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Test of the local form of higher-spin equations via AdS/CFT

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ABSTRACT

The local form of higher-spin equations found recently to the second order [1] is shown to properly reproduce the anticipated AdS/CFT correlators for appropriate boundary conditions. It is argued that consistent AdS/CFT holography for the parity-broken boundary models needs a nontrivial modification of the bosonic truncation of the original higher-spin theory with the doubled number of fields, as well as a nonlinear deformation of the boundary conditions in the higher orders.

1. Introduction

Higher-spin (HS) theories (see e.g. [2] for a review) have attracted much of interest providing a relatively simple playground for AdS/CFT correspondence [3–5]. Studying these models may shed light on the nature of holography itself. Particularly, some dualities relate complicated theory of gravity and infinitely many HS fields in the bulk to simplest CFT duals being just free theories. The HS AdS/CFT story dates back to stringy tensionless limit argument by Sundborg [6] (see also [7–10]) asserting free boundary theory as a HS dual. A concrete proposal of Klebanov and Polyakov [11], was that what is known as HS A-model should be dual to either free or critical $O(N)$-model. The conjecture was later generalized to supersymmetric theories [12] and to HS B-model in [13]. However, due to the lack of conventional action principle for HS theory it was not clear how to test those conjectures at the level of correlation functions until an impressive calculation by Giombi and Yin [14] based on a certain setup for extracting tree-level correlators from equations of motion. In [14] and [15] a substantial piece of evidence in favor of the proposed dualities at the level of three-point correlation functions was given. Later on Maldacena and Zhiboedov showed [16] that the presence of infinitely many exactly conserved HS currents in $d = 3$ constrains CFT theory drastically leaving one with either a theory of free bosons or free fermions. Yet, even if one allows for a slight HS symmetry deformation, the CFT is still highly constrained [17]. Though in this paper we focus on the AdS_4/CFT_3 HS holography, it should be noted that the important proposal on the AdS_3/CFT_2 HS duality was put forward in [18].

Despite noticeable success of the AdS_4/CFT_3 HS holography tests some loose ends still remain even at the level of three-point analysis especially in the sector of holographic duality of parity-noninvariant $3d$ conformal theories proposed in [19,20] exhibiting difficulties in extracting correlation functions from the parity-noninvariant bulk HS theories [21] (where some were nevertheless obtained). The main origin of those problems and inconsistencies can be traced back to the nonlocal setup in HS equations used in the original papers. Indeed, as was noticed in [14] the natural procedure of extracting HS interaction vertices from HS equations results in a nonlocal interaction even at the lowest nontrivial level leading to infinities in the boundary limit. The origin of these nonlocalities is due to the natural freedom in field redefinitions in HS equations. Particularly, the procedure of extracting HS vertices amounts to solving some differential equations in the auxiliary spinorial space which results in unavoidable problem of fixing a representative. More generally, this is the problem of the choice of proper (minimally nonlocal) class of functions respecting physical properties of HS theory.

Partly, the class of functions that respects nonlinear structure of HS equations was proposed in [22] and later further narrowed in [1,23] for the special case of quadratic corrections in the 0-form sector. In [1] it was shown that the proper field redefinition that brings HS equations into a manifestly local form does exist, fixing relative coefficients of the second order HS interaction vertices. In this paper we show that the structure of second-order local HS interactions in four dimensions is in perfect agreement with the CFT expectations.

The paper is organized as follows. In section 2 we briefly review HS equations in four dimensions presenting perturbative expansion up to the second order. Then, in section 3 we discuss boundary conditions and truncations respecting the AdS/CFT duality.

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In section 4 we extract three-point correlation functions from the 0-form sector of HS equations and in section 5 we leave our conclusion.

2. Higher-spin equations

HS equations in four dimensions have the form [24]

$$dW + W * W = 0\,, \tag{2.1}$$

$$dS + [W, S]_* = 0\,, \tag{2.2}$$

$$dB + [W, B]_* = 0\,, \tag{2.3}$$

$$S * S = -i\theta_\alpha \wedge \theta^\alpha (1 + F_*(B) * k * \kappa) - i\bar{\theta}_{\dot\alpha} \wedge \bar{\theta}^{\dot\alpha}(1 + \bar{F}_*(B) * \bar{k} * \bar{\kappa})\,, \tag{2.4}$$

$$[S, B]_* = 0\,. \tag{2.5}$$

Here master fields $W(Z;Y;K|x)$, $B(Z;Y;K|x)$ and $S(Z;Y;K|x)$ depend on spinorial variables $Z_A = (z_\alpha, \bar{z}_{\dot\alpha})$ and $Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$ $(\alpha, \dot\alpha = 1, 2)$, as well as outer Klein operators $K = (k, \bar{k})$. W is a space–time 1-form, B is a 0-form and S is a 1-form in the exterior Z^A-directions with anticommuting differentials θ^A. Functions of spinor variables $Z_A = (z_\alpha, \bar{z}_{\dot\alpha})$ and $Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$, $\alpha, \dot\alpha = 1, 2$ are treated as elements of the star-product algebra with the associative star product

$$(f * g)(Z, Y) = \frac{1}{(2\pi)^4}\int dUdV f(Z+U, Y+U) g(Z-V, Y+V) e^{iU_A V^A} \tag{2.6}$$

$(V^A = (\epsilon^{\alpha\beta}V_\beta, \epsilon^{\dot\alpha\dot\beta}V_{\dot\beta}))$. Inner Klein operators κ and $\bar\kappa$ are

$$\kappa = e^{iz_\alpha y^\alpha}\,, \qquad \bar\kappa = e^{i\bar{z}_{\dot\alpha}\bar{y}^{\dot\alpha}}\,. \tag{2.7}$$

Outer Klein operator k $(\bar{k})$ is defined to anticommute with all (anti)holomorphic variables

$$\{k, V_\alpha\}_* = 0\,, \qquad k * k = 1\,, \tag{2.8}$$

where $V_\alpha = (y_\alpha, z_\alpha, \theta_\alpha)$. This formula extends the star product to $k, \bar{k}$-dependent elements.

