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POLARIZATION ASYMMETRIES IN e^+e^- INCLUSIVE HADRON PRODUCTION*

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ABSTRACT

Interference effects between electromagnetic and weak interactions in e^+e^- hadronic production, which are free from higher order QED corrections, are considered. In particular we calculate the asymmetry between the cross sections when one of the beams is longitudinally polarized positively and negatively, the other unpolarized. This asymmetry is then parity non-conserving. Numerical results are given for total hadronic cross sections, and for π, $K^{\pm}$, $D^{\pm}$ inclusive production.

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Search for interference effects between electromagnetic and neutral weak interactions in energetic electron-positron collisions [1] will soon be underway. This interference is expected to manifest itself in the forward-backward asymmetry between the positively and negatively charged muons in the reactions $e^+ + e^- \to \mu^+ + \mu^-$ in the center-of-mass system.

The purpose of this paper is to point out yet another observable effect of the interference in the inclusive and total hadronic production cross sections in $e^+ + e^- \to$ hadrons, when one of the colliding particles is longitudinally polarized. The effect can be expressed by

$$r = \frac{\sum(+) - \sum(-)}{\sum(+) + \sum(-)} , \tag{1}$$

where $\sum(\pm)$ are the inclusive (energy-weighted) or total hadronic cross sections with $\pm$ signs referring to the positive and negative helicities of the particle, respectively. As is expected, the numerator of (1) is proportional to $(s - m^2_{Z^o})^{-1}$, where $\sqrt{s}$ is the center-of-mass energy. Since the denominator is proportional to s^{-1}, r increases with s as $s/m^2_{Z^o}$ for $s << m^2_{Z^o}$.

Let us begin with the amplitudes that correspond to the diagrams in Fig. 1. The single-photon exchange amplitude is

$$M_\gamma = ie^2 e_Q \bar{v}(p_1) \gamma^\mu u(p_2) \bar{u}_Q(k_2) \gamma_\mu v_Q(k_1) s^{-1} , \tag{2}$$

where e_Q is the electric charge of the quark in units of e. According to the standard model for electromagnetic and weak interactions [2], the Z^o-exchange amplitude is given by

$$M_{Z^o} = \frac{ie^2}{\sin 2\theta_W} \bar{v}(p_1)\gamma^\nu(a+b\gamma_5)u(p_2)\bar{u}_Q(k_2)\gamma_\nu(c+d\gamma_5)v_Q(k_1)\left[s-m^2_{Z^o}\right]^{-1}, \tag{3}$$

where

$$\begin{aligned} a &= -\frac{1}{2} + 2\sin^2\theta_W \\ b &= \frac{1}{2} \\ c &= T_3 - 2e_Q \sin^2\theta_W \\ d &= -T_3 \end{aligned} \quad , \tag{4}$$

where θ_W the weak mixing angle, T_3 the weak isospin (third component) of the quark, and e_Q its electric charge.

It can easily be shown that, when both positron and electron beams are unpolarized or transversely polarized, there exists a parity-conserving forward-backward charge asymmetry in inclusive hadronic production. The higher order QED radiative corrections that give rise to a similar effect [3] have not been worked out in the case of the inclusive hadronic production. Thus, in order to avoid such background corrections, in this work we restrict ourselves to cases in which the numerator of (1) contains only the parity-nonconserving pieces. This occurs when one beam is longitudinally polarized. The case in which one beam is longitudinally polarized and the other transversely polarized is the same as one in which the latter is unpolarized.

For generality we assume that both beams are longitudinally polarized with P_e and $P_{\bar{e}}$, respectively. From (2) and (3) we obtain for the differential cross section for the elementary process $e^+e^- \to$ quark + antiquark that arises from the interference

$$\left[\frac{d\sigma}{d\Omega}\right]_{int} = \frac{e^4 e_Q}{32\pi^2 \sin^2 2\theta_W \left(s - m^2_{Z^o}\right)} \left[c_Q \left\{ a(1 + P_e P_{\bar{e}}) - b(P_e + P_{\bar{e}}) \right\} (1 + \cos^2\theta) \right.$$

$$\left. \pm\, 2d_Q \left\{ a(P_e + P_{\bar{e}}) - b(1 + P_e P_{\bar{e}}) \right\} \cos\theta \right] \qquad (5)$$

where ± signs correspond to the quark $(\vec{k})$ and antiquark $(-\vec{k})$ and their charge-conjugated final states, respectively, and θ refers to the angle that the momentum $\vec{k}$ makes with the direction of the electron beam.

