Towards unified description of shell evolution — Takaharu Otsuka's achievements

Y Utsuno

Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan

E-mail: `utsuno.yutaka@jaea.go.jp`

Abstract. Recent studies on exotic nuclei have rapidly advanced the understanding of the evolution of shell structure, to which Takaharu Otsuka has made remarkable contributions particularly in terms of the nuclear force. This paper presents a brief survey of his achievements, focused on the concept of shell evolution due to the tensor force and the three-nucleon force and on related shell-model works.

1. Introduction

Evolution of the shell structure, often called shell evolution, has been attracting much theoretical and experimental interest in the physics of exotic nuclei, as found in many talks devoted to this subject in this conference. Among various recent findings, Professor Takaharu Otsuka has greatly contributed to unveiling the mechanism of the shell evolution (see [1, 2] for overviews by himself). It is an honor for me, as a former student and a close collaborator of Prof. Otsuka, to have an opportunity to survey his remarkable achievements at the conference celebrating his 60th birthday. This paper concentrates on how he has advanced the idea of shell evolution from personal point of view.

Since Mayer and Jensen accounted for the origin of the magic number by introducing strong spin-orbit coupling more than 60 years ago [3], the shell structure has constituted a robust basis of the nuclear structure. It has been widely accepted that the Woods-Saxon potential consisting of a central and a spin-orbit *one-body* term gives a global property of the shell structure as far as the regions of stable nuclei are concerned [4]. A characteristic behavior of the shell structure created by one-body potentials is a gradual and monotonic evolution of the shell structure: difference of neighboring single-particle energies changes rather flat and increases or decreases monotonically with the mass number. As a result, the conventional magic numbers are quite stable.

In exotic nuclei, on the other hand, several phenomena that cannot be accounted for by the conventional picture are found. An early example is the so-called parity inversion in ^{11}Be. Talmi and Unna proposed [5] that the inversion can be explained by a linear extrapolation of observed single-particle spectra of the $N = 7$ isotones $^{12}_{5}$B and $^{13}_{6}$C into $Z = 4$ and that this linearity is due to the *two-body* force. This particular two-body term causing the shell evolution is often called the monopole interaction acting between nucleons in the orbits i and j whose matrix

element is defined with the j-j coupled two-body matrix elements as

$$v_{i,j}^m = \frac{\sum_J (2J+1)\langle ijJ|V|ijJ\rangle}{\sum_J (2J+1)}, \tag{1}$$

where J runs over possible couplings of angular momentum with a nucleon in i and one in j. The mean (i.e., center-of-mass) energy of a configuration specified by a set of the occupation numbers $\{n_j\}$ is evaluated with the monopole interaction as

$$E(\{n_j\}) = \sum_j \varepsilon_j n_j + \sum_j \frac{1}{2} n_j(n_j-1)v_{j,j}^m + \sum_{j\neq j'} n_j n_{j'} v_{j,j'}^m, \tag{2}$$

where ε_j is the single-particle energy of the orbit j. Hence, the monopole interaction $v_{i,j}^m$ acts as the bond energy between nucleons in i and j. When one defines the effective single-particle energy (ESPE) of an orbit i as a nucleon separation energy from this orbit, it leads to

$$\bar{\varepsilon}_i \equiv \frac{\partial E(\{n_j\})}{\partial n_i} \approx \varepsilon_i + \sum_j v_{i,j}^m n_j. \tag{3}$$

The linearity of the shell evolution during the occupation of j is thus obtained. Equation (3) can also cause a non-monotonic evolution of a shell gap $\bar{\varepsilon}_i - \bar{\varepsilon}_{i'}$ depending on the sign of $v_{i,j}^m - v_{i',j}^m$ for different j. It should be noted that the contribution of the monopole interaction to the effective shell gap is magnified with the occupation number.

Consequently, the monopole interaction plays an essential role in the evolution of the shell structure and also in the resulting many-body properties. For instance, while the $N = 28$ magic structure is rather fragile with the G-matrix interactions, it can be stabilized by tuning the monopole terms [6]. The Strasbourg-Madrid group has demonstrated with large-scale shell-model calculations that an appropriate choice of the monopole interaction leads to a successful description of a wide range of nuclei [7, 8].

