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Featured Article

Charm and hadrons ☆

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1. The November Revolution

The November Revolution was opened by the simultaneous and independent discovery of a new mesonic particle, at Brookhaven and SLAC [1,2], with mass 3.098 GeV and width 93 KeV, decaying into e^+e^-, $\mu^+\mu^-$ and hadrons. The authors named this particle J and Ψ respectively, hence the name J/Ψ adopted since then. A week later, a Frascati collaboration at the electron-positron collider, Adone, confirmed the J/Ψ particle [3]. Remarkably, the three papers followed each other so closely that they could appear in the same issue of the Physical Review Letters.

2. A personal recollection, 1969-1970

The discussion on higher order weak interactions was opened in 1968 with a calculation by Boris Ioffe and Evgeny Shabalin, indicating that $\Delta S = \pm 1$ neutral currents and $\Delta S = 2$ amplitudes would result from higher order weak interactions, even in a theory with only a charged W [4]. The amplitudes were found to be divergent, of order $G(G\Lambda^2)$ and in disagreement with experiments, unless limited by an unreasonably small ultraviolet cut-off $\Lambda = 3 - 4$ GeV (from Δm_K). Similar results were found by R. Marshak and coll. [5] and by F. Low [6]. The exceedingly small value of the cut-off raised a wide discussion.

The Ioffe-Shabalin problem was still on the table in November 1969, when I moved to Harvard and met with John Iliopoulos, at work with Sheldon Glashow on the $G(G\Lambda^2)$ puzzle.

We liked each other and discussed for long, usually two of us arguing against the one at the blackboard, apparently getting nowhere. But during our discussions a change in paradigm occurred. Previous works had been done in the cumbersome framework of the *algebra of currents*, but slowly we began to phrase more and more our discussion in terms of quarks.

In quark language, the Ioffe-Shabalin problem for $K_L \to \mu^+\mu^-$ is represented by the box diagram in Fig. 1(a). The divergent amplitude is proportional to the product of the couplings of quarks d and s to the u quark, as required by the Cabibbo theory.

By January 1970 we got convinced that we had to modify the weak interaction theory. Once we realised that, the solution was just under our eyes.

A fourth quark of charge +2/3, called the charm quark, had been introduced by Bjorken and Glashow (and others), for entirely different reasons. In the weak interaction, the charm quark is coupled to s_C, the quark left out in the Cabibbo theory. The exchange of a c-quark, Fig. 1(b), cancels the singularity and produces an amplitude of order $G[G(m_c^2 - m_u^2)]$, the GIM mechanism [7].

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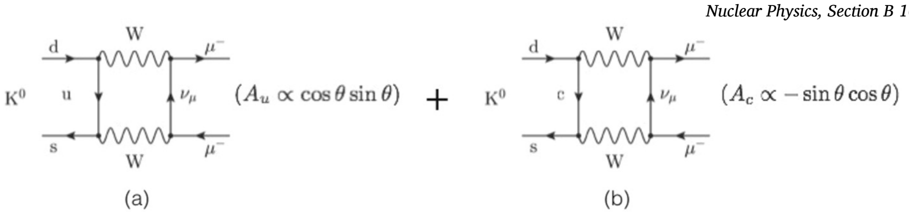

Fig. 1. GIM mechanism for $K^0 \to \mu^+\mu^-$ mixing.

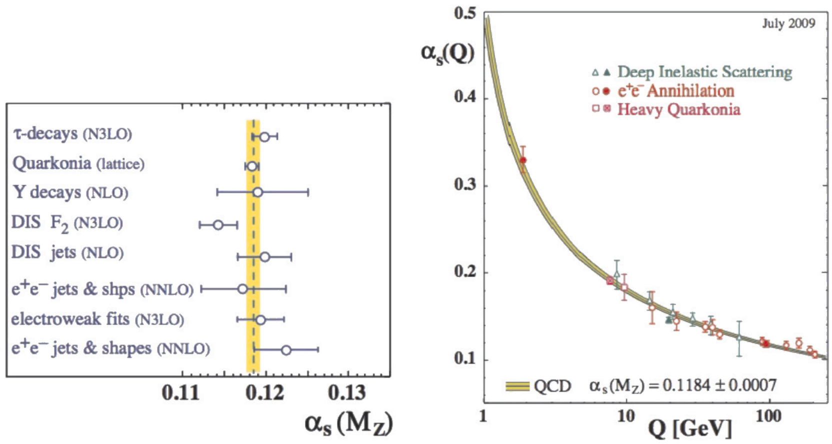

Fig. 2. The running coupling constant of QCD. Figures from Ref. [12].

With two quark generations, Cabibbo weak mixing $d_C = (\cos\theta\ d + \sin\theta\ s)$ is replaced by a unitary 2×2 matrix U

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{1}$$

Charged currents in four-flavour space (u, c, d, s) are given by the matrices C and $C^\dagger$:

$$C = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix};\ C_3 = [C, C^\dagger] = \begin{pmatrix} UU^\dagger & 0 \\ 0 & -U^\dagger U \end{pmatrix} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \tag{2}$$

The neutral current generator, C_3, is flavour diagonal. Thus, a unified gauge theory including charged and neutral vector bosons and flavour conserving neutral currents, is possible [7].