The differential cross section for the inclusive production of a hadronic state h is then obtained by summing (5) over the quarks whose fragmentations could produce this final state hadron, with appropriate fragmentation functions. Ignoring QCD evolution corrections, the fragmentation functions are independent of s, and we get[4]

$$\left[\frac{d^2\sigma^h}{d\Omega dz}\right]_{int} = \frac{3e^4}{32\pi^2 \sin^2 2\theta_w \left(s - m^2_{2^o}\right)}$$

$$\left[\left\{ a\ (1 + P_e P_{\bar{e}}) - b(P_e + P_{\bar{e}}) \right\} (1 + \cos^2\theta) \left\{ \sum_a e_Q c_Q \left(\mathscr{D}^h_Q(z) + \mathscr{D}^h_{\bar{Q}}(z) \right) \right\} \right.$$

$$\left. +\, 2 \left\{ a\ (P_e + P_{\bar{e}}) - b(1 + P_e P_{\bar{e}}) \right\} \cos\theta \left\{ \sum_Q e_Q d_Q \left(\mathscr{D}^h_Q(z) - \mathscr{D}^h_{\bar{Q}}\ (z) \right) \right\} \right]$$

(6)

Here, the scaling variable is defined by $z = 2E_h/\sqrt{s}$, with E_h the hadron energy, and $\mathscr{D}^h_Q(z)$ $\left(\mathscr{D}^h(z)\right)$ the fragmentation function for the quark Q (antiquark $\overline{Q}$). In Eq. (6) the quark color factor has been included.

It is now easy to see the cancellation of the parity-conserving pieces in the numerator of (1) when only one of the beams is longitudinally polarized. Taking $P_{\bar{e}} = 0$, from (6) we get

$$\frac{d^2\sigma^h}{d\Omega dz}(+) - \frac{d^2\sigma^h}{d\Omega dz}(-) = P_e \frac{3\alpha^2}{\sin^2 2Q_W\left(s - m^2_{Z^o}\right)} \left[- b(1 + \cos^2\theta) \left\{ \sum_Q e_Q c_Q \left(\mathscr{D}^h_Q(z) + \mathscr{D}^h_{\bar{Q}}(z) \right) \right\} + 2a \cos\theta \left\{ \sum_Q e_Q d_Q \left(\mathscr{D}^h_Q(z) - \mathscr{D}^h_{\bar{Q}}(z) \right) \right\} \right] \qquad (7)$$

The single-photon exchange does not contribute to the left-hand side of (7) when only one beam is longitudinally polarized.

For the production of a hadronic state h that is not an eigenstate of charge conjugation, the factor $\sum_Q e_Q d_Q \left(\mathscr{D}^h_Q(z) - \mathscr{D}^h_{\bar{Q}}(z) \right)$ in (7) does not vanish in general, giving rise to forward-backward charge asymmetry. This term is parity nonconserving (vector-axial vector couplings). Since the left-hand side of (7) does not contain parity-conserving pieces, the QED radiative corrections [3] do not contribute to this asymmetry. From the angular distributions in (7) one may possibly test separately the axial vector-vector (bc) and the vector-axial vector (ad) couplings [5].

When the inclusively produced states are eigenstates of charge conjugation (i.e., $h = \pi^o, \eta, \rho \ldots$), which implies that for every quark there exists an antiquark of the same flavor, the forward-backward asymmetry vanishes because $\mathscr{D}^h_Q(z) = \mathscr{D}^h_{\bar{Q}}(z)$, as is seen in the second term in (7) ($\sim \cos\theta$). Similarly, for the inclusive production of self-charge conjugate multiparticle states, i.e., $\pi^+ \pi^-$, $\pi^o \pi^+ \pi^-$, K^+K^-...., the sum of the second term ($\sim \cos\theta$) in (7) over h (e.g., π^+ and π^-) vanishes under the condition $\mathscr{D}_Q^h(z) = \mathscr{D}_{\bar{Q}}^{\bar{h}}(z)$

The vanishing of the second term ($\sim \cos\theta$) in (7) which contains the axial vector coupling of Z^o with $h_1 + h_2 \ldots + X$, where $h_1 + h_2 + \ldots$ is an eigenstate of charge conjugation, is consistent with the statement that can be inferred from a more general point of view. By charge conjugation invariance, only the part of the Z^o-exchange amplitude in which Z^o couples with hadrons $h_1 + h_2 + \ldots + X$ through the vector coupling interferes with the single-photon exchange amplitude with the same final state (see Fig. 2).

