that, let us first define what we mean by an infinite distance direction.

An infinite distance direction is a global geodesic which is maximally extended in one direction. By global geodesic, we refer to a geodesic which is the shortest path between every two points on it. An instructive example is type IIB theory. In type IIB, the moduli space is a quotient of the upper-half plane, $\mathcal{H}/SL(2,\mathbb{Z})$. For any point p in the moduli space, one can consider a continuous family of geodesics which leave point p at different angles. However, only countably many of them will go to the infinity of the upper half-plane. Furthermore, only one of them is a global geodesic. Note that this is not true for the upper half-plane before quotienting. However, as a result of the $SL(2,\mathbb{Z})$ identification, some geodesics are no longer the shortest global path between every pair of points on them. According to our definition above, we would say that the type IIB theory has only one infinite distance limit up to dualities.

Now consider that a point p is in the asymptotics of the field space and a geodesic exiting from p, which is a global geodesic. Suppose the corresponding infinite distance limit is a string limit. Therefore, according to the sharpened distance conjecture, the coefficient of distance

[1] This is doable under reasonable physical assumptions laid out in [25].

conjecture in this limit is $1/\sqrt{d-2}$. Now, let us make an infinitesimal change in the velocity vector of the geodesic. The resulting geodesic is either a global geodesic or it is not (like the type IIB example). We want to show that this geodesic cannot be a string infinite distance limit. If this geodesic does not correspond to a global geodesic, there is nothing to prove. However, if this is a new infinite distance limit, we will note that by mixing the tension of the string with other moduli, we have increased the distance we have to travel in the moduli space in order to decrease the tension of the string by a certain amount. Therefore, the λ of the original string tower can no longer be $\lambda = 1/\sqrt{d-2}$. Therefore, even if this limit is a string limit, the corresponding string must be a different string. In other words, every string has a unique infinite distance limit associated with it. This is the direction in which the tension of the string decreases the fastest. Since the number of inequivalent strings is countable, we conclude that the number of string limits must also be countable.

10

Dualities from Swampland principles

One of the most remarkable features of string theory is the emergence of new weakly coupled descriptions at the infinite distance limits of the moduli space. This observation has been formulated more precisely in the Swampland distance conjecture[32] (see, e.g. Refs.[108,35] for tests in string theory), which plays the central role in the Swampland program[244] (see also reviews[245,246,247,56]). The emergent weakly coupled descriptions which appear at different corners of the moduli space are dual to each other. Therefore, the distance conjecture, at its core, is a general quantification of the universality of dualities in string theory. Indeed, the distance conjecture is sometimes referred to as the duality conjecture (e.g. Ref.[248]). However, the precise logical relation between the distance conjecture and the various string dualities is not clear. The purpose of the chapter is to fill this gap. We make this connection more precise by showing that refinements of distance conjecture can in fact explain dualities under several assumptions.

We focus on the higher dimensional supergravity theories with 16 or 32 supercharges. We use a variety of Swampland conjectures to find the relation between the conjectures and all the string dualities in $d \geq 9$. Our arguments center around the sharpened distance conjecture[153], and rely on various other conjectures such as the BPS completeness hypothesis[171]. This work shows that the Swampland principles, with some assumptions, capture the essence of the duality web, and perhaps must be viewed as fundamental.

Note that all the theories we consider are supergravities and not string theories. In other words, we will not assume that the UV completions of the mentioned supergravities are the known string theories.

The chapter is organized as follows. In Section 9, the sharpened distance conjecture and its connection to the emergent string conjecture[152,34], our main tool in the chapter, is reviewed. In Section 10.1, the T-duality between the IIA and IIB theories is derived from the bottom-up perspective. In Section 10.2, other 11d/10d string dualities of theories with 32 supercharges are discussed. Similarly, 10d/9d string dualities of theories with 16 supercharges are argued in Section 10.3. We clarify the relationship between the string dualities and the Swmapland conjectures. The technical details are provided in Appendix H, I and J.

10.1 IIA/IIB T-duality

Theories with $N \geq 16$ supercharges have at least one anti-symmetric two form in their gravity multiplet other than eleven dimension. Each 2-form $B_{\mu\nu}$ is a higher form gauge field with coupling g which is set by the action,

$$\mathcal{L}_B = \frac{1}{2g^2} dB \wedge \star dB. \tag{10.1}$$

A string that is electrically charged under $B_{\mu\nu}$ is called the supergravity string. The supergravity string is special in that it can be BPS. In all the known string theories, in the limit $g \to 0$, a BPS supergravity string becomes the fundamental string. Moreover, if we compactify the limit theory on a circle, when the string is sufficiently weakly coupled, it always has a T-dual in which the supergravity string is also weakly coupled.

In the following, we provide a bottom-up argument for this observation in theories with 32 supercharges in 10 dimension based on Swampland principles. More concretely, we use the BPS completeness hypothesis [171], the sharpened distance conjecture.

Assuming that the BPS supergravity string exists, the tension of the BPS supergravity string is (see Appendix H for derivation)[1]

$$T = M_{10}^2 e^{\hat{\varphi}/\sqrt{2}}, \tag{10.2}$$

where $\hat{\varphi}$ is the canonically normalized dilaton field, M_{10} is the 10d Planck scale, and $g = e^{\sqrt{2}\,\hat{\varphi}}$.

Let us define the string length l_s such that,

$$l_s = T^{-\frac{1}{2}} = l_{10} e^{-\hat{\varphi}/(2\sqrt{2})}, \tag{10.3}$$

where $l_{10} = M_{10}^{-1}$ is the 10d Planck length. We compactify the 10d theory on a circle with radius R and consider a BPS version of a closed string, which is the winding string.

The mass of a winding string is protected by supersymmetry and is given by

$$m_{\text{winding}} = TR = \frac{R}{l_s^2}, \tag{10.4}$$

even for small values of R where the 10d supergravity description breaks down. We can also find the degeneracies of the ground states of a winding string and their representations under the $SO(8)$ rotation group in 9d. This is because the ground states of the BPS string must furnish

[1] There are two $B_{\mu\nu}$ fields in the type IIB supergravity. Here we can choose any of them.

representations of the broken supersymmetries. In Appendix I, we show that based on the chirality of the 10d theory, the ground states of the winding string are given by

- Winding string in type IIA Supergravity: $(8_v \oplus 8_s) \otimes (8_v \oplus 8_s)$ with mass $m = \frac{R}{l_s^2}$.
- Winding string in type IIB Supergravity: $(8_v \oplus 8_c) \otimes (8_v \oplus 8_s)$ with mass $m = \frac{R}{l_s^2}$,

where 8_v, 8_s, and 8_c are the vector, spinor, and conjugate spinor representations of Spin(8), respectively.

Recall that the massless spectrum of the IIA and IIB supergravity is given by $(8_v \oplus 8_c) \otimes (8_v \oplus 8_s)$ and $(8_v \oplus 8_s) \otimes (8_v \oplus 8_s)$. After the compactification on S^1, these become the KK states charged under the KK $U(1)$. On the other hand, the winding strings are charged under the $U(1)$ coming from the dimensional reduction of $B_{\mu\nu}$. Therefore, the first/second line is consistent with the first level of the KK tower of Type IIB/Type IIA supergravity on a circle with radius l_s^2/R. This is remarkable since the BPS completeness hypothesis alone seems to reproduce the well-known T-duality between type IIA and type IIB string theories. In particular, this almost shows that the limit $R \to 0$ of IIA/IIB is the ten-dimensional IIB/IIA theory.

However, since we are not aware of the existence of the other non-BPS towers becoming light in the limit $R \to 0$, there are still following possibilities.

1. The winding strings are the leading tower, corresponding to the decompactification limit to 10d.
2. The winding strings are the leading tower, corresponding to the decompactification limit to 11d.
3. The winding strings are the leading tower, corresponding to the tensionless string limit.
4. The winding strings are not the leading tower, and lighter non-BPS states are the leading tower, corresponding to the decompactification limit.

5. The winding strings are not the leading tower, and lighter non-BPS states are the leading tower, corresponding to the tensionless string limit.

6. The leading tower is neither KK nor string state.

In the following, we show that the possibility 1 is the only option using the sharpened distance conjectures.

First, it is easy to rule out the possibilities 2 and 4. If the possibility 2 is correct, given that we have maximal supersymmetry, the theory must decompactify to the 11d supergravity on T^2. However, in this case, we have to find two BPS towers (the BPS tower other than the winding strings will be discussed in Section 10.2.1). This does not occur at the generic point of θ.

Similarly, the possibility 4 is excluded. Since the decompactification limit is either 10d IIA/IIB supergravity on S^1 or 11d supergravity on T^2, the KK modes are always the BPS states, charged under $U(1)$ gauge symmetries. Therefore, it is impossible that the leading non-BPS tower becomes the KK tower.

Next, the sharpened distance conjecture excludes the possibility 6 by definition.

Then, we use the sharpened distance conjecture to rule out the possibilities 3 and 5. To this end, we clarify what we mean by $R \to 0$ more precisely. There are two moduli fields in 9d supergravity. One is the dilaton $\hat{\varphi}$, and the other is the radion field parameterizing the S^1 radius. The canonically normalized radion modulus in d dimensions, $\hat{\rho}$, is given by[2]

$$\frac{R}{l_9} = \exp\left[\sqrt{\frac{d-1}{d-2}}\hat{\rho}\right], \tag{10.5}$$

which becomes $R/l_9 = e^{\sqrt{8/7}\hat{\rho}}$ for 9d.

We consider the infinite distance in $(\hat{\varphi}, \hat{\rho})$-plane.

$$\hat{\varphi} + i\hat{\rho} =: re^{i\theta}, \qquad r \to \infty, \qquad \theta : \text{fixed}. \tag{10.6}$$

[2] We treat $\hat{\rho}$ as a dimensionless quantity normalized by l_9.

The $R \to 0$ limit corresponds to $\pi < \theta < 2\pi$. In this limit, $\hat{\varphi}$ goes to $-\infty$ for $\pi < \theta < 3\pi/2$, and goes to $+\infty$ for $3\pi/2 < \theta < 2\pi$. For $\theta = 3\pi/2$, $\hat{\varphi}$ remains constant.

As we do not want to change the effective 9d theory, we fix the value of l_9.

$$l_9 = \left(l_s^8 e^{2\sqrt{2}\hat{\varphi}} R^{-1}\right)^{1/7} = \left(l_s^8 e^{2\sqrt{2}\hat{\varphi} - \sqrt{\frac{8}{7}}\hat{\rho}}/l_9\right)^{1/7} = \text{fixed}, \tag{10.7}$$

where the 10d and 9d Planck lengths are related as $(l_9/l_{10})^8 = l_9/R$. This determines the behavior of l_s in this limit:

$$l_s \to l_9\, e^{\frac{1}{2\sqrt{2}}\left(-\hat{\varphi}+\frac{\hat{\rho}}{\sqrt{7}}\right)} = l_9\, e^{\frac{1}{\sqrt{7}}\sin(\theta+\alpha)r}, \qquad \tan\alpha = -\sqrt{7}, \tag{10.8}$$

where $-\pi/2 < \alpha < \pi/2$.

In this limit, the mass of the KK states and the BPS winding strings ($m_w = R/l_s^2$) become

$$\begin{aligned} \frac{1}{R} &= l_9^{-1} e^{-\sqrt{\frac{8}{7}}\hat{\rho}}, \\ \frac{R}{l_s^2} &= l_9^{-1} e^{\left(\frac{\hat{\varphi}}{\sqrt{2}}+\frac{3\hat{\rho}}{\sqrt{14}}\right)} = l_9^{-1} e^{\sqrt{\frac{8}{7}}\sin(\theta+\beta)r}, \qquad \tan\beta = \frac{\sqrt{7}}{3}, \end{aligned} \tag{10.9}$$

where $-\pi/2 < \beta < \pi/2$. We observe that the numerical coefficient λ_{winding} for the BPS winding string is given by

$$\lambda_{\text{winding}} = -\sqrt{\frac{8}{7}}\sin(\theta+\beta). \tag{10.10}$$

The value of λ_{winding} (10.10) as a function of θ is plotted in Fig. 10.1 (solid red line). The dashed blue line corresponds to the minimum value of λ, λ_{min}, dictated by the sharpened distance conjecture. The figure implies that for $\pi < \theta \lesssim 1.65\pi$, λ_{winding} is bigger than λ_{min}. Since the winding tower whose mass is protected by the BPS condition has a decay rate larger than λ_{min}, the sharpened version of the distance conjecture implies that the above limit must be a 10d decompactification limit, and so the possibility 1 is correct.

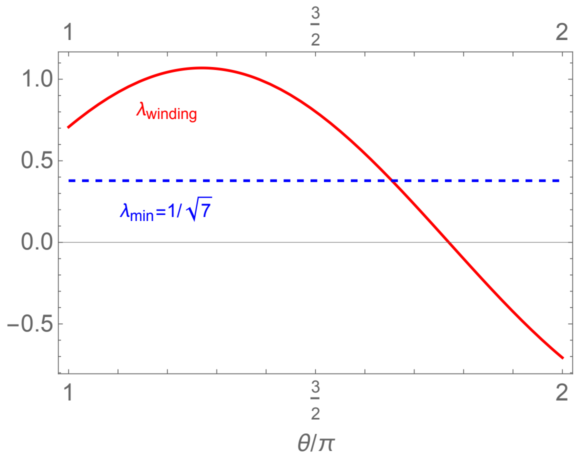

Figure 10.1: Plots of the numerical coefficient λ as a function of θ. Solid red line: The value of λ_{winding} (10.10) as a function of θ. Dashed blue line: The minimum value of λ, λ_{min}, dictated by the sharpened distance conjecture.

Moreover, from the chiralities of the winding states which we listed earlier, we conclude that any weakly coupled type IIA theory is T-dual to a weakly coupled type IIB theory and vice-versa. To be more precise, we showed that if quantum gravity is described by IIB/IIA supergravity on a circle with radius R at low energies, then the same theory at sufficiently small (but finite) R is described by IIA/IIB supergravity at sufficiently small energies. Note that the 9d theory exists for every value of R, even if the 10d description cannot be trusted[3]. The statement of duality holds for **any** radius, and is useful when the radius is sufficiently large in either description.

When the KK/winding state is not the lightest state, the above statement is trivially correct. However, since we have argued that the KK/winding state is lightest for $R \to 0$ or $R \to \infty$, the statement of the T-duality above is non-trivial.

10.2 Other dualities in theories with 32 supercharges

In this section, we argue the relationship between the string dualities of 11d/10d theories (other than IIA/IIB T-duality) with 32 supercharges and the Swampland conjectures.

[3]It is expected that the original supergravity description breaks down for $R \to 0$ because of the species scale[249,250] for a string with tension l_s^{-2} is l_s^{-1}. Any effective field theory description breaks at $R \ll l_s$.

10.2.1 IIA/M-theory

Now let us study the strong coupling limit of a theory which at low energy is described by the type IIA supergravity. In type IIA, assuming the BPS completeness hypothesis[171],[4] we have a tower of BPS particles whose masses are protected by supersymmetry.

$$m_{\text{BPS}} \propto \frac{M_{10}}{g^{\frac{3}{4}}}, \tag{10.11}$$

where g is the coupling constant of the 2-form field in the gravity multiplet (see Eq. (10.1)). Upon expressing this mass scale in terms of the canonically normalized spacetime modulus $\hat{\varphi} = \ln(g)/\sqrt{2}$, we find

$$m_{\text{BPS}} \propto \exp\left(-\frac{3}{\sqrt{8}}\hat{\varphi}\right). \tag{10.12}$$

The BPS particles might not be the lightest tower, however, they provide a tower which has a decay rate faster than $\lambda_{\text{min}} = 1/\sqrt{8}$. Therefore, the sharpened version of the distance conjecture implies that this limit cannot be a string limit and must be a decompactification limit. There is a unique field theory in dimension greater than 10 that has 32 superchrages[252,253,254], which is the 11d supergravity, and the only 1d internal geometry which would preserve all of that supersymmetry is a disjoint union of multiple S^1. However, due to the uniqueness of the lower dimensional graviton, the internal geometry must be connected. Therefore, the only possibility is 11d supergravity on S^1. Note that since the 11d supergravity does not have any 1-form gauge field, there are no fluxes (in this case Willson line) that can be turned on.

After dimensional reduction of the 11d supergravity, it is easy to see that the standard matching[255] of the fields between M-theory on S^1 and type IIA is unique. For example, the 11d supergravity on S^1 produces a unique gauge field, which comes from the KK reduction of the metric. Therefore, the particles charged under the type IIA gauge field must have KK

[4] To be precise, we assume the stronger version of the BPS completeness where the each BPS state is populated by a single particle state. In other words, we assume the existence of the bound state at the threshold[251].

momentum. In fact, from the 11d picture, we can also see that the BPS particles must be the KK particles. Therefore, we can match the mass of the 10d BPS particles with the mass of the KK particles to relate the coupling of IIA with the radius in the 11d supergravity picture in a unique way according to the usual string theory argument.

10.2.2 IIB S-duality

The S-duality of a theory which at low energies is described by type IIB supergravity easily follows from other dualities that we have derived so far.

In particular, we have shown in section 10.1 that such a theory on a circle of radius R is dual to a IIA supergravity on a circle of radius l_s^2/R for sufficiently small R. Moreover, in section 10.2.1, we have shown that type IIA supergravity with $\hat{\varphi} \to \infty$ is described by 11d supergravity on a circle. Therefore, type IIB supergravity on a circle has a dual description as an 11d supergravity on a torus. Since the $SL(2,\mathbb{Z})$ symmetry of the Teichmuller parameter of the torus must be a discrete gauge symmetry in the 11d picture, this must also be the case in the type IIB picture for the duality to hold. Therefore, we find that the low-energy $SL(2,\mathbb{Z})$ symmetry of type IIB supergravity cannot be an accidental symmetry, and must indeed be a duality of the theory.

10.3 Dualities in theories with 16 supercharges

In this section, we argue the relationship between the string dualities of 10d/9d theories with 16 supercharges and the Swampland conjectures.

10.3.1 Duality between Spin$(32)/\mathbb{Z}_2$ and $E_8 \times E_8$ supergravity theories

Similar to the previous duality, the duality between the Spin$(32)/\mathbb{Z}_2$ and $E_8 \times E_8$ supergravity theories follows directly from the sharpened distance conjecture. The key point is that since the gauge group in both theories is maximally enhanced, the charge lattice of the theory on S^1 is fully known. In particular, we show that the charge lattice is even and self-dual.

When we compactify the theory on S^1, the charge lattice is the direct sum of two lattices, i.e. the weight lattice of the 10d theory and $\Gamma^{1,1}$. The first lattice consists of charges under the KK reduction of 10d gauge fields, and the latter lattice is the charge lattice under $B^{\mu 9}$ and $g^{\mu 9}$. A priori, it is non-trivial why the charge lattice must be decomposable this way. However, charges states under $B^{\mu 9}$ are winding strings. The ground state of such winding strings is BPS, and the action of such BPS strings is known from Swampland arguments [18]. In particular, we know that the gauge symmetry of the spacetime induces a current algebra on the string. Therefore, the winding charge decouples from the other charges. Similarly, the KK charge is known to not mix with the higher dimensional gauge charges. Therefore, the full lattice must decompose into a 2d part and the higher dimensional charge lattice in the absence of Wilson lines. Moreover, given that the charges of the KK charge and the winding charge are known, it is easy to verify that the resulting 2d lattice is even and self-dual, just like the remaining 16d lattice, which is either e_8^2 or d_{16}^+, where e_8 and d_{16}^+ are the weight lattices of E_8 and $\mathrm{Spin}(32)/\mathbb{Z}_2$, respectively. Since the direct sum of two even and self-dual lattices is even and self-dual, the overall charge lattice is even and self-dual.

All even and self-dual lattices with the signature $(1, 17)$ are related by a similarity transformation. Since it is natural to assume that variations of the 9d moduli act by a similarity transformation on the charge lattice [256,257] (see Appendix J for an argument), if an even and self-dual charge lattice is realized only at one point of the 9d moduli, all such lattices can be realized. In particular, if we compactify the $E_8 \times E_8$ theory on S^1, we can move in the 9d moduli space to change the charge lattice from $\Gamma^{1,1} \oplus e_8^2$ to $\Gamma^{1,1} \oplus d_{16}^+$. Then we can act with a boost of increasing rapidity on $\Gamma^{1,1}$ by moving to an infinite distance of the moduli space. Assuming the BPS completeness hypothesis, there are BPS particles that lie on both axes of $\Gamma^{1,1}$. Since these particles are BPS, we can calculate their masses and see how they depend on the moduli of the 9d theory. Since the answer is unique, we can find the answer via a trick by looking at the dependence of KK towers in the decompactification limits of 10d supergravities on S^1. We find that the coefficient of the distance conjecture for such particles is $\sqrt{8/7}$. Given that this number is greater than $\lambda_{\min} = 1/\sqrt{7}$, this limit must decompactify. Moreover, from the charge lattice, we know that the gauge lie group contains $\mathrm{Spin}(32)/\mathbb{Z}_2$. Given that the only non-anomalous supergravity with such a gauge algebra is $so(32)$ supergravity, we conclude that $E_8 \times E_8$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$ supergravities are dual to each other.

10.3.2 Duality among heterotic $\mathrm{Spin}(32)/\mathbb{Z}_2$ and type I, and type I′

Here we consider the strong coupling limit of the $\mathrm{Spin}(32)/\mathbb{Z}_2$ supergravity, where the dilaton field in the gravity multiple is taken to infinity. In this limit the supergravity string charged under $B_{\mu\nu}$, which is only BPS state in 10d theories with 16 supercharges, becomes infinitely heavy. Therefore, the tower of the light states must be non-BPS. This is exactly what we know from string theory[186]. This limit corresponds to the perturbative type I theory where a non-supersymmetric type I fundamental string becomes tensionless. The question is how to arrive at the same conclusion in a bottom-up manner.

From the sharpened distance conjecture, the strong coupling limit is either the tensionless string or the decompactification limit. In the latter case, the theory must decompactify to the 11d supergravity. In the following, we argue that there are no compactifications of 11d supergravity which leads to the 10d $\mathrm{Spin}(32)/\mathbb{Z}_2$ supergravity. This indicates that the strong coupling limit is the tensionless non-BPS string limit.

To this end, suppose the strong coupling limit of the 10d supergravity decompactifies to the 11d supegravity. Since the 11d supergravity has 32 supercharges, the internal dimension must have features that break the half of the supersymmetry. A priori, one can imagine any 1d graph with singular points as a viable option of the compactification (see Figure 10.2)[5].

However, from the Swampland conjectures, we can argue that the only background preserving half of the supersymmetry is an interval. To see why we first put the 10d theory on a circle to find a type IIA background on the corresponding 1d graph. Then, we introduce a BPS 4-brane coupled with a 3-form field. The BPS 4-brane is a supergravity solution, and is

[5] The singular points are viewed as positions of the brane.

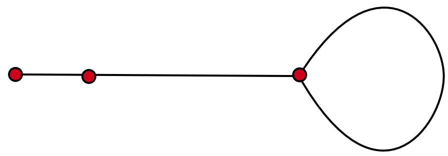

Figure 10.2: A hypothetical exotic internal geometry for 11d supergravity. The consistency of such compactification depends on the existence of appropriate non-perturbative branes sitting at the vertices.

required from the BPS completeness hypothesis. No global symmetry conjecture implies that the number of 4-branes and gauge theory instantons individually is not conserved. However, the overall number of the two is conserved and is protected by a 4-form gauge symmetry. Therefore, the BPS 4-brane solutions placed at a point in the graph must be continuously transformable to zero-sized gauge theory instantons [176] of the 9d theory. The gauge theory instantons correspond to specific points in the Coulomb branch of a BPS 4-brane in 9d. We call the defect which encompasses both the zero-sized gauge theory instanton as well as the BPS 4-brane, a small instanton[6].

The worldvolume theory of small instanton at a generic point of its 1d Coulomb branch is simply a BPS 4-brane placed on the graph.[7] Therefore, the internal geometry must match the Coulomb branch. However, the theory on large scales is described by a 5d SCFT and the vertices of the internal geometry are the Coulomb branch singularities of such 5d SCFT. Given the classification of such theories and their corresponding singularities, we know that the only allowed vertices are the end of the interval points. Therefore, a 1d graph such as Figure 10.2 does not preserve half of the supersymmetry. The only allowed geometry is an interval.