The observed strangeness changing processes appear to one loop (the first weak interaction loop ever computed), are finite and determined by the mass difference $m_c - m_u$. Ioffe's cutoff becomes the prediction: $m_c \sim 1.5$ GeV.

A detailed study made later by B. W. Lee and M. K. Gaillard, in the Glashow-Weinberg-Salam theory with two generations of quark and leptons, confirmed the charm quark mass prediction [8].

3. Asymptotic freedom and heavy quarks

In 1973, David Gross, Franck Wilczeck [9] and, independently, David Politzer [10] proved that strong interactions between quarks mediated by the gauge mesons of the $SU(3)$ colour symmetry, are asymptotically free. The resulting theory, QCD, is the answer to (almost) any Strong Interaction question (see e.g. Ref. [11]).

- QCD is asymptotically free. Quarks carry colour, associated to $SU(3)_{col}$ and flavour, associated to $S(3)_{flavour}$, and are confined inside colour singlet hadrons. The momentum scale which marks the transition from non-perturbative to perturbative QCD is called Λ_{QCD}. Numerically: $\Lambda_{QCD} \simeq 250$ MeV.
- $\Delta^{(++)} = \epsilon^{\alpha\beta\gamma} u^\uparrow_\alpha u^\uparrow_\beta u^\uparrow_\gamma$: Fermi statistics for spin 1/2 quarks is obeyed. These are the *Constituent Quarks*, which determine the structure of baryons and mesons.
- Increasing q^2, quarks radiate gluons (the Altarelli-Parisi picture of scaling violations [13]). At large q^2, we go in the region of Bjorken scaling [14] with quarks and neutral gluons behaving as almost free particles (Feynman's partons [15]).
- The decrease of the colour coupling $\alpha_S(q^2)$ has been observed at LEP and LHC, Fig. 2, and is consistent with the value $\alpha_S(q^2 = M_Z^2) = 0.1185$.

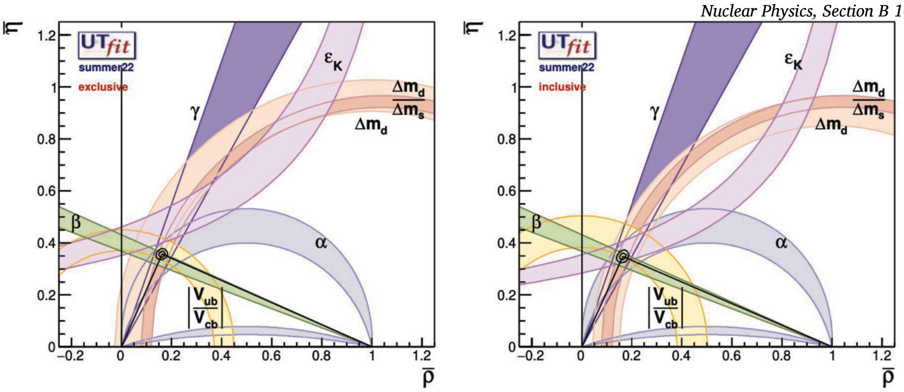

Fig. 3. Inclusive and exclusive determinations of the ratio V_{ub}/V_{cb} in recent lattice QCD calculations [22].

- Quarks u and d have small masses, $m_q \simeq \Lambda_{QCD}$ and fall in the colour strong interaction regime. The strange quark is marginal, but from charm onward we are in the heavy quark region ($m_Q >> \Lambda_{QCD}$). Inclusive semileptonic decays are calculable like deep inelastic processes.
- Bound states involve short distance forces, implying a calculable spectrum of charmonia and bottomonia, as first observed by Politzer and Appelquist [16,17].
- Heavy quark $c\bar{c}$ or $b\bar{b}$ pairs inside hadrons are not easily created or destroyed. A hadron decaying into: J/Ψ or $\Upsilon+$ light hadrons, most likely contains a valence $c\bar{c}$ or $b\bar{b}$ pair: heavy-quark counting is possible.

4. Charm and beauty quarks semileptonic decay

The heavy quark mass may be larger than Λ_{QCD} or quite larger, as is the case of the b quark. If the invariant mass of the hadronic final system is also $>> \Lambda_{QCD}$ we can use parton model and perturbative QCD to compute the inclusive semileptonic width and the energy spectrum of the emerging charged lepton. This is the case of the semileptonic decay of the charm quark [18].

With the observation of B meson decays, we thought our formulae could be used for the energy lepton spectrum near the end point, to determine the Cabibbo-Kobayashi-Maskawa matrix element V_{ub}, not determined by the total rate, which is dominated by the $b \to c$ transition. In this connection, Paolo Franzini (then still in Cornell with CLEO) observed that while in charm decay the lepton energy distribution vanishes at the end point (due to angular momentum conservation) the same probability is non-vanishing in b quark decay. Configurations with small hadron mass are important and perturbative corrections are to be taken into account, to evaluate V_{ub} correctly.