In this paper we focus on the purely HS sector of the theory where B is linear in k and $\bar{k}$ while W and S contain the $k, \bar{k}$-independent part as well as bilinear $k * \bar{k}$. Function $F_*(B)$ is set to be linear with an arbitrary constant complex parameter

$$F = \eta B\,, \qquad \bar{F} = \bar\eta B\,. \tag{2.9}$$

Now, since fields depend on outer Klein operators k and $\bar{k}$, we assume these to enter on the most right, for example,

$$B(Z;Y;k,\bar{k}|x) := B(Z;Y|x)k + \bar{B}(Z;Y|x)\bar{k}\,. \tag{2.10}$$

System (2.1)–(2.5) can be analyzed perturbatively. One starts with the vacuum solution that corresponds to pure AdS_4 space–time

$$B_0 = 0\,, \tag{2.11}$$

$$S_0 = Z_A\theta^A\,, \tag{2.12}$$

$$W_0 = \frac{i}{4}(\omega_{\alpha\alpha}y^\alpha y^\alpha + \bar\omega_{\dot\alpha\dot\alpha}\bar{y}^{\dot\alpha}\bar{y}^{\dot\alpha} + 2e_{\alpha\dot\alpha}y^\alpha\bar{y}^{\dot\alpha})\,. \tag{2.13}$$

We take Poincare coordinates as well adopted for the boundary analysis $(x = (\mathrm{x}, \mathrm{z}))$

$$\omega_{\alpha\alpha} = -\frac{i}{2\mathrm{z}}d\mathrm{x}_{\alpha\alpha}\,, \quad \bar\omega_{\dot\alpha\dot\alpha} = \frac{i}{2\mathrm{z}}d\mathrm{x}_{\dot\alpha\dot\alpha}\,,$$
$$e_{\alpha\dot\alpha} = \frac{1}{2\mathrm{z}}(d\mathrm{x}_{\alpha\dot\alpha} - i\epsilon_{\alpha\dot\alpha}d\mathrm{z})\,, \tag{2.14}$$

where $\mathrm{x}^{\alpha\beta} = \mathrm{x}^{\beta\alpha}$ denote the three boundary coordinates (independently of whether they carry dotted or undotted indices) while z is the Poincaré coordinate.

First-order equations reduce to the twisted-adjoint flatness condition for the 0-form $B_1 = C(Y;k,\bar{k}|x)$

$$DC = D^L C + ie^{\alpha\dot\alpha}(y_\alpha\bar{y}_{\dot\alpha} - \partial_\alpha\partial_{\dot\alpha})C = 0\,, \tag{2.15}$$

and First on-shell theorem for HS potentials $\omega(Y|x)$

$$D\omega = \frac{i}{4}\left(\eta\bar{H}^{\dot\alpha\dot\alpha}\partial^2_{\dot\alpha}\bar{C}(0,\bar{y};k,\bar{k}|x)\bar{k} + \bar\eta H^{\alpha\alpha}\partial^2_\alpha C(y,0;k,\bar{k}|x)k\right)\,. \tag{2.16}$$

At second order the local form of HS equations was extracted from (2.1)–(2.5) in [1] for 0-form $C(Y|x)$ and in [26] for 1-form $\omega(Y|x)$. The equation for 0-form reads

$$DC = \frac{i}{2}\eta e^{\alpha\dot\alpha}\int e^{i\bar{u}_{\dot\alpha}\bar{v}^{\dot\alpha}} y_\alpha(t\bar{u}_{\dot\alpha} + (1-t)\bar{v}_{\dot\alpha}) \times J(ty, -(1-t)y, \bar{y}+\bar{u}, \bar{y}+\bar{v})k + c.c.\,, \tag{2.17}$$

where

$$J(y_1, y_2, \bar{y}_1, \bar{y}_2; k, \bar{k}|x) := C(y_1, \bar{y}_1; k, \bar{k}|x)C(y_2, \bar{y}_2; k, \bar{k}|x)\,, \tag{2.18}$$

and we use the short-hand notation for integrals

$$\int F(t_1, \ldots, t_n; \bar{u}, \bar{v}) := \int_{[0,1]^n} dt_1 \ldots dt_n \int_{\mathbf{R}^4} \frac{1}{(2\pi)^2} d\bar{u}d\bar{v} F(t_1, \ldots, t_n; \bar{u}, \bar{v})\,. \tag{2.19}$$

Similarly for integrals that contain both holomorphic u, v and antiholomorphic $\bar{u}, \bar{v}$ integration variables.

3. Boundary conditions and truncations

HS equations (2.1)–(2.5) admit various truncations. Due to the dependence on Klein operators k and $\bar{k}$ there are two copies of fields of every spin. In the bosonic case, the spectrum can be reduced down to a single copy by setting

$$B(Z, Y; k, \bar{k}|x) \to B(Z, Y|x)(k + \bar{k})\,,$$
$$W(Z, Y; k, \bar{k}|x) \to W(Z, Y|x)(1 + k\bar{k})\,. \tag{3.1}$$

While bosonic truncation (3.1) can be imposed to all orders reducing the spectrum of the theory, it is not *a priori* guaranteed that it has any CFT dual at all in the HS theories with broken parity [25]. Within the perturbation theory however one can impose condition relating fields of the full theory with the doubled spectrum in such a way that the theory becomes bosonic yet different from the one resulting from (3.1). To explain the origin of the modified conditions driven by the AdS/CFT requirement let us analyze the boundary limit of the full fledged HS system in perturbation theory.

3.1. Lowest order

In the free-level analysis carried out in [25] the field-current correspondence is reached via the identification

$$C(y,\bar y;k,\bar k|x) = z\, e^{y_\alpha \bar y^\alpha} T(w,\bar w;k,\bar k|x)\,, \tag{3.2}$$

where

$$w = \sqrt{z}\,y\,, \qquad \bar w = \sqrt{z}\,\bar y\,. \tag{3.3}$$

Eq. (3.2) says that if C is on-shell, that is satisfies (2.15), then T enjoys the unfolded form of the 3d conformal current conservation equation

$$d_x T - \frac{i}{2} dx^{\alpha\alpha}\partial_\alpha \bar\partial_\alpha T = 0\,. \tag{3.4}$$

HS potentials are sourced by the field C in the bulk in accordance with (2.16). Its boundary pushforward reads

$$D_x\omega_x = \frac{1}{4}H^{\alpha\beta}_{xx}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{+\beta}}\left(\bar\eta T(w^+,0|x)k - \eta T(0,iw^+|x)\bar k\right)\,. \tag{3.5}$$

One concludes that, in general, boundary HS fields, which are gauge fields of the boundary conformal HS theory, are sourced by currents. To make AdS/CFT work in the standard sense, *i.e.*, for the usual boundary CFT with the well-defined stress tensor, one has to impose such boundary conditions that make the r.h.s of (3.5) vanish allowing to get rid of boundary HS gauge fields making the boundary stress tensor gauge non-invariant.

For $\eta = 1$ or $\eta = i$ proper conditions read

$$T(w,\bar w|x)k = \pm T(-i\bar w, iw|x)\bar k\,. \tag{3.6}$$

It is important that one can exclude scalar and spinor fields from (3.6) since they do not affect (3.5) (at higher orders this will not be the case) opening the way to alternative boundary conditions in this sector, corresponding to the critical boundary models in accordance with the original proposal of [11–13]. Let us also note that there is no way to include general parameter η into (3.6) demanding $\eta = 1$ or $\eta = i$.