Now we have found that the only way to obtain a 10d theory with 16 supercharges is to compactify the 11d supergravity on the interval. In this case, it is well-known that the anomaly cancellation fixes the gauge group [258] to be $E_8 \times E_8$. Therefore, we conclude that the strong coupling limit of the $\mathrm{Spin}(32)/\mathbb{Z}_2$ supergravity cannot be a KK limit, and according to the

[6] The small instanton was used to classify the possible patterns of the gauge algebra of supergravity theories [174,24].

[7] Moving the internal direction is identified as Coulomb branch as it is real one dimension. We need complex scalars to obtain the Higgs branch.

sharpened distance conjecture, it must be a string limit. As we pointed out earlier, it follows that the corresponding fundamental string is non-supersymmetric, as expected from the type I string theory.

So far, we have explained the duality between the $\mathrm{Spin}(32)/\mathbb{Z}_2$ theory and type I by showing that Swampland conditions imply the existence of the type I string. The duality between the type I and type I′ theory, on the other hand, is more subtle. We will argue for this duality in 10.4.

10.4 Type I and type I′

Using various Swampland conjectures and the conclusion of the last chapter, we will show that the strong coupling limit of the 9d supergravity is always described by type IIA supergravity on an interval. Moreover, there are BPS domain walls along the interval whose location corresponds to the 9d moduli and uniquely determine the unbroken gauge symmetry via Swampland argument.

The 9d supergravity has scalars in the vector multiplets, as well as the dilaton from the gravity multiplet. The moduli space from the scalars in the vector multiplets takes the form

$$M \in SO(1,r)/SO(r), \tag{10.13}$$

where r is the number of vector multiplets. If we move in the moduli space via $\Omega \in SO(1,r)$ as

$$M \to \Omega M \Omega^T, \tag{10.14}$$

the charge lattice transforms in the vector representation of $SO(1,r)$ (see J). If a particle is BPS, its mass is given by

$$m = |Q_R|. \tag{10.15}$$

Since we know how the change in M acts on the charge, we also know how it would act on the mass of the BPS particles. However, the charges are also controlled by the dilaton as

$$Q \propto e^{\frac{9}{2\sqrt{7}}\hat{\varphi}}, \tag{10.16}$$

where $\hat{\varphi}$ is the canonically normalized dilaton. Therefore, if we take the dilaton to go to infinity fast enough, the mass of all of the BPS particles will go to infinity. We will call such limits non-BPS limits. To be more precise, consider the following infinite distance limit in the scalars of the vector multiplets.

$$\lim_{\gamma\to\pm\infty} \Omega(\gamma) M \Omega(\gamma)^T, \tag{10.17}$$

where

$$\Omega_i(\gamma) = \begin{pmatrix} \cosh\gamma & \delta^i_j \sinh\gamma \\ \delta^j_i \sinh\gamma & \delta^i_j \delta^i_{j'} \cosh\gamma \end{pmatrix}, \tag{10.18}$$

such that $i \in \{1, \cdots, r\}$. Then, the mass of BPS particles whose charges go to zero is given by

$$m \simeq e^{\pm\gamma_{a,i}} e^{\frac{d}{2\sqrt{d-2}}\hat{\varphi}} m_p. \tag{10.19}$$

Therefore, any limit in which $\hat{\varphi}$ goes to infinity such that $|\hat{\varphi}|/|\gamma| \geq (2\sqrt{d-2}/d)$ is a non-BPS limit.

Now that we know that these limits are non-BPS, we use the result of the last chapter to conclude that almost all such limits must decompactify. However, since all the light particles must be non-BPS, the decompactification cannot have any toroidal piece which results in a BPS KK tower. This significantly reduces the possibilities. We either decompactify to 11d or 10d. Let us start with the 11d possibility. We will first show that the number of boundaries must be less than or equal to 2. This is due to the fact that as we showed earlier, the moduli space of small instanton in 9d is an interval. However, the 9d small instanton is the wrapped 11d 2-

brane. Therefore, there must be exactly two boundaries which correspond to the endpoints of the small instanton moduli space. The possibilites for internal geometry with less than three boundaries that would preserve 16 supercharges are Mobius strip, cylinder, and torus, all of which have light BPS KK states. Therefore, these decomapctifications cannot arise in non-BPS limits.

If the theory decompactifies to 10d, we have two options for the 10d supergravity. The local theory has either 32 supercharges, half of which are spontaneously broken. Or the theory has 16 supercharges. In the latter case, we cannot afford to lose any supercharges. Therefore, the 10d theory must be compactified on a circle. This again would give rise to light BPS KK states which cannot happen in non-BPS limits. Therefore, the only possibility is to decompactify to a type II supergravity background where half the supersymmetry is broken. Since the compact dimension cannot be circle (due to lack of light BPS KK states), there must be end of the universe walls in the background. However, type IIB is chiral and cannot have BPS end of the universe walls. Therefore, the limit must correspond to a type IIA background on a 1d space with some some boundaries. However, given that the small instanton will correspond to type IIA 4-brane, the internal geometry must match the moduli space of small instanton, which is an interval. Therefore, the internal geometry must be an interval. We find that almost any non-BPS limit in 9d supergravity must decompactify to a type IIA background on an interval. The corresponding background can have a non-trivial profile of dilaton along the interval. This is the type I' supergravity. Furthermore, since the moduli space of the small instanton is matched with the internal geometry, the location of BPS 8-branes must match with the singularities on the small instanton moduli space. We showed earlier that one can read off the gauge theory from the structure of these singularities. Therefore, we can completely match the profile of dilaton along the interval to the gauge group in the bulk. Our bottom-up argument for the gauge group also provides a bottom-up argument for the wolrdvolume theory living on the 8 branes.

Now given the above discussion, we can provide a bottom-up argument for the duality between type I and type I' theory.

Let us compactify the $SO(32)$ supergravity on a circle and take the small radius R limit while taking the dilaton φ to infinity. We can choose different relative rates for them,

$$\hat{\varphi} = \alpha \ln(l_{10}/R). \tag{10.20}$$

where $\hat{\varphi}$ is the canonically normalized dilaton. Since the dilaton goes to infinity, we can think of this theory as type I supergravity on a circle of radius R. If α is sufficiently large, this is a strong coupling limit in 9d supergravity. Therefore, for almost any sufficientley large α, it must decompactify to massive type IIA supergravity on an interval. Such a background is also known as the type I' theory.

10.5 $E_8 \times E_8$ and 11d supergravity

Now we combine the chains of dualities that we have established to identify the strong coupling limit of the $E_8 \times E_8$ theory.

Based on the argument that we will present in 10.4, any 9d supergravity has a strong coupling limit which decompactifies to type I$'$ background (type IIA on an interval). We can apply this knowledge to the $E_8 \times E_8$ supergravity on S^1 and take its strong coupling limit of $\varphi \to \infty$. According to the type IIA picture, this is the limit where the 10d coupling of type IIA is taken to infinity. From the discussion in Section 10.2.1, the theory decompactifies to 11d supergravity on $S^1 \times I^1$. Therefore, we find that the strong coupling limit of the $E_8 \times E_8$ is 11d supergravity on I^1.

10.6 Chain of dualities and 9d Supergravity

Our bottom-up arguments for the dualities between the 10d and 11d supergravities have direct implications for lower dimensions. We will argue below, that in 9d supergravities with 17 vector multiples, we know the structure of almost all infinite distance limits.

In 10.4 we showed that any strong coupling limit of a 9d $\mathcal{N} = 1$ supergravity is type IIA on an interval (type I′ theory). But now that we have a complete web of dualities between the higher dimensional theories, we know that the type I′ theory shares its moduli space with other 9d supergravities such as type I on a circle, Heterotic on a circle, or M-theory on a cylinder. Therefore, we can tell that there are corners of the 9d supergravity with the rank 17 that decompactify to each one of those theories. But we can do even better! Our proof of dualities in the previous sections not only demonstrates that the mentioned theories share a mutual moduli space, but also explains how the moduli of the theories are related. Therefore, we know exactly how the moduli between any two such descriptions are related. For example, the duality between type IIA and 11d supergravity on circle, tells us how the moduli of type I′ are related to the moduli of 11d supergravity on a cylinder. Therefore, if an infinite distance limit is the type I′ supergravity, we can use that matching of the moduli to figure out what limits go to 11d supergravity on the cylinder. In 9d theories of maximal rank ($r = 17$), all the infinite distance limits are already covered by the dualities we derived. Our arguments can be used as a bottom-up derivation that the infinite distance limits of the 9d theory must behave exactly as superstring theory predicts [188].

Note that the reason we can identify the infinite distance limits is that a measure-one subset of infinite distance limits decompactify to higher dimensions. This result, which we showed in 10.4, is based on the sharpened distance conjecture and the finiteness of the black hole entropy. This is a very strong result which demonstrates the non-triviality of the sharpened distance conjecture. As long as we identify only one decompactification limit precisely, we can use the proven dualities to infer the other infinite distance limits.

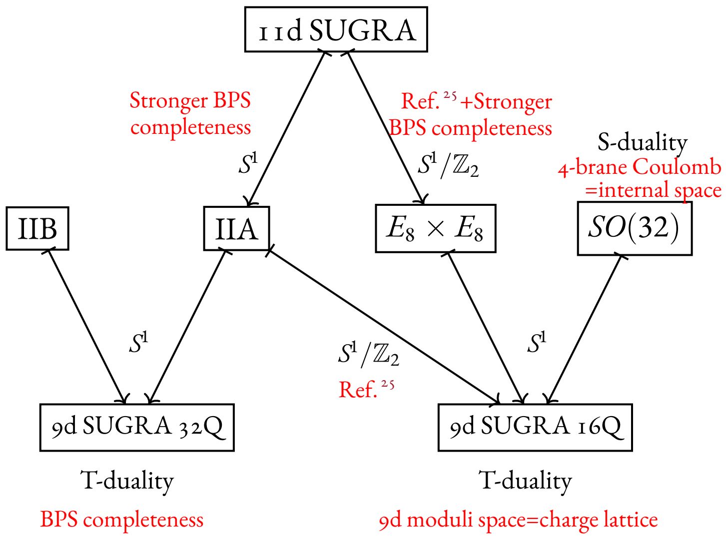

Figure 10.3: Summary of our result. The upper, middle, and lower boxes correspond to 11d, 10d, and 9d supersymmetric theories, respectively. The relation among them and dualities are denoted by the arrows or texts in black color. The assumptions we used to derive the dualities are written in red color. The sharpened distance conjecture are generically used, and is not written in the figure.

A

A Strong Short-Field-Range Inequality

In this appendix, we aim to understand what is the strongest short-field-range statement that TCC would imply for an arbitrary monotonically decreasing positive potential. The conjecture must hold for any physically allowed initial condition (one that $\dot{\varphi}_i < \mathcal{O}(1)$ and $V_i < \mathcal{O}(1)$). To deduce a strong inequality from TCC, we focus on an initial condition that seems to challenge (2.1) the most. As $\dot{\varphi}$ appears in the denominator of the LHS, a natural guess for the initial conditions with the most tension with the TCC would be small $\dot{\varphi}_i$. From (1.2) one can find that H decreases at a rate proportional to $\dot{\varphi}^2$. Thus, small $\dot{\varphi}_i$ could result in an inflationary universe with a slowly-varying Hubble parameter. If $\dot{\varphi}$ does not grow fast enough, the $\frac{a_f}{a_i}$ inflates exponentially leading to a violation of (1.1). With that in mind, we try to obtain an inequality from TCC for small initial field derivative $\dot{\varphi}_i$.

Suppose $\dot{\varphi}_i > 0$ is small enough such that $\ddot{\varphi}_i$ given by the (2.3) is positive. Let φ^* be the smallest $\varphi > \varphi_i$ where $\ddot{\varphi}$ vanishes (later in the appendix we will prove that such a field value exists and we will provide an upper bound for it). Using (2.4), we find

$$\ddot{\varphi} = -V' - (d-1)H\dot{\varphi} < -V' - \sqrt{\frac{d-1}{d-2}}\dot{\varphi}^2. \tag{A.1}$$

Since $\dot{\varphi}$ is increasing in the interval $[\varphi_i, \varphi^*]$, we can use the above inequality to find

$$\frac{d\dot{\varphi}}{d\varphi} = \frac{\ddot{\varphi}}{\dot{\varphi}} < \frac{\ddot{\varphi}}{\dot{\varphi}_i} \leq \frac{-V' - \sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i^2}{\dot{\varphi}_i}. \tag{A.2}$$

By integrating the above inequality, we find the following upper bound on $\dot{\varphi}$ for every $\varphi \in (\varphi_i, \varphi^*]$.

$$\dot{\varphi} < \frac{V(\varphi_i) - V(\varphi)}{\dot{\varphi}_i} - \sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i(\varphi - \varphi_i) + \dot{\varphi}_i. \tag{A.3}$$

Plugging the above upper bound on $\dot{\varphi}$ into the equation of motion (2.3) and using the inequality $H < H_i$, where H_i is the initial Hubble parameter, we find

$$\ddot{\varphi} > \ddot{\varphi}_i + (-V'(\varphi) + V'(\varphi_i)) - (d-1)H_i(\frac{V(\varphi_i) - V(\varphi)}{\dot{\varphi}_i} - \sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i(\varphi - \varphi_i)). \tag{A.4}$$

By setting φ to φ^*, at which $\ddot{\varphi}$ vanishes, we find

$$(d-1)H_i(\frac{V(\varphi_i) - V(\varphi^*)}{\dot{\varphi}_i} - \sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i\Delta\varphi) - (-V'(\varphi^*) + V'(\varphi_i)) > \ddot{\varphi}_i, \tag{A.5}$$

where $\Delta\varphi = \varphi^* - \varphi_i$. According to the mean value theorem, there is a point $\varphi_1 \in [\varphi, \varphi^*]$ such that

$$(d-1)H_i(\frac{V(\varphi_i)-V(\varphi)}{\dot{\varphi}_i})-(V'(\varphi_i)-V'(\varphi^*))=\Delta\varphi[(d-1)H_i(\frac{-V'(\varphi_1)}{\dot{\varphi}_i})+V''(\varphi_1)]. \tag{A.6}$$

We can rewrite the inequality (A.5) in terms of the values of V' and V'' at φ_1 as

$$\Delta\varphi>\frac{-(d-1)H_i\dot{\varphi}_i+|V'(\varphi_i)|}{(d-1)H_i(\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}-\sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i)+V''(\varphi_1)}, \tag{A.7}$$

where we used the equation of motion (2.3) to substitute $\ddot{\varphi}_i$ for the numerator of the right hand side. Suppose $\dot{\varphi}$ is small enough such that

$$\dot{\varphi}_i\leq c_1\sqrt{V(\varphi_i)}\ \&\ \dot{\varphi}\leq c_2\frac{|V'(\varphi_i)|}{\sqrt{V(\varphi_i)}}\ \&\ \dot{\varphi}_i\frac{V''(\varphi_1)}{|V'(\varphi_1)|}\leq c_3\sqrt{V(\varphi_i)}, \tag{A.8}$$

for some non-negative numbers c_1, c_2, and c_3 satisfying $c_2^2(2+c_1^2)<(d-2)/(d-1)$, we have

$$\dot{\varphi}_i\leq c_1\sqrt{V(\varphi_i)}\rightarrow H_i\leq\sqrt{\frac{2+c_1^2}{(d-1)(d-2)}}\sqrt{V(\varphi_i)}, \tag{A.9}$$

$$\dot{\varphi}_i\leq c_2\frac{|V'(\varphi_i)|}{\sqrt{V(\varphi_i)}}\rightarrow -(d-1)H_i\dot{\varphi}_i+|V'(\varphi_i)|\geq|V'(\varphi_i)|(1-c_2\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}}), \tag{A.10}$$

$$\dot{\varphi}_iV''(\varphi_1)\leq c_3\sqrt{V(\varphi_i)}|V'(\varphi_1)|\rightarrow V''(\varphi_1)\leq c_3\frac{\sqrt{V(\varphi_i)}|V'(\varphi_1)|}{\dot{\varphi}_i}, \tag{A.11}$$

where we used the Friedmann equation(2.2) in derivation of (A.9), and we used (A.9) in the derivation of the (A.10). Since $\dot{\varphi}_i>0$ we have

$$(d-1)H_i(\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}-\sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i)+V''(\varphi_1)<(d-1)H_i\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}+V''(\varphi_1). \tag{A.12}$$

By multiplying (A.9) by $(d-1)|V'(\varphi_1)|/\dot{\varphi}_i$ and summing it up with (A.11) we find that

$$(d-1)H_i\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}+V''(\varphi_1)\leq(c_3+\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}})\frac{\sqrt{V(\varphi_i)}|V'(\varphi_1)|}{\dot{\varphi}_i}. \qquad \text{(A.13)}$$

If we combine this with (A.12), we find

$$(d-1)H_i(\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}-\sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i)+V''(\varphi_1)<(c_3+\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}})\frac{\sqrt{V(\varphi_i)}|V'(\varphi_1)|}{\dot{\varphi}_i}. \qquad \text{(A.14)}$$

Dividing (A.10) by the above inequality leads to

$$\begin{aligned}\frac{-(d-1)H_i\dot{\varphi}_i+|V'(\varphi_i)|}{(d-1)H_i(\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}-\sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i)+V''(\varphi_1)}&>\frac{1-c_2\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}}}{c_3+\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}}}(\frac{|V'(\varphi_i)|}{|V'(\varphi_1)|})\frac{\dot{\varphi}_i}{\sqrt{V(\varphi_i)}}\\&\geq\frac{1-c_2\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}}}{c_3+\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}}}(\frac{|V'(\varphi_i)|}{\max_{\varphi\in[\varphi_i,\varphi^*]}(|V'(\varphi)|)})\frac{\dot{\varphi}_i}{\sqrt{V(\varphi_i)}}\\&=c_2f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i}{\sqrt{V(\varphi_i)}}, \qquad \text{(A.15)}\end{aligned}$$

where $f(c_1,c_2,c_3)$ and $g(\dot{\varphi}_i)$ are given by

$$\begin{aligned}f(c_1,c_2,c_3)&=\frac{1-c_2\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}}}{c_2(c_3+\sqrt{\frac{(d-1)(2+c_1^2)}{d-2}})}\\g(\dot{\varphi}_i)&=\frac{|V'(\varphi_i)|}{\max_{\varphi\in[\varphi_i,\varphi^*]}(|V'(\varphi)|)}. \qquad \text{(A.16)}\end{aligned}$$

Using the assumption $\dot{\varphi}\leq c_2|V'(\varphi_i)|/\sqrt{V(\varphi_i)}$ we can lower the right hand side of (A.15) to get

$$\frac{-(d-1)H_i\dot{\varphi}_i+|V'(\varphi_i)|}{(d-1)H_i(\frac{|V'(\varphi_1)|}{\dot{\varphi}_i}-\sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i)+V''(\varphi_1)}>f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}. \qquad \text{(A.17)}$$

By combining the above inequality with (A.7) we find

$$\Delta\varphi > f(c_1, c_2, c_3) g(\dot{\varphi}_i) \frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}. \tag{A.18}$$

For every $\varphi \in [\varphi_i, \varphi_i + \frac{f(c_1,c_2,c_3)g(\dot{\varphi})\dot{\varphi}_i^2}{|V'(\dot{\varphi}_i)|}]$ we have

$$\begin{aligned}
\frac{|V(\varphi) - V(\varphi_i)|}{\dot{\varphi}_i} &\leq \frac{\varphi - \varphi_i}{\dot{\varphi}_i} \max_{\varphi \in [\varphi_i, \varphi_i + \frac{f(c_1,c_2,c_3)g(\dot{\varphi})\dot{\varphi}_i^2}{|V'(\dot{\varphi}_i)|}]} (|V'(\varphi)|) \\
&\leq \frac{\varphi - \varphi_i}{\dot{\varphi}_i} \max_{\varphi \in [\varphi_i, \varphi^*]} (|V'(\varphi)|) \\
&\leq f(c_1, c_2, c_3) g(\dot{\varphi}_i) \frac{\dot{\varphi}_i}{|V'(\dot{\varphi}_i)|} \max_{\varphi \in [\varphi_i, \varphi^*]} (|V'(\varphi)|) \\
&= f(c_1, c_2, c_3) \dot{\varphi}_i,
\end{aligned} \tag{A.19}$$

where in the first line we used the mean value theorem, in the second line we used (A.18), and in the third line we used the (A.16), the definition of $g(\dot{\varphi}_i)$. Using the inequalities we have derived, we find

$$
\begin{aligned}
&\frac{\dot{\varphi}_i\sqrt{2}f(c_1,c_2,c_3)g(\dot{\varphi}_i)}{|V'(\varphi_i)|\sqrt{(d-1)(d-2)}(f(c_1,c_2,c_3)+1)} \\
&\quad= \int_{\varphi_i}^{\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}} \frac{\sqrt{\frac{2}{(d-1)(d-2)}}}{(f(c_1,c_2,c_3)+1)\dot{\varphi}_i} d\varphi \\
&\quad\leq \int_{\varphi_i}^{\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}} \frac{\sqrt{\frac{2}{(d-1)(d-2)}}}{\frac{|V(\varphi)-V(\varphi_i)|}{\dot{\varphi}_i}-\sqrt{\frac{d-1}{d-2}}\dot{\varphi}_i(\varphi-\varphi_i)+\dot{\varphi}_i} d\varphi \\
&\quad\leq \int_{\varphi_i}^{\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}} \frac{\sqrt{\frac{2}{(d-1)(d-2)}}}{\dot{\varphi}} d\varphi \\
&\quad= \int_{\varphi_i}^{\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}} \frac{1}{\sqrt{V(\varphi)}} \frac{\sqrt{\frac{2V(\varphi)}{(d-1)(d-2)}}}{\dot{\varphi}} d\varphi \\
&\quad\leq \frac{1}{\sqrt{V(\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|})}} \int_{\varphi_i}^{\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}} \frac{\sqrt{\frac{2V(\varphi)}{(d-1)(d-2)}}}{\dot{\varphi}} d\varphi \\
&\quad\leq \frac{1}{\sqrt{V(\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|})}} \int_{\varphi_i}^{\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|}} \frac{H}{\dot{\varphi}} d\varphi \\
&\quad< \frac{1}{\sqrt{V(\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|})}} \ln\left(\frac{1}{H(\varphi+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|})}\right) \\
&\quad\leq \frac{1}{\sqrt{V(\varphi_i+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|})}} \ln\left(\frac{\sqrt{\frac{(d-1)(d-2)}{2}}}{\sqrt{V(\varphi+f(c_1,c_2,c_3)g(\dot{\varphi}_i)\frac{\dot{\varphi}_i^2}{|V'(\varphi_i)|})}}\right),
\end{aligned}
\tag{A.20}
$$

where in the third line we used (A.19), in the fourth line we used (A.3), in the sixth line we used the monotonicity of V, in the seventh and the ninth lines we used $V \leq H^2(d-1)(d-2)/2$, and in the eighth line we used the TCC. Below we list the assumptions we made to derive the inequality (A.20).

$$\dot{\varphi}_i \leq \min(c_1\sqrt{V(\varphi_i)}, c_2\frac{|V'(\varphi_i)|}{\sqrt{V(\varphi_i)}}),$$
$$and$$
$$\dot{\varphi}_i \max_{\varphi\in[\varphi_i,\varphi^*]}(\frac{V''(\varphi)}{|V'(\varphi)|}) \leq c_3\sqrt{V(\varphi_i)}. \tag{A.21}$$

Following we find an upper bound for φ^* in terms of φ_i, $\dot{\varphi}_i$ and $V(\varphi_i)$ so that by replacing φ^* in the criteria (A.21) we change them into criteria that only depend on the initial conditions.