Altarelli and Martinelli provided the crucial resummation of perturbative terms and the final result was a valuable tool for the estimate of V_{ub} from inclusive rates [19], see also [20,21].

An alternative method to obtain V_{ub} from exclusive rates is provided by Lattice QCD computations of the form factor for the exclusive transition: B meson $\to$ light flavoured meson, see e.g. [22] (Fig. 3).

5. Unanticipated charmonia: X, Y, Z and more

Unanticipated, hidden charm/beauty resonances not fitting in predicted charmonium/bottomonium spectra have been observed, classified initially as **X**, **Y** and **Z** particles.

- X, e.g. $X(3872)$ (BELLE, BaBar, 2003): neutral, typically seen in $\Psi + 2\,\pi$, positive parity: $J^{PC} = 0^{++},\ 1^{++},\ 2^{++}$.
- Y, e.g. $Y(4260)$ (BaBar, 2005): neutral, seen in e^+e^- annihilation with Initial State Radiation (ISR): $e^+e^- \to e^+e^- + \gamma_{ISR} \to Y + \gamma_{ISR}$, therefore $J^{PC} = 1^{--}$.
- Z, e.g. $Z(4430)$ (BELLE, 2007; confirmed by LHCB, 2014): typically $J^{PC} = 1^{+-}$, charged or neutral; mostly seen to decay in $\Psi + \pi$ ($Z(3900)$, BESIII 2013) and in $h_c(1P) + \pi$ ($Z(4020)$, BESIII, 2013). 4 valence quarks manifest in the charged Z: ($c\bar{c}u\bar{d}$). Z_b observed ($b\bar{b}u\bar{d}$).

Fig. 4 reports, in black, a recent determination of the spectrum of charmonia, Ref. [23,24], using the Cornell potential [25–27], see also [28]. In red, the lowest lying unexpected charmonia.

Unexpected electrically neutral states differ from the predicted charmonia also by their decay modes, e.g. $X(3872) \to J/\Psi + \rho^0/\omega^0$ with a substantial violation of isospin symmetry, not expected for a pure $c\bar{c}$ bound state.

We know by now about one hundred meson resonances that contain two quark pairs, tetraquarks ($c\bar{c}q\bar{q}$ or $cc\bar{q}\bar{q}$), and a few baryons with ($c\bar{c}qqq$) composition, pentaquarks: a new spectroscopy is showing up.

Multiquark hidden charm hadrons, in hadron colliders, originate mostly from the decays of mesons and baryons containing b-quark, via the weak decay $b \to c + (\bar{c}s)$. For B^+ decay see Fig. 5, taken from [29]. A similar diagram for Λ_b gives rise to pentaquark production: $\Lambda_b \to K + \mathcal{P} \to K + J/\Psi + p$.

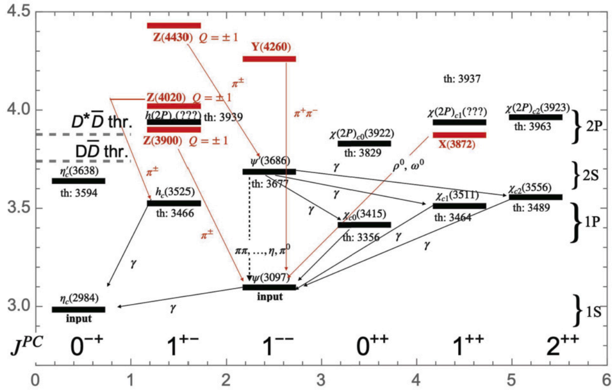

Fig. 4. Predicted and observed charmonia, $S1,2$ and $P1,2$ states in (black). In red the first discovered unanticipated charmonia. Figure from Ref. [23].

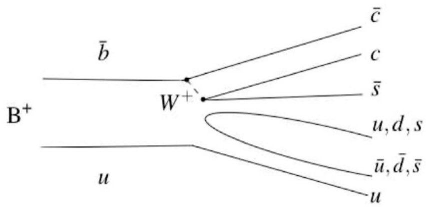

Fig. 5. Quark diagram for $B^+ \to K + X$, with $X = (c\bar{c}q\bar{q}')$. Figure from [29].

The challenge, after X, Y and Z particle discovery, was to reconcile their structure with what we know about the binding of the classical mesons ($q\bar{q}$) and baryons (qqq) by QCD interactions.

There is no consensus yet. A few alternatives still under discussion can be described as follows, see also [30].

