

TEST OF PHYSICS BEYOND THE STANDARD MODEL IN NUCLEI * **

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Grand Unifications of the electroweak and the strong interaction prefer that the neutrino is a Majorana particle and therefore essentially identical with its own antiparticle. In such grand unified models the neutrino has also a finite mass and a slight right-handed weak interaction, since the model is left-right symmetric. These models have vector bosons mediating the left- and the right-handed weak interactions. If these models are correct, the neutrinoless double beta-decay is feasible. Although the neutrinoless double beta-decay has not been seen it is possible to extract from the lower limits of the lifetime upper limits for the effective electron-neutrino mass and for the effective mixing angle of the vector bosons mediating right-handed and the left-handed weak interaction. One also can obtain an effective upper limit for the mass ratio of the light and the heavy vector bosons. A condition for obtaining reliable limits for these fundamental quantities from the measured lower limits of the half lives of the $0\nu\beta\beta$ decay are that the nuclear matrix can be calculated correctly. These nuclear structure calculations can be tested by calculating the two neutrino double beta decay ($2\nu\beta\beta$) for which we have experimental data and not only lower limits as for the $0\nu\beta\beta$ decay. The $2\nu\beta\beta$ decay is dominated by the Gamow Teller (GT) transitions. The intermediate 1^+ states in the odd-odd mass nucleus are usually calculated within the Quasi-Particle Random Phase Approximation (QRPA). The QRPA treats Fermion pairs as bosons. This overestimates the ground state correlations and leads to the collapse of the $2\nu\beta\beta$ decay probability for the physical $J^\pi = 1^+, T = 1$ particle-particle interaction. We have extended the QRPA including the Fermi commutation relations. One finds now agreement (in almost all cases) for the $2\nu\beta\beta$ decay probability. This increases also the reliability of the conclusions extracted from the upper limits of the $0\nu\beta\beta$ decay for the neutrino mass, the left-right mixing angle and the lower limit for the mass of the “heavy” vector boson.

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1. Introduction

Grand unified theories predict mostly that the neutrino is a Majorana particle [1], that means it is up to a phase identical with its antiparticle. Left-right symmetric theories inaugurated by Mohapatra, Pati and Senjanovic [2] and especially theories based on SO10 which have first been proposed by Fritzsch and Minkowski [2] predict in improved versions [1] not only, that the neutrino is a Majorana particle, but automatically predict also that the neutrino has a mass and a weak right-handed interaction. The basic idea behind grand unified models is an extension of the local gauge invariance from quantum chromodynamics (SU3) involving only the coloured quarks also to electrons and neutrinos. The presently favoured models are left-right symmetric models. They contain left- and right-handed vector bosons $W_L^\pm$ and $W_R^\pm$.

$$\begin{aligned} W_1^\pm &= \cos \zeta W_L^\pm + \sin \zeta W_R^\pm, \\ W_2^\pm &= -\sin \zeta W_L^\pm + \cos \zeta W_R^\pm. \end{aligned} \quad (1)$$

The vector bosons mediating the left and right-handed interaction are mixed if the mass eigenstates are not identical with the weak eigenstates. The left-right symmetry is broken since the vector bosons $W_1^\pm$ and $W_2^\pm$ obtain different masses by the Higgs mechanism. Since we have not seen a right-handed weak interaction the mass of the heavy, mainly “right-handed” vector boson must be much larger than the mass of the light (81 GeV) vector boson, which is responsible for the left-handed force.

The weak interaction Hamiltonian must now be generalized.

$$\begin{aligned} H_W &\approx \frac{G_F}{\sqrt{2}} \left[(L \cdot l) + \tan \zeta (R \cdot l) + \tan \zeta (L \cdot r) + \left(\frac{M_1}{M_2} \right)^2 (R \cdot r) \right], \\ L/R &= \bar{\psi}_p (g_V \gamma_\mu \mp g_A \gamma_\mu \gamma_5) \psi_n, \\ g_v &= 1; \quad g_A = 1.25, \\ l/r &= \bar{e} (\gamma_\nu \mp \gamma_\nu \gamma_5) \nu. \end{aligned} \quad (2)$$

The capital L and R indicate the hadronic right- and left-handed currents changing a neutron into a proton. The lower case l and r are the left and right handed leptonic currents which annihilate a neutrino (or create an antineutrino) and create an electron (or annihilate a positron). ζ is the mixing angle of the vector bosons (2) and M_1 and M_2 are the light and heavy vector boson masses. The weak interaction Hamiltonian (3) is given for $\zeta \ll 1$ and $M_2 \gg M_1$.