$$\begin{aligned} H_i &> H_i - H(\varphi^*) \\ &= -\int_{\varphi_i}^{\varphi^*} \frac{\dot{H}}{\dot{\varphi}} d\varphi \\ &= \int_{\varphi_i}^{\varphi^*} \frac{\dot{\varphi}}{d-2} d\varphi \\ &\geq \frac{\dot{\varphi}_i}{d-2}(\varphi^* - \varphi_i), \end{aligned} \tag{A.22}$$

which can be rearranged into the form

$$\varphi^* < \frac{(d-2)H_i}{\dot{\varphi}_i} + \varphi_i. \tag{A.23}$$

By replacing φ^* in (A.21) with this upper-bound, our criteria change into

$$\dot{\varphi}_i \leq \min(c_1\sqrt{V(\varphi_i)}, c_2\frac{|V'(\varphi_i)|}{\sqrt{V(\varphi_i)}}),$$
$$and$$
$$\dot{\varphi}_i \max_{\varphi\in[\varphi_i,\frac{(d-2)H_i}{\dot{\varphi}_i}+\varphi_i]}(\frac{V''(\varphi)}{|V'(\varphi)|}) \leq c_3\sqrt{V(\varphi_i)}. \tag{A.24}$$

We can view the last inequality as an inequality for c_3 rather than a criterion for $\dot{\varphi}_i$. Moreover, it seems that to get the most non-trivial result from the inequality (A.20), we should pick the largest $\dot{\varphi}$ possible. We can choose $\dot{\varphi}$ such that $\dot{\varphi}_i = \min(c_1\sqrt{V(\varphi_i)}, c_2\frac{|V'(\varphi_i)|}{\sqrt{V(\varphi_i)}})$ and then we can pick c_3 accordingly as follows to make sure that all of the criteria are satisfied.

$$\dot{\varphi}_i = \min(c_1\sqrt{V(\varphi_i)}, c_2\frac{|V'(\varphi_i)|}{\sqrt{V(\varphi_i)}}),$$
$$c_3 = \max(0, \dot{\varphi}_i \max_{\varphi\in[\varphi_i, \frac{(d-2)H_i}{\dot{\varphi}_i}+\varphi_i]}(\frac{V''(\varphi)}{|V'(\varphi)|})). \tag{A.25}$$

From this point on, we take the above identities as definitions of $\dot{\varphi}_i$ and c_3. Note that for a given potential $V(\varphi)$, c_3, and $\dot{\varphi}_i$ are now functions of φ_i, c_1 and c_2. Therefore from now on, we show them as $c_3(c_1, c_2, \varphi_i)$ and $\dot{\varphi}(c_1, c_2, \varphi_i)$. By plugging (A.25) into the inequality (A.20), we find the following two-parameter family of inequalities for non-negative pair of numbers (c_1, c_2) where $c_2^2(2 + c_1^2) < (d-2)/(d-1)$. For every φ we have

$$\min(\frac{V(\varphi)}{|V'(\varphi)|}c_1, c_2)A_1(c_1, c_2, \varphi) < \sqrt{\frac{V(\varphi)}{V(\varphi + A_3(c_1, c_2, \varphi))}}\ln\left(\frac{A_2}{\sqrt{V(\varphi + A_3(c_1, c_2, \varphi))}}\right), \tag{A.26}$$

where,

$$A_1 = \frac{f(c_1, c_2, c_3(c_1, c_2, \varphi))g(\dot{\varphi}(c_1, c_2, \varphi))\sqrt{2}}{\sqrt{(d-1)(d-2)}(1 + f(c_1, c_2, c_3(c_1, c_2, \varphi)))},$$
$$A_2 = \sqrt{\frac{(d-1)(d-2)}{2}},$$
$$A_3 = f(c_1, c_2, c_3(c_1, c_2, \varphi))g(\dot{\varphi}_i(c_1, c_2, \varphi))\frac{\min(c_1\sqrt{V(\varphi)}, c_2\frac{|V'(\varphi)|}{\sqrt{V(\varphi)}})^2}{|V'(\varphi)|^2}. \tag{A.27}$$

The inequality (A.26), although complicated, is very strong. It is almost local in the sense that it mostly depends on the values of V and its derivatives at point φ, and provides a good way to see if an arbitrary potential violates TCC. This inequality does not depend on initial conditions since we used TCC for the initial conditions that seem to challenge TCC the most to find it. This feature makes it easy to be applied to an arbitrary potential numerically or a class of potentials analytically. For example, for convex potentials (A.26) takes much simpler form since $g(c_1, c_2, \varphi) = 1$. Note that in the case which c_2 is large enough such that $V/|V'|$ comes out of the $\min$ function on the LHS of (A.26), we get an inequality very similar to the dS conjecture except an extra logarithmic term. In fact, most of the local results that we find from TCC share this feature.

B

Unstable Critical Points

In this appendix we prove the inequality (2.49) which can be stated as in the following form.

Suppose φ_0 is a critical point (local maximum) of $V(\varphi)$, such that $V' < 0$ and $|V''(\varphi)| \leq |V''|_{\max}$ over the field range $\varphi_0 \leq \varphi \leq \varphi_0 + \Delta\varphi$. Then, either

$$\Delta\varphi < \frac{B_1(d) B_2(d)^{\frac{3}{4}} V_{max}^{\frac{d-1}{4}} V_{min}^{\frac{3}{4}} \ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{\frac{1}{2}}}{V_{min} B_2(d) - |V''|_{max} \ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^2}, \quad or \quad \frac{|V''|_{max}}{V_{min}} \geq B_2(d) \ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{-2}, \tag{B.1}$$

where $V_{max} = V(\varphi_0)$ and $V_{min} = V(\varphi_0 + \Delta\varphi)$ are respectively the maximum and the minimum of V over $\varphi \in [\varphi_0, \Delta\varphi]$, and $B_1(d)$, $B_2(d)$, and $B_3(d)$ are $\mathcal{O}(1)$ numbers given by

$$B_1(d) = \frac{\Gamma(\frac{d+1}{2})^{\frac{1}{2}} 2^{1+\frac{d}{4}}}{\pi^{\frac{d-1}{4}}((d-1)(d-2))^{\frac{d-1}{4}}},$$
$$B_2(d) = \frac{4}{(d-1)(d-2)},$$
$$B_3(d) = \sqrt{\frac{(d-1)(d-2)}{2}}. \tag{B.2}$$

To show the above result, we prove the following one parameter family of inequalities for $0 \leq c \leq 1$.

$$\Delta\varphi < \frac{c^{1/2}}{1-c^2} B_1(d) \frac{V_{max}^{\frac{d-1}{4}}}{|V''|_{max}^{\frac{1}{4}}} \quad or \quad \frac{|V''|_{max}}{V_{min}} \geq c^2 B_2(d) \ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{-2}. \tag{B.3}$$

One can check that by setting c equal to $\min(1, \varepsilon + \sqrt{\frac{|V''|_{max} \ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^2}{V_{min} B_2(d)}})$ and taking the limit $\varepsilon \to 0^+$, we can recover the statement (B.1).

Proof of (B.3)*:*

We start by assuming that first inequality in the (B.3) is violated, and will prove that for TCC to hold, the second inequality must be true. Violation of the first inequality implies

$$\Delta\varphi \geq \frac{c^{1/2}}{1-c^2} B_1(d) \frac{V_{max}^{\frac{d-1}{4}}}{|V''|_{max}^{\frac{1}{4}}}. \tag{B.4}$$

We treat the problem semi-classically in the sense that we demand the TCC to hold for all classical evolutions with initial conditions

$$\varphi(t=0) = \varphi_0 + \delta\varphi_i \;\;\&\;\; \dot{\varphi}(t=0) = \delta\dot{\varphi}_i, \tag{B.5}$$

where $\delta\varphi_i = \sqrt{\langle(\varphi - \varphi_0)^2\rangle}$ and $\delta\dot{\varphi}_i = \sqrt{\langle\dot{\varphi}^2\rangle}$. In the appendix C we study the quantum fluctuations to find the lower bound on the product $\delta\varphi_i \delta\dot{\varphi}_i$. Later, we will optimize our choice of initial conditions among all those that satisfy that uncertainty principle. Until then, we express all of our results in terms of arbitrary initial conditions $\delta\varphi_i$ and $\delta\dot{\varphi}_i$.

From the equation of motion (2.3), we have

$$\ddot{\varphi} \leq \ddot{\varphi} + (d-1)H\dot{\varphi} = -V' \leq |V''|_{max}(\varphi - \varphi_0), \tag{B.6}$$

where in the last inequality we used the mean value theorem. If we use the mean value theorem again, we find

$$\ddot{\varphi} \leq |V''|_{max}(\varphi - \varphi_0) \leq |V''|_{max} t \dot{\varphi}_{max} + \delta\varphi_i |V''|_{max}, \tag{B.7}$$

where $\dot{\varphi}_{max}(t) = max_{t' \in [0,t]}\{\dot{\varphi}\}$. If we integrate this inequality from $t' = 0$ to $t' = t$, using $\dot{\varphi}_{max}(t') \leq \dot{\varphi}_{max}(t)$ we find

$$\dot{\varphi} \leq \frac{|V''|_{max}}{2} t^2 \dot{\varphi}_{max} + |V''|_{max}\delta\varphi_i t + \delta\dot{\varphi}_i. \tag{B.8}$$

Since the right hand side is monotonic in t, and the left hand side is equal to $\dot{\varphi}_{max}$ for some $t' \in [0,t]$, we have

$$\dot{\varphi}_{max} \leq \frac{|V''|_{max}}{2} t^2 \dot{\varphi}_{max} + |V''|_{max}\delta\varphi_i t + \delta\dot{\varphi}_i. \tag{B.9}$$

Suppose c is a positive number smaller than 1, for $t \leq \sqrt{2/|V''|_{max}}c$, the above inequality gives us

$$\dot{\varphi}_{max} \leq \frac{|V''|_{max}\delta\varphi_i t + \delta\dot{\varphi}_i}{1 - \frac{|V''|_{max}t^2}{2}} \leq \frac{|V''|_{max}\delta\varphi_i t + \delta\dot{\varphi}_i}{1 - c^2}. \tag{B.10}$$

From $\dot{\varphi} \leq \dot{\varphi}_{max}$ we find

$$\dot{\varphi} \leq \frac{|V''|_{max}\delta\varphi_i t + \delta\dot{\varphi}_i}{1 - c^2}. \tag{B.11}$$

Integrating this inequality gives

$$\varphi - \varphi_0 \leq \frac{|V''|_{max} t^2 \delta\varphi_i}{2(1-c^2)} + \frac{\delta\dot{\varphi}_i t}{1-c^2} + \delta\varphi_i. \tag{B.12}$$

Using $t \leq c\sqrt{2/|V''|_{max}}$ again, we find

$$\begin{aligned} \varphi - \varphi_0 &\leq (1 + \frac{c^2}{1-c^2})\delta\varphi_i + \frac{\delta\dot{\varphi}_i}{1-c^2}\frac{c\sqrt{2}}{\sqrt{|V''|_{max}}} \\ &= \frac{2}{1-c^2}\delta\varphi_i + \delta\dot{\varphi}_i \frac{c\sqrt{2}}{(1-c^2)\sqrt{|V''|_{max}}}. \end{aligned} \tag{B.13}$$

The above inequality is true for all $t \leq c\sqrt{2/|V''|_{max}}$ such that $\varphi(t) \leq \varphi_0 + \Delta\varphi$. If the right hand side in (B.13) is less than $\Delta\varphi$, that would mean φ is in $[\varphi_0, \varphi + \Delta\varphi]$ for every $t \leq c\sqrt{2/|V''|_{max}}$. We show that initial conditions could be optimized to make sure that this happens without violating the uncertainty principle (C.7) $\delta\varphi_i \delta\dot{\varphi}_i \geq \frac{\Gamma((d+1)/2)H_i^{d-1}}{2\pi^{d-1/2}}$.

For the initial conditions

$$\begin{aligned} \delta\varphi &= \frac{c^{\frac{1}{2}}\Gamma(\frac{d+1}{2})^{\frac{1}{2}} H^{\frac{d-1}{2}}}{2^{\frac{3}{4}}\pi^{\frac{d-1}{4}}|V''|_{max}^{\frac{1}{4}}}, \\ \delta\dot{\varphi} &= \frac{\Gamma(\frac{d+1}{2})^{\frac{1}{2}} H^{\frac{d-1}{2}} |V''|_{max}^{\frac{1}{4}}}{c^{\frac{1}{2}} 2^{\frac{1}{4}} \pi^{\frac{d-1}{4}}}, \end{aligned} \tag{B.14}$$

the uncertainty principle gets saturated and the right hand side of (B.13) becomes equal to

$$B_1(d)\frac{c^{1/2}}{1-c^2}\frac{V_{max}^{\frac{d-1}{4}}}{|V''|_{max}^{\frac{1}{4}}}, \tag{B.15}$$

where we used the Friedmann equation $(d-1)(d-2)H_i^2/2 = V_{max}$. According to (B.4), the above expression is less than $\Delta\varphi$. Therefore, for these initial conditions, $\varphi \in [\varphi_0, \varphi_0 + \Delta\varphi]$ for every $t \leq c\sqrt{2/|V''|_{max}}$. If we set $t = c\sqrt{2/|V''|_{max}}$, from (1.4) we find

$$c\sqrt{\frac{2}{|V''|_{max}}} \leq -\frac{1}{H}\ln(H)$$
$$\leq \sqrt{\frac{(d-1)(d-2)}{2V_{min}}}\ln\left(\frac{\sqrt{\frac{(d-1)(d-2)}{2}}}{\sqrt{V_{min}}}\right), \tag{B.16}$$

which can be rearranged into

$$\frac{|V''|_{max}}{V_{min}} \geq c^2 B_2(d)\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{-2}, \tag{B.17}$$

which is our desired result.

Now we use the inequality (B.1) that we just proved to obtain a result for quadratic potentials. Suppose the quadratic potential $V(\varphi)$ has local maximum $V(\varphi_0) = V_0$ and second derivative $-|V''|$ over a field range $[\varphi_0, \varphi_0 + \sqrt{\frac{2(1-c)V_0}{|V''|}}]$ for some $0 \leq c \leq 1$. This field range corresponds to the potential range $[V_{\min}, V_0]$ where $V_{\min} = cV_0$. Let k be positive number smaller than 1. We can weaken the (B.1) by multiplying the right hand side of the second inequality by k as

$$\Delta\varphi < \frac{B_1(d)B_2(d)^{\frac{3}{4}}V_{max}^{\frac{d-1}{4}}V_{min}^{\frac{3}{4}}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{\frac{1}{2}}}{V_{min}B_2(d) - |V''|_{max}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{2}}, \quad or \quad \frac{|V''|_{max}}{V_{min}} \geq kB_2(d)\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{-2}. \tag{B.18}$$

If the second inequality gets violated, we get an upper bound on $|V''|$ in terms of V. Plugging this upper bound in the first inequality in (B.18) would weaken the above statement to

$$\Delta\varphi < \frac{B_1(d)B_2(d)^{\frac{3}{4}}V_{max}^{\frac{d-1}{4}}V_{min}^{\frac{3}{4}}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{\frac{1}{2}}}{(1-k)V_{min}B_2(d)}, \quad or \quad \frac{|V''|_{max}}{V_{min}} \geq KB_2(d)\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{-2}. \tag{B.19}$$

By plugging $\Delta\varphi = \sqrt{\frac{2(1-c)V_0}{|V''|}}$ and $V_{\min} = cV_0$ into the above inequalities we find either

$$\frac{|V''|}{V_0} > \frac{2(1-k)^2(1-c)c^{\frac{1}{2}}B_2(d)^{\frac{1}{2}}}{B_1(d)^2} V_0^{\frac{2-d}{2}} \ln\left(\frac{B_3(d)}{\sqrt{cV_0}}\right)^{-1}, \quad or \quad \frac{|V''|_{max}}{V_0} \geq kcB_2(d)\ln\left(\frac{B_3(d)}{\sqrt{cV_0}}\right)^{-2}. \tag{B.20}$$

In other words,

$$\frac{|V''|}{V_0} \geq \min(kcB_2(d)\ln\left(\frac{B_3(d)}{\sqrt{cV_0}}\right)^{-2}, \frac{2(1-k)^2(1-c)c^{\frac{1}{2}}B_2(d)^{\frac{1}{2}}}{B_1(d)^2} V_0^{\frac{2-d}{2}} \ln\left(\frac{B_3(d)}{\sqrt{cV_0}}\right)^{-1}) \tag{B.21}$$

We can optimize the above inequality by setting $k = 1 + D(V_0, d) - \sqrt{D(V_0, d)^2 + 2D(V_0, d)}$ where

$$D(V_0, d) = \frac{c^{\frac{1}{2}}B_2(d)^{\frac{1}{2}}B_1(d)^2 V_0^{\frac{d-2}{2}}}{4(1-c)} \ln\left(\frac{B_3(d)}{\sqrt{cV_0}}\right)^{-1}, \tag{B.22}$$

so that the two expressions in the $\min(,)$ become equal to each other. This gives

$$\frac{|V''|}{V_0} \geq (1 + D(V_0, d) - \sqrt{D(V_0, d)^2 + 2D(V_0, d)})cB_2(d)\ln\left(\frac{B_3(d)}{\sqrt{cV_0}}\right)^{-2}. \tag{B.23}$$

Note that the right hand side only depends on V_0. This is a potential dependent lower bound on $|V''|/V_0$ for quadratic potentials defined over a potential range $[cV_0, V_0]$ for some number $0 \leq c \leq 1$.

C

Uncertainty Principle

In this appendix we derive the uncertainty inequality for $\delta\varphi\delta\dot{\varphi}$ where $\delta\varphi = \sqrt{\langle(\varphi - \varphi_0)^2\rangle}$ and $\delta\dot{\varphi} = \sqrt{\langle\dot{\varphi}^2\rangle}$. Note that since we study the evolution of a Hubble patch, the field values that we work with are not the local field values $\varphi(x)$, instead they are averaged over a $(d-1)$-ball of radius $1/H$.

If we quantize a scalar field in a generic background, using a foliation $\Sigma(t)$ such that Σ's are Cauchy surfaces, for every $x_{d-1} \in \Sigma(t)$ and every function f on $\Sigma(t)$, the commutation relations would look like,

$$\int_{\Sigma(t)} f(x'_{d-1})[\hat{\varphi}(x_{d-1}), \partial_\mu\hat{\varphi}(x'_{d-1})]da^\mu_\Sigma(x'_{d-1}) = if(x_{d-1}), \tag{C.1}$$

where a^μ is the area vector with respect to the background metric. Suppose the metric take the form

$$ds^2 = dt^2 - g_\Sigma(t)dx_{d-1}^2. \tag{C.2}$$

The equation (C.1) would take the form

$$\int_{\Sigma(t)} \sqrt{g_\Sigma} f(x'_{d-1})[\hat{\varphi}(x_{d-1}), \partial_t\hat{\varphi}(x'_{d-1})]dx'_{d-1} = if(x_{d-1}), \tag{C.3}$$

which can be written as

$$[\varphi(x), \dot{\varphi}(x')] = i\delta_{\mu_\Sigma}(x - x'), \tag{C.4}$$

where δ_{μ_Σ} is the Dirac delta distribution on Σ with respect to the measure μ_Σ induced by g_Σ. If we define $\bar{\varphi}$, and $\bar{\dot{\varphi}}$ to be the average of φ and $\dot{\varphi}$ respectively over $M \subset \Sigma$ with respect to μ_Σ, integrating (C.4) over $\{(x, x') \in M \times M\}$ leads to

$$[\bar{\varphi}, \bar{\dot{\varphi}}] = \frac{i}{\mu_\Sigma(M)}. \tag{C.5}$$

If we take M to be a $(d-1)$-ball of Hubble radius $1/H$ in a spatially flat FRW background, we find

$$[\bar{\varphi}, \bar{\dot{\varphi}}] = \frac{i}{\frac{\pi^{d-1/2}}{\Gamma((d+1)/2)}\left(\frac{1}{H}\right)^{d-1}}, \tag{C.6}$$

which would result in the uncertainty principle

$$\delta\varphi_i\delta\dot{\varphi}_i \geq \frac{\Gamma((d+1)/2)H^{d-1}}{2\pi^{d-1/2}}. \tag{C.7}$$

D

Subtleties of the thin-wall approximation

In the main text we presented a simplified version of the thin-wall discussion to make the presentation easier to follow, but in such a way that the main conclusions are unaltered. The actual calculations are more complicated, and we discuss them here, in order of appearance in the main text.

D.0.1 Gravitational effects in thin-wall formulae

In the discussion in section 4.1, we neglected gravitational effects that are relevant when the bubble radius is comparable to the Hubble scale. This turns out a posteriori to be a good approximation since the results only change by an $\mathcal{O}(1)$ factors, but one needs to check the full result, which we do here.

Taking into account gravitational effects, the euclidean bounce solution[259,78] is given by

$$S = 2\pi^2 T r^3 + \frac{12\pi^2}{\kappa^2}\left\{\frac{1}{V_f}\left[(1-\frac{1}{3}\kappa V_f r^2)^{\frac{3}{2}}-1\right]-\frac{1}{V_i}\left[(1-\frac{1}{3}\kappa V_i r^2)^{\frac{3}{2}}-1\right]\right\}. \quad \text{(D.1)}$$

where T is the tension of the membrane (the wall), and the initial and final vacuum energies are V_i and V_f respectively, with $V_f < V_i$ (we will discuss the possibility of up-tunneling below). The critical radius corresponds to the value of r that minimizes the above action, which implies solving the following equation,

$$\gamma r = -T\sqrt{1-r^2\Lambda_i}, \quad \gamma \equiv \left(\frac{T^2}{4}+\Lambda_f-\Lambda_i\right). \quad \text{(D.2)}$$

We can see that non-trivial solutions exist only for $\gamma < 0$, namely for bubble radius smaller than the de Sitter horizon $r < \Lambda_i^{-1/2} = H_i^{-1}$ where H is the Hubble scale. We have defined $\Lambda \equiv \kappa V/3$ and set $\kappa = 1$ to work in Planck units. Notice that the case $\gamma = 0$ corresponds to a bubble of Hubble radius, while $\gamma < 0$ implies the following critical radius

$$R^2 = \frac{1}{\left(\frac{\gamma}{T}\right)^2+\Lambda_i} \quad \text{(D.3)}$$

Plugging this back into (D.1) one gets the final result for the action S. For later convenience, it is useful to define the parameters,

$$p \equiv \frac{T}{\sqrt{\Lambda}}, \quad q \equiv \sqrt{\Lambda}\left(\frac{T}{\Delta\Lambda}\right) = R_0 H \quad \text{(D.4)}$$

where we have renamed $\Lambda \equiv \Lambda_i$ and $R_0 \equiv T/\Delta\Lambda$ is the critical radius of a bubble in flat space. When the parameter p becomes small, gravitational corrections become subleading, and we can expand the instanton action for small p to obtain:

$$S = w(q)\frac{T}{\Lambda^{3/2}} + \mathcal{O}(p^2), \qquad \frac{w(q)}{2\pi^2} = \frac{1+2/q^2}{\sqrt{1+1/q^2}} - \frac{2}{q} \quad \text{(D.5)}$$

In this limit, the bubble radius reads

$$R^2 \simeq \frac{1}{\Lambda} \frac{1}{1+(1/q)^2} \;\to\; (RH)^{-2} \simeq 1 + (R_0 H)^{-2} \tag{D.6}$$

Hence, the size of the bubble is parametrised by the value of q, which yields two limits of interest. On one hand, if $q \ll 1$, the critical radius is small compared to the de Sitter horizon and we recover the result for the transition rate in the flat space limit,

$$S \simeq \frac{T}{\Lambda^{3/2}} q^3 = \frac{T^4}{\Delta\Lambda^3} \equiv S_0\,, \qquad R \simeq R_0 = \frac{T}{\Delta\Lambda} \tag{D.7}$$

On the other hand, if $q \simeq \mathcal{O}(1)$, the bubble radius is of Hubble size and we get

$$S \simeq \frac{T}{\Lambda^{3/2}}\,, \qquad R \simeq H^{-1} \tag{D.8}$$

Notice that $q = 1$ is the largest value that this parameter can take which is still consistent with a solution to (D.2), so (D.8) gives the largest possible radius and the smallest possible euclidean action of an instanton solution describing bubble nucleation in de Sitter space in the thin wall approximation.

We also comment briefly on the prefactor[78]. The full expression taking into account gravitational backreaction is

$$P = \frac{e^{-\zeta_R(-2)}}{4} T^2 R^2 \simeq \frac{T^2 R_0^2}{1 + (R_0 H)^2} \sim \frac{S^2}{R^4} \tag{D.9}$$

where the last equality is true modulo a $\mathcal{O}(1)$ function of HR only which goes to 1 at zero, and $\zeta_R(-2) = -0.0394\ldots$.