- The first guess advanced for $X(3872)$ was that of a *hadron molecule*, pioneered by E. Braaten [31]: a ($D\bar{D}^*$ + C-conjugate) meson pair bound by the same nuclear forces that bind nucleons in atomic nuclei. The rationale was the closeness of X mass to the $D\bar{D}^*$ threshold, reminding the deuteron pn bound state, see [32] for a more recent review. In the first papers, pion exchange was considered to provide the binding force, but this is incompatible with exotic mesons such as $J/\Psi - \phi$ or Z_{cs}, which cannot be bound by pion exchange. A recently advanced hypothesis [33] to connect exotic hadrons to the known mesons is that *contact interactions* described by a chiral Effective Field Theory, produce exotic hadrons as real or virtual poles in the scattering amplitude of meson pairs.
- A different scheme is provided by the *compact diquark-antidiquark* picture proposed in 2005 and further specified in 2014 [34,35]. It describes the exotic hadrons as multiquark states bound by QCD forces, in addition to but independent from $q\bar{q}$ mesons or qqq baryons. Closeness to meson pair thresholds of the lowest lying exotic hadron masses would be the obvious consequence of the fact that these exotic hadrons are made of the same quarks as Gell-Mann Zweig mesons and baryons.[1]
- Models based on pure QCD interactions and specific to hidden charm or hidden beauty exotic hadron, called Hadrocharmonia, have been considered by Voloshin and coll. [38] and by Braaten and coll. [39], in which a colour octet, heavy quark pair is formed by QCD forces, with colour being shielded by a cloud of light quarks and antiquarks. They are dominated by QCD interactions, and are, in fact, simple variations of the last mentioned scheme.

6. Flavour SU(3) broken symmetry and hidden charm nonets

QCD interactions are approximately symmetric under flavour $SU(3)$ and the compact tetraquark model makes a firm prediction: hidden charm tetraquarks must form complete nonets of flavour $SU(3)$, broken by the quark mass difference $m_s - m_{u,d} \sim 150$ MeV.

Starting from 2016, new kinds of exotic hadrons have been discovered that go along this way:

- $J/\Psi - \phi$ resonances, valence structure ($c\bar{c}s\bar{s}$); di-J/Ψ resonances, valence structure ($c\bar{c}c\bar{c}$);
- open strangeness exotics: $Z_{cs}(3082)$ [40], $Z_{cs}(4003)$ [41] and a possible higher energy state $Z_{cs}(4220)$, also reported in [41].

[1] It is interesting to consider the relation between compact tetraquarks and the chiral EFT theory envisaged in [33]. The appearance of the $X(3872)$ pole in the $D\bar{D}^*$ scattering amplitude holds in the compact tetraquark scheme as well. The issue would then be reduced to the question posed by Weinberg for deuterium [36], namely if the Hilbert space of $q\bar{q}$ meson states is complete or there are other independent "elementary" states, namely the $qq\bar{q}\bar{q}$ states. Following Weinberg, the discrimination could be decided by the sign of the effective radius r_0, to be measured (or predicted) in the same scattering process. In [37] we find for $X(3872)$ a negative r_0, favouring tetraquarks. More precise experimental data are needed, for a more conclusive evidence. But then, what is the prediction of [33] for the r_0 of $X(3872)$?

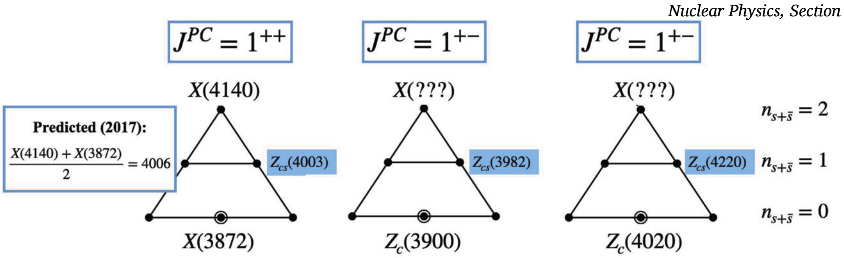

Fig. 6. Nonet particles can be represented in function of the total number of strange quarks/antiquarks, with equal mass spacing in the ladder. The observed hidden charm tetraquarks can be fitted in three nonets. See text for the still missing states.

No pion exchange forces could bind the new exotics as hadron molecules made by colour singlet mesons. Molecular models applied to the new hadrons have to stand on phenomenological forces with undetermined parameters.

On the other hand, $J/\Psi - \phi$ and Z_{cs} exotics are very naturally described as diquark-antidiquarks $[cq]^{\bar{3}}[\bar{c}\bar{q}']_{3}, (q, q' = u, d, s)$ bound in colour singlet by QCD forces (same for di-J/Ψ states). Indeed, with the observed charm-strange $J/\Psi - \phi$ resonances and $Z_{cs}(3082), Z_{cs}(4003)$ and $Z_{cs}(4220)$ one is able to almost fill three tetraquark nonets, with mass differences as expected [42]. In addition, mass and decay properties of the missing particles can be determined with reasonable confidence.

Nonet particles can be represented in function of the total number of strange quarks/antiquarks, see Fig. 6. Octet breaking implies the equal spacing rule of the masses in the ladder. The rule is very well satisfied for the $J^{PC} = 1^{++}$ octet. From the mass difference $M[X(4140)] - M[X(3872)] = 275$ MeV we predict a $M(Z_{cs}) = 2006$ MeV vs an observed mass of 2003 MeV (for comparison, $M[\phi] - M[\rho] = 244$ MeV).