However, for general η conditions (3.6) can be modified as follows. Having two fields in the decomposition

$$C(Y;k,\bar k|x) = C(Y|x)k + \bar C(Y|x)\bar k \tag{3.7}$$

one can identify the Weyl module $C(Y|x)$ with positive helicity part $\mathcal{C}^+(Y|x)$ of some bosonic module $\mathcal{C}(Y|x)$, while $\bar C(Y|x)$ with its negative helicity part $\mathcal{C}^-(Y|x)$, i.e.

$$C(Y|x) := \mathcal{C}^+(Y|x)\,, \qquad \bar C(Y|x) := \mathcal{C}^-(Y|x)\,, \tag{3.8}$$

where by the doubled helicity of a spin s field we mean the difference between the number of y and $\bar y$ variables, i.e. the eigenvalue of the following operator

$$n = y^\alpha\frac{\partial}{\partial y^\alpha} - \bar y^{\dot\alpha}\frac{\partial}{\partial \bar y^{\dot\alpha}}\,. \tag{3.9}$$

Particularly, $\mathcal{C}^+$ carries more y variables than $\bar y$ and $\mathcal{C}^-$ other way around. Let us stress that this way one truncates the spectrum to the bosonic system in a way different from (3.1) allowing to get rid of the sources in (3.5) in the parity broken case by setting

$$\bar\eta T^+(w,\bar w|x) = \eta T^-(-i\bar w, iw|x)\,. \tag{3.10}$$

To make contact of the introduced boundary conditions with those usually imposed in the HS literature consider HS boundary to bulk propagators. In the 0-form sector the positive and negative helicity parts have the following form [14][1]

$$\mathcal{C}^+ = \eta\mathcal{K}e^{if_{\alpha\dot\alpha}y^\alpha\bar y^{\dot\alpha}+i\xi^\alpha y_\alpha}\,, \quad \mathcal{C}^- = \bar\eta\mathcal{K}e^{if_{\alpha\dot\alpha}y^\alpha\bar y^{\dot\alpha}+i\bar\xi^{\dot\alpha}\bar y_{\dot\alpha}} \tag{3.12}$$

(no η-factors for a scalar), where

$$\mathcal{K} = \frac{z}{(x-x_0)^2+z^2}\,, \tag{3.13}$$

$$f_{\alpha\dot\alpha} = -\frac{2z}{(x-x_0)^2+z^2}(x-x_0)_{\alpha\dot\alpha} - i\frac{(x-x_0)^2-z^2}{(x-x_0)^2+z^2}\epsilon_{\alpha\dot\alpha}\,, \tag{3.14}$$

$$\xi_\alpha = \Pi_\alpha{}^\beta\mu_\beta\,, \qquad \Pi_{\alpha\beta} = \mathcal{K}\left(\frac{1}{\sqrt{z}}(x-x_0)_{\alpha\beta} - i\sqrt{z}\epsilon_{\alpha\beta}\right)\,, \tag{3.15}$$

and the reality conditions for polarization spinors are

$$\mu_\alpha = i\bar\mu_\alpha\,. \tag{3.16}$$

Scalar part of the propagator (3.12) corresponds to the $\Delta = 1$ solution. Another scalar branch associated with $\Delta = 2$ reads

$$C_{\Delta=2} = \mathcal{K}^2(1 + if_{\alpha\dot\alpha}y^\alpha\bar y^{\dot\alpha}) \times e^{if_{\alpha\dot\alpha}y^\alpha\bar y^{\dot\alpha}}\,. \tag{3.17}$$

Let us show how these propagators match the reality conditions just spelled out. Using boundary prescription (3.2) one finds for (3.12)

$$T^+ = \frac{\eta}{|x-x_0|^2}e^{-2i(x-x_0)^{-1}_{\alpha\alpha}w^\alpha\bar w^\alpha + i(x-x_0)_{\alpha\beta}\mu^\beta w^\alpha}\,, \tag{3.18}$$

$$T^- = \frac{\bar\eta}{|x-x_0|^2}e^{-2i(x-x_0)^{-1}_{\alpha\alpha}w^\alpha\bar w^\alpha + i(x-x_0)_{\alpha\beta}\bar\mu^\beta\bar w^\alpha} \tag{3.19}$$

and

$$T_{\Delta=2} = \frac{w_\alpha\bar w^\alpha}{|x-x_0|^4}\times e^{-2i(x-x_0)^{-1}_{\alpha\alpha}w^\alpha\bar w^\alpha}\,. \tag{3.20}$$

One can see now that condition (3.6) for $\eta = 1$ is fulfilled for (3.18), (3.19) while for $\eta = i$ one has to use (3.20) in accordance with parity-odd scalar condition for HS B-model. For generic η (3.18) and (3.19) as well as (3.20) for the alternative scalar boundary condition satisfy (3.10).

Having HS equations to the second order one may wish to examine them in the boundary limit. Particularly, the expectation for (3.6) boundary condition for the A and B HS theories is that in these cases HS symmetry remains undeformed leading to conservation of boundary currents yet leaving no HS gauge fields at the boundary. We will show that this is indeed the case. For boundary conditions like (3.10) or for alternative scalar like in the critical case on the contrary it turns out that HS potentials get sourced on the boundary and one should introduce certain nonlinear completion for (3.10) at higher orders to make them vanish. This implies among other things that without such a nonlinear completion the tree-level correlation functions extracted from the bulk are anticipated to differ from boundary expectation starting from the 4-point functions.

[1] While we will not use it in this paper, let us give for completeness the explicit formula for the 1-form ω propagator.

$$\omega = -\frac{i}{2}\mathcal{K}e^{\alpha\dot\alpha}\xi_\alpha\bar\xi_{\dot\alpha}\int_0^1 dt e^{it\xi^\alpha y_\alpha + i(1-t)\bar\xi^{\dot\alpha}\bar y_{\dot\alpha}}\,. \tag{3.11}$$

This form of the propagator was found by one of us (V.D.) with Zhenya Skvortsov in 2014 but was never published.

3.2. Second order

Let us carry out the boundary limit for (2.17). This will give us the deformed version of current equation (3.4). The limit is quite straightforward using (3.2). The final result is

$$\begin{aligned} & d_{\mathrm{x}} T - \frac{i}{2} d\mathrm{x}^{\alpha\alpha} \partial_\alpha \bar{\partial}_\alpha T \\ & = -\frac{\eta}{4} d\mathrm{x}^{\alpha\alpha} w_\alpha \int_0^1 (t\bar{\partial}_{2\alpha} - (1-t)\bar{\partial}_{1\alpha}) \\ & \quad \times I\big(tw, -(1-t)w, \bar{w} + i(1-t)w, \bar{w} - itw\big)k \\ & \quad + c.c.\,, \end{aligned} \tag{3.21}$$

where

$$I(w_1, w_2, \bar{w}_1, \bar{w}_2|\mathrm{x}) = T(w_1, \bar{w}_1; k, \bar{k}|\mathrm{x}) T(w_2, \bar{w}_2, k, \bar{k}|\mathrm{x})\,. \tag{3.22}$$