A precise determination of the prefactor outside of the thin-wall approximation requires the calculation of a one-loop determinant around the Euclidean saddle, and it is both complicated and detail-dependent (see[260] for an example). As shown in[78], in the thin-wall approximation it is possible to determine the prefactor since the only low-energy degrees of freedom that contribute to the prefactor are fluctuations of its local position (fluctuations of the Goldstone associated to translational invariance). These will have energies of the order of $1/R$, while we will assume that the next excitation, corresponding to internal worldvolume degrees of freedom, will appear at a much higher energy scale.

Even though this computation only took into account Goldstone modes, we expect our conclusions to hold modulo $\mathcal{O}(1)$ corrections if a finite number of worldvolume degrees of freedom at the scale of the Goldstones are included. For instance, if the domain wall is a D-brane, we would expect to have worldvolume gauge fields and gauginos as well. Since we are not sensitive to $\mathcal{O}(1)$ coefficients, we will drop it in (D.9).

Combining (D.9) and (D.5) we get that the transition rate per unit time and volume is given by

$$\Gamma = H^4 \left(\frac{T}{H^3}\right)^2 \frac{(R_0 H)^2}{1+(R_0 H)^2} \exp\left(-\frac{T}{H^3} w(R_0 H)\right) \tag{D.10}$$

The above thin-wall expressions can be easily generalised to any dimension[78]. We use this generalization in subsection 4.3.5.

D.0.2 Up-tunneling

In de Sitter space, up-tunneling is allowed due to the gravitational effects, but it is more suppressed than down-tunneling. More precisely, as discussed in[78], up-tunneling is described by the same kind of CdL instanton that down-tunneling, considering an anti-membrane rather than a membrane. In four dimensions, the difference in action between these two tunneling rates is

$$S_{\text{up}} - S_{\text{down}} \propto \frac{\Delta\Lambda}{\Lambda^2}, \tag{D.11}$$

which will be large in most of our parameter space, but can be significant near the eternal inflation point. Notice that the difference (D.11) is also essentially the difference between the entropies of the down-tunneling and up-tunneling de Sitters. The reason up-tunneling is suppressed is entropic.

Uptunnelling is responsible for the last term in the formula (E.19) for the effective potential. It is never dominant in the region allowed by the swampland conjectures.

D.0.3 Regime of validity of CdL formulae

In most of the region of interest to us, the action of the Euclidean instanton (4.5) is small. However, the usual lore is that semiclassical expressions such as these are only accurate as long as the instanton action is large and so there is exponential suppression. So how come that we can use it more generally? CdL is essentially an application of the WKB formula to field theory, and this is controlled not by whether there is exponential suppression or not, but by whether the perturbation parameter is small. These two notions can differ in a theory that has more than one parameter.

An illustrative example is Schwinger's original calculation of the decay via emission of charged pairs in (1+1) dimensions. Schwinger obtained a vacuum decay amplitude (later reinterpreted as a pair production rate[261,262]) given by

$$\Gamma = \frac{(qE)^2}{4\pi^3} e^{-\frac{\pi m^2}{qE}} . \tag{D.12}$$

This calculation was done in the semiclassical limit $q \to 0, E \to \infty$, with qE fixed. The small parameter is therefore the electron charge q and we can expect that (4.5) is just the first in a series of corrections suppressed in higher powers of q. The classical instanton action is becoming small when $m \to 0$, yet the result is still trustworthy because the expansion parameter is q, which remains small.

This is not always the case. When computing the potential generated for the θ parameter in a Yang-Mills theory, there is only one small coupling, the Yang-Mills coupling g. The instanton action $S = 8\pi^2/g^2$ only depends on g, and sending $g \to \infty$ means both that the instanton action becomes small and the perturbative expansion fails.

The CdL scenario is very similar to the Schwinger example above. Instead of the particle mass, we have the tension T, and the parameter qE in the Schwinger model, which is just the difference in vacuum energy before/after pair nucleation, is replaced by $\Delta\Lambda$ in our example. The change in vacuum energy $\Delta\Lambda$ is then related to the background flux density in the false vacuum as

$$\Delta\Lambda = g_3^2\, n, \tag{D.13}$$

where n is an integer parametrizing the background "top-form flux density". The parameter g_3 is the 3-form gauge coupling[263] which controls the strength of interactions and backscattering between the domain walls.

The thin-wall computations above should be understood as taking place in a formal limit where g_3 is going to zero and n diverges in such a way that

$$\Delta\Lambda = g_3^2 n \to \text{const.} \tag{D.14}$$

More physically, this should be thought of as a limit in which brane-brane interactions are switched off, but branes still respond to the background difference in vacuum energies. In such a limit, just as in the Schwinger case, we expect to be able to trust the thin-wall expression even when the membrane tension T is small and there is no exponential suppression since it is just the leading piece of the small $\Delta\Lambda$ expansion. In this case, the physics is dictated by the prefactor. But we also emphasize that this is an assumption we make and which we cannot prove. Ultimately, the reason for this is that the rigorous argument for the Schwinger effect above relied on the fact that we have a Lagrangian description of the system, which we are lacking in the higher-dimensional case since extended objects are an essential ingredient[1].

[1] We could try to compactify to $(1+1)$ dimensions to recover a Lagrangian description, or we could resort to the effective action for a probe brane/particle, which is available also in higher dimensions. But none of these arguments are conclusive since the probe brane approach we do not know how to compute corrections systematically, and compactification to $(1+1)$ would involve talking about wrapped branes, with very different kinematic properties.

E

Derivation of the effective potential

Here we discuss in detail the derivation of the effective potential introduced in the main text. The basic quantity we are interested in is

$$dN/dt, \tag{E.1}$$

the number of membranes that hit an observer per unit time. We will now do so by geometric means, as in[264].

Take d-dimensional de Sitter space in conformal flat slicing:

$$ds^2 = \frac{1}{\tau^2}\left(-d\tau^2 + \sum_{i=1}^{d-1} dx_i^2\right), \tag{E.2}$$

where the coordinates x_i parametrize flat space. Here $-\infty < \tau < 0$ parametrizes half of a dS_d (see Figure E.2). If the observer sets her clock such that $t = 0$ is at $\tau = -1$, then in general her proper time is related to τ as

$$\tau = -e^{-t}. \tag{E.3}$$

Let

$$\xi = \frac{dN}{dt} = -\tau\frac{dN}{d\tau} \tag{E.4}$$

be the number of bubbles that hit the worldline of a timelike static observer at $\vec{x} = \text{const}$. Our task is to determine ξ. We will do this by calculating ξ in two different ways, and then demanding they are both equal:

- ξ is related to the mean free path traversed by a bubble. Suppose one has a bubble whose wall is expanding at the speed of light. In the above coordinate system, it moves along a straight line $x = \tau$. When traversing a time interval $\Delta\tau$, there is a probability

  $$P_{\text{hit}} = -\frac{\xi}{2\tau}\Delta\tau \tag{E.5}$$

  that the bubble gets hit by another bubble entering its lightcone (see Figure E.1).

  If it gets hit, the bubble will annihilate and disappear. The probability that the bubble survives these collisions at $\tau + \Delta\tau$, $P_{\tau+\Delta\tau}$, is equal to the probability P_τ that it made it to τ, times the probability that it does not get hit by the bubble, so

  $$P_{\tau+\Delta\tau} = P_\tau + \Delta P_\tau = P_\tau\left(1 - \frac{\xi}{2\tau}\Delta\tau\right), \tag{E.6}$$

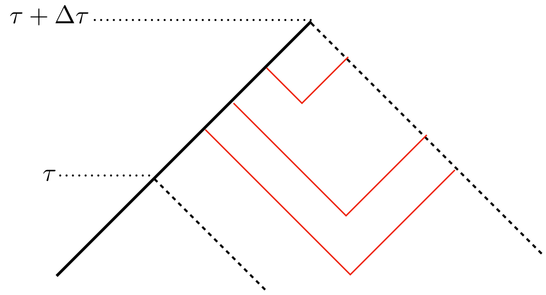

Figure E.1: In black, we have the worldline of an expanding domain wall. As it moves in a lightlike fashion through an interval $\Delta\tau$, there is a probability that it gets hit by a bubble. This probability depends on ξ, the number of membranes that arrive per unit time the worldline of static observers.

which has solution

$$P_\tau = \left(\frac{\tau_0}{\tau}\right)^{\frac{\xi}{2}}. \tag{E.7}$$

This equation gives the probability that a bubble which was born at conformal time τ_0 actually makes it to conformal time τ (see[264]). In terms of proper time, we have

$$P_t = P_{t_0} e^{-\frac{\xi}{2}(t-t_0)}, \tag{E.8}$$

which implies that the quantity ξ we are actually looking for is just the inverse of the mean free path of a bubble.

- Similarly, to compute the number of bubbles that hit an observer at conformal time τ_0 in a conformal time window $\Delta\tau$, we just need to integrate the bubble production rate on the past light cone, and correct by the factor (E.7) which establishes that only a fraction of the bubbles produced at conformal time τ actually make it to τ_0.

  Taking these two things into account, the number of bubbles we get is

$$\Delta N = \Gamma S_{d-2} \int_{-\infty}^{\tau_0} d\tau \int_{r(\tau,\tau_0)}^{r(\tau,\tau_0+\Delta\tau_0)} dV \left(\frac{\tau_0}{\tau}\right)^{\frac{\xi}{2}}, \tag{E.9}$$

  where S_{d-2} is the volume of S^{d-2}, and

$$r(\tau, \tau_0) = \tau_0 - \tau \tag{E.10}$$

parametrizes the null radial geodesic from (τ, r) to $(\tau_0, 0)$. Plugging everything back in, one gets

$$\frac{\Delta N}{\Delta \tau_0} = \Gamma S_{d-2} \int_{-\infty}^{\tau_0} \frac{(\tau_0 - \tau)^{d-2}}{\tau^d} \left(\frac{\tau_0}{\tau}\right)^{\frac{\xi}{2}}. \tag{E.11}$$

Evaluating the integral for $d = 4$, one obtains

$$-\frac{\xi}{\tau_0} = \frac{dN}{d\tau_0} = -\Gamma S_{d-2} \frac{16}{\tau_0(\xi+2)(\xi+4)(\xi+6)}, \tag{E.12}$$

which allows one to get (restoring the Hubble constant)

$$\frac{\xi}{H} = -3 + \sqrt{4\sqrt{\frac{\Gamma S_{d-2}}{H^d}+1}+5}. \tag{E.13}$$

On the other hand, we could write ξ as

$$\xi = \frac{dN}{dt} = \Gamma \mathcal{V}_{\text{eff}} \tag{E.14}$$

where $\mathcal{V}_{\text{eff}}$ is some effective volume. Notice that this effective volume

$$\mathcal{V}_{\text{eff}} = \frac{\xi}{\Gamma} = \frac{H}{\Gamma}\left(-3 + \sqrt{4\sqrt{\frac{\Gamma S_{d-2}}{H^d}+1}+5}\right) \tag{E.15}$$

is always below 1 in Hubble units. This works in general, but due to TCC we are interested in the regime $\Gamma/H^d \gg 1$, in which (E.13) is just

$$\frac{\xi}{H} \sim \frac{\Gamma^{1/4}}{H^{d/4}}. \tag{E.16}$$

and the effective volume becomes of order $\mathcal{V}_{\text{eff}} \sim \Gamma^{-3/4}$. Equation (E.16) can also be easily understood: in the regime where Γ is large and bubbles are efficiently produced, a bubble will die of a collision way before it notices the expansion of the universe. As a result, it will hit another bubble by the time its volume is such that the probability of bubble nucleation is of order one, in other words $\Gamma\xi^4 \sim 1$, which is precisely (E.16).

Once we have the number of bubbles that hit the observer per unit time, it is a simple task to relate this to the potential. Assuming slow-roll inflation, we get

$$\frac{dV}{dt} = \Delta\Lambda \frac{dN}{dt} = \Delta\Lambda\xi \approx \frac{(V')^2}{\sqrt{V}}, \tag{E.17}$$

We should also take into account that in dS there can be up-tunneling on top of down-tunneling, as discussed in appendix D.0.2. The effect will be small away from the curve $\beta = 2$, but significant for $\beta > 2$. up-tunneling membranes may annihilate with up-tunneling membranes and vice-versa, but down-tunneling and up-tunneling membranes just go through each other. As a result, to compute the change in vacuum energy, one simply has to replace (E.17) by

$$\frac{dV}{dt} = \Delta\Lambda \frac{dN}{dt}\left(1 - e^{-\Delta\Lambda/\Lambda^2}\right) \approx \frac{(V')^2}{\sqrt{V}}, \tag{E.18}$$

where the last equation is again due to slow-roll. The general effective scalar potential then reads

$$\frac{V'}{V} \simeq \Delta\Lambda^{1/2}\Lambda^{-3/4}\Gamma^{1/8}\sqrt{1 - e^{-\Delta\Lambda/\Lambda^2}}. \tag{E.19}$$

This has (4.10) and (4.11) as limits when the decay rate is very small or very large compared to Hubble. We use the full expression (E.19) to make the figures in the main text.

As a final comment, we should explain why we used the open slicing (E.2) which, as illustrated in Figure E.2, only covers half of de Sitter space. But clearly, bubbles that nucleate in the lower half of the diagram can reach the upper half! So why don't we take them into account? The answer is related to the phenomenon of "persistence of memory", beautifully discussed in [265,266]. It is often stated that, due to the exponential expansion, a de Sitter universe soon "forgets" any initial condition; indeed, this property is crucial for the success of the inflationary mechanism. It is, however, not true in general. Certain fields, such an electric current in two dimensions [266] and a 3-form current in four (the case we consider here) can develop vevs that partially break the de Sitter isometries and are never diluted by the exponential expansion (any nonzero vev will do this). While the top-form field-strength preserves all of the de Sitter isometries, the current it generates via quantum effects does not; it always picks a preferred reference frame, where e.g. it is purely spatial. This current vev in a sense retains information about what happened in the early universe, which didn't dilute away completely; hence, "persistence of memory".

Let us describe this in more detail. To do this, it is convenient to use conformal global coordinates, which cover all of de Sitter. In these, the metric is given by

$$ds^2 = \frac{1}{\cos^2(\chi)}\left(-d\chi^2 + d\Omega_{d-1}^2\right), \tag{E.20}$$

where the conformal time coordinate χ lives in $(-\pi/2, \pi/2)$ and the spatial slices are $(d-1)$ spheres.

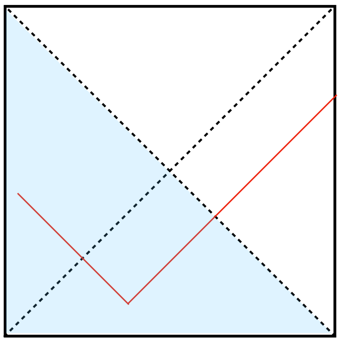

Figure E.2: A depiction of the conformal diagram of de Sitter space. The flat coordinates (E.2) only cover half of the spacetime (they do not cover the part shaded in blue). Global coordinates (E.20) cover all of spacetime. Surfaces of constant χ would be horizontal lines in the picture, while surfaces of constant τ asymptote to the past cosmological horizon of the patch they cover (the top left to down right diagonal line).

Suppose the universe nucleates into existence (has a Big Bang) at some particular initial timeslice. We will take this to happen at a constant global timeslice χ_{BB}, but the main lesson is actually independent of the choice of timeslice. This choice breaks the dS isometries, and when computing the bubble nucleation rate, one should stop integrating at the Big Bang. The number of bubbles that reach a static observer at a later time χ (ignoring backreaction, for simplicity) can be computed as the volume of the lightcone times the nucleation rate Γ, so (in $d=4$),

$$\frac{dN}{dt} = 4\pi\Gamma\cos(\chi)\int_{\chi_{\mathrm{BB}}}^{\chi}\frac{\sin^2(\chi-\chi_{\mathrm{BB}})}{\cos^4(\chi)} = \frac{4\pi}{3}\Gamma\frac{\sin^3(\chi-\chi_{\mathrm{BB})}}{\cos^3(\chi_{\mathrm{BB}})}, \tag{E.21}$$

where we have stated the result again in terms of proper time t. Notice that, with finite $\chi_{\rm BB}$, the result at late times is actually independent of $\chi_{\rm BB}$ and finite,

$$\lim_{\chi\to\pi/2}\frac{dN}{dt}=\frac{4\pi}{3}\Gamma, \tag{E.22}$$

while if we take $\chi_{\rm BB}\to -\pi/2$ first (which would correspond to a truly eternal de Sitter, with no Big Bang), the result diverges due to the volume factor in the denominator. This noncommutativity of limits is the technical manifestation of persistence of memory. Sending $\chi_{\rm BB}\to -\pi/2$ first corresponds to integrating the bubble nucleation rate over all of the past light cone of a point. This is a manifestly de Sitter-invariant way of computing dN/dt, so of course, it must produce a dS-invariant answer; but the only dS-invariant "values" of dN/dt are zero or infinity, so that's why the result diverges.

By contrast, sending $\chi\to\infty$ yields a finite value; at very late times, all the information about the Big Bang has been diluted away, except for the fact that nonzero dN/dt tells us that there *was* a Big Bang in the first place (or, at the very least, that the system does not enjoy de Sitter invariance). We focus on this possibility; because of the Big Bang, or for whatever other reason, there is a nonzero but finite dN/dt, which breaks de Sitter invariance, but the magnitude of dN/dt forgets the details about the initial timeslice. This justifies using open slicing (E.2) and only integrating over half of the dS; it corresponds to having a Big Bang at $\tau=-\infty$, and it leads to particularly simple computations. As we have just explained, the result at late times is independent of this choice.

F

Constraints on Hawking-Moss

In the Hawking-Moss (HM) transition[79] the universe tunnels from a local minimum to a local maximum and it happens everywhere at once. This spatial homogeneous transition can be interpreted[267] as a thermal fluctuation of a horizon-sized region up to the top of the barrier, followed by the rolling of the field down to the true vacuum. It also has an entropic interpretation[268] since it can be shown to be completely determined by the gravitational entropy of the system. Another characteristic of the HM transition is its relatively small decay rate given by[79,267]

$$\Gamma \lesssim H^4 \exp\left(\frac{1}{V_{\rm i}} - \frac{1}{V_{\rm top}}\right) \sim \Lambda^2 \exp\left(-\frac{\delta\Lambda}{\Lambda^2}\right), \tag{F.1}$$

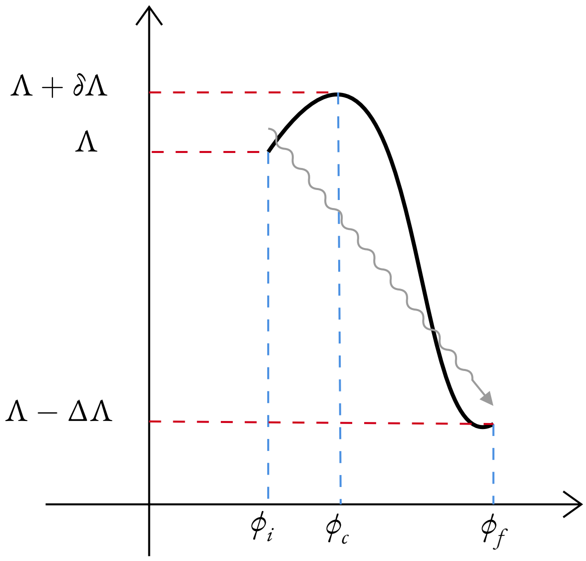

Figure F.1

where in the last step we have used that the height of the potential barrier $\delta\Lambda \equiv V_{top} - V_i$ is small compared to the initial vacuum energy $\Lambda = V_i \simeq V_{top}$. Otherwise, the transition will be dominated by thin-walls. This decay rate corresponds to a transition time which is greater than the Hubble time. This suggests that a HM transition in which the physics does not change drastically can only be marginally consistent with TCC, since it might still allow for an originally subplanckian mode of the first vacuum to become Hubble-sized after the transition. One might attempt to resolve this tension by requiring the physics in the two vacua to be sufficiently different so the fluctuations in one can no longer be expressed in terms of long-wavelength fluctuations of light degrees of freedom in the other. The distance conjecture suggests that for this to be true, the field must traverse a trans-Planckian range. In the following we show that even trans-Planckian field ranges do not mitigate the tension between HM transition and TCC.

We consider the potentials of the form shown in figure F.1. Suppose $\Delta\Lambda$ is the energy difference between the initial and final vacua and $\delta\Lambda$ is the height of the potential as shown in figure F.1.

Under some circumstances, the TCC implies the refined dS conjecture up to some logarithmic corrections which can be neglected for order of magnitude analysis. We will come back to the required condition and check them later. For now, we assume the refined dS conjecture is true,

$$\frac{|V''|}{V} > \mathcal{O}(1). \tag{F.2}$$

If we estimate the potential interpolating between the two vacua with an inverted parabola, we find

$$\phi_f - \phi_i \simeq \frac{\sqrt{2}(\sqrt{\delta\Lambda + \Delta\Lambda} + \sqrt{\delta\Lambda})}{\sqrt{|V''|}}. \tag{F.3}$$

As we discussed in the previous section, $\Delta\Lambda \lesssim \delta\lambda$ corresponds to the thin-wall approximation. Therefore, for Hawking-Moss transition we need $\Delta\Lambda \gtrsim \delta\lambda$. Using this inequality we can simplify (F.3) to find

$$\phi_f - \phi_i \sim \sqrt{\frac{\Delta\Lambda}{|V''|}}. \tag{F.4}$$

Plugging this into (F.2) leads to

$$\Delta\varphi = \phi_c - \phi_i < \phi_f - \phi_i \lesssim \sqrt{\frac{\Delta\Lambda}{\Lambda}}, \tag{F.5}$$

and from $\Delta\Lambda < \Lambda$, we find

$$\Delta\varphi \leq \mathcal{O}(1). \tag{F.6}$$

Thus, for the potential to be consistent with the TCC the field range must be sub-Planckian, but as we discussed in the beginning of the section, this poses a tension with the other swampland conjectures, in particular the Distance Conjecture.

Now we go back and check the assumptions we made to get (F.2). In [19] it was shown that in d spacetime dimensions, the TCC would imply the refined dS conjecture if

$$\Delta\varphi \geq \frac{B_1(d)B_2(d)^{\frac{3}{4}}V_{max}^{\frac{d-1}{4}}V_{min}^{\frac{3}{4}}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^{\frac{1}{2}}}{V_{min}B_2(d)-|V''|_{max}\ln\left(\frac{B_3(d)}{\sqrt{V_{min}}}\right)^2}, \tag{F.7}$$

where V_{max} and V_{min} are respectively the maximum and the minimum of V over $\varphi \in [\varphi_i, \varphi_c]$, and $B_1(d)$, $B_2(d)$, and $B_3(d)$ are $\mathcal{O}(1)$ numbers given by

$$\begin{aligned}
B_1(d) =& \frac{\Gamma(\frac{d+1}{2})^{\frac{1}{2}}2^{1+\frac{d}{4}}}{\pi^{\frac{d-1}{4}}((d-1)(d-2))^{\frac{d-1}{4}}},\\
B_2(d) =& \frac{4}{(d-1)(d-2)},\\
B_3(d) =& \sqrt{\frac{(d-1)(d-2)}{2}}.
\end{aligned} \tag{F.8}$$

We show that either the above condition holds and henceforth (F.2) is true, or the field range is sub-Planckian. Since we proved (F.2) leads to a sub-Planckian field range, this would prove that in either case the field range must be sub-Planckian which is our final desired result. We prove this claim by contradiction. Suppose $\frac{|V''|}{V} << \mathcal{O}(1)$ and $\Delta\varphi$ is trans-Planckian, in particular $\Delta\varphi >> \Lambda^{\frac{1}{2}}$. We show that this leads to a contradiction.