Of the other two Z_{cs}, to be associated to the $J^{PC} = 1^{+-}$ nonets, the $Z_{cs}(3982)$ is lighter than expected for a partner of $Z_c(3900)$ and $Z_c(4220)$ is heavier than expected, to be associated to $Z_c(4020)$. The discrepancy could be explained by a mixing of the two nonets with the same quantum numbers, given that mixing implies a widening of the unmixed levels. Indeed, a mixing angle of 30^0 would bring the unmixed masses in line with the mass difference observed for the 1^{++} nonet.

Only few particles are missing to complete the multiplets of Fig. 6, and they are expected in well defined mass regions with well identified decay modes. The shopping list goes as follows.

- Two $X_{[\bar{c}\bar{s}][cs]}$ with: $M \sim 4170$ MeV, associated to $Z_c(3900)$; $M \sim 4290$ MeV, associated to $Z_c(4020)$, with decays into: $\eta J/\Psi$, $\phi\ \eta_c$, $D_s^* \bar{D}_s$ (threshold: 4080 MeV).
- The I = 1 partner of X(3872), with decays into: $X^+ \to J/\psi\ \rho^\pm \to J/\psi\ \pi^+\pi^0$. X^+ is expected to be produced in B non leptonic decays within the bounds [43]

$$0.057 < \frac{\Gamma(B^0 \to K^+X^- \to K^+\psi\ \pi^0\pi^-)}{\Gamma(B^0 \to K^0X(3872) \to K^0\psi\ \pi^+\pi^-)} < 0.50 \tag{3}$$

- The I = 0 partners of $Z_c(3900)$ and $Z_c(4020)$, expected to decay into $J/\psi + f_0(500)$.

7. Tetraquarks with $J^P = 0^+$, open charm and strangeness

In a recent lattice QCD calculation the flavour $SU(3)$ configurations of possible bound states in the $\bar{D}K, J^P = 0^+$ channel are studied. The allowed channels are those appearing as irreducible components of the tensor product $\bar{D}K = \mathbf{3} \otimes \mathbf{8} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$. Yeo et al. [44] find attraction in $\mathbf{3}$ and $\bar{\mathbf{6}}$ but not in $\mathbf{15}$.

Prompted by this observation, we have examined single charm tetraquarks in the simplest diquark-antidiquark model [45], restricting to the all-spin-zero case:

$$\begin{aligned} &[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}} [q_1 q_2]^{\bar{\mathbf{3}}}_{S_{12}}\ (q, q_1, q_2 = u, d, s); \\ &S_{c3} = S_{12} = 0;\ J^P = S_{c3} + S_{12} = 0^+. \end{aligned} \tag{4}$$

The product $[q_1 q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$ is antisymmetric in spin (to get total spin 0) and colour (to obtain a $\bar{\mathbf{3}}$ of colour).

Due to Fermi statistics, quarks in the light diquark must be antisymmetric in flavour, i.e. must be in a $\bar{\mathbf{3}}$ of $SU(3)_{flavor}$. Taking into account the light antiquark $\bar{q} \subset \bar{\mathbf{3}}$, the tetraquark must be in the $SU(3)_{flavor}$ multiplets: $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}$. No $\mathbf{15}$ comes in, in agreement with the lattice indication.

The situation is different for the molecular structure $(\bar{c}q_1)(\bar{q}_3 q_2)$. Colours of q_1 and q_2 are not correlated and there are no apparent reasons for spin 0 molecules not to display flavours in all the representations appearing in the decomposition of the product $\bar{D}K$. Presence or absence of single charm mesons of the $\mathbf{15}$, $J^P = 0^+$, could discriminate between the two models. For a most recent molecular point of view, see [33,46,47].

Analyzing the flavour $SU(3)$ properties of the $\mathbf{3} \oplus \bar{\mathbf{6}}$, Fig. 7, we encounter a remarkable regularity.

Like the masses of single charm baryons, masses of single charm tetraquarks must be equally spaced in Strangeness, with a slope given by a parameter α. However, unlike charmed baryons, the lower indices of S_{11} in Fig. 7 correspond to the antiquark-diquark

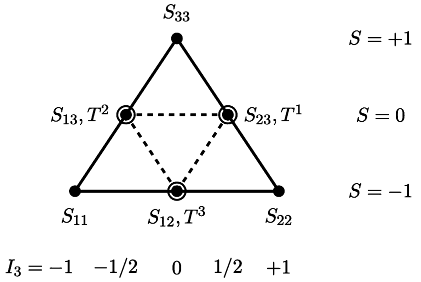

Fig. 7. The $\mathbf{3} \oplus \bar{\mathbf{6}}$ representation in the I_3-Strangeness plane. Electric charges are: $Q(S_{11}) = -2$, $Q(S_{13}) = Q(S_{12}) = -1$, $Q(S_{33}) = Q(S_{23}) = Q(S_{22}) = 0$. States with $S = 0$ do not have a fixed number of $s\bar{s}$ pairs, e.g. $S_{13} = ([\bar{c}\bar{u}][ud] + [\bar{c}\bar{s}][ds])/\sqrt{2}$, $T^2 = ([\bar{c}\bar{u}][ud] + [\bar{c}\bar{s}][sd])/\sqrt{2}$. It is assumed that the mixing matrix diagonalizes the number of strange quarks as in $\omega - \phi$ mixing (magic mixing).