In the quark-parton model, the vanishing of the axial vector coupling at the hadronic vertex when the inclusively produced hadrons are in an eigenstate of charge conjugation is realized in the cross section rather than in the amplitude.

Let us now proceed to the longitudinal polarization asymmetries of the total hadronic and π,K,D-inclusive energy-weighted cross sections. We assume that the positron beam retains its transverse polarization and the electron beam is longitudinally polarized. The cross section is the same as if the positron beam is unpolarized. We obtain from (7) for the difference between the two helicity states of the electron [6]

$$\sum^h(+) - \sum^h(-) \equiv \int z dz d\Omega \left[\frac{d^2\sigma^h(+)}{d\Omega dz} - \frac{d^2\sigma^h(-)}{d\Omega dz}\right]$$

$$= - P_e \frac{4\pi\alpha^2}{\sin^2 2\theta_W \left(s - m_{Z^o}^2\right)} \int_0^1 z dz \left\{\sum_Q e_Q c_Q \left(\mathscr{D}_Q^h + \mathscr{D}_{\bar{Q}}^h\right)\right\}, \quad (8)$$

This formula applies to any arbitrary hadron h.

Upon summation over all hadrons, we get

$$\sum_h \left[\Sigma^h(+) - \Sigma^h(-)\right] = \frac{-8\pi\alpha^2 P_e}{\sin^2 2\theta_W \left(S - m^2_{Z^o}\right)} \sum_Q e_Q \left[(T_3)_Q - 2e_Q \sin^2 Q_W\right] \tag{9}$$

by virtue of the momentun conservation relation

$$\sum_h \int_0^1 zdz\, \mathcal{D}^h_Q(z) = 1 \quad . \tag{10}$$

At energies $s << m^2_{Z^o}$, we can neglect the Z^o contribution in the sum of energy-weighted cross sections, and obtain for the inclusive hadronic cross section [6]

$$\Sigma^h(+) + \Sigma^h(-) = \frac{4\pi\alpha 2}{s} \int_0^1 zdz \sum_Q e^2_Q \left[\mathcal{D}^h_Q(z) + \mathcal{D}^h_{\bar{Q}}(z)\right] \quad . \tag{11}$$

For the total hadronic cross section, from (10) to (11) we get

$$2\sigma_{tot}(e^+e^- \to \text{hadrons}) = \sum_h \left[\Sigma^h(+) + \Sigma^h(-)\right] = 2R\, \frac{4\pi\alpha^2}{3s} \quad , \tag{12}$$

where

$$R = 3 \sum_Q e^2_Q$$

Note that the use of the energy-weighted inclusive cross sections makes the connection with total hadronic cross sections simple. This may also be advantageous from an experimental point of view.

Referring to (1), we obtain from (9) and (12) for $s << m^2_{Z^o}$

$$r_t = \frac{\sum_h \left[\Sigma^h(+) - \Sigma^h(-)\right]}{\sum_h \left[\Sigma^h(+) + \Sigma^h(-)\right]}$$

$$\doteq P_e \frac{s}{m_{Z^o}^2 \sin^2 2\theta_W} \frac{\sum_Q e_Q \left(T_3 - 2e_Q \sin^2\theta_W\right)}{\sum_Q e_Q^2} \quad . \tag{13}$$

Using the experimental value $\sin^2\theta_W = 0.23$, and taking $m_{Z^o} = 90$ GeV, we have evaluated r_t/P_e from (13) at $\sqrt{s} = 15$, 30 and 40 GeV. As has been shown experimentally [7], these energies are sufficiently above the $b\bar{b}$-quark pair production threshold so that the scaling of the cross sections can be assumed. Summing over u,d,c,s, and b quarks, the r_t/P_e obtained is given in Table I.

We now turn to the computation of r^h for certain special cases from (8) and (11). If h is a pion $(\pi^o, \pi^\pm)$ or a kaon $(K^\pm)$, we can use the phenomenological calculation of the fragmentation functions by Feynman and Field [8]. Charge conjugation and isospin invariance reduce the number of fragmentation functions [8]. Also we have assumed that these functions are equal for different quarks that are not valence quarks of the π or K (for example $\mathscr{D}_s^{\pi^o}(z) = \mathscr{D}_c^{\pi^o} = \mathscr{D}_b^{\pi^o}(z)$).