Grand unified theories with Majorana neutrinos allow the double beta-decay without neutrinos. Or inversely: The existence of the double neutrinoless beta-decay would establish that the neutrino is a Majorana particle. Figure 1 shows the diagram for the neutrinoless double beta-decay. rys.1

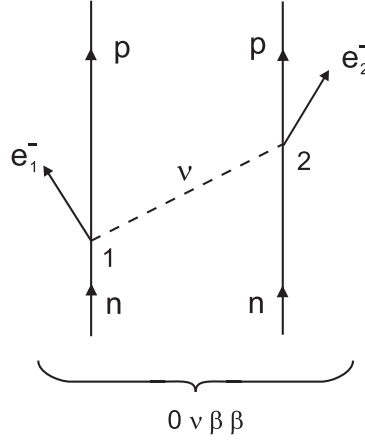


Fig. 1. Diagrams for the neutrinoless double beta decay with a Majorana neutrino. By having only two particles in the final states in the continuum, the phase space is increased by a factor of about 10^6 compared to the $2\nu\beta\beta$ decay. Even with a Majorana neutrino this process is only possible if the neutrino has a finite mass and it is also favored with a right-handed weak interaction. But a right-handed weak current must be accompanied by a finite neutrino mass to yield a finite $0\nu\beta\beta$ decay.

But even if the neutrino is a Majorana particle, the process in Figure 1 can not happen since for a pure left-handed weak interaction theory, the emitted neutrino must be right-handed (positive helicity), while the absorbed neutrino must be left-handed (negative helicity). But grand unified theories predict also that the neutrino has a mass and a slight right-handed weak interaction. With a finite mass the neutrino has not any more a good helicity and the interference term between the leading helicity and the small admixtures allows a neutrinoless double beta-decay.

2. Description of the two-neutrino double beta-decay in the renormalized QRPA

Since there are measurements available for the two neutrino double beta-decay with the geochemical method [3–5], and for five nuclei even laboratory measurements [6–16], one could try to calculate for a test of the theory the double beta-decay with two neutrinos and compare them with the data.

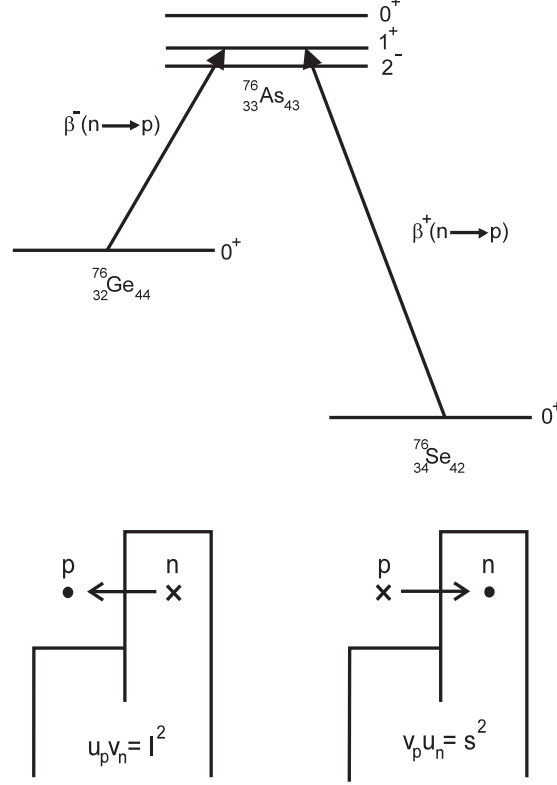


Fig. 2. The upper part shows the way how in the Random Phase Approximation (RPA) the $2\nu\beta\beta$ decay is calculated. For the Fermi transitions the $\beta^-(n \rightarrow p)$ amplitude moves just a neutron into the same proton level and the $\beta^+(p \rightarrow n)$ amplitude moves a proton into the same neutron level. For the Gamow–Teller transitions it can also involve a spin flip, but the orbital part remains the same. One immediately realizes that the occupation and non-occupation amplitudes favour the β^- amplitude, but disfavour the β^+ amplitude. There one has a transition from an unoccupied to an occupied single particle state, which is two-fold small (s^2) first by the fact that the occupation amplitude for the proton v_p and secondly that the unoccupation amplitude for the neutron state u_n are both small. Therefore, the $2\nu\beta\beta$ is drastically reduced.