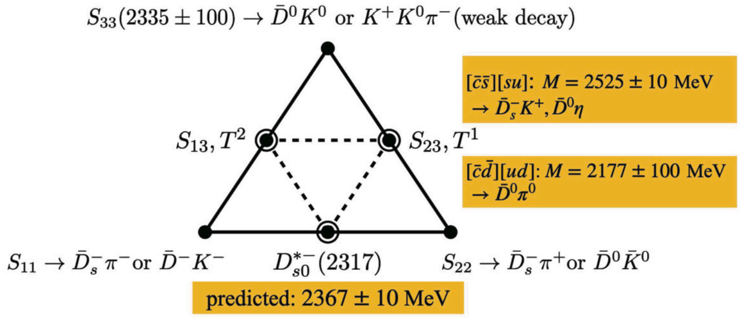

Fig. 8. The $J^P = 0^+, n = 1$, single charm multiplet.

configuration $\bar{u} \otimes [ds]_A$ (diquark ds antisymmetric in flavour), while the lower indices in S_{33} correspond to $\bar{s} \otimes [ud]_A$. The two cases have obviously the same content in quark masses, two light and one heavy.

The exact equality $M(S_{33}) = M(S_{11})$ corresponds to $\alpha = 0$: same masses at the upper vertex and lower corners of the triangle in the figure. Symmetry breaking is restricted to the mass difference between the two $S = 0$ isospin multiplets induced by $\mathbf{3} - \bar{\mathbf{6}}$ mixing of order $\delta \sim 2(m_s - m_q)$, with all other masses degenerate at M. A small, non vanishing value of α may arise from differences in the hyperfine interactions, which are between different pairs in the two cases.,

It is worth noticing that in charmed baryons two light quarks in spin one are in a $\mathbf{6}$ representation. In this case, however, indices 1 or 3 correspond univocally to u or s quarks, α is non vanishing and indeed $M(\Omega_c) - M(\Sigma_c) \simeq 270$ MeV. Group theory disentangles efficiently the ambiguity in the two cases making use of the parameter α allowed by the Wigner-Eckart theorem.

Data (2024). We restrict to particles with Charm C = -1. PdG 2024 [48] reports four entries with $J^P = 0^+$:

- $D_{s0}^*(2317)^\pm$, observed decay: $\bar{D}_{s0}^*(2317)^- \to \bar{D}_s^- + \pi^0$, classified as $I = 0$, (at present there are no known isospin partners);
- $X(2900)^0$, required by LHCb for the full amplitude analysis of $B^+ \to D^+ D^- K^+$, quark composition: $(\bar{c}\bar{s}ud)$;
- $T_{c\bar{s}0}^*(2900)^{--,0}$, $I = 1$, $(I_3 = -1, +1)$, called $D_{s0}^{--}(2900)$, $D_{s0}^0(2900)$ in LHCb notation. Decays: $D_{s0}^{--}(2900) \to \bar{D}_s^- \pi^- (\bar{c}s\bar{u}d)$, $D_{s0}^0(2900) = \bar{D}_s^- \pi^+$ $(\bar{c}s\bar{d}u)$.

The lightest particle $D_{s0}^*(2317)$ goes in the basic $\mathbf{3} \oplus \bar{\mathbf{6}}$ multiplet. However X and T^* are too heavy to be included in the same multiplet. In fact, the difference:

$$M[D_{s0}(2900)] - M[D_{s0}^*(2317)] = 583 \text{ MeV} \tag{5}$$

is similar to the mass gaps:

$$M(J/\Psi) - M(J/\Psi') = 590 \text{ MeV}, \; M[Z_c(4430)] - M[Z_c(3900)] = 530 \text{ MeV} \tag{6}$$

(see Fig. 4). We interpret the LHCb resonances as the first radial excitations of the basic multiplet, to be allocated in a different $\mathbf{3} \oplus \bar{\mathbf{6}}$.

The n = 1 multiplet. Assuming additivity of tetraquark masses with respect to diquark and anti-diquark masses (a "constituent diquark-antidiquark model", see [45]), we may estimate the masses of all single charm tetraquarks in the basic, $n = 1$, multiplet from the masses of the hidden charm tetraquarks discussed in Sect. 6 and the masses of the lightest scalar mesons, assumed to be Jaffe's tetraquarks [49–51].

In this approximation, we obtain a value not far from reality: $M(D_{s0}^*(2317)) = 2367 \pm 10$ and can estimate the masses of all other multiplet components reported in Fig. 8 and predict the decay modes indicated in Fig. 8.