Since $|V''| << V$ the denominator of RHS in (F.7) is dominated by V. Moreover, $\delta\Lambda \lesssim \Delta\Lambda < \Lambda$, thus $V_{\max} \sim V_{\min} \sim \Lambda$. Plugging in $V_{max} \simeq V_{min} \simeq \Lambda$ and neglecting the $\mathcal{O}(1)$ terms, including the logarithmic terms, makes (F.7) take the following form.

$$\Delta\varphi \lesssim \Lambda^{\frac{1}{2}}, \tag{F.9}$$

which is in contradiction with $\Delta\varphi > \mathcal{O}(1)$. This proves our claim by contradiction. To summarize, we showed that for the Hawking-Moss transition to be consistent with the TCC the field range traversed during the transition must be sub-Planckian. But then, there is no reason to expect the physics to drastically change after the HM transition, and we run into the problems explained at the beginning of the section: the lifetime associated with HM is of order or greater than Hubble, which might exceed the TCC time getting into tension with the conjecture.

Note that we assumed that second-order expansion around the peak of the potential reasonably approximates the ridge of the potential. One could argue that this makes our derivation somewhat model-dependent.

G

Thought experiment: Meeting beyond the Hubble horizon

We present the de Sitter version of the thought experiment discussed in subsection 5.2. This thought experiment was developed in conversations with Cumrun Vafa and Georges Obied. We show that $\tau_s \gtrsim \tau_{TCC}$.

We assume that the vacuum is BD after a scrambling time. As we will see, this is consistent with our final result $\tau_s \gtrsim \tau_{TCC}$, because as we showed in subsection 5.2, any vacuum evolves into BD in τ_{TCC}. Since BD vacuum is maximally entangled across the Hubble horizon, we can use it to perform the Hayden-Perskill protocol[117] to recover the information. Therefore, the information of a system that has exited the Hubble horizon can be recovered after the scrambling time.

Consider Alice and Bob carrying two fully entangled q-bits. The idea is to have Bob cross Alice's horizon and see if Alice can get two copies of Bob's state; one through Hawking radiation and another via a null signal from Bob. If Alice succeeds, both the no-cloning theorem and the monogamy theorem would be violated.

Alice is initially stationary with respect to the comoving frame. Bob crosses Alice's stretched horizon located at $\sim l_P$ from the Hubble horizon and the spacetime point X. After he is one l_P outside the Hubble horizon, he ,makes a measurement on the q-bit and sends the outcome by a null ray toward Alice. We consider an extra l_P since any emergent phenomenon from quantum gravity such as a horizon has a Planckian resolution.

We call the spacetime point at which Bob sends the signal Y. As Bob jumps in, the information of the qbit he is carrying will thermalize and radiate back to Alice from the stretched horizon after a scrambling time $t_s \simeq 1/H \log(1/H)$. We denote the point of radiation by Z. Suppose Alice moves toward the horizon on a null ray and catches the signal midway at spacetime point T.

We consider a de Sitter space with flat coordinates such that $t_X = 0$ and the scale factor at X is set to 1.

The metric takes the form

$$ds^2 = dt^2 - a(t)^2[dr^2 + d\Omega^2], \tag{G.1}$$

where $a(t) = e^{Ht}$. Therefore, $r_X = \frac{1}{H} - 1$ and $t_X = 0$. From $ds^2 \geq 0$ we find $t_Y - t_X \geq r_Y - r_X$.By plugging in $r_X = \frac{1}{H} - 1$ and $r_Y = \frac{1}{H} + 1$ we find $t_Y \geq 2$. Bob's information radiate back off of the stretched horizon after a scrambling time τ_s. Thus $t_Z \sim \tau_s$. The physical distance of Z

which is on the stretched horizon from the Hubble horizon is $\sim l_P$, therefore

$$a(t_Z)r_Z = \frac{1}{H} - 1 \rightarrow r_Z = e^{-H\tau_s}(\frac{1}{H} - 1). \tag{G.2}$$

Now we solve the null ray equation to find when the radiation will reach Alice at T.

$$\begin{aligned}
&ds = 0 \rightarrow dt = -a(t)dr \\
&\rightarrow \Delta \frac{1}{H} e^{-Ht} = \Delta r \\
&\rightarrow e^{-Ht_T} - e^{-H\tau_s} = -H \frac{e^{-H\tau_s}(\frac{1}{H} - 1)}{2} \\
&\rightarrow e^{-Ht_T} = e^{-H\tau_s}(H + \frac{1}{2}).
\end{aligned} \tag{G.3}$$

Now to see if there is any cloning paradox we should see if the future lightcones of T and Y intersect. For points in the future light cone of T, we have,

$$\begin{aligned}
dt \geq a(t)dr \rightarrow \frac{1}{H}(e^{-Ht_T} - e^{-Ht}) \geq r \\
\rightarrow r \leq \frac{1}{H}(e^{-H\tau_s}(H + \frac{1}{2}) - e^{-Ht}).
\end{aligned} \tag{G.4}$$

For points in the future light cone of Y, we have,

$$\begin{aligned}
dt \geq -a(t)dr \rightarrow \frac{1}{H}(-e^{-Ht_Y} + e^{-Ht}) \leq r - r_Y \\
\rightarrow r \geq \frac{1}{H} + 1 - \frac{1}{H}(e^{-2H} - e^{-Ht}) = \frac{e^{-Ht}}{H} + 1 + \frac{1 - e^{-2H}}{H}.
\end{aligned} \tag{G.5}$$

From (G.4) and (G.5) we find the following inequality must hold to prevent the future lightcones of T and Y from intersecting so that the no-cloning theorem is not violated.

$$e^{-H\tau_s}(1 + \frac{1}{2H}) \leq 1 + \frac{1 - e^{-2H}}{H}. \tag{G.6}$$

For sub-planckian energy dinsities $H < 1$, the LHS is $e^{-H\tau_s}\mathcal{O}(1)$ and the RHS is $\mathcal{O}(1)$.

$$e^{-H\tau_s} \lesssim H \to \tau_s \gtrsim \frac{1}{H}\ln\left(\frac{1}{H}\right). \tag{G.7}$$

Note that if Bob could send his signal right after exiting the horizon instead of l_P beyond the horizon, Alice could catch it and the experiment would fail. The Planckian resolution of the stretched horizon plays an important role in preventing a cloning paradox.

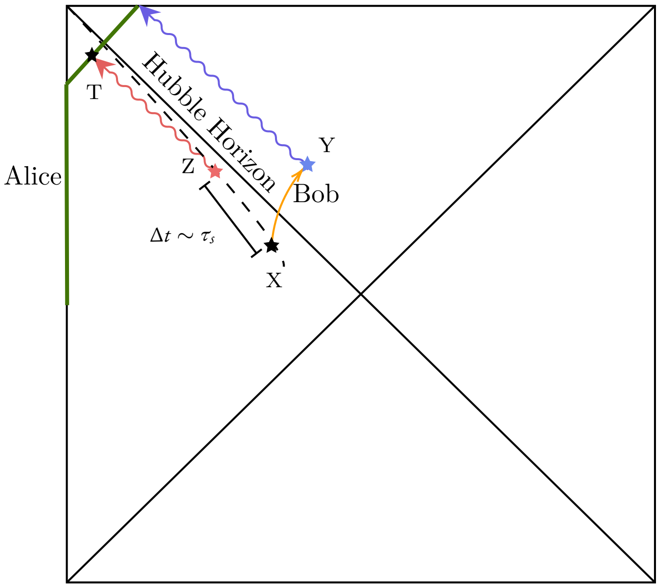

Figure G.1: Penrose diagram of the thought experiment in de Sitter space. Alice (green curve) and Bob (orange curve) carry entangled q-bits. The red and blue squiggly lines respectively represent the Hawking radiation and Bob's message both of which carry a copy of Bob's q-bit's information

H

Tension of the supergravity string

We use supersymmetry to find the tension of a BPS supergravity string. We present the calculation in 7d where there is a nice way of labeling the vectors in the gravity multiplet. However, as we will explain later, the argument applies to any theory with 16 supercharges.

The gravity multiplet in $7d$ has three graviphotons and the smallest irreducible spinor representation is a pseudo Majorana spinor which can be viewed as two sets of 8 real grassman numbers. These two are actually related. The gravity multiplet can be packaged in representations of $Sp(1)$. The pseudo Majorana spinors carry a 2 dimensional representation and are labeled by $i \in \{1, 2\}$ and the graviphotons furnish a three dimensional representation A^{ij} where A is antisymmetric in i and j. The theory also has a pseudo Majorana spinor χ^i which can be viewed as a doublet in $Sp(1)$.

In this notation, the supersymmetry transformation rules take a simple form. The fields are tetrads e^{μ}_{A}, gravitino $\psi^{\mu}_{a,i}$, graviphotons $A^{\mu i}_{j}$, 2-form $B^{\mu\nu}$, fermions χ^{a}_{i}, and the dilaton φ. Greek indices are spacetime indices, $\{A, B, ...\}$ are Lorentz indices, $\{i, j, ...\}$ are $Sp(1)$ indices, and $\{a, b, ...\}$ are spinor indices. We will often drop the spinor indices in the calculations. Also, $\gamma^{\mu} = \gamma^{A} e^{\mu}_{A}$ where γ^{A} are 7d Dirac matrices.

Now let us go back to our question of interest. Consider the string electrically coupled to $B^{\mu\nu}$. This string has an action term

$$S \propto i \int \star B \wedge dX \wedge dX + ... \tag{H.1}$$

Now we consider the action of supersymmetry on the field $B_{\mu\nu}$ to see how this action transforms. The full list of possible supersymmetry transformations of the fields in the gravity multiplet is given as below[269] [1].

[1] We have used the fact that the spinors are pseudo Majorana to rewrite the terms in a slightly different way that makes the calculations simpler. In particular, we have expressed the transformation rules of all of the bosonic fields in terms of $\bar{\varepsilon}$ rather than ε.

$$
\begin{aligned}
\delta e^A &= \kappa\bar{\varepsilon}^i\gamma^A \wedge \psi_i \\
\delta\psi_i &= \frac{2}{\kappa}D\varepsilon_i + c_1 \star [\star(\gamma\wedge\gamma\wedge\gamma\varepsilon_j)\wedge F^i_j]e^{q\kappa\varphi} \\
&+ d_1 \star (\gamma\varepsilon_j \wedge \star F^j_i)e^{q\kappa\varphi} \\
&+ c_2 \star [\star(\gamma\wedge\gamma\wedge\gamma\wedge\gamma\varepsilon_i)\wedge G]e^{q\kappa\varphi} \\
&+ d_2 \star (\gamma\wedge\gamma\varepsilon_i \wedge \star G)e^{r\kappa\varphi} + \text{bilinear fermions} \\
\delta\chi_i &= c_3 \star [(\gamma\wedge\gamma\varepsilon_i)\wedge\star F^i_j]e^{q\kappa\varphi} \\
&+ c_4 \star [(\gamma\wedge\gamma\wedge\gamma\varepsilon_i)\wedge\star G]e^{r\kappa\varphi} \\
&+ c_5 \star (\gamma\varepsilon_i \wedge \star d\varphi) + \text{bilinear ferimoins} \\
\delta A^j_i &= f_1(\bar{\varepsilon}^j\psi - \frac{1}{2}\delta^j_i\bar{\varepsilon}^k\psi)e^{-q\kappa\varphi} \\
&+ f_3(\bar{\varepsilon}^j\gamma\chi_i - \frac{1}{2}\delta^j_i\bar{\varepsilon}^k\gamma\chi_k)e^{-q\kappa\varphi} \\
\delta B &= (f_2\bar{\varepsilon}^i\gamma\wedge\psi_i + f_4\bar{\varepsilon}^i\gamma\wedge\gamma\chi_i)e^{-r\kappa\varphi} + p_2 A^j_i \wedge \delta A^i_j \\
\delta\varphi &= f_5\bar{\varepsilon}^i\chi_i,
\end{aligned}
\tag{H.2}
$$

where $G = dB$, $F^i_j = dA^i_j$, γ^A is a 0-form, γ is a 1-form, ψ_i is a 1-form, κ is proportional to the 7d Newton constant, and $\{c_i, d_i, f_i, p_i, r, q\}$ are all non-zero numerical coefficients that are determined by the closure of the supersymmetry algebra. Note that, modulo the $Sp(1)$ index $\{i, j\}$, the above expressions hold for all dimensions. Note that the exponential factor $e^{-r\kappa\varphi}$ is the fundamental charge of the supergravity string, because the kinetic term for B takes the following form

$$
S^{7d}_B = -\int \frac{1}{2} dB \wedge \star dB e^{2r\kappa\varphi} \tag{H.3}
$$

The supersymmetry transformation of the action (H.1) is $\delta B = (f_2\bar{\varepsilon}^i\gamma\wedge\psi_i + f_4\bar{\varepsilon}^i\gamma\wedge\gamma\chi_i)e^{-r\kappa\varphi_0} + p_2 A^j_i \wedge \delta A^i_j + \ldots$. For the string action to be supersymmetric, this variation must be canceled by the supersymmetry transformation of other terms in the action. Note that φ_0 is the asymptotic value of φ which sets the coupling constant of the theory. It is almost clear how to cancel the last term since it already contains the supersymmetry transformation of $A^j_i \wedge A^i_j$. What is less clear, is how to cancel the other two terms. We will focus on $\bar{\varepsilon}^i\gamma \wedge \psi_i e^{-r\kappa\varphi_0}$ term. It is easy to see that if the supersymmetric variation of a term does not depend on anything other than e and ψ and has a single γ matrix, that term must only depend on e.

Therefore, we are looking for a geometric 2-form. Moreover, our 2-form must be a function of e and not its derivatives. The only such area form is the induced volume form on the worldsheet. Let us verify that the induced area gives the correct variation under supersymmetry transformation. Suppose h is the induced metric. We can express $h_{\alpha\beta}$ in terms of the tetrads as

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu e^M_\mu e^N_\nu \eta_{MN}. \tag{H.4}$$

Moreover, we can write the determinant h as

$$h = \frac{1}{2} h_{\alpha\beta} h_{\alpha'\beta'} \varepsilon^{\alpha\alpha'} \varepsilon^{\beta\beta'}. \tag{H.5}$$

and its supersymmetry variation is

$$\delta\sqrt{-h} = -\frac{h_{\alpha'\beta'}\varepsilon^{\alpha\alpha'}\varepsilon^{\beta\beta'}}{\sqrt{-h}} \partial_\alpha X^\mu \partial_\beta X^{nu} e^N_\nu \delta e^M_\mu \eta_{MN}. \tag{H.6}$$

In conformal coordinates, the first term simplifies as

$$\frac{h_{\alpha'\beta'}\varepsilon^{\alpha\alpha'}\varepsilon^{\beta\beta'}}{\sqrt{-h}} = \varepsilon^{\alpha\beta}. \tag{H.7}$$

Working in conformal coordinates, and plugging in δe^M_μ from (H.2) gives

$$\delta\sqrt{-h} d^2\sigma = -e^j_\nu \delta e^i_\mu \eta_{ij} dx^\mu dx^\nu = -\kappa\bar{\varepsilon} \wedge \gamma\psi. \tag{H.8}$$

which is coordinate independent and holds in every coordinate system. Therefore, we find that to cancel the supersymmetry transformation of (H.1) we must add

$$i\frac{f_2}{\kappa} \int \sqrt{-h} e^{-r\kappa\varphi}. \tag{H.9}$$

The prefactor $\frac{f_2}{\kappa}$ is a pure imaginary number that is determined by the closure of supersymmetry algebra. Similarly, the number r is determined by supersymmetry. For example, in 7d, we have

$$f_2 = \frac{i}{\sqrt{2}}, \ \ r = -\frac{2}{\sqrt{5}}. \tag{H.10}$$

In general, we find that supersymmetry dictates the term

$$S_{\text{Nambu-Goto}} = -\frac{A_d}{2\pi} \int \sqrt{-h} e^{c_d \varphi} \tag{H.11}$$

for some dimension dependent positive constants A_d and c_d. This also implies that the tension of the supergravity string is given by

$$T = A_d e^{c_d \varphi_0}, \tag{H.12}$$

where $e^{c_d \varphi_0}$ is the coupling of the 2-form B in the gravity multiplet.

In 10 dimensions, $r = 1/\sqrt{2}$. Therefore, the tension of the string is given by

$$T = A e^{\hat{\varphi}_0/\sqrt{2}}, \tag{H.13}$$

for some constant A. We can simplify our equations by shifting $\hat{\varphi}$ by a constant to absorb the coefficient A. This shift will make the normalization of B non-canonical, but the normalization of $\hat{\varphi}$ will remain canonical.

$$T = e^{\hat{\varphi}_0/\sqrt{2}} M_{10}^2. \tag{H.14}$$

Note that in our convention,

$$S = \frac{T}{2\pi} \cdot \text{Area}. \tag{H.15}$$

I

Ground states of winding BPS string

In this section, we determine the Spin(8) representation of the ground state of the static winding BPS supergravity string[1]. After fixing the coordinates on the worldsheet, the low energy action on the string would be in terms of spacetime coordinates X^i with $i \in \{1, 2, \dots, 8\}$. In Appendix A, we found that the action around the ground state of a BPS supergravity string is

$$S = -T \int \sqrt{-g} d^2 \sigma. \tag{I.1}$$

where $T = A_d e^{\epsilon_d \varphi}$ and g is the induced metric on the string worldsheet. The above action is trustable for perturbations around BPS configurations as well.

[1] The partial supersymmetry breaking by the winding string was discussed, e.g., in Refs. [270,271].

The action (I.1) is not the full worldsheet action. Any spacetime supercharge that is preserved by the string, maps to a global charge on the worldsheet. The corresponding worldsheet charges can be found by the Green-Schwarz formalism[272] for studying supersymmetric branes[273]. However, the detail of the worldsheet action is not important for us.

We compactify the X^9 direction with the radius R, $X^9 \in [0, 2\pi R)$. We consider a static string winding around a circle. We choose the worldsheet coordinates as

$$(\sigma, \tau) = \left(\frac{X^9}{R}, X^0\right), \tag{I.2}$$

and we have $g_{\tau\tau} = 1$ and $g_{\sigma\sigma} = R^2$. This gauge choice is known as the static gauge or unitary gauge[274,275,276].

The winding string along the X^9 direction breaks the Lorentz symmetry as

$$SO(9,1) \to SO(8) \times SO(1,1). \tag{I.3}$$

Let us take a closer look at the supercharges in 10d non-compact spacetime and their action on the BPS string. Suppose we start with a supergravity with 32 supercharges. The BPS string preserves half of the supersymmetry. These supercharges act on the worldsheet fields and must be in fermionic representations of $SO(8)$ corresponding to rotations in the transverse coordinates to the string. This symmetry manifests itself as the R-symmetry of the worldsheet theory. The smallest spinor representation of $SO(8)$ is 8 dimensional. The supercharges must also furnish representations of $SO(1,1)$ which is the Lorentz group of the worldsheet. The irreducible representations of $SO(1,1)$ are one-dimensional and can be left or right handed. However, since the R-symmetry maps the supercharges in an irreducible representation of $SO(8)$ to each other, they must all have the same worldsheet handedness. Therefore, the supercharges lead to worldsheet charges that come in groups of 8 that all have the same worldsheet handedness and are in the vector representation of $SO(8)$. For theories with

32 spacetime supercharges, we find the following two possibilities:

We consider the dimensionally reduced 9d supersymmetry algebra. In Spin(8) notation corresponding to the spatial rotation, we have[277][2]

$$\{Q_A^a, Q_B^{\dot{b}}\} \sim \gamma^j_{a\dot{a}} p^i \delta_{AB}, \tag{I.4}$$

where $i = 1, \cdots, 8$ is the direction transverse to the string, $A, B = 1, 2$ is the label of the supersymmetry charges, and $a = 1, \cdots, 8$ and $\dot{a} = 1, \cdots, 8$ are the indexes for 8_S and 8_C representations, respectively. The string states are not invariant under p^i action since this generates translation along the transverse direction to the string. This indicates that the BPS string can preserve either Q^a or $Q^{\dot{a}}$, but not both.

Suppose that Q^a is preserved and $Q^{\dot{a}}$ is broken. Then, we can normalize $Q^{\dot{a}}$ in such a way that

$$\{Q^{\dot{a}}, Q^{\dot{b}}\} = \delta^{\dot{a}\dot{b}}, \tag{I.5}$$

is satisfied. This is analog to the Clifford algebra, and by utilizing the triality relation in Spin(8), we see that the representation of the algebra above is

$$8_v \oplus 8_s. \tag{I.6}$$

Similarly, if $Q^{\dot{a}}$ is preserved and Q^a is broken, then the algebra of the broken supersymmetry tells us that the representation is

$$8_v \oplus 8_c. \tag{I.7}$$

Therefore, by looking at the broken supersymmetry, we can find the Spin(8) representation of the BPS supergravity string.

[2] The minimal spinor in 9d is 16 components Majorana spinor, which can be decomposed into $8_s + 8_c$ of Spin(8). Since we are working in the theories with 32 supercharges, we have two copies of the Majorana spinor.

- 10d $\mathcal{N} = (1,1)$ supergravity (IIA supergravity):

  The supercharges are the two 10d Majorana-Weyl spinors with different chirality. These supercharges are decomposed as

$$(8_s, 1/2) \oplus (8_c, -1/2), \qquad \text{and} \qquad (8_s, -1/2) \oplus (8_c, 1/2), \tag{I.8}$$

  under $SO(8) \times SO(1,1)$. Here $-1/2$ and $1/2$ correspond to the left and right-handed fermions, respectively.

  Depending on the symmetry breaking pattern, we obtain either $\mathcal{N} = (0,16)$ or the vector $\mathcal{N} = (8,8)$ as a 2d worldsheet theory. In the following, we argue that it is not possible to obtain the 2d $\mathcal{N} = (0,16)$ worldsheet theory assuming that the theory flows to an SCFT without the symmetry enhancement.

  First, as the perturbative anomaly of the bulk theory is canceled without Green-Schwarz mechanism [278], there is no anomaly inflow [279] to the BPS string. This indicates that the central charges of the left-mover and right-mover satisfy the relation [18]

$$c_L - c_R = 0. \tag{I.9}$$

  Moreover, we use the knowledge of $\mathcal{N} = (0,2)$ superconformal subalgebra in the $\mathcal{N} = (0,16)$ theory. By identifying the R-symmetry $U(1)_R$ of the $\mathcal{N} = (0,2)$ superconformal algebra as an $SO(2)$ subgroup of the $SO(8)$ rotation group, we obtain the relation $c_R = 0$ [18]. In total, we have

$$c_L = c_R = 0. \tag{I.10}$$

  However, this contradicts the fact that there must be the center of mass degrees of freedom.

  Therefore, we conclude that the worldsheet theory possesses $\mathcal{N} = (8,8)$.[3] The

[3] In this case, Eq. (I.9) is still correct, but Eq. (I.10) is modified.

supercharge preserved by the BPS string is either

$$(8_s, 1/2), \qquad \text{and} \qquad (8_s, -1/2), \tag{I.11}$$

or

$$(8_c, -1/2), \qquad \text{and} \qquad (8_c, 1/2). \tag{I.12}$$

The worldsheet supersymmetry is $\mathcal{N} = (8_s, 8_s)$ or $(8_c, 8_c)$. In both cases, the Spin(8) chirality is the same. This means that the Spin(8) representation of the BPS string is

$$(8_v \oplus 8_{s(c)}) \otimes (8_v \oplus 8_{s(c)}). \tag{I.13}$$

Note that BPS states contain 2^8 states while non-BPS states contain 2^{16} states.

- 10d $\mathcal{N} = (2, 0)$ supergravity (IIB supergravity):

  The supercharges are the two 10d Majorana-Weyl spinors with the same chirality. Both supercharges are decomposed as

$$Q_{A=1,2} : (8_s, 1/2) \oplus (8_c, -1/2), \tag{I.14}$$

  under $SO(8) \times SO(1,1)$.

  As in the previous case, the supercharge preserved by the BPS string is

$$Q_{A=1} : (8_s, 1/2), \qquad Q_{A=2} : (8_c, -1/2), \tag{I.15}$$

  or vice versa.

  Therefore the worldsheet supersymmetry is $\mathcal{N} = (8_s, 8_c)$ or $(8_c, 8_s)$, and the Spin(8) representation of the BPS string is

$$\left(8_v \oplus 8_{s(c)}\right) \otimes \left(8_v \oplus 8_{c(s)}\right). \tag{I.16}$$

J

Massive vector multiplets in supergravity

In this appendix, we argue that $N = 16$ supersymmetry determines the transformation of the charge lattice under variation of the moduli. The idea is to show that the coupling of any massive particle to massless vectors depends on the scalars in the same vector multiplet in a specific way due to supersymmetry. Since these couplings set the charges, the charges are set by the spacetime moduli.

