



# A New Approach to Nuclear Form Factors for Direct Dark Matter Searches

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## Abstract

We present a new approach to determine the nuclear form factors which are important for the direct dark matter experiments. We perform a systematic global determination of the form factors covering a wide range of nuclei, from  $^9\text{Be}$  to  $^{209}\text{Bi}$ . The commonly-used Lewin-Smith approach is improved by fitting both parameters of the Fermi proton-density distributions directly to the experimental data. Our procedure allows to extract the widely-used Helm form factor, providing for the first time realistic (conservative) uncertainties for the parameters. In addition, we rely on recent measurements of antiprotonic atoms to constrain the neutron-density distributions. Systematics errors are estimated and possible correlations are explored.

**Keywords:** Dark matter, dark matter direct detection, nuclear form factor.

## 1. Introduction

The goal of several ongoing and future direct detection experiments is to discover the dark matter present in our galactic halo in the form of Weakly Interacting Massive Particles (WIMPs). These experiments attempt to isolate from various backgrounds the signal of nuclear recoils from the elastic scattering of WIMPs with the target nuclei inside the detector [1–16]. The expected differential rate of nuclear recoils in a detector is given by (see for instance Ref. [17]):

$$\frac{dR}{dE_{nr}} = \frac{\rho_\chi}{2m_\chi\mu^2} \sigma^{SI} F^2(q) \int_{v_{min}}^{v_{escape}} \frac{f(\vec{v}, t)}{v} d^3v \quad (1)$$

where  $E_{nr}$  is the energy of the recoiling nucleus,  $\rho_\chi$  is the local halo WIMP density,  $m_\chi$  is the WIMP mass and  $\mu = m_\chi M / (m_\chi + M)$  the WIMP-nucleus reduced mass;  $f(\vec{v}, t)$  is the WIMP velocity distribution in the reference frame of the detector and  $\sigma^{SI}$  the spin-independent WIMP-nucleus elastic scattering cross section off a point-like nucleus;  $F(q)$  is the nuclear form factor which depends on the recoil momentum  $q = \sqrt{2ME_{nr}}$ .

The nuclear form factor critically determines the spectrum of the recoil nuclei. Therefore its precise determination and error estimation is crucial to establish the bounds on the WIMP-nucleon cross section from running experiments and to plan future ones.

In direct dark matter searches and related studies, it has been customary to describe the nuclear form factors using the Helm ansatz [18], which leads to an analytic expression for the form factor. On the other hand, the charge density distributions have been extracted from muon spectroscopy [19] using two-parameter Fermi (2PF) distributions. The widespread strategy to deal with this dichotomy is to convert the 2PF parameters into Helm ones adopting an *ad hoc* value for the nuclear thickness [20].

The present approach improves the one of Ref. [20] (Lewin-Smith), taking also into account the information about neutron-density distributions recently extracted in measurements of antiprotonic atoms [21, 22]. We provide realistic (conservative) uncertainties for the parameters, estimating the systematic errors and exploring possible correlations. At the same time, we keep the simple and analytic expressions intrinsic of the Helm parameterization.

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## 2. The Nuclear Form Factor

The spatial extension of a nucleus is described by the nuclear form factor. The role of the form factor is easily understood by looking, for example, at the elastic scattering of electrons off nuclei [23]. The scattering of electrons from a point-like target is simply described by the Rutherford/Mott formula (Coulomb scattering). However, the nucleus is not point-like, but has a structure. It is observed that the Rutherford formula agrees with the experimental cross sections only for small momentum transfers  $q$ , i.e., for scattering angles very close to  $0^\circ$ . At larger  $q$  the experimental cross sections are systematically smaller and show typical diffraction patterns, which reflect the internal structure of the nucleus. The location of the minima is related to the size of the target nucleus. For light nuclei the form factor falls off slowly with  $q^2$ , while for heavier ones the more extended density distributions cause a stronger fall-off of the form factor. In the limit of a point-like target nucleus  $F(q) \rightarrow 1$ .

The form factor is the Fourier transform of the nuclear density:

$$F(\vec{q}) = \frac{1}{A} \int \rho_{nucl}(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^3r \quad (2)$$

normalized so that  $F(0) = 1$ . Dark-matter studies have traditionally assumed that

$$\frac{\rho_{nucl}}{A} = \frac{\rho_{charge}}{Z}, \quad (3)$$

relying on the well measured nuclear charge density distributions to determine the form factor and bypassing the fact that neutron distributions are, in general, different.