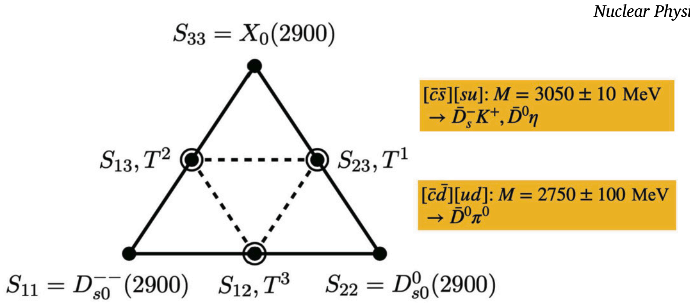

Fig. 9. The $J^P = 0^+, n = 2$, single charm multiplet.

The state D_{s0}^* is predicted to appear in a complex of $I = 0, 1$ similar to the $\omega - \rho$ system. The I = 0 component should decay into $\bar{D}_s^- \eta$, which however is forbidden by phase space. There are two independent mechanisms for the observed decay, both related to the mass difference $m_d - m_u \sim 5$ MeV: mixing $T^3 - S_{12}$, or $\eta - \pi^0$ mixing. It would be interesting to observe the decay $D_{s0}^* \to \bar{D}_s \gamma\gamma$, quoted in PdG with the upper bound $B(\gamma\gamma) < 0.18$, to compare with $\bar{D}_{s0}^*(2317) \to \bar{D}_s^- \eta^* \to \bar{D}_s^- \gamma\gamma$ via a virtual η.

The n = 2 multiplet. We indicate in Fig. 9 the position in the multiplet of the three resonances reported by LHC.

The mass degeneracy between $X_0(2900)$(S = +1) and $D_{s0}^{--,0}(2900)$(S = -1) is the footprint of the tetraquark composition: $[\bar{c}\bar{s}]_0[ud]_0$ and $[\bar{c}\bar{u}]_0[sd]_0$.

We report in the figure the masses of the $S = 0$ still missing states[2] and the expected decay modes.

The $J^P = 0^+, n = 2$, single charm multiplet is tantalizingly close to completion. The missing states are:

- the very likely $D_{s0}^-(2900)$ ($I_3 = 0,\ I = 1$), to fill the isotriplet together with $D_{s0}^{--,0}(2900)$;
- its (almost degenerate) partner with $I = 0$;
- two other I = 1/2 multiplets of composition (for $I_3 = +1/2$): $[\bar{c}\bar{s}][su]$ and ($[\bar{c}\bar{d}][ud]$ with predicted masses and decay modes.

8. Doubly heavy tetraquarks: toward stability?

A doubly charmed baryon has been observed by LHCb in 2018 [52]. The first doubly heavy tetraquark, T_{cc}^+, anticipated theoretically in [53], was observed by LHCb in 2021 as a peak in the invariant mass of $D^0 D^0 \pi^+$ mesons, with mass 3874.83 ± 0.11 MeV, very close to the DD^* threshold.

The $DD\pi$ final state was reconstructed using the $D^0 \to K^-\pi^+$ decay channel with two D^0 mesons and a pion all produced promptly in the same pp collision [54]. The quantum numbers of T_{cc}^+ are $I = 0,\ J^P = 1^+$, valence composition $cc\bar{u}\bar{d}$.

The possibility has been raised that the double beauty T_{bb}^+ ($bb\bar{u}\bar{d}$) be stable under strong and e.m. decays [56–58]. The limiting behaviour of the doubly heavy tetraquark for large heavy quark mass has been most clearly derived by Eichten and Quigg. The argument runs as follows.

In the large mass limit, the heavy quarks go to short distance, where coulomb-like QCD potential dominates. The QQ binding energy is then given by the QCD Rydberg ($\bar{M}_Q$ is the reduced mass):

$$B_{QQ} = -\frac{1}{2}(\frac{2}{3}\alpha_s)^2 \bar{M}_Q \tag{7}$$

In the same limit, the mass of the PS $\bar{Q}q$ meson is of order M_Q plus small corrections of order m, M_Q^{-1} (the latter from hyperfine interactions). In the difference $M(T_{QQ}) - 2M(\bar{Q}q)$ large rest masses cancel and we are left with

$$M(T_{QQ}) - 2M(\bar{Q}q) = -\frac{1}{2}(\frac{2}{3}\alpha_s)^2 \bar{M}_Q + \mathcal{O}(m, M_Q^{-1}) \tag{8}$$

which becomes negative for sufficiently large M_Q.

Is the b-quark mass heavy enough?

Several calculations have been done of the Q-value with respect to the PS-PS threshold:

$$Q - \text{value} = M(T_{QQ'}) - M(\bar{Q}q) - M(\bar{Q}'q) \tag{9}$$

using the constituent quark model, the Born-Oppenheimer approximation and lattice QCD, with the results reported in Table 1.

The results, particular Refs. [55–57], reproduce the position of T_{cc} with respect to the DD^* threshold but leave some doubt about the stability of T_{bb}.

Searching for $\mathbf{T_{cb}^0}$ ($\mathbf{J^P = 1^+, 0^+}$).