Thus, the monopole interaction is one of the most important but undetermined parts of the effective interaction. The empirical monopole shift works well only for the regions with plenty of experimental data to fix monopole matrix elements. Universal understanding of the monopole interaction, or shell evolution, is strongly needed for constructing a general theory of shell formation and for enhancing predictive power for unexplored regions. Otsuka has proposed several novel mechanisms of the shell evolution in terms of the nuclear forces which is severely tested by experiment and microscopic theory. In the following sections, I attempt to display those ideas as comprehensively as possible. In Sect. 2, the importance of the tensor force is demonstrated. It was overlooked for a long time but is now regarded as one of the dominant proton-neutron interactions causing the shell evolution. Section 3 is focused on the three-nucleon force. It gives a solution to the long-standing issue why effective interactions based on microscopic theories failed to describe neutron-rich nuclei due to an incorrect shell structure. Section 4 is directed to shell-model studies to show how they play an indispensable role in clarifying the shell evolution.

2. Shell evolution due to the tensor force
2.1. Importance of spin-isospin dependence
At the beginning of the 2000s, Otsuka has found a phenomenology of shell evolution common to various major shells [9]. Figure 1 shows how this evolution occurs schematically. Here, the case of light nuclei is considered where the β-stability line lies around $N = Z$. For stable nuclei shown in Fig. 1 (a), the splitting of the orbits belonging to the same harmonic oscillator quantum number
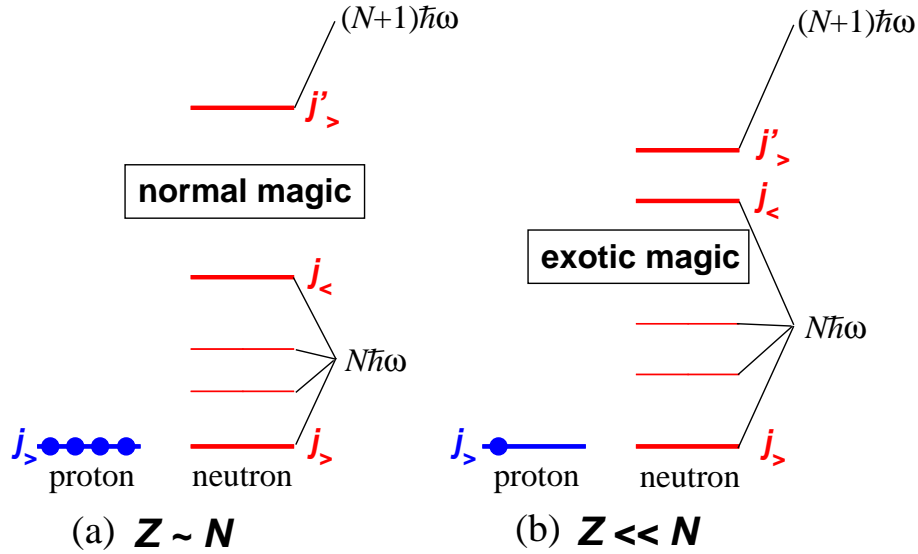
Figure 1. Phenomenological shell evolution compared between (a) stable nuclei with $Z \sim N$ and (b) exotic nuclei with $Z \ll N$.

is not so large that the normal magic number is made up between the harmonic-oscillator shell gap. For exotic nuclei illustrated in Fig. 1 (b), on the other hand, the shell structure seems rather different. The highest orbit of the $N\hbar\omega$ shell denoted as $j_<$ is located too high to stabilize the normal magic number. A new magic number can emerge below $j_<$ instead. This accounts for the appearance of a new magic number $N = 16$ in oxygen isotopes pointed out earlier [12, 13]. When applied to the $0p$ shell, this leads to the parity inversion in ^{11}Be. When one moves to the $0f$-$1p$ shell, a new magic number $N = 34$ could appear around calcium isotopes [14], attracting much experimental interest.

The difference between Fig. 1 (a) and (b) is the proton number. In any case mentioned above, the $j_<$ orbit is sharply lowered especially when protons occupy the $j_>$ orbit, i.e., $Z = 2$-6 for the $0p$ shell, $Z = 8$-14 for the $0d$-$1s$ shell, and $Z = 20$-28 for the $0f$-$1p$ shell. This indicates that the proton-neutron monopole interaction v_{k_1,k_2}^m with $k_1 = j_>$ and $k_2 = j_<$ is quite a strongly attractive one. This effect includes the so-called Federman-Pittel mechanism causing deformation in medium-heavy nuclei [15]. Otsuka has noticed that the interaction depending on spin and isospin is responsible for this property. While the spin-isospin dependent central interaction $(\tau \cdot \tau)(\sigma \cdot \sigma)f(r)$ was introduced originally [9], he proposed the tensor force later [16] as the source of the strong proton-neutron monopole interaction between $j_>$ and $j_<$.