Note, that while field-current correspondence (3.2) contains a potentially dangerous projector $e^{iy_\alpha \bar{y}^\alpha}$ which may cause infinities at the boundary [25] it turns out that no divergencies actually appear due to specific dependence on the homotopy parameter t in (2.17). One observes that currents receive contributions originated from current-current interaction that may lead to nonconservation. Indeed, from (3.21) it follows that

$$\begin{aligned} & \frac{\partial}{\partial w^\alpha} \frac{\partial}{\partial w^\beta} \frac{\partial}{\partial \mathrm{x}_{\alpha\beta}} T = \\ & = -\frac{\eta}{4} \int_0^1 dt \left(3 + w^\alpha \frac{\partial}{\partial w^\alpha}\right) \left(t\bar{\partial}_2^\beta - (1-t)\bar{\partial}_1^\beta\right) \\ & \quad \times \frac{\partial}{\partial w^\beta}\Big[T(tw, \bar{w} + i(1-t)w) T(-(1-t)w, \bar{w} - itw)\Big]k \\ & \quad + c.c. \end{aligned} \tag{3.23}$$

which in general implies

$$\partial \cdot J_s \neq 0\,. \tag{3.24}$$

Let us analyze this issue starting from the parity preserving models. In this case with the boundary conditions (3.6) one finds that despite the deformation is nonlinear the boundary currents remain conserved

$$\frac{\partial}{\partial w^\alpha} \frac{\partial}{\partial w^\beta} \frac{\partial}{\partial \mathrm{x}_{\alpha\beta}} T = 0 \qquad \Rightarrow \qquad \partial \cdot J_s = 0\,. \tag{3.25}$$

This is most easily seen from noting that under (3.6) the *r.h.s.* of (3.21) gets rewritten as

$$\begin{aligned} & d_{\mathrm{x}} T - \frac{i}{2} d\mathrm{x}^{\alpha\alpha} \partial_\alpha \bar{\partial}_\alpha T \\ & = -\frac{\eta}{4} d\mathrm{x}^{\alpha\alpha} w_\alpha \frac{\partial}{\partial w^\alpha} \\ & \quad \times \int_0^1 dt I\big(tw, -(1-t)w, \bar{w} + i(1-t)w, \bar{w} - itw\big)\,, \end{aligned} \tag{3.26}$$

from where (3.25) immediately follows. The fact that for free theories (3.26) results in current conservation means that there is a local field redefinition that brings (3.26) to the canonical conserved current form (3.4). For parity broken boundary condition (3.10) as well as for the alternative ones corresponding to critical models the HS currents are not conserved.[2] In obtaining (3.26) the structure of (3.21) was important. Particularly one uses the symmetry with respect to the exchange $t \to 1-t$. The check carried out for parity preserving boundary conditions (3.25) alone is sufficient to justify the agreement between bulk vertices given in (2.17) and boundary free theory 3pt correlation functions. Indeed, according to the Maldacena–Zhiboedov theorem [16] the conservation of HS currents inevitably implies free boundary theory.

A soft spot in this argument is the following. As a matter of principle it may happen that while HS currents are conserved the theory still contains sources for the boundary HS connections, in which case the standard AdS/CFT correspondence can be lost. So let us check out the conditions at which sources for HS connections do vanish. To do so we should analyze the 1-form sector found in [26] in the boundary limit.

It is easy to perform boundary limit for current interaction equation in the 1-form sector of [26] (Eq. (3.4) of [26]). Following the logic of [25] this gives

$$\begin{aligned} & D_{\mathrm{x}} \omega_{\mathrm{x}}(w^+, v^-|\mathrm{x}) \\ & = \frac{i}{8} \eta\bar{\eta} \int d^2 t \delta'(1 - t_1 - t_2) H^{\alpha\alpha}_{\mathrm{xx}} \left(\frac{\partial}{\partial u^\alpha}\right)^2 \times \\ & \quad \times \Big\{ I(t_1(w^+ + u), -t_2(w^+ + u), it_2 w^+, -it_1 w^+) \\ & \quad - I(t_1 w^+, -t_2 w^+, it_2(w^+ + u), -it_1(w^+ + u)) \Big\}\Big|_{u=0}\,, \end{aligned} \tag{3.27}$$

where the following variables have been introduced

$$w = w^+ + izv^-\,, \qquad \bar{w} = iw^+ + zv^-\,. \tag{3.28}$$

Just as well as at the linearized level, one observes that sources for HS connections do not vanish in general (although they almost do since the two terms on the *r.h.s.* of (3.27) are equal to each other at $u = 0$). However, imposing free theory boundary conditions (3.6) one finds exact cancellation and the theory becomes free of boundary HS connections in accordance with the AdS/CFT expectation.

An important comment is now in order. One may expect that for parity breaking boundary conditions (3.10) or for those of critical theories one has vanishing sources for boundary connections too. This is not the case as (3.6) is likely to be the only linear relation that cancels out sources. This implies that on the way of proposed dualities with critical models and vectorial models with Chern–Simons matter one has to modify boundary conditions by nonlinear terms to compensate the nonlinear corrections.

Also it should be noted that in the CFT-based HS literature terms on the r.h.s. of (3.21) are usually interpreted as “slight breaking” of HS symmetry [17]. From the perspective of the original HS equations, however, they are naturally interpreted as a deformation rather than breaking of HS symmetry. Indeed, consistency of nonlinear terms on the r.h.s. of HS field equations implies that the HS gauge symmetry transformations receive nonlinear corrections as well. The tricky point is that, at the boundary, the resulting deformation may go beyond the standard class of CFTs with well defined (gauge invariant) stress tensor because the deformed HS gauge transformation in most cases mixes HS 0-forms, that have clear meaning from the boundary CFT perspective, with the HS 1-forms at the boundary, which are conformal HS gauge fields on the boundary not allowed in the standard CFTs.

[2] Recall that according to the HS AdS/CFT expectation (see e.g. [21]) the HS currents are supposed to be conserved for the $\Delta = 1$ boundary condition in A-model and $\Delta = 2$ in B-model only. Other cases result in HS current non-conservation.

4. Boundary correlators

In this section we venture to extract correlation functions of a dual theory from the 0-form sector of the bulk field equations. We restrict ourselves to the case with two sources on the boundary s_1 and s_2 that generate spin s such that

$$s \geq s_1 + s_2 \,. \tag{4.1}$$

This constraint comes from the fact that so far we have taken into account only current interactions given in (2.17), which is only consistent when restriction (4.1) is imposed since otherwise the contribution of HS 1-forms also has to be taken into account. For the opposite case of three spins obeying the triangle inequalities, the original current interaction is supported by the HS 1-forms and is local in the original setup of Giombi and Yin [14], giving the proper answer.

According to the proposal of [14] a solution to the second order equation for Weyl 0-form (2.17) generated by two boundary sources can be associated with a properly normalized 3pt function via

$$\langle JJJ \rangle \sim \lim_{\mathrm{z}\to 0} \mathrm{z}^{-1} G(w\mathrm{z}^{-\frac{1}{2}}, \bar{w}\mathrm{z}^{-\frac{1}{2}})\Big|_{\bar{w}=0} \,, \tag{4.2}$$

where $G(y, \bar{y})$ is a Green's function for equation (2.17). The remaining w-variable is to be associated with a polarization spinor for the outgoing leg of spin s. Though such a prescription for the computation of correlation functions may need some further justification, for a time being we take it as a working tool. Before going into technical details of the computation we give general arguments on the dependence of the boundary correlators on the phase parameter in the HS theory.