The state with $J^P = 1^+$ ($S_{\bar{q}\bar{q}} = 0,\ S_{cb} = 1$) could be produced in LHC collisions, in association with one $\bar{B}\bar{D}$ pair. Most likely, it decays strongly:

[2] The diagonal masses are degenerate with the $S = +1$ and $S = -1$ masses and the mixing is fixed so as to diagonalize the number of strange quarks.

Table 1
Q values in MeV for decays into $PS+PS+\gamma$ of doubly heavy tetraquarks. MPPR is our result in the Born-Oppenheimer approximation with string tension $1/4\,k$ (in parentheses with string tension k) with k the charmonium string tension [55]. Models in [56–58] are elaborations of the constituent quark model, for details see the original references. In the last column the lattice QCD results [59–63].

$QQ'\bar{u}\bar{d}$	MPPR[2022] [55]	K&R[2017] [56]	E&Q[2017] [57]	L[2017] [58]	Lattice QCD
$cc\bar{u}\bar{d}$	+136(+111)	+140	+102	+39	-23 ± 11 [59]
$cb\bar{u}\bar{d}$	+72(+48)	~ 0	+83	−108	$+8\pm 23$ [60]
$bb\bar{u}\bar{d}$	−8(−38)	−170	−121	−75	-143 ± 34 [59] $-143(1)(3)$ [61] $-82\pm 24\pm 10$ [62] -13^{+38}_{-30} [63]

$$T^0_{cb}(J^P=1^+)\to D^0B^{*0},\ D^+B^{*-} \tag{10}$$

followed by $B^*\to B+\gamma$.

The state with $J^P=0^+$, $(S_{\bar{q}\bar{q}}=0,\ S_{cb}=0)$ can be searched in the collisions: $p+p\to \bar{D}\bar{B}+DB+\dots$ with no gamma ray in the final state, corresponding to the decay:

$$T^0_{cb}(J^P=0^+)\to D^0B^0,\ D^+B^- \tag{11}$$

Searching for a stable $\mathbf{T^-_{bb}}$ ($\mathbf{J^P=1^+}$).

T^-_{bb} should be produced at LHCb, together with a $\bar{B}\bar{B}$ pair.

If stable, T^-_{bb} should decay weakly with the b-quark lifetime, at a detectable distance from the p-p interaction point. The expected weak decay $b\to c+\bar{c}+s$ (or $c+\bar{u}+d$) gives rise to the chain of Cabibbo allowed decays

$$T^-_{bb}\to \bar{D}_s+T^0_{bc}\to \bar{D}_s+D^{+/0}+B^{*-/0}\to \bar{D}_s+D+B+\gamma$$

or

$$T^-_{bb}\to\ \pi^-+T^0_{bc}\to \pi^-+D^{+/0}+B^{*-/0}\to \pi^-+D+B+\gamma \tag{12}$$

in total:

$$p+p\to \bar{B}\bar{B}+\cdots+(\bar{D}_s+D+B+\gamma)_{(\text{at a distance})}$$

or

$$p+p\to \bar{B}\bar{B}+\cdots+(\pi^-+D+B+\gamma)_{(\text{at a distance})} \tag{13}$$

The weak decay could produce also the tetraquark T^0_{bc} ($J^P=0^+$), which would decay into ($\bar{D}_s$ or π^-) $+D+B$ without the gamma ray (an early discussion is found in [64], see [65] for a recent, more extended discussion).

Observation of T_{bc} at LHC is being pursued by LHCb. Production of two $b\bar{b}$ pairs at the LHC is probably below the observable luminosity.

Production and observation of T_{bb} will be among the targets of future colliders in project at CERN, FCC, and in China, CepC, as discussed in [66].

9. Conclusions

In conclusion, I would propose a few crucial questions that I hope could be answered in a not-too-distant future.

Are $Z_{cs}(3986)$ and $Z(4003)$ two different states?

Can one confirm the third $Z_{cs}(4220)$?

Can X^+ near $X(3872)$ be found in B decays?

Can we find the missing partners of the $\mathbf{3}\oplus\bar{\mathbf{6}}$, (n = 2) multiplet?

LHCb has used efficiently the channel $B\to(J/\Psi)\phi K$+hadrons to study the states $X_{s\bar{s}}(4140)$ etc., and Z_{cs} etc.

Can the study of the $B\to\bar{D}_sD\phi$ channel be similarly used to study single charm tetraquarks of the interesting $\mathbf{15}\oplus\mathbf{3}$, $J^P=1^+$ multiplet?

One should reconsider K-like states, e.g. $K_1(2650)$, which decay into $K\phi$ and, to me, are unlikely to be $(\bar{q}s)$ excited Kaons.

Could these “anomalous” excited Kaons be $[\bar{u}\bar{s}][ss]$ tetraquarks?

All these are tough orders indeed: more luminosity, better energy definition, detectors with exceptional qualities, a lot of work. Close exchange between theory and experiments is essential, as it was in the ’50s and ’60s when the basic hadron spectroscopy was deciphered.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

All data appearing in the text are from in publications quoted in the paper.

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