It is worthwhile mentioning here the possibility of the $N = 34$ magic number in further detail. Since a drastic lowering of $0f_{5/2}$ in moving from calcium ($Z = 20$) [10] to nickel ($Z = 28$) [11] is observed on top of the $N = 28$ core, the mechanism discussed above must be valid also for the $0f$-$1p$ shell. It is thus a matter of the neutron-neutron interaction, which is discussed in Sect. 3, whether the new $N = 34$ magic number really survives or not in going from $N = 28$ to 34.

2.2. Tensor force
The tensor force is well known to play a crucial role in the binding of nuclei. However, before Otsuka's work, it was almost ignored as the *effective* interaction as exemplified by the absence of the tensor force from standard Skyrme and Gogny interactions. This is possibly because the short-range tensor force, which is rather important in the gain of binding energy, serves predominantly as the effective central force for the independent particle picture. Namely, the

expectation value of the tensor force vanishes when a spin-saturated Slater determinant such as an L-S closure ^{16}O is taken, and then a strong effective central force is required for keeping the binding energy. Indeed, according to an analysis of realistic interactions [17] by using the spin-tensor decomposition [18], the magnitude of the effective tensor force is much smaller than that of the effective central force.

Hence, Otsuka's finding [16] that the tensor force is vital for the shell evolution, a sort of the mean-field effect, was a surprise. The significance of the tensor force in spite of being relatively small comes from its monopole property that is opposite between $j_>$ and $j_<$. The monopole matrix elements of any tensor operator satisfy the equation

$$(2j_> + 1)v_{j_>,j'}^m + (2j_< + 1)v_{j_<,j'}^m = 0 \tag{4}$$

for j' not equal to $j_>$ or $j_<$. This represents that the net effect from a spin saturated wave function ($2j_> + 1$ nucleons in $j_>$ plus $2j_< + 1$ nucleons in $j_<$) on the ESPE of j' vanishes. As a result, $v_{j_>,j'}^m$ and $v_{j_<,j'}^m$ are opposite in sign. As illustrated in his paper (see Fig. 2 of [16]), the realistic tensor force gives, on average, attraction between $j_<$ and $j'_>$ and thus repulsion between $j_>$ and $j'_>$.

Therefore, the tensor force changes the spin-orbit splitting as depicted in Fig. 1 of [16]. The shift of the magic number illustrated in Fig. 1 is one of the manifestations of the shell evolution due to the tensor force. As presented in [16], many experimental data are described in this manner with a single choice of the tensor force of the $\pi + \rho$ exchange potential with cutoff at 0.7 fm, including a famous example of the energy levels in antimony isotopes [19]. Otsuka has successfully introduced the tensor force to the mean-field interaction soon after [20], as the pioneering work of the same kind.

2.3. Renormalization persistency of the tensor force and monopole-based universal interaction
The success of the "bare" $\pi + \rho$ tensor force in the description of the shell evolution should not be taken for granted because the effective interaction is in general modified from the bare interaction due to short-range correlation and rather limited model space. Otsuka has demonstrated [21, 22] that the monopole part of the tensor force is almost unchanged due to the renormalization procedures involving the short-range correlation and also involving the reduction of the model space. This property is referred to as renormalization persistency [22].

Since the $\pi + \rho$ tensor force is thus justified microscopically, it makes sense to examine the remaining part of the effective interaction. From analyses of empirical interactions for the sd and pf shells, Otsuka has found [21] that the monopole interaction after subtracting the tensor force has a very simple structure: rather weak dependence on spin and strong dependence on node. The $T = 0$ monopole matrix elements are much strongly attractive between the orbits of the same node than between those of different nodes. This property is well simulated by the Gaussian central force. It is amazing that despite the complexity of the effective central force comprising various effects it is expressed so simply at least as far as the monopole part is concerned. As a result, a simple and comprehensive effective interaction, called monopole-based universal interaction ($V_{\rm MU}$) [21], is obtained for describing the shell evolution in a unified way. The $V_{\rm MU}$ is very useful particularly for regions where a reliable interaction is not available such as for those far from stability. Some applications of the $V_{\rm MU}$ to exotic nuclei are shown in Sect. 4.