Figure 2 explains why the $2\nu\beta\beta$ decay amplitude is so drastically reduced. Therefore the small effects which normally do not play a major role can affect the $2\nu\beta\beta$ transition probability. If one looks to the second leg of the double beta-decay which is calculated backwards as a $\beta^+(p \rightarrow n)$ decay from the final nucleus to the intermediate nucleus one finds that the matrix elements involved in these diagrams are Pauli suppressed by a factor

$(u_n v_p)^2 = (\text{small})^4$. The neutron–particle proton-hole force in the isovector channel, which is usually included is repulsive while the particle–particle force usually neglected is attractive. Therefore both excitations tend to cancel each other and therefore the amplitude β^+ is drastically reduced. To show the dependence on the particle–particle matrix element we multiply this matrix element derived from the realistic Bonn potential by solving the Bethe Goldstone equation (Brueckner reaction matrix element) with a factor g_{pp} .

Although one can obtain agreement in this way with the measured $2\nu\beta\beta$ data multiplying the particle–particle matrix elements with a factor g_{pp} in a range of $0.8 \leq g_{pp} \leq 1.2$, the strong dependence on g_{pp} (which renormalizes the Brueckner reaction matrix element of the Bonn potential) does not allow a reliable prediction of the $2\nu\beta\beta$ decay probability.

The reason for this cancellation is that for the second leg the backgoing amplitudes and thus groundstate correlations cancel the leading forward going terms.

The quasi-particle Random Phase Approximation (QRPA) is derived by using the Quasi-Boson Approximation (QBA), where one requests for the quasi-particle Fermion pairs boson commutation relations.

$$\begin{aligned} & \left[\{a_l a_k\}_{\text{JM}}; \{a_k^+, a_l^+\}_{\text{JM}} \right] = n(k, l) n(k', l') \left[\delta_{k, k'} \delta_{l, l'} - \delta_{l, k'} \delta_{k, l'} (-)^{j_k + j_l - J} \right] \\ & \times \left\{ 1 - \frac{1}{\hat{j}_l} \langle 0_{\text{RPA}}^+ | \{a_l^+ a_l\}_{00} | 0_{\text{RPA}}^+ \rangle - \frac{1}{\hat{j}_k} \langle 0_{\text{RPA}}^+ | \{a_k^+ a_k\}_{00} | 0_{\text{RPA}}^+ \rangle \right\} \quad (3) \end{aligned}$$

with:

$$\begin{aligned} n(k, l) &= (1 + (-)^J \delta_{k, l}) / (1 + \delta_{k, l})^{3/2} \\ \hat{j} &= \sqrt{2j + 1}. \end{aligned}$$

In the QBA one puts

$$D_{k, l, J\pi} = 1 - \frac{1}{\hat{j}_l} \langle 0_{\text{RPA}}^+ | \{a_l^+ a_l\}_{00} | 0_{\text{RPA}}^+ \rangle - \frac{1}{\hat{j}_k} \langle 0_{\text{RPA}}^+ | \{a_k^+ a_k\}_{00} | 0_{\text{RPA}}^+ \rangle,$$

equal to unity. Here we include the Fermion character of the quasi-particles a_k^+ (which include proton–neutron mixing and therefore also proton–neutron $T = 1$ pairing [17, 18]) in an approximation determined by the QRPA ground state expectation value [19, 20]. Using the commutation relations (3), which include the Fermion nature of the quasi-particles in the nucleon pair operators, one obtains the renormalized QRPA (RQRPA) equations [19].

Figures 3 and 4 show the second order GT matrix element as function of the particle–particle strength parameter g_{pp} for the $2\nu\beta\beta$ decay calculated in QRPA (and RQRPA, see below) for ^{76}Ge and ^{82}Se , including pn -pairing.