4.1. Phase dependence

In this section we reconsider the analysis of HS holography of [1] in a more conventional setup leading to the same conclusions. To this end, consider HS equations of [26]

$$D\omega = \frac{i}{4}\Big(\eta \bar{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial \bar{y}^{\dot\alpha}\partial \bar{y}^{\dot\beta}} C_-(0, \bar{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta} C_+(y, 0|x)\Big) + \eta\bar\eta \Gamma^{loc}(J) \,, \tag{4.3}$$

where Γ^{loc} is the second-order current interaction, $C_\pm$ denote positive and negative helicity parts of $C(y, \bar{y})$ and the dependence on the Klein operators is discarded. Though as shown in [1,26] the quadratic J-dependent deformation is independent of the phase of $\eta = |\eta| \exp i\varphi$, the linear part is phase-dependent. Introducing the new fields

$$C'_- = \eta C_- \,, \qquad C'_+ = \bar\eta C_+ \,, \tag{4.4}$$

Eq. (4.3) takes the form

$$D\omega(y, \bar{y}|x) = \frac{i}{4}\Big(\bar{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial \bar{y}^{\dot\alpha}\partial \bar{y}^{\dot\beta}} C'_-(0, \bar{y}|x) + H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta} C'_+(y, 0|x)\Big) + \Gamma^{loc}(J(C(C'))) \,, \tag{4.5}$$

where, setting for simplicity $|\eta| = 1$,

$$C(C') = \exp i\varphi\, C'_+ + \exp -i\varphi\, C'_- \,. \tag{4.6}$$

Clearly, redefinition (4.6) is an $U(1)$ electromagnetic duality transformation with the phase φ.

The linear term in Eq. (4.5) tells us that it is the 0-form C' that has to be identified with the generalized Weyl (Faraday for $s = 1$) tensor associated with the curvatures of the Fronsdal HS fields contained in $\omega(y, \bar{y})$. In these terms, the vertex that was φ-independent in terms of C acquires the nontrivial φ-dependence in terms of C'

$$\Gamma^{loc}(J) = \Gamma^{loc}\left(\exp 2i\varphi J_{++}(C') + \exp -2i\varphi J_{--}(C') + 2J_{+-}(C')\right) . \tag{4.7}$$

Since the A-model with $\varphi = 0$ and B-model with $\varphi = \frac{\pi}{2}$ are known to correspond to bosonic and fermionic parity-invariant boundary vertices, we set

$$J_b := J_{++}(C') + J_{--}(C') + 2J_{+-}(C') \,,$$
$$J_f := -J_{++}(C') - J_{--}(C') + 2J_{+-}(C') \,. \tag{4.8}$$

The remaining parity-odd boundary vertex is associated with

$$J_o = i(J_{++}(C') - J_{--}(C')) \,. \tag{4.9}$$

In terms of these currents, $\Gamma^{loc}(J)$ acquires the form

$$\Gamma^{loc}(J) = \cos^2(\varphi) J_b + \sin^2(\varphi) J_f + \frac{1}{2}\sin(2\varphi) J_o \tag{4.10}$$

coinciding with the expression obtained in [1] by slightly different arguments. This expression precisely matches the dependence on the phase φ anticipated from the HS holography [17,19–21].

To summarize, the proper phase dependence of the current interactions for the phase-independent vertex results from that in the terms linear in the 0-forms upon the identification of the genuine HS Weyl tensors.

This simple analysis is useful in many respects. In particular it shows that, to find the phase dependence of the boundary correlators it suffices to know it for any three different data in φ. For instance it is enough to find the boundary correlators in the A-model with $\varphi = 0$, B-model with $\varphi = \pi/2$ to identify the parity-even part, and, say, the first φ-derivative at its B-model value $\varphi = \pi/2$ to identify its parity-odd part. Interestingly, the latter definition is somehow reminiscent of the interpretation of the odd $3d$ conformal structure proposed in [27] as a massive deformation of $3d$ fermionic currents. Indeed, from the boundary perspective the parameter η is closely related to the $3d$ massive boundary deformation though at a nonzero VEV of the 0-form B, which, though making sense in the model including topological fields not considered in this paper, has to be set to zero in the end of the computation.

4.2. 0-form Green's function

The *r.h.s.* of equation (2.17) contains two pieces proportional to η and $\bar\eta$, respectively. Therefore the Green's function can be found as a sum of two

$$G = G_\eta + G_{\bar\eta} \,, \tag{4.11}$$

where G_η (similarly $G_{\bar\eta}$) obeys the equation

$$DG_\eta = \frac{i}{2}\eta e^{\alpha\dot\alpha} \int e^{i\bar{u}_{\dot\alpha}\bar{v}^{\dot\alpha}} y_\alpha (t\bar{u}_{\dot\alpha} + (1-t)\bar{v}_{\dot\alpha}) \times J(ty, -(1-t)y, \bar{y} + \bar{u}, \bar{y} + \bar{v})k \,. \tag{4.12}$$

In terms of power series, the Green's function was analyzed in [23] for the case with constituent fields $C_1(y, \bar{y})$ and $C_2(y, \bar{y})$ having opposite chiralities (3.9), *i.e.*,

$$n(C_1)n(C_2) < 0 \,. \tag{4.13}$$

Its representation suitable for practical calculations results from the following Ansatz

$$G_\eta = \eta \int f(t_1,t_2,t_3)e^{iu_A v^A} J(u+t_1 y, t_3 v - t_2 y, \bar{y}+\bar{u}, \bar{y}+\bar{v})k\,, \tag{4.14}$$

which is most general in the holomorphic sector of spinor variables. Since the measure in t_i – integration is compact and assuming that a function (distribution) $f(t_1,t_2,t_3)$ is well defined we will be freely integrating by parts. Substituting (4.14) into (2.17) one finds

$$f(t_1,t_2,t_3) = \frac{1}{2}\delta'(1-t_1-t_2-t_3) \tag{4.15}$$

and, therefore,

$$G_\eta = \frac{\eta}{2}\int \delta'(1-t_1-t_2-t_3)e^{iu_A v^A} \times J(u+t_1 y, t_3 v - t_2 y, \bar{y}+\bar{u}, \bar{y}+\bar{v})k\,. \tag{4.16}$$

So defined Green's function does not satisfy (2.17) in general in the first place because (2.17) is not everywhere consistent since the contribution of 1-forms has to be taken into account if constraint (4.1) is not respected. But even for $s \geq s_1 + s_2$ Green's function (4.16) is only valid for those sources in which (4.13) is true. Since the extension of the Green's function to the general case with arbitrary signs of chiralities yet remains to be constructed, our strategy will be as follows. Assuming that the coefficients in correlation function depend solely on spins in the vertex, *i.e.,* modules of helicities, we will take sources (*i.e.,* constituent fields) of opposite chiralities in the calculation.