The charge distribution of protons in nuclei can be extracted precisely and, to a large extent, model independently. It has been extensively determined by elastic scattering of electrons [24] and, more recently, also by muonic atom spectroscopy [19]. The present knowledge of the neutron distributions is far more uncertain. Therefore, realistic error estimates should take their larger errors into account. This is particularly important in scenarios where the cross sections on protons and neutrons are different.

## 3. The Form Factor in the Lewin-Smith Approach

The approach commonly used to determine the form factor for direct dark matter searches for the spin-independent case is reported in Ref. [20]. The nuclear density distribution is assumed to be the same as the

charge distribution of protons. This is available for a large set of nuclei [19] in the form of 2PF distributions:

$$\rho(r) = \rho_0 \left[ 1 + \exp \left( \frac{r-c}{a} \right) \right]^{-1}, \quad (4)$$

where  $c$  is the nuclear radius at half of the central density  $\rho_0$ , and  $a$  is the diffuseness of the nuclear surface. The latter is related to the surface thickness  $t = 4 \ln 3 a$ , defined as the distance over which the density decreases from 90% to 10% of  $\rho_0$ .

As there is no analytical expression for the Fourier transform of the 2PF distribution, most dark matter studies adopt the Helm expression [18] for the spin-independent form factor

$$F(qR) = 3 \frac{J_1(qR)}{qR} e^{-(qs)^2/2}, \quad (5)$$

where  $R$  is the effective nuclear radius,  $s$  is the nuclear skin thickness and  $J_1(qR)$  is the spherical Bessel function.

The strategy of Lewin and Smith is to convert the 2PF parameters into Helm ones. This is achieved by equating the root mean square radii

$$r_{rms}^2(2PF) = \frac{3}{5}c^2 + \frac{7}{5}\pi^2a^2, \quad (6)$$

$$r_{rms}^2(\text{Helm}) = \frac{3}{5}R^2 + 3s^2. \quad (7)$$

The diffuseness is fixed at  $a \simeq 0.52$  fm [19], while  $c$  is derived from a not-weighted fit to the muon spectroscopy data of Ref. [19]:  $c \simeq (1.23 A^{1/3} - 0.60)$  fm. Finally, an *ad hoc* value is taken for  $s \simeq 0.9$  fm. According to Ref. [20], this value is chosen to improve the matching between the form factors obtained with the Helm ansatz and from numerically-integrated 2PF distributions.

This approach is qualitative, with only the radius parameter  $c$  fitted to the data, while both  $a$  and  $s$  are fixed. Moreover, it only accounts for the proton distribution and the errors are not estimated.

## 4. A New Approach to the Nuclear Form Factor

We have performed a systematic global determination of nuclear form factors for nuclei ranging from  $^9\text{Be}$  to  $^{209}\text{Bi}$ . Concerning the proton distributions, we improve the Lewin-Smith approach [20] by fitting directly the measured nuclear radii [19] and diffuseness [24]. We also add the information on the neutron distributions, obtained from antiprotonic atoms [21, 22].

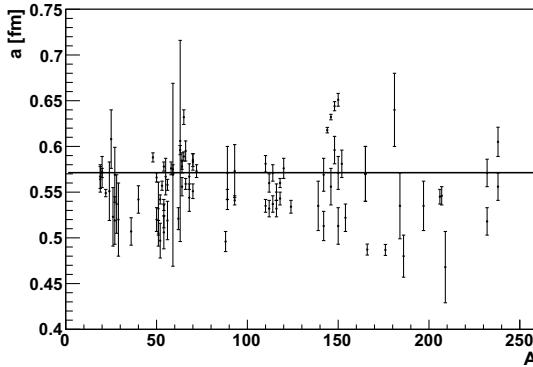


Figure 1: Electron scattering data of the 2PF diffuseness  $a$  from Ref. [24] and our weighted fit.