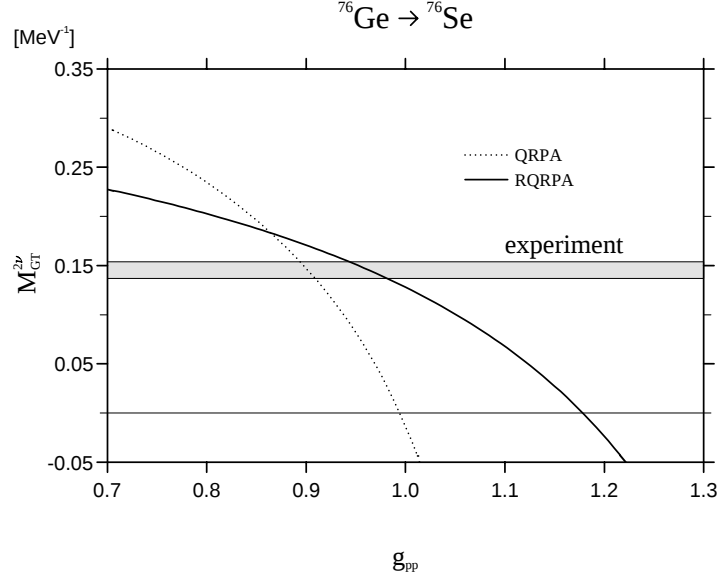


Fig. 3. The Gamow–Teller transition matrix element $M_{\text{GT}}^{2\nu}$ of the 2ν $\beta\beta$ decay of ^{76}Ge is plotted as function of particle–particle coupling constant g_{pp} . The solid line corresponds to full-RQRPA (with p – n pairing), the dashed line to full QRPA (with p – n pairing)

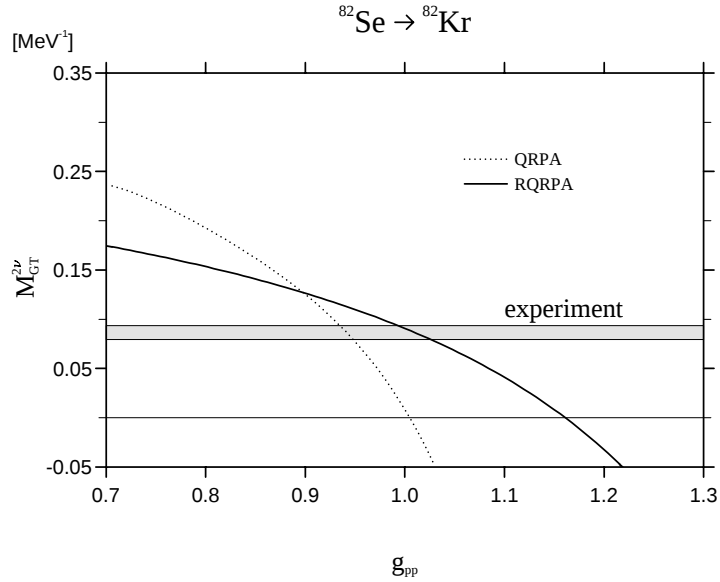


Fig. 4.

$$\begin{pmatrix} \bar{\mathcal{A}} & \bar{\mathcal{B}} \\ \bar{\mathcal{B}} & \bar{\mathcal{A}} \end{pmatrix} \begin{pmatrix} \bar{X} & \bar{Y} \end{pmatrix} = \Omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{X} & \bar{Y} \end{pmatrix}, \\
\begin{aligned} \bar{\mathcal{A}} &= D^{-1/2} \mathcal{A} D^{-1/2}, \\ \bar{\mathcal{B}} &= D^{-1/2} \mathcal{B} D^{-1/2}, \\ \bar{X} &= D^{1/2} X; \bar{Y} = D^{1/2} Y. \end{aligned} \tag{4}$$

Here D is the renormalization matrix given in Eq. (3) and $\mathcal{A}, \mathcal{B}$ and X, Y are the usual QRPA matrices and amplitudes, respectively. Explicit expressions are given in Ref. [19]

3. Calculation and discussion of the results for the $2\nu\beta\beta$ decay

We applied the full-RQRPA method to the $2\nu\beta\beta$ -decay of ^{76}Ge , ^{82}Se , ^{128}Te and ^{130}Te . We assumed the single particle model space both for protons and neutrons as follows.