First, let us remind ourselves of the supermultiplets in theories with 16 supercharges and dimensions greater than 6. The only massless multiplets are the gravity multiplet and the vector multiplet. However, we can also have massive multiplets. For example, if we start with a theory with a non-Abelian gauge symmetry and move in the Coulomb branch, we can Higgs some of the previously massless vector multiplets into massive vector multiplets. From the Higgsing argument, it is easy to see that the massive vector multiplets always have $9-d$ scalars in d dimensions. For example, in 9 dimensions, the massless vector multiplet has one real scalar, which gets absorbed into the vector, turning it into a massive vector. The same thing happens in all other dimensions in $N=16$ theories.

If the mass of the massive vector field is sufficiently small, we must be able to incorporate it into the field theory. In order to get some intuition about the supersymmetric coupling of the massive vector multiplet to massless multiplets, let us start with the example of a Higgsed gauge group.

It is helpful to work with the $O(10-d,k)$ formulation of supergravities[280] where d is the dimension of spacetime and k is the dimension of the gauge group. We use the convention where the $(10-d,k)$ metric is

$$\eta = \begin{pmatrix} 0 & 1_{10-d} & 0 \\ 1_{10-d} & 0 & 0 \\ 0 & 0 & 1_{k-(10-d)} \end{pmatrix}. \tag{J.1}$$

Note that the $10-d$ graviphotons are excluded from k. Let us take the gauge group to be $G=U(1)^{r-1}\times SU(2)$, and move in its Coulomb branch to Higgs it into $U(1)^r$. This allows us to see how the two massive vector multiplets $W_\pm$ can supersymmetrically couple to the other massless multiplets. In this example, $k=r+2$. The bosonic part of the action is given by[280]

$$S=\int d^n x\sqrt{-g}e^{-2\varphi}[\mathcal{R}(g)+4\partial_\mu\varphi\partial^\mu\varphi-\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} \\ +\frac{1}{8}D^\mu\hat{\mathcal{H}}^{\hat{M}\hat{N}}D_\mu\hat{\mathcal{H}}_{\hat{M}\hat{N}}-\frac{1}{4}\hat{\mathcal{H}}_{\hat{M}\hat{N}}\hat{\mathcal{F}}^{\mu\nu\hat{M}}\hat{\mathcal{F}}_{\mu\nu}{}^{\hat{N}}-V(\mathcal{H})], \tag{J.2}$$

where

$$\begin{aligned}
D_\mu \hat{\mathcal{H}}^{\hat{M}\hat{N}} &= \partial_\mu \hat{H}^{\hat{M}\hat{N}} - 2\hat{A}_\mu{}^{\hat{K}} f_{\hat{K}}{}^{(\hat{M}}{}_{\hat{L}} \hat{\mathcal{H}}^{\hat{N})\hat{L}}. \\
\hat{\mathcal{F}}_{\mu\nu}{}^{\hat{M}} &= 2\partial_{[\mu}\hat{A}_{\nu]}{}^{\hat{M}} + f^{\hat{M}}{}_{\hat{K}\hat{L}} \hat{A}_\mu{}^{\hat{K}} \hat{A}_\nu{}^{\hat{L}}, \\
H_{\mu\nu\rho} &= 3(\partial_{[\mu]} B_{\nu\rho]} - \hat{A}_{[\mu}{}^{\hat{M}} \partial_\nu \hat{A}_{\rho]\hat{M}} - \frac{1}{3} f_{\hat{M}\hat{K}\hat{L}} \hat{A}_{[\mu}{}^{\hat{M}} \hat{A}_\nu{}^{\hat{K}} \hat{A}_{\rho]}{}^{\hat{L}}), \\
V(\mathcal{H}) &= f^{\hat{M}}{}_{\hat{K}\hat{P}} f^{\hat{N}}{}_{\hat{L}\hat{Q}} \hat{\mathcal{H}}_{\hat{M}\hat{N}} \hat{\mathcal{H}}^{\hat{K}\hat{L}} \hat{\mathcal{H}}^{\hat{P}\hat{Q}} + \frac{1}{4} f^{\hat{M}}{}_{\hat{N}\hat{K}} \hat{\mathcal{H}}_{\hat{M}\hat{N}} f^{\hat{N}}{}_{\hat{M}\hat{L}} \hat{\mathcal{H}}^{\hat{K}\hat{L}} + \frac{1}{6} f_{\hat{M}\hat{N}\hat{K}} f^{\hat{M}\hat{N}\hat{K}},
\end{aligned} \tag{J.3}$$

where the scalars $\hat{\mathcal{H}}^{\hat{K}\hat{L}}$ are in $O(10-d, r+2)/[O(10-d) \times O(r+2)]$, and $\hat{M} = 1, \cdots, 10-d+r+2$.

Suppose that $SU(2)$ corresponds to indices $10-d+r \leq \hat{M} \leq 10-d+r+2$. After Higgsing the $SU(2)$, we can use the quotient group $O(10-d) \times O(r+2)$ to make all the scalars in $\ln[\hat{\mathcal{H}}]^{\hat{M}\hat{N}}$ where $\max\{\hat{M}, \hat{N}\} \geq 10-d+r$ except the following vanish.

$$\begin{aligned}
(\hat{M}, \hat{N}) \in &\{1, \dots 2(10-d)\} \times \{10-d+r, 10-d+r+1, 10-d+r+2\} \\
&\cup \{10-d+r, 10-d+r+1, 10-d+r+2\} \times \{1, \dots 2(10-d)\}.
\end{aligned} \tag{J.4}$$

such that for $10-d < \hat{M} \leq 2(10-d)$ and $\hat{N} \in \{10-d+r, 10-d+r+1, 10-d+r+2\}$ we have

$$\ln[\hat{\mathcal{H}}^{\hat{M}\hat{N}}] = -\ln[\hat{\mathcal{H}}^{\hat{M}-(10-d)\ \hat{N}}] = \ln[\hat{\mathcal{H}}^{\hat{N}\hat{M}}] = -\ln[\hat{\mathcal{H}}^{\hat{N}\ \hat{M}-(10-d)}]. \tag{J.5}$$

We use $\ln\left(\hat{\mathcal{H}}\right)$ because it belongs to the Lie algebra of $O(d) \times O(r+2)$, which has a simpler description.

We can also use the gauge symmetry to impose the unitary gauge $\ln[\hat{\mathcal{H}}]^{\hat{N}\,11-d} = \ln[\hat{\mathcal{H}}]^{11-d\,\hat{N}} = 0$ for $\hat{N} > 10-d+r$ and $\ln[\hat{\mathcal{H}}]^{10-d+r\,11-d} = C$ is the Higgsing parameter that sets the mass scale of the Higgsed vector multiplets $\hat{M}_{\text{Massive}} \in \{10-d+r+1, 10-d+r+2\}$. Note that these massive vector multiplets now each have $9-d$ scalars corresponding to $\ln[\hat{\mathcal{H}}]^{\hat{M}\hat{N}}$ where $11-d < \hat{N} \leq 2(10-d)$.

Looking at the action, one can see that the charge of the two Higgsed vector multiplet $\hat{A}^{\hat{N}}$ under the massless vector $\hat{A}^{\hat{M}}$ comes from the terms quadratic in $\mathcal{F}$ and is proportional to $\hat{\mathcal{H}}^{\hat{M}\hat{N}}$. This scalar depends on scalars in the massless and massive multiplets through exponentiation of $\ln[\hat{\mathcal{H}}]$. Even though the dependence of the gauge couplings on the scalars in the massless multiplets is complicated, its change under the change of them is easy. Changing the scalars in the massless multiplets corresponds to a similarity transformation on $\hat{\mathcal{H}}$ by an element of $O(10-d,r)$, which acts on the first $10-d+r$ indices. Therefore, the charges of the massive multiplets which are given by $\hat{\mathcal{H}}^{\hat{M}\hat{N}}$ with $\hat{M} \leq 10-d+r < \hat{N}$ transform in the fundamental representation of $O(10-d,r)$.

$$\Lambda \in O(10-d,r) : \hat{\mathcal{H}}^{\hat{M}\hat{N}} \to \Lambda^{\hat{M}'}_{\hat{M}} \hat{\mathcal{H}}^{\hat{M}'\hat{N}}. \tag{J.6}$$

In fact, this argument holds for any Higgsed massive multiplet. Consequently, if the theory has a point of maximal enhancement where the gauge algebra becomes simple, this argument tells us that the charge lattice transforms covariantly under $O(10-d,r)$ changes of the moduli. We are particularly interested in 9d supergravities, which according to Swampland arguments[24], always have such a point of symmetry enhancement. However, even if such a point did not exist, such a constraint is generally expected from supersymmetry. For the coupling terms $\hat{A}^{\hat{M}} \times \ldots$ or $\partial\hat{A}^{\hat{M}} \times \ldots$ to be supersymmetrically invariant, we need to cancel the second order variation of $\hat{A}^{\hat{M}}$ under supersymmetry with terms which involve scalars in the same multiplet as $\hat{A}^{\hat{M}}$, which is why we typically end up with a term like $\mathcal{H}_{\hat{M}\hat{N}}\hat{A}^{\hat{M}} \times \ldots$ or $\mathcal{H}_{\hat{M}\hat{N}}\partial\hat{A}^{\hat{M}} \times \ldots$. Even though the rest of the terms (such as the mass term) can have different scalar dependencies, the coupling to the massless vector multiplets is expected to have fixed dependence on the scalars in the massless multiplets due to supersymmetry.

K

Rolling backgrounds

In this Appendix we study FRW solutions that are driven by a scalar potential that exponentially decays at infinity. We show that in expansionary solution, exponential potentials lead to polynomial expansion at $t \to \infty$ and in contracting solutions, they lead to polynomial contraction at $t \to -\infty$. We also show that unless a smooth bounce happens, there is always a space-like singularity at finite past or future (i.e. big bang or big crunch).

We focus on FRW solution with zero spatial curvature. Let us start with the asymptotic future. We later use the time reversal symmetry of Einstein-Hilbert action to deduce similar conclusions about the past boundary. We assume that the evolution is driven by an potential that dies off exponentially in all directions of the field space. In that case, the vev of the scalar fields will always roll towards the infinity of the field space in the asymptotic future.

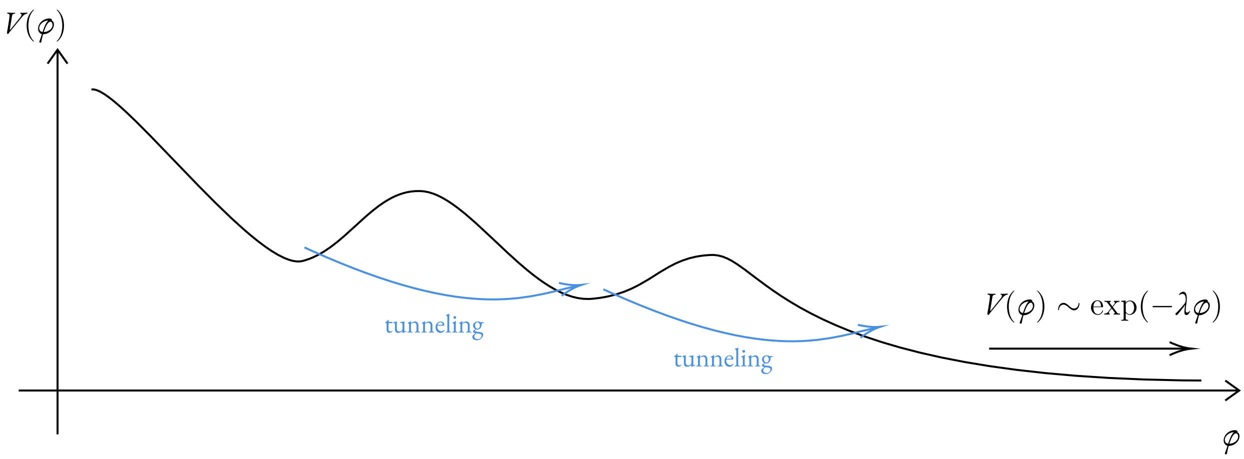

Figure K.1: If the potential always dies of exponentially, the universe cannot stay in any local minimum of the potential forever and it will eventually tunnel to a lower vacuum energy. Therefore, the at future infinity, the evolution of the universe is given by a scalar field rolling in an exponential potential.

Therefore, we just consider the quintessence solutions of the form $\varphi \to \infty$ for an exponential potential $V(\varphi) = V_0 \exp(-\lambda\varphi)$. We have two options: expanding universe or a contracting universe. As we will shortly see, the fate of these two universes is very different. In a contracting universe, the energy will blow up at finite time. Therefore, we will have a big crunch. While, in the expanding universe, the universe smoothly expands and dilutes. However, depending on the value of λ, the expansion can be accelerating or decelerating which changes the asymptotic boundary of spacetime.

Let us start by setting up the equations of motion.

$$\frac{(d-1)(d-2)}{2}H^2 = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$
$$\ddot{\varphi} + (d-1)H\dot{\varphi} + V'(\varphi) = 0. \tag{K.1}$$

First thing to notice from the first equation is that H can never vanish. Therefore, an expanding universe remains expanding and a contracting universe remains contracting[1]. From the above

[1]Note that here we are assuming there is no thermal background of other fields which is usually assumed in bounding cosmologies. We will come back to this assumption later.

equations one can also find

$$\dot{H} = -\frac{\dot{\varphi}^2}{d-2}. \tag{K.2}$$

If we start with a contracting solution ($H < 0$), then the absolute value of H will keep increasing while φ rolls down the potential. Therefore, after some point, the Hubble energy must be mainly sourced by kinetic energy, which implies

$$\frac{(d-1)(d-2)}{2} H^2 \simeq \frac{1}{2}\dot{\varphi}^2 \gg V(\varphi). \tag{K.3}$$

From this we find that V' is suppressed in the second equation of motion in (K.1). So, we find,

$$\ddot{\varphi} \simeq -(d-1)H\dot{\varphi} \simeq \sqrt{\frac{d-1}{d-2}}\dot{\varphi}^2. \tag{K.4}$$

The solution to this equation is

$$\varphi(t) \simeq -\sqrt{\frac{d-1}{d-2}} \ln\left(c_1 - \sqrt{\frac{d-1}{d-2}} \cdot t\right) + c_2, \tag{K.5}$$

where c_1 and c_2 are constants. As one can see, this solution diverges at finite time $t = c_1\sqrt{\frac{d-2}{d-1}}$. At this time, the kinetic energy diverges and drives the field to $\varphi \to \infty$. This is a big crunch. Now let us look at the scale factor. We have

$$\frac{\dot{a}}{a} = H \simeq -\frac{1}{\sqrt{(d-1)(d-2)}}\dot{\varphi} = -\frac{1}{\sqrt{(d-1)(d-2)}(c_1 - \sqrt{\frac{d-1}{d-2}} \cdot t}. \tag{K.6}$$

After integrating we find

$$a \propto (c_1 - \sqrt{\frac{d-1}{d-2}} \cdot t)^{\frac{1}{d-2}}. \tag{K.7}$$

To see if the asymptotic boundary is spacelike or null, we should see if the contraction is slow enough to create a horizon[2]. In other words, if we take two points, can they communicate with each other before hitting the asymptotic boundary of spacetime? If any two points can, the asymptotic boundary is null, and if some points cannot, the asymptotic boundary is spacelike.

The FRW metric is given by $ds^2 = -dt^2 + a(t)^2 dX^2$. Therefore, the furthest coordinate distance that a signal can travel is $\int a^{-1}$. If this integral diverges, the asymptotic boundary is null. In the case of (K.7), this integral clearly converges. Thus, the Penrose diragram looks like the following.

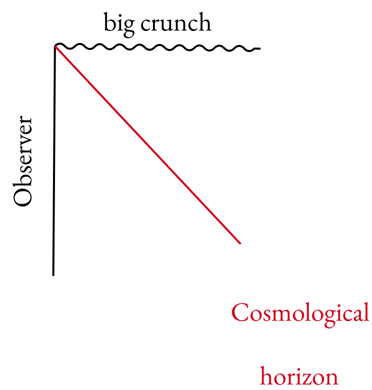

Figure K.2: Penrose diagrams of contracting universes that are driven by an exponentially decaying scalar potential. The universe ends with a spacelike big-crunch.

In the above analysis we neglected creation of matter near the singularity. The inclusion of a thermal background would not have changed the qualitative behavior of the solution. To see this, note that in the above solution $H^2 \propto a^{-2(d-2)}$. In dimensions greater than 3 where gravity is dynamical, both matter and radiation increase slower than, or equal to, this rate. The energy density of a relativistic matter increases as $H^2_{rad} \propto a^{-d}$ and the energy density of a non-relativistic matter increases as $H^2_{matter} \propto a^{-(d-1)}$. Therefore, the qualitative behavior of the solution remains the same.

Note that the asymptotic behavior of the solution at $t \to \infty$ is also independent from λ. So we find that, contracting solutions have a universal behavior at the asymptotic future and they end with a spacelike big crunch.

Now let us consider the expanding solutions. In the expanding solutions, depending on

[2] Fast expansions and slow contractions create cosmological horizons.

exponent λ, the solution behaves qualitatively differently. For large enough λ, the expansion is so fast that the kinetic term dominates the Hubble energy. On the other hand, for small λ, all the terms are of the same order.

Case I: $\lambda < 2\sqrt{\frac{d-1}{d-2}}$

The attractor solution is

$$\varphi = \frac{2}{\lambda} \ln \left(\sqrt{\frac{\lambda V_0}{\frac{4(d-1)}{\lambda^2(d-2)} - 1}} t + c_1 \right), \tag{K.8}$$

where c_1 is a constant. For this solution, all the terms in the Hubble energy are of the same order.

$$H^2 \propto \dot{\varphi}^2 \propto V(\varphi). \tag{K.9}$$

As $t \to \infty$ we have

$$\frac{\dot{a}}{a} \propto \frac{4}{(d-2)\lambda^2 t}. \tag{K.10}$$

Therefore, at future infinity, the scale factor goes like

$$a \propto t^p; \;\; p = \frac{4}{(d-2)\lambda^2}. \tag{K.11}$$

Case II: $\lambda > 2\sqrt{\frac{d-1}{d-2}}$

The attractor solution is

$$\varphi = \sqrt{d-2}d - 1 \ln(t + c_1) + c_2, \tag{K.12}$$

where c_1 and c_2 are constants. For this solution, the kinetic term dominates and we have

$$H^2 \propto \dot{\varphi}^2 \gg V(\varphi). \tag{K.13}$$

As $t \to \infty$ we have

$$\frac{\dot{a}}{a} \propto \frac{1}{(d-1)t}. \tag{K.14}$$

Therefore, at future infinity, the scale factor goes like

$$a \propto t^p; \quad p = \frac{1}{(d-1)}. \tag{K.15}$$

Now we study the shape of the Penrose diagram at future infinity. As we discussed earlier, the shape of the future boundary depends on the integral $\int a^{-1}$. For all the expanding solutions we found a polynomial dependence $a \sim t^p$. For $p > 1$, the integral $\int a^{-1}$ converges which signals the existence of a spacelike boundary with a null cosmological horizon. While for $p < 1$, there is no cosmological horizon and the future boundary must be null.

Note that $p < 1$ corresponds to $\lambda > \frac{2}{\sqrt{d-2}}$. Therefore, we find the following Penrose diagrams depending on the value of λ.

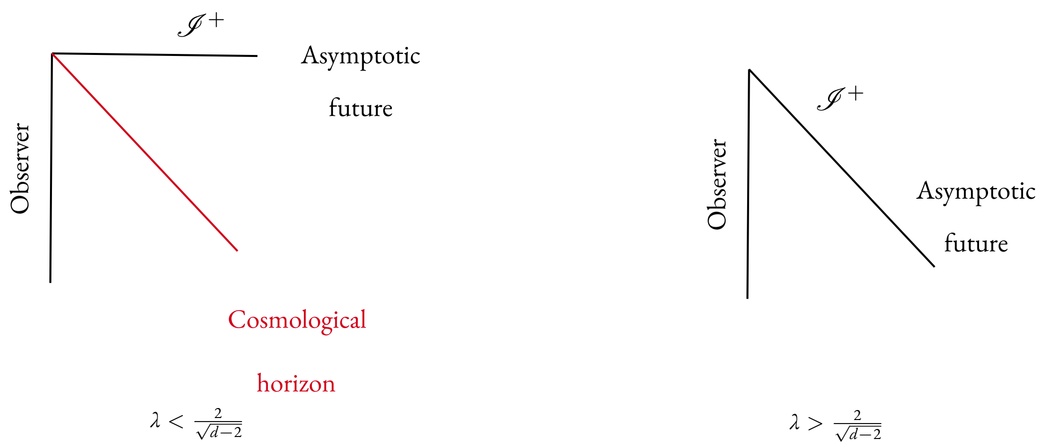

Figure K.3: Depending on the value of λ, the Penrose diagram takes a different shape in $t \to \infty$. For $\lambda > \frac{2}{\sqrt{d-2}}$, the future infinity is decelerating and the asymptotic future $\mathscr{I}^+$ is null. However, for $\lambda < \frac{2}{\sqrt{d-2}}$, the asymptotic future has an accelerating expansion which creates a cosmological horizon.

Now, we can time reverse the above analysis and make similar statements about the past infinity of the expanding and contracting universes.

We find that the past infinity of the expanding universes is always spacelike and the past infinity of contracting universes depends on the coefficient λ that drives the evolution. Therefore, we find the following four options (Fig. K.4).

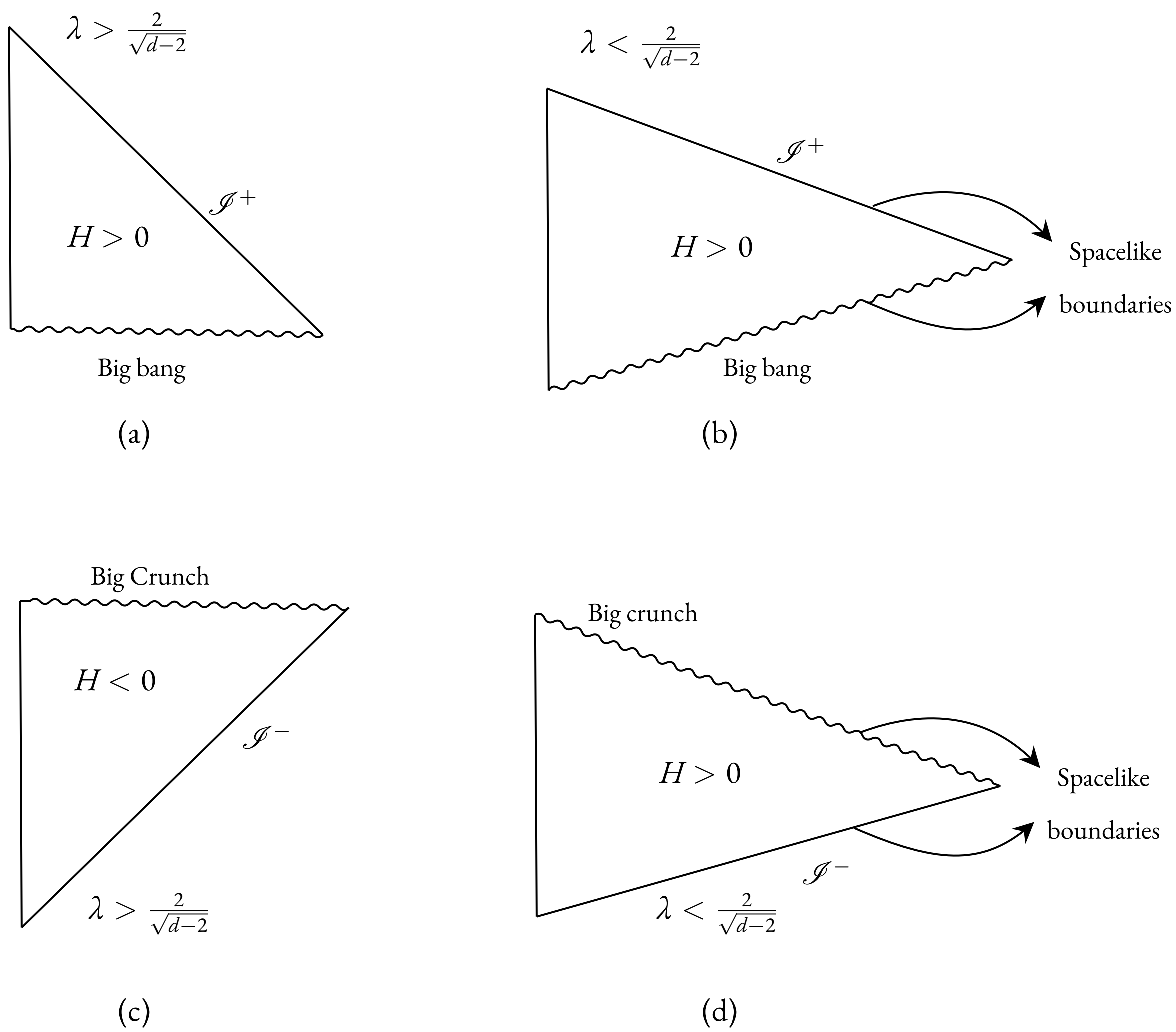

Figure K.4: Penrose diagrams of: (a) expanding universe with deccelerating asymptotic future, (b) expanding universe with accelerating asymptotic future, (c) contracting universe with deccelerating asymptotic past, and (d) contracting universe with accelerating asymptotic past.