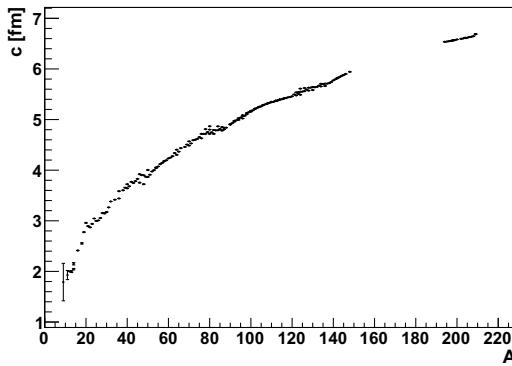
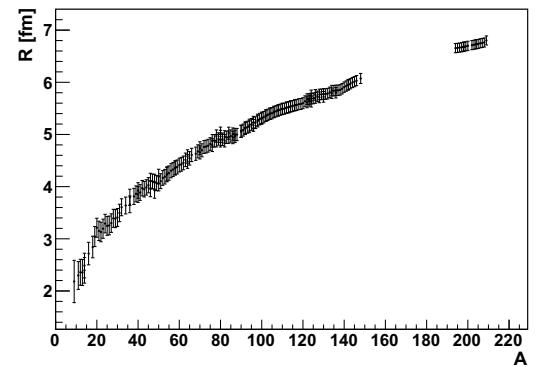


Figure 2: Muon spectroscopy data of the 2PF radius  $c$  from Ref. [19].

#### 4.1. The proton distribution

In our approach the parameters of the 2PF distribution (Eq. 4) are determined as follows. The diffuseness  $a$  is extracted from a weighted fit to all the available electron scattering data [24]. From this fit, shown in Fig. 1, we obtain:

$$a = (0.57 \pm 0.04) \text{ fm}. \quad (8)$$

We verified that the assumption of  $a(A) \approx \text{const.}$  [23] is statistically valid. Indeed the values of  $a$  for the different nuclei are Gaussian-distributed. For the 2PF radius  $c$  we take the values obtained in Ref. [19] from muonic atom spectroscopy, including 192 nuclei from  ${}^9\text{Be}$  to  ${}^{209}\text{Bi}$  (Fig. 2).

To relate the 2PF parameters  $(c, a)$  to the Helm ones  $(R, s)$  we use an additional condition in comparison to Ref. [20]; we demand that both parameterizations have the same thickness and equate

$$t(\text{2PF}) = 4(\ln 3)a \quad (9)$$

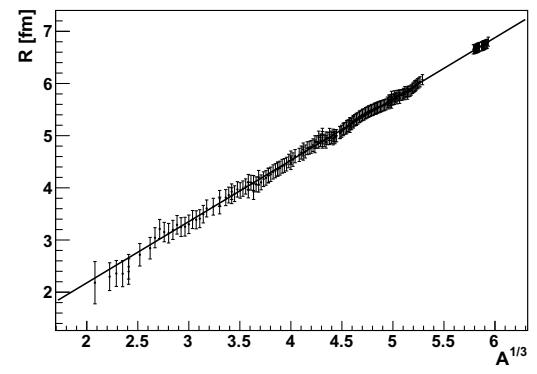


Figure 3: Helm radius  $R$  shown as a function of  $A$  (top) and  $A^{1/3}$  (bottom). Our weighted fit is also shown in the bottom plot.

to

$$t(\text{Helm}) \simeq 2.6s, \quad (10)$$

obtaining  $s$  from the fitted value of  $a$  (Eq. 8):

$$s = \frac{4(\ln 3)a}{2.6} = (0.97 \pm 0.07) \text{ fm}. \quad (11)$$

This procedure avoids to adopt an *ad-hoc* value for the nuclear thickness  $s$  as done in Ref. [20]. The approximation of Eq. 10 is valid for the whole range of nuclei under consideration. The systematic error intrinsic of this approximation, estimated to be of  $\sim 3\%$ , has been included in the quoted uncertainty of  $s$ .

By equating the root mean square radii (Eqs. 6 and 7), we obtain the  $R$  parameter of the Helm formula (Eq. 5) for each nucleus  $i$ :

$$R_i^2 = \frac{5}{3} r_{rms,i}^2(\text{2PF}) - 5s^2. \quad (12)$$

The error on  $R_i$  is determined from the uncertainties in  $s$  and  $r_{rms,i}$  by standard error propagation. The results, shown in Fig. 3, are fitted by a weighted fit. In this way,

one finds the following parameterization of the effective nuclear radius  $R$  as a function of the nuclear mass  $A$ :

$$\begin{aligned} R &= \alpha A^{1/3} + \beta \\ &= [(1.17 \pm 0.05)A^{1/3} - (0.17 \pm 0.26)] \text{ fm} \end{aligned} \quad (13)$$

with the advantage that Eq. 13 is compact and incorporates all the errors. As the correlation between the fit coefficients  $\alpha$  and  $\beta$  turns out to be negligible, being the covariance  $\sigma_{\alpha\beta} = -5.8 \times 10^{-4}$ , the error of  $R$  can be calculated according to the standard error propagation for independent quantities:

$$\sigma_R^2 = A^{2/3} \sigma_\alpha^2 + \sigma_\beta^2. \quad (14)$$

#### 4.2. The neutron distribution

Antiprotons can test the matter distribution of nuclei, in contrast to electromagnetic probes which only test the charge distribution. To constrain the neutron-density distributions we rely on recent measurements of antiprotonic atoms [21], where the difference between the root mean square radii of the neutron and proton distributions

$$\Delta r_{np} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} \quad (15)$$

has been determined.