There are two sets of primary currents stored in the boundary limit of the Green's function: those depending only on w or only on $\bar{w}$. We focus on the w-dependent ones which makes it possible extracting correlation functions from the holomorphic part G_η. In accordance with the general analysis of Section 4.1, there are three different structures that arise upon substituting propagators (3.12) that satisfy boundary conditions (3.10)

$$\langle JJJ\rangle_{boson} \sim G^{++} + G^{--} + G^{+-} + G^{-+}\,, \tag{4.17}$$

$$\langle JJJ\rangle_{fermion} \sim G^{++} + G^{--} - G^{+-} - G^{-+}\,, \tag{4.18}$$

$$\langle JJJ\rangle_{odd} \sim G^{++} - G^{--}\,, \tag{4.19}$$

where pluses and minuses refer to the chirality signs. The dependence on the phase parameter η is fixed according to (4.10). Particularly, it follows that $G^{++}+G^{--}$ and $G^{+-}+G^{-+}$ correspond to free theories correlators. Substituting propagators (3.12) into (4.16) and performing simple Gaussian integration leads to the following result in the leading order in z

$$G_{12}^{+-} = \int d^3t \frac{\mathcal{K}_1\mathcal{K}_2}{\Delta}\delta'(1-t_1-t_2-t_3) \times e^{2\frac{t_1t_2}{\Delta}Q + \frac{t_1}{\Delta}((1-t_3)P_1 + zt_3\tilde{S}_1) - \frac{t_1}{\Delta}((1-t_3)P_2+zt_3\tilde{S}_2)}\,, \tag{4.20}$$

$$G_{12}^{-+} = \int d^3t \frac{\mathcal{K}_1\mathcal{K}_2}{\Delta}\delta'(1-t_1-t_2-t_3) \times e^{2\frac{t_1t_2}{\Delta}Q + \frac{t_2}{\Delta}((1-t_3)P_1 - zt_3\tilde{S}_1) - \frac{t_2}{\Delta}((1-t_3)P_2-zt_3\tilde{S}_2)}\,. \tag{4.21}$$

Here, indices 1 and 2 label points at the boundary and

$$\Delta = (1-t_3)^2 + z^2\epsilon^2 t_3 + O(z^4)\,, \qquad \epsilon = \frac{2\,\mathrm{x}_{12}}{\mathrm{x}_{01}\mathrm{x}_{02}}\,, \tag{4.22}$$

where the outgoing leg x is denoted by x_0. The parity-preserving conformal structures P_i and Q, and parity-odd S_i read

$$P_1 = i\frac{(\mathrm{x}_{01})_{\alpha\alpha}w^\alpha\mu_1^\alpha}{|\mathrm{x}_{01}|^2}\,, \qquad P_2 = i\frac{(\mathrm{x}_{02})_{\alpha\alpha}w^\alpha\mu_2^\alpha}{|\mathrm{x}_{02}|^2}\,;$$

$$Q = \left(\frac{\mathrm{x}_{01}}{|\mathrm{x}_{01}|^2} - \frac{\mathrm{x}_{02}}{|\mathrm{x}_{02}|^2}\right)_{\alpha\alpha} w^\alpha w^\alpha\,, \tag{4.23}$$

$$S_1 = \frac{(\mathrm{x}_{02})^{\beta\alpha}(\mathrm{x}_{12})_\alpha{}^\gamma\mu_{1\gamma}w_\beta}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|}\,,$$

$$S_2 = \frac{(\mathrm{x}_{01})^{\beta\alpha}(\mathrm{x}_{12})_\alpha{}^\gamma\mu_{2\gamma}w_\beta}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|}\,. \tag{4.24}$$

The result in (4.20)–(4.21) is not yet conformal and we have introduced $\tilde{S}_i = \epsilon S_i$.

An important comment is that, naively, it looks like in the boundary limit $z\to 0$ all parity-odd structures $\tilde{S}$ vanish, because they are accompanied by a factor of z. This is not the case due to the pole at $z=0$ resulting from Δ upon integration over t_3. Indeed, careful analysis shows that the terms $(1-t_3)$ and zt_3 in exponentials (4.20) and (4.21) give the same contribution in z as $z\to 0$. Carrying out the boundary limit $z\to 0$ and integrating over t_1 and t_2 one arrives at the following result

$$G_{12}^{++} = \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|} \times \int_0^\infty d\tau \frac{\tau^{2s}(\tau P_1 + S_1)^{2s_1}(-\tau P_2 + S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,, \tag{4.25}$$

$$G_{12}^{--} = \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|} \times \int_0^\infty d\tau \frac{\tau^{2s}(\tau P_1 - S_1)^{2s_1}(-\tau P_2 - S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,, \tag{4.26}$$

$$G_{12}^{+-} = \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|} \times \int_0^\infty d\tau \frac{\tau^{2s}(\tau P_1 + S_1)^{2s_1}(-\tau P_2 - S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,, \tag{4.27}$$

$$G_{12}^{-+} = \frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|\mathrm{x}_{01}||\mathrm{x}_{02}||\mathrm{x}_{12}|} \times \int_0^\infty d\tau \frac{\tau^{2s}(\tau P_1 - S_1)^{2s_1}(-\tau P_2 + S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,, \tag{4.28}$$

where

$$K_{s_1s_2s} = \frac{2^{s-s_1-s_2}(s+s_1+s_2)!}{(2s)!(2s_1)!(2s_2)!}\,. \tag{4.29}$$

Note that for equal chirality signs, i.e. for G^{++} and G^{--}, the coefficient $K_{s_1s_2s}$ would be different should we still used (4.16) in this case as a Green's function and had the following form

$$K'_{s_1s_2s} = \frac{2^{s-s_1-s_2}(s+s_1-s_2)!(s+s_2-s_1)!}{(2s)!(2s_1)!(2s_2)!(s-s_1-s_2)!}\,. \tag{4.30}$$

Both coefficients $K_{s_1s_2s}$ and $K'_{s_1s_2s}$ coincide whenever one of two spins s_1 or s_2 is equal to zero.

To identify three-point correlation functions from (4.25)–(4.28) one uses prescription (4.17)–(4.19) and symmetrization over the sources at x_1 and x_2.

As noted above, $G^{++}+G^{--}$ and $G^{+-}+G^{--}$ correspond to the parity-preserving three-point functions. To verify these against free theory correlators let us start with

$$G^{+-}_{12}+G^{-+}_{12}=\frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}}{|x_{01}||x_{02}||x_{12}|}\times\int_{-\infty}^{\infty}d\tau\,\frac{\tau^{2s}(\tau P_1+S_1)^{2s_1}(\tau P_2+S_2)^{2s_2}}{(1+\tau^2)^{s+s_1+s_2+1}}\,. \quad (4.31)$$

Though (4.31)depends on the parity-odd structure S, the integration in (4.31) is carried out along the real axis and therefore they appear in bilinear combinations leading to a parity-even result. Since conformal structures (4.23)–(4.24) are not algebraically independent (see e.g., [29] for a list of identities on these structures), it is hard to identify in this expression the product of cosines and sines found in [15]. (The form of the final result is sensitive to a particular representation choice.) For a simple check showing that the result matches free theory correlators it is convenient to fix boundary points as follows

$$x_0=0\,,\quad x_1=x\,,\quad x_2=x-\delta\,,\quad |\delta|\ll|x| \quad (4.32)$$

and take equal polarization spinors

$$w_\alpha=i\mu_{1\alpha}=i\mu_{2\alpha}=\lambda_\alpha\,. \quad (4.33)$$

In addition it is convenient to require

$$\vec{x}\cdot\vec{\lambda}=0\,. \quad (4.34)$$

In this limit, which was also used in [14] and is similar to the light cone limit of [16], the 3pt correlators calculated in a free theory amount to [14]

$$\langle J_{s_1}(x,\lambda)J_{s_2}(x-\delta,\lambda)J_s(0,\lambda)\rangle_{\lambda\cdot x=0}\sim\frac{\Gamma(s_1+s_2+\frac12)\Gamma(s+\frac12)}{\pi s_1!s_2!s!}\frac{(\lambda\cdot\delta)^{s_1+s_2+s}}{|x|^{2s+2}|\delta|^{2s_1+2s_2+1}}\,. \quad (4.35)$$