For the case in which h is a heavy meson, for example D^+, we assume that $\mathscr{D}_c^{D^+}$ is strongly peaked around $z \simeq \frac{m_c}{m_{D^+}} \simeq 1$ [9], and all other fragmentation functions are negligible. This approximation is based on the fact that the binding energy in this case is small compared to the masses.

The computed values of r^h are given in Table I, where $r^{\pi} = r^{\pi^0} = r^{\pi^+} = r^{\pi^-}$. We have also included for comparison the values for the ratio for $\mu^+\mu^-$-pair production evaluated from[10]

$$r(\mu^+\mu^-) = P_e \frac{s}{\sin^2 2\theta_W \, m^2_{z^o}} \left(\frac{1}{2} - 2 \sin^2 Q_W\right) . \tag{14}$$

As can be seen in Table I, the values for r_t/P_e, r^{π}/P_e, $r^{K^{\pm}}/P_e$ and $r^{D^{\pm}}/P_e$ are not negligibly small, especially for the range $\sqrt{s} \approx$ 30-40 GeV.

It should be remarked, however, that, while the absence of the QED radiative corrections in this work may be an advantage, the experimental test of the predictions must wait for the availability of longitudinally polarized high-energy electron or positron beam of the future.

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TABLE I

Observable ratios for the total hadron, pion, $K^{\pm}$, $D^{\pm}$ and $\mu^+\mu^-$ pair productions, and the total hadron cross section.

$\sqrt{s}$(GeV)	r_t/P_e	σ^t(nb)	r^{π}/P_e	$r^{K^{\pm}}/P_e$	$r^{D^{\pm}}/P_e$	$r(\mu^+\mu^-)/P_e$
15	0.019	1.42	0.019	0.020	0.011	-0.0016
30	0.078	0.35	0.077	0.082	0.045	-0.0063
40	0.139	0.20	0.139	0.145	0.078	-0.0114

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5. For $\pi^\pm$ inclusive productions, the forward-backward charge asymmetries obtained from (7) are found to be ∓ 0.015. The fact that the asymmetries are proportional to the parameter a (see (4)) renders the values small.

6. The factor ½ needed to avoid double counting in the integrated cross section is incorporated.

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10. Equation (14) is obtained directly from (5) by setting $c_Q = a$.

FIGURE CAPTIONS

1. Diagrams for the elementary process $e^+ + e^- \rightarrow Q + \overline{Q}$ via single photon and Z^o exchanges.

2. When the inclusively produced hadronic state is an eigenstate of charge conjugation, charge conjugation invariance admits only the vector coupling at the Z^o-hadron vertex in the interference.

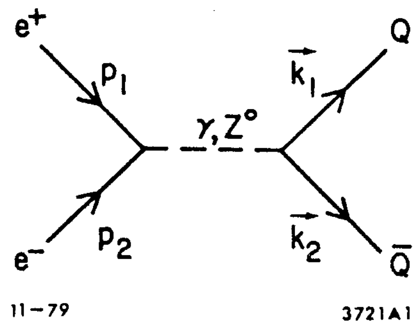

Fig. 1

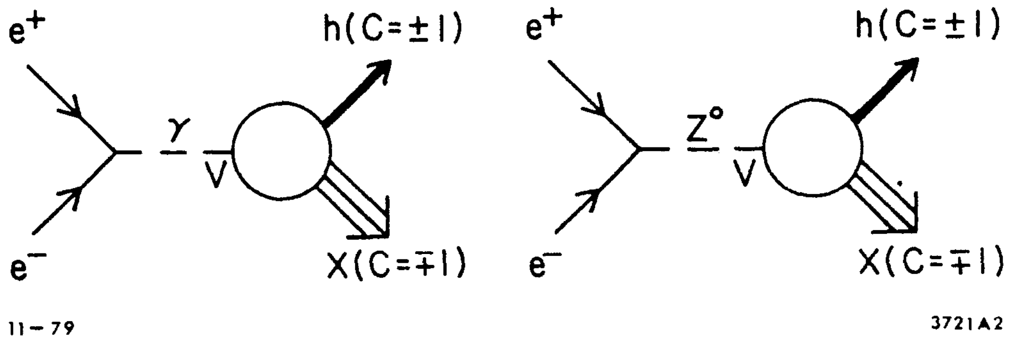

Fig. 2