3. Shell evolution due to the three-nucleon force
As recognized earlier [6, 7], effective interactions derived from microscopic NN forces are not useful as a *universal* interaction in many cases. This is attributed largely to the $T = 1$ monopole interaction. From comparison to empirically fitted interactions, the $T = 1$ monopole matrix

elements of microscopic effective interactions for the pf shell are known to be systematically too attractive [14]. This applies also to the sd shell [21]. From comparison to experiment, for instance, the $N = 28$ magic number for the $\nu(0f_{7/2})^8$ configuration collapses with a microscopic effective interaction because its $T = 1$ monopole interaction between $0f_{7/2}$ and $1p_{3/2}$ is too attractive to stabilize the shell gap between them. Another example is found in the drip line of oxygen isotopes. Microscopic effective interactions predicts the drip line of oxygen isotopes to be located at $N = 20$ or beyond contrary to the experimental drip line located at $N = 16$. The discrepancy for heavy oxygen isotopes is due to a bound neutron $0d_{3/2}$ orbit affected by $T = 1$ attraction from other occupied neutron sd orbits. Thus, correct understanding of the $T = 1$ monopole interaction is essential for describing the neutron shell structure evolving as the neutron number increases.

As an origin of the overbinding of the $T = 1$ monopole interaction, Otsuka has proposed a new mechanism due to the three-nucleon force (3NF) [23]. According to his explanation, the dominant source of the $T = 1$ repulsion is the Fujita-Miyazawa (FM) 3NF in which a virtual excitation of a Δ-particle nucleon-hole occurs as an intermediate state. The 3NF of this type among two valence nucleons and one core nucleon is reduced to a two-body interaction between valence nucleons by summing up all the contributions of the core states. Since this reduced two-body interaction is repulsive as illustrated in [23], the inclusion of this 3NF process gives rise to a repulsive shift of the $T = 1$ monopole interaction as desired. Otsuka has demonstrated also that the FM 3NF accounts for the main part of the repulsive shift by including other types of the chiral 3NFs.

As a result, he has succeeded in reproducing the drip line of oxygen isotopes in a microscopic way without any empirical tuning. This feature has recently been confirmed by a more elaborate coupled-cluster calculation [24] with the inclusion of the chiral 3NFs. A similar effect is demonstrated to occur also in calcium isotopes [25].

4. Shell-model calculations for probing the shell evolution
Although the shell structure is often crucial for the nuclear structure, it does not appear in general through pure single-particle states but through strongly correlated many-body states. Thus, solving a many-body problem to a good accuracy is needed to trace the nuclear property back to the shell evolution. The shell model is one of the most suitable approaches for this purpose. Otsuka has led the shell-model group of the University of Tokyo, producing substantial important results. Among them is advancing large-scale shell-model calculations mainly by developing the Monte Carlo shell model (MCSM) [26, 27]. An overview of this aspect is presented in this conference by Shimizu [28].

Shell-model studies conducted by Otsuka have been useful for clarifying the shell evolution. A unified pf-shell interaction, called GXPF1(A) [14, 29], was completed at the beginning of the 2000s with the help of the development of large-scale shell-model calculations. This interaction is used as one of the standard pf-shell interactions, and its monopole properties provide a basis for the shell-evolution picture as a reference interaction. Studies of the "island of inversion" with the SDPF-M interaction [30] are also strongly connected to constructing the shell evolution. Its feature that the magic number is shifted from $N = 16$ to 20 was phenomenologically introduced [30], and now is regarded as a manifestation of the shell evolution predominantly due to the tensor force for its Z dependence and due to the three-nucleon force for its N dependence.

The concept of the shell evolution has been exported to the construction of effective interactions, which provide good tests for the concept. Spin-isospin properties such as the Gamow-Teller transition in the p-sd shell region are well described by a new interaction [31] incorporating stronger spin-isospin dependent monopole interaction of [9]. The V_{MU} interaction [21] has recently been applied to shell-model interactions with success [32, 33]. For instance, a large deformation in ^{42}Si with the $N = 28$ magic number arises naturally as a consequence of

the tensor-force driven Jahn-Teller effect [33].

Towards shell-model descriptions of medium-heavy nuclei, the barium region [34] and the $28 \leq N(Z) \leq 50$ region [35] were explored already, and the neutron-rich nickel region is under investigation [27]. This direction should be accompanied by the development of large-scale calculations as well as reliable theory of shell evolution.