- (i) For $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ and $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ the model space comprises 13 levels: $1s_{1/2}, 0d_{5/2}, 0d_{3/2}, 1p_{3/2}, 1p_{1/2}, 0f_{7/2}, 0f_{5/2}, 2s_{1/2}, 1d_{5/2}, 1d_{3/2}, 0g_{9/2}, 0g_{7/2}, 0h_{11/2}$.
- (ii) For $^{128}\text{Te} \rightarrow ^{128}\text{Xe}$ and $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ we used 16 levels: $1p_{3/2}, 1p_{1/2}, 0f_{7/2}, 0f_{5/2}, 2s_{1/2}, 1d_{5/2}, 1d_{3/2}, 0g_{9/2}, 0g_{7/2}, 0h_{11/2}, 0h_{9/2}, 1f_{7/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 0i_{13/2}$.

The single particle energies have been calculated with a Coulomb-corrected Wood-Saxon potential. For the two body interaction we used the nuclear G-matrix calculated from Bonn one-boson exchange potential. The single quasiparticle energies and occupation amplitudes have been found by solving the HFB equation with p - n pairing for both the parent and the daughter nuclei in the above mentioned space. The renormalization of the proton-proton, neutron-neutron and proton-neutron pairing interaction has been determined according to Ref. [17].

In the calculation of the full-RQRPA equation we renormalized particle-particle and particle-hole channels of the G-matrix interaction by introducing parameters g_{pp} and g_{ph} , which in principle should be equal to unity.

It is worthwhile mentioning that the calculation of $M_{\text{GT}}^{2\nu}$ within full-RQRPA needs a great computational effort. The RQRPA self-consistent scheme requires the solution of the RQRPA equation (9) for all multiplicities J^π in each iteration for the initial and the final nuclei. In comparison with the RQRPA, the QRPA calculation of M_{GT} requires to solve the QRPA equation only for the multipolarity 1^+ once for the initial and once for the

final nucleus. The iterative procedure of the RQRPA have been found to converge rapidly.

Our results for $2\nu\beta\beta$ -decay of ^{76}Ge and ^{82}Se are presented in Fig. 3 and Fig. 4, respectively.

4. $0\nu\beta\beta$ Decay and the Renormalized Quasi-Particle Random Phase Approximation (RQRPA)

After we established in chapter three that the Renormalized Quasi-Particle Random Phase Approximation (RQRPA) gives a reliable description for the $2\nu\beta\beta$ decay and removes the collapse discussed since 1986 in the literature, we are applying this approach to the $0\nu\beta\beta$ decay [21] which gives information as discussed in the introduction about Grand Unified theories. The $0\nu\beta\beta$ decay is only possible if the neutrino is a Majorana particle and by that has automatically a mass. The favored left-right symmetric theories allow also for a right-handed weak interaction and have a heavy vector boson which is mediating this interaction.

The $0\nu\beta\beta$ decay can be calculated with Fermi's golden rule.

$$w_{0\nu\beta\beta} = \frac{2\pi}{\hbar} \left| \mathcal{M}_m \langle m_\nu \rangle + \mathcal{M}_J \langle \tan \zeta \rangle + \mathcal{M}_W \left\langle \frac{M_1^2}{M_2^2} \right\rangle \right|^2 \times \rho_{\text{final}} \leq w_{0\nu\beta\beta}^{\text{exp}} \quad (\text{upper limit}). \quad (5)$$

Since one has not yet seen the $0\nu\beta\beta$ decay experimentally, the data only allow to give an upper limit for the $0\nu\beta\beta$ decay probability. The three terms in equation (10) turn out to be all of the same sign and thus the upper experimental limit measured for the $0\nu\beta\beta$ decay probability allows to give upper limits for an averaged electron neutrino mass $\langle m_\nu \rangle$ an averaged mixing angle of the vector bosons responsible for the left- and the right-handed weak interaction $\langle \tan \zeta \rangle$ and the ratio of the masses squared for the light over the heavy vector boson $\langle M_1^2/M_2^2 \rangle$.