To see what (4.31) gives in this limit one finds from (4.23)–(4.24)

$$P_1=0\,,\qquad P_2=\frac{\delta\cdot\lambda}{|x|^2}\,,\qquad Q=-\frac{\delta\cdot\lambda}{|x|^2}\,, \quad (4.36)$$

$$S_1=S_2=i\frac{(x\cdot\delta)_{\alpha\alpha}\lambda^\alpha\lambda^\alpha}{|x|^2|\delta|}\,. \quad (4.37)$$

Since $\delta\ll x$ we can neglect P_2 in (4.31) and therefore

$$G^{+-}_{12}+G^{-+}_{12}=\frac{z}{2}K_{s_1s_2s}\frac{Q^{s-s_1-s_2}S_1^{2s_1+2s_2}}{|x|^2|\delta|}\int_{-\infty}^{\infty}d\tau\,\frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}}\,. \quad (4.38)$$

Using Fierz (*i.e.*, Schouten) identities it is easy to see, that (note, that (4.25)–(4.28) do not apply for half-integer spins)

$$S_1^2=-\frac{(\delta\cdot\lambda)^2}{|x|^2|\delta|^2} \quad (4.39)$$

leading to

$$G^{+-}_{12}+G^{-+}_{12}\sim K_{s_1s_2s}\frac{(\lambda\cdot\delta)^{s_1+s_2+s}}{|x|^{2s+2}|\delta|^{2s_1+2s_2+1}}\int_{-\infty}^{\infty}d\tau\,\frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}}\,. \quad (4.40)$$

Integrating by residues,

$$\int_{-\infty}^{\infty}d\tau\,\frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}}=\frac{\Gamma(s+\frac12)\Gamma(s_1+s_2+\frac12)}{(s+s_1+s_2)!} \quad (4.41)$$

and substituting $K_{s_1s_2s}$ one finds

$$G^{+-}_{12}+G^{-+}_{12}\sim\frac{\Gamma(s+\frac12)\Gamma(s_1+s_2+\frac12)}{(2s)!(2s_1)!(2s_2)!}\frac{(\lambda\cdot\delta)^{s_1+s_2+s}}{|x|^{2s+2}|\delta|^{2s_1+2s_2+1}}\,, \quad (4.42)$$

which is consistent with the free theory prediction (4.35) upon an appropriate 2pt-normalization. Same is true for $G^{++}_{12}+G^{--}_{12}$.

The parity-odd contribution resides in $G^{++}_{12}-G^{--}_{12}$ and the corresponding three-point function can be obtained from that expression by symmetrizing sources at x_1 and x_2. Up to the two-point function normalization $\langle J_sJ_s\rangle$ the final result reads

$$\begin{aligned}&\langle J_{s_1}J_{s_2}J_s\rangle_{odd} \quad (4.43)\\&\sim\frac12\frac{K_{s_1s_2s}}{|x_{01}||x_{02}||x_{12}|}\int_0^{\infty}d\tau\,\frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}}Q^{s-s_1-s_2}\times\\&\times\Big((\tau P_1+S_1)^{2s_1}(-\tau P_2+S_2)^{2s_2}\\&-(\tau P_1-S_1)^{2s_1}(-\tau P_2-S_2)^{2s_2}\Big)\\&+(x_1,\mu_1,s_1)\leftrightarrow(x_2,\mu_2,s_2)\,.\end{aligned}$$

Recall that spins are restricted by (4.1). To see that the result is nonzero it is enough to consider the case of $s_1=1$, $s_2=0$ which gives

$$\langle O_{\Delta=1}(x_2)J_1(x_1)J_s(x_0)\rangle\sim\frac{s!}{(2s)!}\frac{2^{s-2}}{|x_{01}||x_{02}||x_{12}|}(Q^{s-1}+(-Q)^{s-1})P_1S_1\,. \quad (4.44)$$

Similarly, using (3.20) propagator one can calculate correlation functions corresponding to critical models. We do not perform this calculation in our paper. Note that from the boundary side non-conservation of HS currents in the parity-broken case was recently studied in [28], where some correlators were explicitly found. We have checked (4.43) against those few examples of correlation functions for quasi-bosonic scalar given in the appendix of [28] and find them match. For example, using (4.43) one can easily calculate

$$\langle J_4J_0J_6\rangle\sim\bar{Q}_3^2\bar{S}_2(107\bar{P}_2^4\bar{Q}_1\bar{Q}_3+40\bar{P}_2^2\bar{Q}_1^2\bar{Q}_3^2+102\bar{P}_2^6+3\bar{Q}_1^3\bar{Q}_3^3)\,, \quad (4.45)$$

which agrees with [28], where barred conformal structures from [28] are related to (4.23)–(4.24) as follows

$$\bar{Q}_3=Q\,,\quad\bar{S}_2=P_2S_2\,,\quad\bar{Q}_1\bar{Q}_3=P_2^2+S_2^2\,. \quad (4.46)$$

Let us also note that calculation of the parity odd structure using (4.16) for equal chiralities leads to different spin dependent coefficient in (4.43). The coefficient would be given by (4.30). Interestingly, it coincides with (4.29) for $s_1=0$ or $s_2=0$.

As stressed earlier, the form of the final result (4.43) depends on the freedom in using relations on conformal structures (4.23)–(4.24). We expect (4.43) to admit a simpler representation. In this respect it is interesting to note that typical integrals that show up in the boundary limit of a Green's function

$$G(C_{s_1},C_{s_2})\sim\int_0^{\infty}d\tau\,\frac{\tau^{2s}}{(1+\tau^2)^{s+s_1+s_2+1}}(\tau a+b)^{2s_1}(\tau c+d)^{2s_2}\,, \quad (4.47)$$

where a, b, c, d are some conformal structures among list (4.23)–(4.24), can be rewritten upon the change of integration variable $\tau = \tan\phi$ as

$$R \int_0^{\pi/2} d\phi \, \sin^{2s}\phi \, \sin^{2s_1}(\phi+\phi_1) \sin^{2s_2}(\phi+\phi_2) \,, \tag{4.48}$$

where

$$R = (a^2+b^2)^{s_1}(c^2+d^2)^{s_2} \,, \quad \tan\phi_1 = \frac{b}{a} \,, \quad \tan\phi_2 = \frac{d}{c} \,. \tag{4.49}$$

This representation may be useful for finding a simpler representation for the parity-odd three-point functions.