The experimental data for  $\Delta r_{np}$  show an approximately linear behavior as a function of the asymmetry parameter  $I = (N - Z)/A$  [21, 22] (see Fig. 4). We have performed a weighted fit, also shown in Fig. 4, obtaining:

$$\begin{aligned} \Delta r_{np} &= \kappa I + \gamma \\ &= [(0.82 \pm 0.54)I - (0.02 \pm 0.08)] \text{ fm}. \end{aligned} \quad (16)$$

The covariance of the fit coefficients  $\kappa$  and  $\gamma$ ,  $\sigma_{\kappa\gamma} = -5 \times 10^{-3}$ , is not negligible, thus the error on  $\Delta r_{np}$  has to be calculated according to the complete formula:

$$\sigma_{\Delta r}^2 = I^2 \sigma_\kappa^2 + \sigma_\gamma^2 + 2I \sigma_{\kappa\gamma}. \quad (17)$$

By substituting the Helm root mean square radius, Eq. 7, into Eq. 15 one obtains

$$\sqrt{R_n^2 + 5s_n^2} = \sqrt{R_p^2 + 5s_p^2} + \sqrt{\frac{5}{3}} \Delta r_{np}, \quad (18)$$

where  $R_p$  and  $s_p$  are given by Eqs. 13 and 11, respectively.

In nuclei two mechanisms can generate the neutron skin (see for instance Ref. [25]). One is a *bulk* effect, consisting in a displacement between the positions of the neutron and proton sharp surfaces but keeping the same thickness, i.e.,  $R_n > R_p$  and  $s_n = s_p$ . The other

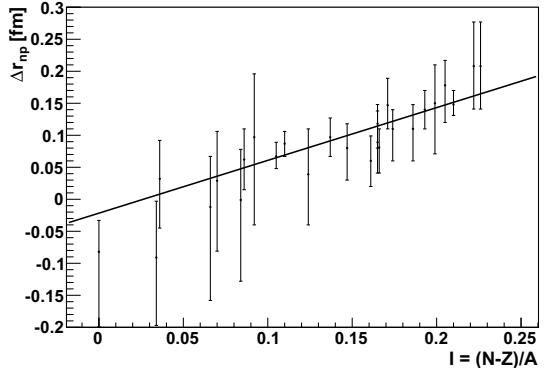


Figure 4: Weighted fit of the difference between the root mean square radii of the neutron and proton distributions, measured in antiprotonic atoms [21, 22], shown as a function of the asymmetry  $I$ .

one is a *surface* effect, consisting in a different surface diffuseness between the neutron and proton density profiles while maintaining the same radius, i.e.,  $R_n = R_p$  and  $s_n > s_p$ . Intermediate density distributions having  $R_n > R_p$  and  $s_n > s_p$  are, of course, also possible.

Considering the argument above, we derive the equations for the parameters  $R_n$  and  $s_n$  of the Helm's neutron distribution:

$$R_n = R_p + \frac{1}{2} \sqrt{\frac{5}{3}} \Delta r_{np}, \quad (19)$$

$$s_n = \frac{1}{2} \left( s_p + \sqrt{s_p^2 + \frac{2}{\sqrt{15}} R_p \Delta r_{np}} \right). \quad (20)$$

The statistical errors on  $R_n$  and  $s_n$  can be calculated by propagating the errors on  $R_p$  and  $s_p$ . One has also to add in quadrature the systematic errors, which account for all the possible scenarios from the extreme *bulk* to *surface* distributions:

$$\delta R_n^{\text{syst}} = \frac{1}{2} \sqrt{\frac{5}{3}} \Delta r_{np}, \quad (21)$$

$$\delta s_n^{\text{syst}} = \frac{1}{2} \left( s_p - \sqrt{s_p^2 + \frac{2}{\sqrt{15}} R_p \Delta r_{np}} \right). \quad (22)$$

The larger errors of the neutron distributions in comparison to the proton ones reflect the experimental difficulties in their determinations. Progress is expected from new parity-violating electron scattering experiments (PREX-II [26], CREX [27] and Qweak [28]).