For the $2\nu\beta\beta$ decay a single particle basis of 9 levels for the protons and 9 levels for the neutrons turned out to be enough. For the $0\nu\beta\beta$ decay the transition operator depends on the distance between the two vertices and one needs a larger basis of at least 12 levels [21]. In addition the pn pairing turns out to be essential [21].

Table I shows the upper limits obtained for the averaged electron neutrino mass [21]. Here one should stress, that the quantities derived are averaged in a special way over the six neutrinos in a left-right symmetric Grand Unified theory. If the weak and the mass eigenstates of this six neutrinos are identical, the averaged electron neutrino mass is equal to the bare

electron neutrino mass but the mixing angle and the mass ratio are averaged to zero. To derive from the averaged quantities, the bare quantities, one needs the six dimensional unitary transformation from the weak eigenstates to the mass eigenstates of the six neutrinos. This is naturally highly model dependent and thus the bare quantities can only be derived if one specializes to a specific Grand Unified model.

TABLE I

Matrix elements of the RQRPA with proton-neutron pairing ($M_{\text{mass}}^{0\nu}$), lower experimental limit of the halfives, derived upper limit for the averaged electron neutrino mass.

Nucleus	$M_{\text{mass}}^{0\nu}$	$T_{1/2}^{0\nu-\text{exp}}$ [years]		$ \langle m_\nu \rangle $ [eV]	
^{76}Ge	1.86	$\geq 7.4 \times 10^{24}$	(90 % C. L.)	[38]	≤ 1.1
^{100}Mo	4.22	$\geq 4.4 \times 10^{22}$	(68 % C. L.)	[47]	≤ 2.4
^{116}Cd	2.47	$\geq 2.9 \times 10^{22}$	(90 % C. L.)	[48]	≤ 4.9
^{128}Te	3.28	$\geq 7.3 \times 10^{24}$	(68 % C. L.)	[49]	≤ 1.2
^{136}Xe	0.96	$\geq 6.4 \times 10^{23}$	(90 % C. L.)	[50]	≤ 3.7

The upper limit for the averaged mixing angle $\langle \tan \zeta \rangle$ and the mass ratio squared of the vector bosons $\langle M_1^2/M_2^2 \rangle$ are about 10^{-8} and 10^{-6} , respectively.

5. Summary

In this contribution we studied the effect of the Pauli principle on the two neutrino and the zero neutrino double beta decay. In the Quasi Particle Random Phase Approximation (QRPA) one is using the Quasi-Boson-Approximation (QBA). This means one requests for a Fermion pair boson commutation relations. We discussed that this neglect of the Pauli principle is drastically overestimating the ground state correlations in nuclei. Due to this effect, the $\nu\beta\beta$ decay probability is collapsing for the realistic particle-particle Gamow-Teller-Matrix element of forces which reproduce the two-body data like the Bonn potential. We included here [19] the Pauli principle for the nucleon pairs at least as the ground state expectation value. In this approximation one can derive again a RPA type equation which we call renormalized QRPA (RQRPA). Using this approach, the collapse of the $2\nu\beta\beta$ decay probability is removed from the physical region ($g_{pp} = 1$). Apart of the Xe isotopes we obtain a very good description of the experimental data for the $2\nu\beta\beta$ probability. This gives us the conviction that also the $0\nu\beta\beta$ decay probability calculation is reliable at least for the nuclei apart of Xe. This allows now to deduce reliable upper limits for the averaged electron

neutrino mass, the averaged mixing angle ζ and the mass ratio of the vector bosons squared as given in table one.

The inclusion of the Pauli principle for the Fermion pairs of nucleons in the RPA is violating the Ikeda sum rule. One obtains a value for the Ikeda sum rule which is about 15 % different from $3(N - Z)$. This is probably due to the fact, that in the commutation relation we include terms which contain a quasi-particle creation and a quasi-particle annihilation operator. To be consistent, one should also include such terms in the excitation operator which excites the states in the intermediate nucleus from the initial nucleus and from final nucleus. We expect that this will give agreement with the Ikeda sum rule. We are working on this problem.

The work which I presented here has been obtained in collaboration with J. Schwieger, Prof. F. Simkovic, Prof. J. Vergados and Prof. G. Pantis.

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