References

[1] Michael Dine and Nathan Seiberg. Is the Superstring Weakly Coupled? *Phys. Lett. B*, 162:299–302, 1985.

[2] Juan Martin Maldacena and Carlos Nunez. Supergravity description of field theories on curved manifolds and a no go theorem. *Int. J. Mod. Phys. A*, 16:822–855, 2001.

[3] Mark P. Hertzberg, Shamit Kachru, Washington Taylor, and Max Tegmark. Inflationary Constraints on Type IIA String Theory. *JHEP*, 12:095, 2007.

[4] Paul J. Steinhardt and Daniel Wesley. Dark Energy, Inflation and Extra Dimensions. *Phys. Rev. D*, 79:104026, 2009.

[5] Timm Wrase and Marco Zagermann. On Classical de Sitter Vacua in String Theory. *Fortsch. Phys.*, 58:906–910, 2010.

[6] David Andriot. Open problems on classical de Sitter solutions. *Fortsch. Phys.*, 67(7):1900026, 2019.

[7] Luis Alvarez-Gaume, Paul H. Ginsparg, Gregory W. Moore, and C. Vafa. An O(16) x O(16) Heterotic String. *Phys. Lett. B*, 171:155–162, 1986.

[8] Lance J. Dixon and Jeffrey A. Harvey. String theories in ten-dimensions without space-time supersymmetry. *Nucl. Phys.*, B274:93–105, 1986. 93(1986).

[9] J. Polchinski. *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 12 2007.

[10] Shamit Kachru, Renata Kallosh, Andrei Linde, and Sandip P Trivedi. De sitter vacua in string theory. *Physical Review D*, 68(4):046005, 2003.

[11] Fernando Quevedo. Local string models and moduli stabilisation. In *PERSPECTIVES ON STRING PHENOMENOLOGY*, pages 219–243. World Scientific, 2015.

[12] Xin Gao, Arthur Hebecker, and Daniel Junghans. Control issues of KKLT. *Fortsch. Phys.*, 68:2000089, 2020.

[13] Severin Lüst, Cumrun Vafa, Max Wiesner, and Kai Xu. Holography and the KKLT scenario. *JHEP*, 10:188, 2022.

[14] Georges Obied, Hirosi Ooguri, Lev Spodyneiko, and Cumrun Vafa. De Sitter Space and the Swampland. 6 2018.

[15] Hirosi Ooguri, Eran Palti, Gary Shiu, and Cumrun Vafa. Distance and de Sitter Conjectures on the Swampland. *Phys. Lett. B*, 788:180–184, 2019.

[16] Matthew J Dolan, Patrick Draper, Jonathan Kozaczuk, and Hiren Patel. Transplanckian censorship and global cosmic strings. *Journal of High Energy Physics*, 2017(4):133, 2017.

[17] Patrick Draper and Szilard Farkas. Gravitational instabilities and censorship of large scalar field excursions. *Journal of High Energy Physics*, 2019(5):158, 2019.

[18] Hee-Cheol Kim, Houri-Christina Tarazi, and Cumrun Vafa. Four-dimensional $\mathcal{N} = 4$ SYM theory and the swampland. *Phys. Rev. D*, 102(2):026003, 2020.

[19] Alek Bedroya and Cumrun Vafa. Trans-Planckian Censorship and the Swampland. *JHEP*, 09:123, 2020.

[20] Alek Bedroya, Robert Brandenberger, Marilena Loverde, and Cumrun Vafa. Trans-Planckian Censorship and Inflationary Cosmology. *Phys. Rev. D*, 101(10):103502, 2020.

[21] Alek Bedroya, Miguel Montero, Cumrun Vafa, and Irene Valenzuela. de sitter bubbles and the swampland. *Fortschritte der Physik*, 68(11-12):2000084, 2020.

[22] Alek Bedroya. de Sitter Complementarity, TCC, and the Swampland. *Letters in High Energy Physics*, 2021(187), 2021.

[23] Alek Bedroya. Holographic origin of TCC and the Distance Conjecture. 11 2022.

[24] Alek Bedroya, Yuta Hamada, Miguel Montero, and Cumrun Vafa. Compactness of brane moduli and the String Lamppost Principle in d > 6. *JHEP*, 02:082, 2022.

[25] Alek Bedroya, Sanjay Raman, and Houri-Christina Tarazi. Non-BPS path to the string lamppost. 3 2023.

[26] Alek Bedroya and Yuta Hamada. Dualities from Swampland principles. 3 2023.

[27] Jerome Martin and Robert H Brandenberger. Trans-planckian problem of inflationary cosmology. *Physical Review D*, 63(12):123501, 2001.

[28] Robert H Brandenberger and Jerome Martin. The robustness of inflation to changes in super-planck-scale physics. *Modern Physics Letters A*, 16(15):999–1006, 2001.

[29] Robert H Brandenberger and Jerome Martin. Trans-planckian issues for inflationary cosmology. *Classical and Quantum Gravity*, 30(11):113001, 2013.

[30] Nemanja Kaloper, Matthew Kleban, Albion Lawrence, Stephen Shenker, and Leonard Susskind. Initial conditions for inflation. *JHEP*, 11:037, 2002.

[31] Richard Easther, Brian R Greene, William H Kinney, and Gary Shiu. Generic estimate of trans-planckian modifications to the primordial power spectrum in inflation. *Physical Review D*, 66(2):023518, 2002.

[32] Hirosi Ooguri and Cumrun Vafa. On the Geometry of the String Landscape and the Swampland. *Nucl. Phys. B*, 766:21–33, 2007.

[33] Tom Rudelius. Asymptotic scalar field cosmology in string theory. *JHEP*, 10:018, 2022.

[34] Seung-Joo Lee, Wolfgang Lerche, and Timo Weigand. Emergent strings, duality and weak coupling limits for two-form fields. *JHEP*, 02:096, 2022.

[35] Seung-Joo Lee, Wolfgang Lerche, and Timo Weigand. Tensionless Strings and the Weak Gravity Conjecture. *JHEP*, 10:164, 2018.

[36] Prateek Agrawal, Georges Obied, Paul J. Steinhardt, and Cumrun Vafa. On the Cosmological Implications of the String Swampland. *Phys. Lett.*, B784:271–276, 2018.

[37] Prateek Agrawal, Georges Obied, and Cumrun Vafa. H_0 tension, swampland conjectures, and the epoch of fading dark matter. *Phys. Rev. D*, 103(4):043523, 2021.

[38] Leonard Susskind. Addendum to Fast Scramblers. 1 2011.

[39] Tom Rudelius. Conditions for (no) eternal inflation. *Journal of Cosmology and Astroparticle Physics*, 2019(08):009, 2019.

[40] K. M. Gorski, A. J. Banday, C. L. Bennett, G. Hinshaw, A. Kogut, George F. Smoot, and E. L. Wright. Power spectrum of primordial inhomogeneity determined from the four year COBE DMR sky maps. *Astrophys. J. Lett.*, 464:L11, 1996.

[41] C. L. Bennett et al. Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results. *Astrophys. J. Suppl.*, 208:20, 2013.

[42] N. Aghanim et al. Planck 2018 results. V. CMB power spectra and likelihoods. *Astron. Astrophys.*, 641:A5, 2020.

[43] Alan H. Guth. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Phys. Rev. D*, 23:347–356, 1981.

[44] Andrei D. Linde. A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems. *Phys. Lett. B*, 108:389–393, 1982.

[45] R. Brout, F. Englert, and E. Gunzig. The Creation of the Universe as a Quantum Phenomenon. *Annals Phys.*, 115:78, 1978.

[46] Alexei A. Starobinsky. A New Type of Isotropic Cosmological Models Without Singularity. *Phys. Lett. B*, 91:99–102, 1980.

[47] K. Sato. First Order Phase Transition of a Vacuum and Expansion of the Universe. *Mon. Not. Roy. Astron. Soc.*, 195:467–479, 1981.

[48] Robert H. Brandenberger. Alternatives to the inflationary paradigm of structure formation. *Int. J. Mod. Phys. Conf. Ser.*, 01:67–79, 2011.

[49] Viatcheslav F. Mukhanov and G. V. Chibisov. Quantum Fluctuations and a Nonsingular Universe. *JETP Lett.*, 33:532–535, 1981.

[50] Alexei A. Starobinsky. Spectrum of relict gravitational radiation and the early state of the universe. *JETP Lett.*, 30:682–685, 1979.

[51] Jerome Martin and Robert H. Brandenberger. The TransPlanckian problem of inflationary cosmology. *Phys. Rev. D*, 63:123501, 2001.

[52] Jens C. Niemeyer. Inflation with a Planck scale frequency cutoff. *Phys. Rev. D*, 63:123502, 2001.

[53] Richard Easther, Brian R. Greene, William H. Kinney, and Gary Shiu. Inflation as a probe of short distance physics. *Phys. Rev. D*, 64:103502, 2001.

[54] Achim Kempf and Jens C. Niemeyer. Perturbation spectrum in inflation with cutoff. *Phys. Rev. D*, 64:103501, 2001.

[55] V. Bozza, Massimo Giovannini, and G. Veneziano. Cosmological perturbations from a new physics hypersurface. *JCAP*, 05:001, 2003.

[56] Nathan Benjamin Agmon, Alek Bedroya, Monica Jinwoo Kang, and Cumrun Vafa. Lectures on the string landscape and the Swampland. 12 2022.

[57] Robert H. Brandenberger and C. Vafa. Superstrings in the Early Universe. *Nucl. Phys. B*, 316:391–410, 1989.

[58] Fabio Finelli and Robert Brandenberger. On the generation of a scale invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase. *Phys. Rev. D*, 65:103522, 2002.

[59] M. Gasperini and G. Veneziano. Pre - big bang in string cosmology. *Astropart. Phys.*, 1:317–339, 1993.

[60] Justin Khoury, Burt A. Ovrut, Paul J. Steinhardt, and Neil Turok. The Ekpyrotic universe: Colliding branes and the origin of the hot big bang. *Phys. Rev. D*, 64:123522, 2001.

[61] Viatcheslav F. Mukhanov, H. A. Feldman, and Robert H. Brandenberger. Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions. *Phys. Rept.*, 215:203–333, 1992.

[62] Robert H. Brandenberger. Lectures on the theory of cosmological perturbations. *Lect. Notes Phys.*, 646:127–167, 2004.

[63] N. Aghanim et al. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020. [Erratum: Astron.Astrophys. 652, C4 (2021)].

[64] Ali Nayeri, Robert H. Brandenberger, and Cumrun Vafa. Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology. *Phys. Rev. Lett.*, 97:021302, 2006.

[65] Robert H. Brandenberger, Ali Nayeri, Subodh P. Patil, and Cumrun Vafa. Tensor Modes from a Primordial Hagedorn Phase of String Cosmology. *Phys. Rev. Lett.*, 98:231302, 2007.

[66] Arjun Berera. Warm inflation. *Phys. Rev. Lett.*, 75:3218–3221, 1995.

[67] Suratna Das. Warm Inflation in the light of Swampland Criteria. *Phys. Rev. D*, 99(6):063514, 2019.

[68] Meysam Motaharfar, Vahid Kamali, and Rudnei O. Ramos. Warm inflation as a way out of the swampland. *Phys. Rev. D*, 99(6):063513, 2019.

[69] Robert H. Brandenberger and J. H. Kung. Chaotic Inflation as an Attractor in Initial Condition Space. *Phys. Rev. D*, 42:1008–1015, 1990.

[70] Hume A. Feldman and Robert H. Brandenberger. Chaotic Inflation With Metric and Matter Perturbations. *Phys. Lett. B*, 227:359–366, 1989.

[71] William E. East, Matthew Kleban, Andrei Linde, and Leonardo Senatore. Beginning inflation in an inhomogeneous universe. *JCAP*, 09:010, 2016.

[72] Katy Clough, Eugene A. Lim, Brandon S. DiNunno, Willy Fischler, Raphael Flauger, and Sonia Paban. Robustness of Inflation to Inhomogeneous Initial Conditions. *JCAP*, 09:025, 2017.

[73] Dalia S. Goldwirth and Tsvi Piran. Initial conditions for inflation. *Phys. Rept.*, 214:223–291, 1992.

[74] Katherine Freese and Douglas Spolyar. Chain inflation: 'Bubble bubble toil and trouble'. *JCAP*, 07:007, 2005.

[75] Katherine Freese, James T. Liu, and Douglas Spolyar. Chain inflation via rapid tunneling in the landscape. 2006.

[76] Sidney R. Coleman. The Fate of the False Vacuum. 1. Semiclassical Theory. *Phys. Rev.*, D15:2929–2936, 1977. [Erratum: Phys. Rev.D16,1248(1977)].

[77] Sidney R. Coleman and Frank De Luccia. Gravitational Effects on and of Vacuum Decay. *Phys. Rev.*, D21:3305, 1980.

[78] Jaume Garriga. Nucleation rates in flat and curved space. *Phys. Rev. D*, 49:6327–6342, 1994.

[79] Stephen William Hawking and Ian L Moss. Supercooled phase transitions in the very early universe. *Physics Letters B*, 110(1):35–38, 1982.

[80] Jose J. Blanco-Pillado, Handhika S. Ramadhan, and Benjamin Shlaer. Decay of flux vacua to nothing. *JCAP*, 1010:029, 2010.

[81] Jose J. Blanco-Pillado and Benjamin Shlaer. Bubbles of Nothing in Flux Compactifications. *Phys. Rev. D*, 82:086015, 2010.

[82] Adam R. Brown and Alex Dahlen. On 'nothing' as an infinitely negatively curved spacetime. *Phys. Rev. D*, 85:104026, 2012.

[83] Dieter Lüst, Eran Palti, and Cumrun Vafa. AdSand the Swampland. *Physics Letters B*, 797:134867, 2019.

[84] Nima Arkani-Hamed, Lubos Motl, Alberto Nicolis, and Cumrun Vafa. The String landscape, black holes and gravity as the weakest force. *JHEP*, 06:060, 2007.

[85] Qing-Guo Huang, Miao Li, and Wei Song. Weak gravity conjecture in the asymptotical dS and AdS background. *JHEP*, 10:059, 2006.

[86] Miguel Montero, Thomas Van Riet, and Gerben Venken. Festina Lente: EFT Constraints from Charged Black Hole Evaporation in de Sitter. *JHEP*, 01:039, 2020.

[87] Ignatios Antoniadis and Karim Benakli. Weak Gravity Conjecture in de Sitter Space-Time. *Fortsch. Phys.*, 68(9):2000054, 2020.

[88] Ben Michel, Eric Mintun, Joseph Polchinski, Andrea Puhm, and Philip Saad. Remarks on brane and antibrane dynamics. *JHEP*, 09:021, 2015.

[89] Stefano Lanza, Fernando Marchesano, Luca Martucci, and Irene Valenzuela. Swampland Conjectures for Strings and Membranes. *JHEP*, 02:006, 2021.

[90] Hirosi Ooguri and Cumrun Vafa. Non-supersymmetric AdS and the Swampland. *Adv. Theor. Math. Phys.*, 21:1787–1801, 2017.

[91] J.David Brown and C. Teitelboim. Neutralization of the Cosmological Constant by Membrane Creation. *Nucl. Phys. B*, 297:787–836, 1988.

[92] David Andriot. On the de Sitter swampland criterion. *Phys. Lett. B*, 785:570–573, 2018.

[93] Sumit K. Garg and Chethan Krishnan. Bounds on Slow Roll and the de Sitter Swampland. *JHEP*, 11:075, 2019.

[94] Ulf H. Danielsson and Thomas Van Riet. What if string theory has no de Sitter vacua? *Int. J. Mod. Phys. D*, 27(12):1830007, 2018.

[95] Thomas W. Grimm, Chongchuo Li, and Irene Valenzuela. Asymptotic Flux Compactifications and the Swampland. *JHEP*, 06:009, 2020. [Erratum: JHEP 01, 007 (2021)].

[96] David Andriot, Niccolò Cribiori, and David Erkinger. The web of swampland conjectures and the TCC bound. *JHEP*, 07:162, 2020.

[97] Ulf Danielsson. The quantum swampland. *JHEP*, 04:095, 2019.

[98] Arthur Hebecker and Timm Wrase. The Asymptotic dS Swampland Conjecture - a Simplified Derivation and a Potential Loophole. *Fortsch. Phys.*, 67(1-2):1800097, 2019.

[99] Daniel Klaewer, Dieter Lüst, and Eran Palti. A Spin-2 Conjecture on the Swampland. *Fortsch. Phys.*, 67(1-2):1800102, 2019.

[100] David Andriot and Christoph Roupec. Further refining the de Sitter swampland conjecture. *Fortsch. Phys.*, 67(1-2):1800105, 2019.

[101] Jose J. Blanco-Pillado, Heling Deng, and Alexander Vilenkin. Eternal Inflation in Swampy Landscapes. *JCAP*, 05:014, 2020.

[102] Don N Page. Space for both no-boundary and tunneling quantum states of the universe. *Physical Review D*, 56(4):2065, 1997.

[103] Kai Schmitz. Trans-planckian censorship and inflation in grand unified theories. *Physics Letters B*, 803:135317, 2020.

[104] Jacob McNamara and Cumrun Vafa. Cobordism Classes and the Swampland. 2019.

[105] Leonard Susskind. Entanglement and Chaos in De Sitter Space Holography: An SYK Example. *JHAP*, 1(1):1–22, 2021.

[106] Robert H. Brandenberger and Jerome Martin. The Robustness of inflation to changes in superPlanck scale physics. *Mod. Phys. Lett. A*, 16:999–1006, 2001.

[107] David Polarski and Alexei A. Starobinsky. Semiclassicality and decoherence of cosmological perturbations. *Class. Quant. Grav.*, 13:377–392, 1996.

[108] Thomas W. Grimm, Eran Palti, and Irene Valenzuela. Infinite Distances in Field Space and Massless Towers of States. *JHEP*, 08:143, 2018.

[109] Naomi Gendler and Irene Valenzuela. Merging the weak gravity and distance conjectures using BPS extremal black holes. *JHEP*, 01:176, 2021.

[110] Gia Dvali and Cesar Gomez. Quantum Compositeness of Gravity: Black Holes, AdS and Inflation. *JCAP*, 01:023, 2014.

[111] Gia Dvali and Cesar Gomez. Quantum Exclusion of Positive Cosmological Constant? *Annalen Phys.*, 528:68–73, 2016.

[112] Gia Dvali, Cesar Gomez, and Sebastian Zell. Quantum Break-Time of de Sitter. *JCAP*, 06:028, 2017.

[113] Min-Seok Seo. The entropic quasi-de Sitter instability time from the distance conjecture. *Phys. Lett. B*, 807:135580, 2020.

[114] Leonard Susskind, Larus Thorlacius, and John Uglum. The Stretched horizon and black hole complementarity. *Phys. Rev. D*, 48:3743–3761, 1993.

[115] Yasuhiro Sekino and Leonard Susskind. Fast Scramblers. *JHEP*, 10:065, 2008.

[116] Don N. Page. Information in black hole radiation. *Phys. Rev. Lett.*, 71:3743–3746, 1993.

[117] Patrick Hayden and John Preskill. Black holes as mirrors: Quantum information in random subsystems. *JHEP*, 09:120, 2007.

[118] Tom Banks and W. Fischler. M theory observables for cosmological space-times. 2 2001.

[119] Tom Banks. Cosmological breaking of supersymmetry? *Int. J. Mod. Phys. A*, 16:910–921, 2001.

[120] W. Fischler. Taking de sitter seriously. *Role of Scaling Laws in Physics and Biology (Celebrating the 60th Birthday of Geoffrey West)*, Santa Fe, Dec. 2000.

[121] T. Banks, W. Fischler, and S. Paban. Recurrent nightmares? Measurement theory in de Sitter space. *JHEP*, 12:062, 2002.

[122] Raphael Bousso. Positive vacuum energy and the N bound. *JHEP*, 11:038, 2000.

[123] Lisa Dyson, James Lindesay, and Leonard Susskind. Is there really a de Sitter/CFT duality? *JHEP*, 08:045, 2002.

[124] Leonard Susskind. Twenty years of debate with Stephen. In *Workshop on Conference on the Future of Theoretical Physics and Cosmology in Honor of Steven Hawking's 60th Birthday*, pages 330–347, 4 2002.

[125] Lisa Dyson, Matthew Kleban, and Leonard Susskind. Disturbing implications of a cosmological constant. *JHEP*, 10:011, 2002.

[126] Maulik K. Parikh, Ivo Savonije, and Erik P. Verlinde. Elliptic de Sitter space: dS/Z(2). *Phys. Rev. D*, 67:064005, 2003.

[127] Juan Maldacena and Leonard Susskind. Cool horizons for entangled black holes. *Fortsch. Phys.*, 61:781–811, 2013.

[128] Ping Gao, Daniel Louis Jafferis, and Aron C. Wall. Traversable Wormholes via a Double Trace Deformation. *JHEP*, 12:151, 2017.

[129] Juan Maldacena, Douglas Stanford, and Zhenbin Yang. Diving into traversable wormholes. *Fortsch. Phys.*, 65(5):1700034, 2017.

[130] Maulik K. Parikh and Jan Pieter van der Schaar. Not One Bit of de Sitter Information. *JHEP*, 09:041, 2008.

[131] Lars Aalsma and Gary Shiu. Chaos and complementarity in de Sitter space. *JHEP*, 05:152, 2020.

[132] Hao Geng. Non-local entanglement and fast scrambling in de-Sitter holography. *Annals Phys.*, 426:168402, 2021.

[133] Bruce Allen. Vacuum States in de Sitter Space. *Phys. Rev. D*, 32:3136, 1985.

[134] N. D. Birrell and P. C. W. Davies. *Quantum Fields in Curved Space*. Cambridge Monographs on Mathematical Physics. Cambridge Univ. Press, Cambridge, UK, 2 1984.

[135] E. Mottola. Particle Creation in de Sitter Space. *Phys. Rev. D*, 31:754, 1985.

[136] Raphael Bousso, Alexander Maloney, and Andrew Strominger. Conformal vacua and entropy in de Sitter space. *Phys. Rev. D*, 65:104039, 2002.

[137] Paul R. Anderson, Emil Mottola, and Dillon H. Sanders. Decay of the de Sitter Vacuum. *Phys. Rev. D*, 97(6):065016, 2018.

[138] Ulf H. Danielsson. A Note on inflation and transPlanckian physics. *Phys. Rev. D*, 66:023511, 2002.

[139] Ulf H. Danielsson. Transplanckian energy production and slow roll inflation. *Phys. Rev. D*, 71:023516, 2005.

[140] Ulf H. Danielsson. Inflation as a probe of new physics. *JCAP*, 03:014, 2006.

[141] Ahmed Almheiri, Thomas Hartman, Juan Maldacena, Edgar Shaghoulian, and Amirhossein Tajdini. The entropy of Hawking radiation. *Rev. Mod. Phys.*, 93(3):035002, 2021.