Acknowledgments

References

[1] Otsuka T and Schwenk A 2012 *Nuclear Physics News* vol 22 (London: Taylor & Francis) pp 12-17
[2] Otsuka T 2013 *Phys. Scr.* **T152** 014007
[3] Mayer M G 1949 *Phys. Rev.* **75** 1969; Haxel O, Jensen J H D and Suess H E 1949 *Phys. Rev.* **75** 1766
[4] Bohr A and Mottelson B R 1969 *Nuclear Structure* vol 1 (New York: Benjamin)
[5] Talmi I and Unna I 1960 *Phys. Rev. Lett.* **4** 469
[6] McGrory J B, Wildenthal B H and Halbert E C 1970 *Phys. Rev.* C **2** 186
[7] Poves A and Zuker A 1981 *Phys. Rep.* **70** 235
[8] Caurier E, Martínez-Pinedo G, Nowacki F, Poves A and Zuker A. P. 2005 *Rev. Mod. Phys.* **77** 427
[9] Otsuka T, Fujimoto R, Utsuno Y, Brown B A, Honma M and Mizusaki T 2001 *Phys. Rev. Lett.* **87** 082502
[10] Uozumi Y *et al.* 1994 *Nucl. Phys.* A **576** 123
[11] Rehm K E *et al.* 1998 *Phys. Rev. Lett.* **80** 676
[12] Utsuno Y, Otsuka T, Mizusaki T and Honma M 1999 *Phys. Rev.* C **60** 051315
[13] Ozawa A, Kobayashi T, Suzuki T, Yoshida K and Tanihata I 2000 *Phys. Rev. Lett.* **84** 5493
[14] Honma M, Otsuka T, Brown B A and Mizusaki T 2004 *Phys. Rev.* C **69** 034335; Honma M, Otsuka T, Brown
    B A and Mizusaki T 2002 *Phys. Rev.* C **65** 061301(R)
[15] Federman P and Pittel S 1977 *Phys. Lett.* **69B** 385
[16] Otsuka T, Suzuki T, Fujimoto R, Grawe H and Akaishi Y 2005 *Phys. Rev. Lett.* **95** 232502
[17] Brown B A, Richter W A, Julies R E and Wildenthal B H 1988 *Ann. Phys.* **182** 191
[18] Kirson M W 1973 *Phys. Lett.* **47B** 110
[19] Schiffer J P *et al.* 2004 *Phys. Rev. Lett.* **92** 162501
[20] Otsuka T, Matsuo T and Abe D 2006 *Phys. Rev. Lett.* **97** 162501
[21] Otsuka T, Suzuki T, Honma M, Utsuno Y, Tsunoda N, Tsukiyama K and Hjorth-Jensen M 2010 *Phys. Rev. Lett.* **104** 012501
[22] Tsunoda N, Otsuka T, Tsukiyama K and Hjorth-Jensen M 2011 *Phys. Rev.* C **84** 044322
[23] Otsuka T, Suzuki T, Holt J D, Schwenk A and Akaishi Y 2010 *Phys. Rev. Lett.* **105** 032501
[24] Hagen G, Hjorth-Jensen M, Jansen G R, Machleidt R and Papenbrock T 2012 *Phys. Rev. Lett.* **108** 242501
[25] Holt J D, Otsuka T, Schwenk A and Suzuki T 2012 *J. Phys.* G **39** 085111
[26] Otsuka T, Honma M, Mizusaki T, Shimizu N and Utsuno Y 2001 *Prog. Part. Nucl. Phys.* **47** 319
[27] Shimizu N, Abe T, Tsunoda Y, Utsuno Y, Yoshida T, Mizusaki T, Honma M and Otsuka T 2012 *Prog. Theor. Exp. Phys.* **2012** 01A205
[28] Shimizu N *et al.* contribution to this symposium
[29] Honma M, Otsuka T, Brown B A and Mizusaki T 2005 *Eur. Phys. J.* A **25** Suppl. 1 499
[30] Utsuno Y, Otsuka T, Mizusaki T and Honma M 1999 *Phys. Rev.* C **60** 05431; Utsuno Y, Otsuka T, Glasmacher T, Mizusaki T and Honma M 2004 *Phys. Rev.* C **70** 044307
[31] Suzuki T, Fujimoto R and Otsuka T 2003 *Phys. Rev.* C **67** 044302
[32] Yuan C, Suzuki T and Otsuka T 2012 *Phys. Rev.* C **84** 064324
[33] Utsuno Y, Otsuka T, Brown B A, Honma M, Mizusaki T and Shimizu N 2012 *Phys. Rev.* C **86** 051301(R)
[34] Shimizu N, Otsuka T, Mizusaki T and Honma M 2001 *Phys. Rev. Lett.* **86** 1171
[35] Honma M, Otsuka T, Mizusaki T and Hjorth-Jensen M 2009 *Phys. Rev.* C **80** 064323