5. Conclusion

We have examined local form of HS equations to the second order at the level of equations of motion in its most sensitive part of the current interaction sector with spins obeying $s \geq s_1 + s_2$, *i.e.*, outside the triangle inequality region. We have checked whether the coefficients obtained in [1] and [26] are consistent with the boundary theory expectations and found perfect agreement. Particularly, the boundary limit that describes deformation to current conservation condition is consistent with the requirement for free theories to have exactly conserved HS currents. For these boundary conditions we have also checked that, in agreement with the conventional AdS/CFT prescription, no HS connections survive at the boundary. These facts, being crucially dependent on the structure of vertices obtained in [1], confirm that the prescription of [1] is the only proper one. Still we have carried out some calculation at the level of three point functions extracted from the 0-form sector *a la* Giombi and Yin [14]. Though details of the prescription of extracting correlators from the 0-form sector of HS equations is not entirely clear to us and perhaps needs some further analysis (particularly, this concerns the argument on the linear relation between the Weyl module and HS connections) we found perfect agreement in case of free theories. For parity broken case we have calculated correlation functions $\langle J_{s_1} J_{s_2} J_{s_3}\rangle_{odd}$ for $s_3 \geq s_1 + s_2$ using the same approach. The result is nonzero which seemingly contradicts to the analysis of [16] where parity-odd three-point functions were found within the triangle identity $s_i \leq s_j + s_k$ and it was claimed that for $s_3 \geq s_1 + s_2$ the result is zero. However, it is important to note, that in paper [16] all HS currents were supposed to be conserved, while in our case we do not have current conservation for parity-odd case which is in agreement with general analysis of [29].

Another observation of our work highlighting the conjecture of [25] on the role of the boundary conditions is that, apart from the case of free boundary theories, no boundary conditions linear in the HS 0-forms make the sources to the boundary HS connection vanish. Particularly, for critical models and parity-broken models a nonlinear correction to the source for boundary connections always springs out. This implies that boundary conditions consistent with the standard AdS/CFT prescription may need a nonlinear deformation anticipated to become important starting from the 4pt correlation functions. This deformation is similar to the one observed recently in $\mathcal{N} = 8$ supergravity theory which requires boundary supersymmetry modification in order to match superconformal correlation functions [30].

Finally, the analysis carried out in this paper suggests that even in the purely bosonic case there exist perturbatively different reductions of the full nonlinear HS equations with the doubled set of fields compared to the naive reduction with a single set of bosonic fields of any spin.

6. Note added

After completion of our work we learned that closely related problem was considered in [31] by E. Sezgin, E.D. Skvortsov and Y. Zhu.

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References

[1] M.A. Vasiliev, Current interactions and holography from the 0-form sector of nonlinear higher-spin equations, arXiv:1605.02662 [hep-th].
[2] M.A. Vasiliev, in: M.A. Shifman (Ed.), The Many Faces of the Superworld, pp. 533–610, arXiv:hep-th/9910096.
[3] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, Int. J. Theor. Phys. 38 (1999) 1113, arXiv:hep-th/9711200.
[4] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105, arXiv:hep-th/9802109.
[5] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, arXiv:hep-th/9802150.
[6] B. Sundborg, Nucl. Phys. Proc. Suppl. 102 (2001) 113, arXiv:hep-th/0103247.
[7] E. Witten, Talk at the John Schwarz 60-th birthday symposium, http://theory.caltech.edu/jhs60/witten/1.html.
[8] S.E. Konstein, M.A. Vasiliev, V.N. Zaikin, J. High Energy Phys. 0012 (2000) 018, arXiv:hep-th/0010239.
[9] A. Mikhailov, arXiv:hep-th/0201019.
[10] E. Sezgin, P. Sundell, Nucl. Phys. B 644 (2002) 303;
E. Sezgin, P. Sundell, Nucl. Phys. B 660 (2003) 403 (Erratum), arXiv:hep-th/0205131.
[11] I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 550 (2002) 213, arXiv:hep-th/0210114.
[12] R.G. Leigh, A.C. Petkou, J. High Energy Phys. 0306 (2003) 011, arXiv:hep-th/0304217.
[13] E. Sezgin, P. Sundell, J. High Energy Phys. 0507 (2005) 044, arXiv:hep-th/0305040.
[14] S. Giombi, X. Yin, J. High Energy Phys. 1009 (2010) 115, arXiv:0912.3462 [hep-th].
[15] S. Giombi, X. Yin, J. High Energy Phys. 1104 (2011) 086, arXiv:1004.3736 [hep-th].
[16] J. Maldacena, A. Zhiboedov, J. Phys. A 46 (2013) 214011, arXiv:1112.1016 [hep-th].
[17] J. Maldacena, A. Zhiboedov, Class. Quantum Gravity 30 (2013) 104003, arXiv:1204.3882 [hep-th].
[18] M.R. Gaberdiel, R. Gopakumar, Phys. Rev. D 83 (2011) 066007, arXiv:1011.2986 [hep-th].
[19] O. Aharony, G. Gur-Ari, R. Yacoby, J. High Energy Phys. 1203 (2012) 037, arXiv:1110.4382 [hep-th].
[20] S. Giombi, S. Minwalla, S. Prakash, S.P. Trivedi, S.R. Wadia, X. Yin, Eur. Phys. J. A 72 (2012) 2112, arXiv:1110.4386 [hep-th].
[21] S. Giombi, X. Yin, J. Phys. A 46 (2013) 214003, arXiv:1208.4036 [hep-th].
[22] M.A. Vasiliev, J. High Energy Phys. 1506 (2015) 031, arXiv:1502.02271 [hep-th].
[23] M.A. Vasiliev, On the local frame in nonlinear higher-spin equations, arXiv:1707.03735 [hep-th].
[24] M.A. Vasiliev, Phys. Lett. B 285 (1992) 225.
[25] M.A. Vasiliev, J. Phys. A 46 (2013) 214013, arXiv:1203.5554 [hep-th].
[26] O.A. Gelfond, M.A. Vasiliev, Current interactions from the one-form sector of nonlinear higher-spin equations, arXiv:1706.03718 [hep-th].
[27] L. Bonora, M. Cvitan, P. Dominis Prester, B. Lima de Souza, I. Smolic, J. High Energy Phys. 1605 (2016) 072, arXiv:1602.07178 [hep-th].
[28] S. Giombi, V. Gurucharan, V. Kirilin, S. Prakash, E. Skvortsov, J. High Energy Phys. 1701 (2017) 058, arXiv:1610.08472 [hep-th].

[29] S. Giombi, S. Prakash, X. Yin, J. High Energy Phys. 1307 (2013) 105, arXiv:1104.4317 [hep-th].
[30] D.Z. Freedman, K. Pilch, S.S. Pufu, N.P. Warner, J. High Energy Phys. 1706 (2017) 053, arXiv:1611.01888 [hep-th].
[31] E. Sezgin, E.D. Skvortsov, Y. Zhu, J. High Energy Phys. 1707 (2017) 133, arXiv:1705.03197 [hep-th].