[142] Ulf H. Danielsson, Daniel Domert, and Martin E. Olsson. Miracles and complementarity in de Sitter space. *Phys. Rev. D*, 68:083508, 2003.

[143] Prateek Agrawal, Sergei Gukov, Georges Obied, and Cumrun Vafa. Topological Gravity as the Early Phase of Our Universe. 9 2020.

[144] Nima Arkani-Hamed, Yu-tin Huang, Jin-Yu Liu, and Grant N. Remmen. Causality, unitarity, and the weak gravity conjecture. *JHEP*, 03:083, 2022.

[145] Yuta Hamada, Toshifumi Noumi, and Gary Shiu. Weak Gravity Conjecture from Unitarity and Causality. *Phys. Rev. Lett.*, 123(5):051601, 2019.

[146] Stefano Andriolo, Daniel Junghans, Toshifumi Noumi, and Gary Shiu. A Tower Weak Gravity Conjecture from Infrared Consistency. *Fortsch. Phys.*, 66(5):1800020, 2018.

[147] Daniel Harlow and Hirosi Ooguri. Symmetries in quantum field theory and quantum gravity. *Commun. Math. Phys.*, 383(3):1669–1804, 2021.

[148] David Andriot. On classical de Sitter and Minkowski solutions with intersecting branes. *JHEP*, 03:054, 2018.

[149] David Andriot, Paul Marconnet, Muthusamy Rajaguru, and Timm Wrase. Automated consistent truncations and stability of flux compactifications. 9 2022.

[150] David Andriot and Ludwig Horer. (Quasi-) de Sitter solutions across dimensions and the TCC bound. 8 2022.

[151] Pierre Corvilain, Thomas W. Grimm, and Irene Valenzuela. The Swampland Distance Conjecture for Kähler moduli. *JHEP*, 08:075, 2019.

[152] Seung-Joo Lee, Wolfgang Lerche, and Timo Weigand. Emergent strings from infinite distance limits. *JHEP*, 02:190, 2022.

[153] Muldrow Etheredge, Ben Heidenreich, Sami Kaya, Yue Qiu, and Tom Rudelius. Sharpening the Distance Conjecture in Diverse Dimensions. 6 2022.

[154] Andrew Strominger. The dS / CFT correspondence. *JHEP*, 10:034, 2001.

[155] Edward Witten. Quantum gravity in de Sitter space. In *Strings 2001: International Conference*, 6 2001.

[156] Raphael Bousso. Cosmology and the S-matrix. *Phys. Rev. D*, 71:064024, 2005.

[157] Tom Rudelius. Asymptotic observables and the swampland. *Phys. Rev. D*, 104(12):126023, 2021.

[158] Steven Weinberg. Minimal fields of canonical dimensionality are free. *Phys. Rev. D*, 86:105015, 2012.

[159] Yoh Tanimoto. Massless Wigner particles in conformal field theory are free. *SIGMA*, 2:e21, 2014.

[160] Bryce S. DeWitt. Quantum Theory of Gravity. 2. The Manifestly Covariant Theory. *Phys. Rev.*, 162:1195–1239, 1967.

[161] Juan Martin Maldacena. The Large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.*, 2:231–252, 1998.

[162] Holmfridur Hannesdottir and Matthew D. Schwartz. S-Matrix for massless particles. *Phys. Rev. D*, 101(10):105001, 2020.

[163] Michael E. Peskin and Daniel V. hroeder. *An Introduction to quantum field theory*. Addison-Wesley, Reading, USA, 1995.

[164] Edward Witten. Anti-de Sitter space and holography. *Adv. Theor. Math. Phys.*, 2:253–291, 1998.

[165] Sidney R. Coleman. The Uses of Instantons. *Subnucl. Ser.*, 15:805, 1979.

[166] Simeon Hellerman and Ian Swanson. Cosmological solutions of supercritical string theory. *Phys. Rev. D*, 77:126011, 2008.

[167] John Stout. Infinite Distances and Factorization. 8 2022.

[168] Robbert Dijkgraaf, Ben Heidenreich, Patrick Jefferson, and Cumrun Vafa. Negative Branes, Supergroups and the Signature of Spacetime. *JHEP*, 02:050, 2018.

[169] Dieter Lüst, Eran Palti, and Cumrun Vafa. AdS and the Swampland. *Phys. Lett. B*, 797:134867, 2019.

[170] Joseph Polchinski. Monopoles, duality, and string theory. *Int. J. Mod. Phys. A*, 19S1:145–156, 2004.

[171] Hee-Cheol Kim, Gary Shiu, and Cumrun Vafa. Branes and the Swampland. *Phys. Rev. D*, 100(6):066006, 2019.

[172] Seung-Joo Lee and Timo Weigand. Swampland Bounds on the Abelian Gauge Sector. *Phys. Rev. D*, 100(2):026015, 2019.

[173] Sheldon Katz, Hee-Cheol Kim, Houri-Christina Tarazi, and Cumrun Vafa. Swampland Constraints on 5d $\mathcal{N} = 1$ Supergravity. *JHEP*, 07:080, 2020.

[174] Yuta Hamada and Cumrun Vafa. 8d supergravity, reconstruction of internal geometry and the Swampland. *JHEP*, 06:178, 2021.

[175] Houri-Christina Tarazi and Cumrun Vafa. On the Finiteness of 6D Supergravity Landscape. 6 2021.

[176] Edward Witten. Small instantons in string theory. *Nucl. Phys. B*, 460:541–559, 1996.

[177] Ori J. Ganor and Amihay Hanany. Small E_8 Instantons and Tensionless Noncritical Strings. *Nucl. Phys. B*, 474:122–140, 1996.

[178] N. Seiberg and Edward Witten. Comments on String Dynamics in Six-Dimensions. *Nucl. Phys. B*, 471:121–134, 1996.

[179] Sergio Cecotti, Michele Del Zotto, Mario Martone, and Robert Moscrop. The Characteristic Dimension of Four-dimensional $\mathcal{N} = 2$ SCFTs. 8 2021.

[180] Joseph A. Minahan and Dennis Nemeschansky. An $\mathcal{N} = 2$ Superconformal Fixed Point with E_6 Global Symmetry. *Nucl. Phys. B*, 482:142–152, 1996.

[181] Joseph A. Minahan and Dennis Nemeschansky. Superconformal Fixed Points with E_N Global Symmetry. *Nucl. Phys. B*, 489:24–46, 1997.

[182] Edward Witten. Sigma Models and the ADHM Construction of Instantons. *J. Geom. Phys.*, 15:215–226, 1995.

[183] Masashi Hamanaka and Toshio Nakatsu. ADHM Construction of Noncommutative Instantons. In *20th International Colloquium on Integrable Systems and Quantum Symmetries*, 11 2013.

[184] Sergio Cecotti. Swampland geometry and the gauge couplings. *JHEP*, 09:136, 2021.

[185] Ben Heidenreich, Jacob McNamara, Miguel Montero, Matthew Reece, Tom Rudelius, and Irene Valenzuela. Chern-Weil global symmetries and how quantum gravity avoids them. *JHEP*, 11:053, 2021.

[186] Joseph Polchinski and Edward Witten. Evidence for heterotic - type I string duality. *Nucl. Phys. B*, 460:525–540, 1996.

[187] Ashoke Sen and Barton Zwiebach. Tachyon condensation in string field theory. *JHEP*, 03:002, 2000.

[188] Ofer Aharony, Zohar Komargodski, and Assaf Patir. The Moduli space and M(atrix) theory of 9d N=1 backgrounds of M/string theory. *JHEP*, 05:073, 2007.

[189] Edward Witten. Phase transitions in M theory and F theory. *Nucl. Phys. B*, 471:195–216, 1996.

[190] Nathan Seiberg. Five-dimensional SUSY field theories, nontrivial fixed points and string dynamics. *Phys. Lett. B*, 388:753–760, 1996.

[191] Michael R. Douglas, Sheldon H. Katz, and Cumrun Vafa. Small instantons, Del Pezzo surfaces and type I-prime theory. *Nucl. Phys. B*, 497:155–172, 1997.

[192] Ofer Aharony, Amihay Hanany, and Barak Kol. Webs of (p,q) five-branes, five-dimensional field theories and grid diagrams. *JHEP*, 01:002, 1998.

[193] Michele Del Zotto, Jonathan J. Heckman, and David R. Morrison. 6D SCFTs and Phases of 5D Theories. *JHEP*, 09:147, 2017.

[194] Patrick Jefferson, Hee-Cheol Kim, Cumrun Vafa, and Gabi Zafrir. Towards Classification of 5D Scfts: Single Gauge Node. 5 2017.

[195] Patrick Jefferson, Sheldon Katz, Hee-Cheol Kim, and Cumrun Vafa. On Geometric Classification of 5d SCFTs. *JHEP*, 04:103, 2018.

[196] Lakshya Bhardwaj and Patrick Jefferson. Classifying 5d SCFTs via 6d SCFTs: Rank one. *JHEP*, 07:178, 2019. [Addendum: JHEP 01, 153 (2020)].

[197] Lakshya Bhardwaj and Patrick Jefferson. Classifying 5D SCFTs via 6D Scfts: Arbitrary Rank. *JHEP*, 10:282, 2019.

[198] Fabio Apruzzi, Ling Lin, and Christoph Mayrhofer. Phases of 5d SCFTs from M-/F-theory on Non-Flat Fibrations. *JHEP*, 05:187, 2019.

[199] Cyril Closset, Michele Del Zotto, and Vivek Saxena. Five-dimensional SCFTs and gauge theory phases: an M-theory/type IIA perspective. *SciPost Phys.*, 6(5):052, 2019.

[200] Fabio Apruzzi, Craig Lawrie, Ling Lin, Sakura Schäfer-Nameki, and Yi-Nan Wang. 5d Superconformal Field Theories and Graphs. *Phys. Lett. B*, 800:135077, 2020.

[201] Lakshya Bhardwaj. On the classification of 5d SCFTs. *JHEP*, 09:007, 2020.

[202] Fabio Apruzzi, Craig Lawrie, Ling Lin, Sakura Schäfer-Nameki, and Yi-Nan Wang. Fibers add Flavor, Part I: Classification of 5d SCFTs, Flavor Symmetries and BPS States. *JHEP*, 11:068, 2019.

[203] Fabio Apruzzi, Craig Lawrie, Ling Lin, Sakura Schäfer-Nameki, and Yi-Nan Wang. Fibers add Flavor, Part II: 5d SCFTs, Gauge Theories, and Dualities. *JHEP*, 03:052, 2020.

[204] Lakshya Bhardwaj, Patrick Jefferson, Hee-Cheol Kim, Houri-Christina Tarazi, and Cumrun Vafa. Twisted Circle Compactifications of 6d SCFTs. *JHEP*, 12:151, 2020.

[205] Lakshya Bhardwaj. Do all 5d SCFTs descend from 6d SCFTs? *JHEP*, 04:085, 2021.

[206] Lakshya Bhardwaj and Gabi Zafrir. Classification of 5d $\mathcal{N} = 1$ gauge theories. *JHEP*, 12:099, 2020.

[207] Lakshya Bhardwaj. More 5d KK theories. *JHEP*, 03:054, 2021.

[208] Kenneth A. Intriligator, David R. Morrison, and Nathan Seiberg. Five-dimensional supersymmetric gauge theories and degenerations of Calabi-Yau spaces. *Nucl. Phys. B*, 497:56–100, 1997.

[209] David R. Morrison and Nathan Seiberg. Extremal transitions and five-dimensional supersymmetric field theories. *Nucl. Phys. B*, 483:229–247, 1997.

[210] Edward Witten. Toroidal compactification without vector structure. *JHEP*, 02:006, 1998.

[211] Miguel Montero and Cumrun Vafa. Cobordism Conjecture, Anomalies, and the String Lamppost Principle. *JHEP*, 01:063, 2021.

[212] Anamaría Font, Bernardo Fraiman, Mariana Graña, Carmen A. Núñez, and Héctor Parra De Freitas. Exploring the landscape of heterotic strings on T^d. *JHEP*, 10:194, 2020.

[213] Anamaria Font, Bernardo Fraiman, Mariana Graña, Carmen A. Núñez, and Héctor Parra De Freitas. Exploring the landscape of CHL strings on T^d. *JHEP*, 08:095, 2021.

[214] Jan de Boer, Robbert Dijkgraaf, Kentaro Hori, Arjan Keurentjes, John Morgan, David R. Morrison, and Savdeep Sethi. Triples, fluxes, and strings. *Adv. Theor. Math. Phys.*, 4:995–1186, 2002.

[215] Arjan Keurentjes. Discrete moduli for type 1 compactifications. *Phys. Rev. D*, 65:026007, 2002.

[216] Freddy Alexander Cachazo and Cumrun Vafa. Type I' and Real Algebraic Geometry. 1 2000.

[217] Shyamoli Chaudhuri, George Hockney, and Joseph D. Lykken. Maximally supersymmetric string theories in D $<$ 10. *Phys. Rev. Lett.*, 75:2264–2267, 1995.

[218] Shyamoli Chaudhuri and Joseph Polchinski. Moduli space of CHL strings. *Phys. Rev. D*, 52:7168–7173, 1995.

[219] Jaemo Park. Orientifold and F theory duals of CHL strings. *Phys. Lett. B*, 418:91–97, 1998.

[220] Atish Dabholkar and Jaemo Park. Strings on orientifolds. *Nucl. Phys. B*, 477:701–714, 1996.

[221] Shigeki Sugimoto. Anomaly cancellations in type I D-9 - anti-D-9 system and the USp(32) string theory. *Prog. Theor. Phys.*, 102:685–699, 1999.

[222] Cumrun Vafa. Evidence for F theory. *Nucl. Phys. B*, 469:403–418, 1996.

[223] David R. Morrison and Cumrun Vafa. Compactifications of F theory on Calabi-Yau threefolds. 1. *Nucl. Phys. B*, 473:74–92, 1996.

[224] David R. Morrison and Cumrun Vafa. Compactifications of F theory on Calabi-Yau threefolds. 2. *Nucl. Phys. B*, 476:437–469, 1996.

[225] Philip Argyres, Matteo Lotito, Yongchao Lü, and Mario Martone. Geometric constraints on the space of $\mathcal{N}$ = 2 SCFTs. Part I: physical constraints on relevant deformations. *JHEP*, 02:001, 2018.

[226] Mirjam Cvetič, Markus Dierigl, Ling Lin, and Hao Y. Zhang. String Universality and Non-Simply-Connected Gauge Groups in 8d. *Phys. Rev. Lett.*, 125(21):211602, 2020.

[227] Markus Dierigl and Jonathan J. Heckman. Swampland Cobordism Conjecture and Non-Abelian Duality Groups. *Phys. Rev. D*, 103(6):066006, 2021.

[228] Mirjam Cvetic, Markus Dierigl, Ling Lin, and Hao Y. Zhang. Gauge group topology of 8D Chaudhuri-Hockney-Lykken vacua. *Phys. Rev. D*, 104(8):086018, 2021.

[229] Iñaki García-Etxebarria, Hirotaka Hayashi, Kantaro Ohmori, Yuji Tachikawa, and Kazuya Yonekura. 8d gauge anomalies and the topological Green-Schwarz mechanism. *JHEP*, 11:177, 2017.

[230] Christopher Beem, Madalena Lemos, Pedro Liendo, Wolfger Peelaers, Leonardo Rastelli, and Balt C. van Rees. Infinite Chiral Symmetry in Four Dimensions. *Commun. Math. Phys.*, 336(3):1359–1433, 2015.

[231] Hiroyuki Shimizu, Yuji Tachikawa, and Gabi Zafrir. Anomaly Matching on the Higgs Branch. *JHEP*, 12:127, 2017.

[232] Alfred D. Shapere and Yuji Tachikawa. Central Charges of $\mathcal{N} = 2$ Superconformal Field Theories in Four Dimensions. *JHEP*, 09:109, 2008.

[233] Nathan Seiberg and Edward Witten. Gauge dynamics and compactification to three-dimensions. In *Conference on the Mathematical Beauty of Physics (In Memory of C. Itzykson)*, pages 333–366, 6 1996.

[234] Bernardo Fraiman and Héctor Parra De Freitas. Symmetry Enhancements in 7d Heterotic Strings. 6 2021.

[235] Nathan Seiberg. IR Dynamics on Branes and Space-Time Geometry. *Phys. Lett. B*, 384:81–85, 1996.

[236] Kenneth A. Intriligator. Compactified Little String Theories and Compact Moduli Spaces of Vacua. *Phys. Rev. D*, 61:106005, 2000.

[237] Yuji Tachikawa. Frozen singularities in M and F theory. *JHEP*, 06:128, 2016.

[238] P. K. Townsend. A New Anomaly Free Chiral Supergravity Theory from Compactification on K3. *Phys. Lett. B*, 139:283–287, 1984.

[239] S. J. Gates, Jr., C. M. Hull, and M. Roček. Twisted Multiplets and New Supersymmetric Nonlinear Sigma Models. *Nucl. Phys. B*, 248:157–186, 1984.

[240] G. W. Gibbons, G. Papadopoulos, and K. S. Stelle. Hkt and Okt Geometries on Soliton Black Hole Moduli Spaces. *Nucl. Phys. B*, 508:623–658, 1997.

[241] Duiliu-Emanuel Diaconescu and Nathan Seiberg. The Coulomb Branch of (4,4) Supersymmetric Field Theories in Two-Dimensions. *JHEP*, 07:001, 1997.

[242] Tohru Eguchi, Hirosi Ooguri, Anne Taormina, and Sung-Kil Yang. Superconformal Algebras and String Compactification on Manifolds with $SU(N)$ Holonomy. *Nucl. Phys. B*, 315:193–221, 1989.

[243] Bernardo Fraiman and Héctor Parra de Freitas. Freezing of gauge symmetries in the heterotic string on T^4. *JHEP*, 04:007, 2022.

[244] Cumrun Vafa. The String landscape and the swampland. 9 2005.

[245] Eran Palti. The swampland: Introduction and review. *Fortschritte der Physik*, 67(6):1900037, June 2019.

[246] Marieke van Beest, José Calderón-Infante, Delaram Mirfendereski, and Irene Valenzuela. Lectures on the Swampland Program in String Compactifications. *Phys. Rept.*, 989:1–50, 2022.

[247] Mariana Graña and Alvaro Herráez. The Swampland Conjectures: A Bridge from Quantum Gravity to Particle Physics. *Universe*, 7(8):273, 2021.

[248] Miguel Montero, Cumrun Vafa, and Irene Valenzuela. The dark dimension and the Swampland. *JHEP*, 02:022, 2023.

[249] Gia Dvali and Michele Redi. Black Hole Bound on the Number of Species and Quantum Gravity at LHC. *Phys. Rev. D*, 77:045027, 2008.

[250] Gia Dvali. Black Holes and Large N Species Solution to the Hierarchy Problem. *Fortsch. Phys.*, 58:528–536, 2010.

[251] Savdeep Sethi and Mark Stern. D-brane bound states redux. *Commun. Math. Phys.*, 194:675–705, 1998.

[252] W. Nahm. Supersymmetries and their Representations. *Nucl. Phys. B*, 135:149, 1978.

[253] E. Cremmer, B. Julia, and Joel Scherk. Supergravity Theory in Eleven-Dimensions. *Phys. Lett. B*, 76:409–412, 1978.

[254] Daniel Z. Freedman and Antoine Van Proeyen. *Supergravity*. Cambridge Univ. Press, Cambridge, UK, 5 2012.

[255] Edward Witten. String theory dynamics in various dimensions. *Nucl. Phys. B*, 443:85–126, 1995.

[256] Jnanadeva Maharana and John H. Schwarz. Noncompact symmetries in string theory. *Nucl. Phys. B*, 390:3–32, 1993.

[257] Ashoke Sen. Black hole solutions in heterotic string theory on a torus. *Nucl. Phys. B*, 440:421–440, 1995.

[258] Petr Horava and Edward Witten. Heterotic and type I string dynamics from eleven-dimensions. *Nucl. Phys. B*, 460:506–524, 1996.

[259] Luis E. Ibanez, Miguel Montero, Angel Uranga, and Irene Valenzuela. Relaxion Monodromy and the Weak Gravity Conjecture. *JHEP*, 04:020, 2016.

[260] Gerald V. Dunne and Hyunsoo Min. Beyond the thin-wall approximation: Precise numerical computation of prefactors in false vacuum decay. *Phys. Rev. D*, 72:125004, 2005.

[261] A.I. Nikishov. Barrier scattering in field theory removal of klein paradox. *Nucl. Phys. B*, 21:346–358, 1970.

[262] Thomas D. Cohen and David A. McGady. The Schwinger mechanism revisited. *Phys. Rev. D*, 78:036008, 2008.

[263] Raphael Bousso and Joseph Polchinski. Quantization of four form fluxes and dynamical neutralization of the cosmological constant. *JHEP*, 06:006, 2000.

[264] Sitender Pratap Kashyap, Swapnamay Mondal, Ashoke Sen, and Mritunjay Verma. Surviving in a Metastable de Sitter Space-Time. *JHEP*, 09:139, 2015.

[265] Jaume Garriga, Alan H. Guth, and Alexander Vilenkin. Eternal inflation, bubble collisions, and the persistence of memory. *Phys. Rev. D*, 76:123512, 2007.

[266] Markus B. Fröb, Jaume Garriga, Sugumi Kanno, Misao Sasaki, Jiro Soda, Takahiro Tanaka, and Alexander Vilenkin. Schwinger effect in de Sitter space. *JCAP*, 04:009, 2014.

[267] Erick J. Weinberg. Hawking-Moss bounces and vacuum decay rates. *Phys. Rev. Lett.*, 98:251303, 2007.

[268] Naritaka Oshita and Jun'ichi Yokoyama. Entropic interpretation of the Hawking–Moss bounce. *PTEP*, 2016(5):053E02, 2016.

[269] Seung Kee Han, I. G. Koh, and Hyung-Won Lee. Gauged $d = 7N = 2$ Pure Supergravity With Two Form Potential and Its Compactifications. *Phys. Rev. D*, 32:3190, 1985.

[270] James Hughes and Joseph Polchinski. Partially Broken Global Supersymmetry and the Superstring. *Nucl. Phys. B*, 278:147–169, 1986.

[271] Atish Dabholkar, Gary W. Gibbons, Jeffrey A. Harvey, and Fernando Ruiz Ruiz. Superstrings and Solitons. *Nucl. Phys. B*, 340:33–55, 1990.

[272] Michael B. Green and John H. Schwarz. Supersymmetrical String Theories. *Phys. Lett. B*, 109:444–448, 1982.

[273] Michael B. Green and John H. Schwarz. Covariant Description of Superstrings. *Phys. Lett. B*, 136:367–370, 1984.

[274] Martin Luscher and Peter Weisz. String excitation energies in SU(N) gauge theories beyond the free-string approximation. *JHEP*, 07:014, 2004.

[275] Ofer Aharony and Eyal Karzbrun. On the effective action of confining strings. *JHEP*, 06:012, 2009.

[276] Ofer Aharony and Zohar Komargodski. The Effective Theory of Long Strings. *JHEP*, 05:118, 2013.

[277] Michael B. Green, J. H. Schwarz, and Edward Witten. *SUPERSTRING THEORY. VOL. 1: INTRODUCTION*. Cambridge Monographs on Mathematical Physics. 7 1988.

[278] Michael B. Green and John H. Schwarz. Anomaly Cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory. *Phys. Lett. B*, 149:117–122, 1984.

[279] Curtis G. Callan, Jr. and Jeffrey A. Harvey. Anomalies and Fermion Zero Modes on Strings and Domain Walls. *Nucl. Phys. B*, 250:427–436, 1985.

[280] Olaf Hohm, Ashoke Sen, and Barton Zwiebach. Heterotic Effective Action and Duality Symmetries Revisited. *JHEP*, 02:079, 2015.

THIS THESIS WAS TYPESET using LaTeX, originally developed by Leslie Lamport and based on Donald Knuth's TeX. The body text is set in 11 point Egenolff-Berner Garamond, a revival of Claude Garamont's humanist typeface. The above illustration, "Science Experiment 02", was created by Ben Schlitter and released under CC BY-NC-ND 3.0. A template that can be used to format a PhD thesis with this look and feel has been released under the permissive MIT (X11) license, and can be found online at github.com/suchow/Dissertate or from its author, Jordan Suchow, at suchow@post.harvard.edu.