## 5. Conclusions

We have developed a new approach to the determination of the nuclear form factors relevant to the di-

rect dark matter searches. The qualitative procedure of Ref. [20] has been considerably improved. In our approach both the diffuseness and radius of the proton-density distributions are fitted directly to the measurements. This procedure provides for the first time realistic (conservative) uncertainties for the parameters, allowing in the meantime to keep the analytic Helm parameterization. A wide range of nuclei, from  ${}^9\text{Be}$  to  ${}^{209}\text{Bi}$ , is considered. Moreover, we have determined the Helm parameters for the neutron-density distributions using the experimental data from antiprotonic atoms. For both the proton and neutron distributions, the covariance of the fit parameters is provided and the systematics errors are taken into account.

## References

- [1] R. Agnese, et al., SuperCDMS Collaboration, Phys.Rev.Lett. 112 (2014) 241302.
- [2] D. S. Akerib, et al., LUX Collaboration, Phys. Rev. Lett. 112 (2014) 091303.
- [3] C. E. Aalseth, et al., CoGeNT Collaboration, Phys. Rev. D 88 (2013) 012002.
- [4] K. Abe, et al., XMASS Collaboration, Phys.Lett. B719 (2013) 78–82.
- [5] E. Aprile, XENON1T collaboration, Springer Proc.Phys. C12-02-22 (2013) 93–96.
- [6] E. Aprile, et al., XENON100 Collaboration, Phys.Rev.Lett. 109 (2012) 181301.
- [7] L. Baudis, DARWIN Consortium, J.Phys.Conf.Ser. 375 (2012) 012028.
- [8] G. Angloher, et al., CRESST-II Collaboration, The European Physical Journal C 72 (2012) 1971.
- [9] E. Behnke, et al., COUPP Collaboration, Phys.Rev. D86 (2012) 052001.
- [10] S. Archambault, et al., PICASSO Collaboration, Phys.Lett. B711 (2012) 153–161.
- [11] M. Felizardo, et al., SIMPLE Collaboration, Phys.Rev.Lett. 108 (2012) 201302.
- [12] D. Y. Akimov, et al., ZEPLIN-III Collaboration, Phys.Lett. B709 (2012) 14–20. arXiv:1110.4769.
- [13] Z. Ahmed, et al., CDMS Collaboration, Phys. Rev. Lett. 106 (2011) 131302.
- [14] E. Armengaud, et al., EDELWEISS Collaboration, Phys.Lett. B702 (2011) 329–335.
- [15] J. Angle, et al., XENON10 Collaboration, Phys.Rev.Lett. 107 (2011) 051301.
- [16] R. Bernabei, et al., DAMA Collaboration, LIBRA Collaboration, Eur.Phys.J. C67 (2010) 39–49.
- [17] G. Duda, A. Kemper, P. Gondolo, JCAP 0704 (2007) 012.
- [18] R. H. Helm, Phys.Rev. 104 (1956) 1466–1475.
- [19] G. Fricke, et al., Atom.Data Nucl.Data Tabl. 60 (1995) 177.
- [20] J. Lewin, P. Smith, Astropart.Phys. 6 (1996) 87–112.
- [21] A. Trzcinska, et al., Phys.Rev.Lett. 87 (2001) 082501.
- [22] J. Jastrzebski, et al., Int.J.Mod.Phys. E13 (2004) 343–351.
- [23] B. Povh, K. Rith, C. Scholz, F. Zetsche, M. Lavelle, Particles and Nuclei, Springer-Verlag, Berlin Heidelberg, 2008.
- [24] H. De Vries, C. De Jager, C. De Vries, Atom.Data Nucl.Data Tabl. 36 (1987) 495–536.
- [25] M. Warda, M. Centelles, X. Vinas, X. Roca-Maza, Phys.Rev. C89 (2014) 064302.
- [26] The PREX-II proposal, available at [hallaweb.jlab.org/parity/prex](http://hallaweb.jlab.org/parity/prex).
- [27] The CREX proposal, available at [hallaweb.jlab.org/parity/prex](http://hallaweb.jlab.org/parity/prex).
- [28] D. Androic, et al., Qweak Collaboration, Phys.Rev.Lett. 111 (14) (2013) 141803.