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Stealth field in FLRW spacetime with non minimal derivative coupling

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Abstract. In this work, we show a solution for the stealth scalar field arising from a non-minimal derivative coupling in a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime coupled to the simplest case of a perfect fluid, namely, dust.

1. Introduction

Recently in modified theories of gravitation, particularly in special cases of Hordenski theory it has been studied that these have implications on cosmological models in order to provide late mechanisms of self tuning [1]. As it was reported in [1] the theory rised by the Lagrangians L_{jhon} and L_{paul} provide a self tuning mechanism, being these the strongest terms of the so-called four Fabs. Within the context of the modified theories and the Hordenski one, there are interesting solutions for scalar fields that emerge from theories with non minimal coupling between the scalar fields and gravity, named *stealths*. The main feature of the stealth is that they do not backreact on gravitational fields, so its existence leave intact the evolution of underlying spacetime. As is well mentioned in [2], stealths may be perceived as a mathematical curiosity, however, their behavior could alleviate the cosmological constant problem: basically the predictions about the energy density of the quantum vacuum is approximately 120 orders of magnitude larger than the measured cosmological energy density. Also in the same reference, it was analyzed the stability of solutions finding three stables cases against five unstable ones and showing the sensibility of the integration constants and the coupling parameter in the analyzed cases.

The stealth fields are being studied in different contexts. For instance, in vector-tensor theories [3, 4], three dimensional black-holes [5, 6], Galileons [7]. The existence of these configurations have been found in [6] for the well known $(2+1)$-dimensional BTZ static black hole and later on in $(3+1)$-dimensions, for the Minkowski flat spacetime [8], in (A)dS spacetime [9]; in the cosmology context [10, 11, 12, 13], and on a charged dilatonic $(1+1)$-D black hole [14]. In a similar spirit, in n dimensions, for higher-order gravity theories [15, 16], in the Lovelock gravity context [17] as well as in the non-relativistic version of the gauge/gravity correspondence in Lifshitz spacetime [18].

In this work, we study the term L_{john} non minimally coupled to the scalar field in order to find solutions for stealth fields in a Friedmann-Lemaître-Robertson-Walker spacetime in presence of sources, choosing as a source the simplest case of a perfect fluid, namely, dust. That is consistent with the scenario described in [19] about the Universe modeled as a fluid and its evolution going over different stages. From recent observations, it is known that ordinary matter accounts for $4\% \pm 1$ of the total mass/energy content, with dark matter and dark energy accounting 96%; and its expansion is accelerating and not decelerating. So in a good approximation, the stress energy-momentum tensor in the present Universe is described by the dust energy-momentum tensor.

2. Field equations and stealth configurations

As is well known, the cosmological principle suggest a homogeneous and isotropic Universe and its evolution is given by the cosmological standard model. A spacetime which agrees with this principle is the line element Friedmann-Lemaître-Robertson-

Walker;

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sen^2\theta d\varphi^2)\right]. \quad (1)$$

where $a(t)$ is the scale factor, and the constant k determines the topology of the Universe, for $k = 0$ flat, $k = -1$ open and $k = 1$ closed Universe. In the dust case $p = 0$, the energy density evolves as $\rho = \rho_0 a^{-3}$ and its scale factor has the functional form $a(t) = a_0 t^{2/3}$.

Stealth configurations have been found when one considers some coupling between a scalar field and gravity, which is described by the action

$$S[g_{\mu\nu}, \phi] = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}R + \frac{1}{2}(\eta G_{\mu\nu} - g_{\mu\nu})\nabla^\mu\phi\nabla^\nu\phi - V(\phi)\right) \quad (2)$$

where R is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor, $\phi(t)$ is a scalar field, $V(\phi)$ is the scalar field's potential, and η is the coupling parameter, from this action the term L_{john} is easily recognized.

After performing the variation with respect to metric, the field equations can be rewritten as,

$$G_{\mu\nu} - \kappa T^m_{\mu\nu} = \kappa T^s_{\mu\nu}, \quad (3)$$

where $T^m_{\mu\nu}$ and $T^s_{\mu\nu}$ are the energy-momentum tensor of matter and stealth, respectively.

Explicitly, the stealth stress-energy tensor for the non minimal derivative coupling

$$\begin{aligned} T^s_{\mu\nu} &= \nabla_\mu\phi\nabla_\nu\phi - \left[\frac{1}{2}\nabla_\lambda\phi\nabla^\lambda\phi + V(\phi)\right]g_{\mu\nu} + \eta\Bigg\{\frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi R \\ &- 2\nabla_\lambda\phi\nabla_{(\mu}\phi R^\lambda_{\nu)} - \nabla^\lambda\phi\nabla^\rho\phi R_{\mu\lambda\nu\rho} - (\nabla_\mu\nabla^\lambda\phi)(\nabla_\nu\nabla_\lambda\phi) \\ &+ (\nabla_\mu\nabla_\nu\phi)\Box\phi + \frac{1}{2}G_{\mu\nu}(\nabla\phi)^2 - g_{\mu\nu}\Bigg[-\frac{1}{2}(\nabla^\lambda\nabla^\rho\phi)(\nabla_\lambda\nabla_\rho\phi) \\ &+ \frac{1}{2}(\Box\phi)^2 - \nabla_\lambda\phi\nabla_\rho\phi R^{\lambda\rho}\Bigg]\Bigg\} = 0, \end{aligned} \quad (4)$$

the stealth solutions are found imposing the vanishing of the above expression. By inserting (1) into (4), one finds that the system of equations which leaves (4) are just two linearly independent components, namely

$$T_t^{\ t} = \frac{1}{2}(9\eta H^2 - 1)\left(\frac{d\phi}{dt}\right)^2 - V(\phi), \quad (5)$$

$$T_r^{\ r} = \frac{1}{2}[\eta(2H' + 3H^2) + 1]\left(\frac{d\phi}{dt}\right)^2 + \eta H\frac{d}{dt}\left(\frac{d\phi}{dt}\right)^2 - V(\phi), \quad (6)$$

here $H = \frac{a'}{a}$ is the Hubble parameter and H' its temporal derivate.

The problem now is to solve the system of equations given by (5) and (6) for ϕ and $V(\phi)$, so combining both components we have the equation for the scalar field

$$\frac{1}{\eta H}(T_r^{\ r} - T_t^{\ t}) = \frac{d}{dt}\left(\frac{d\phi}{dt}\right)^2 + \left(\frac{\eta H' - 3\eta H^2 + 1}{\eta H}\right)\left(\frac{d\phi}{dt}\right)^2 = 0, \quad (7)$$

which integrated for $(\frac{d\phi}{dt})^2$ gives the solution

$$\left(\frac{d\phi(t)}{dt}\right)^2 = \int \exp\left(-\frac{\eta H' - 3\eta H^2 + 1}{\eta H}\right) dt = \phi_0 t^3 \exp\left(-\frac{3t^2}{4\eta}\right) \tag{8}$$

where $\phi_0 > 0$ is an integration constant. Integrating the above equation we get;

$$\phi(t) = -\frac{2}{9}\eta(3t^2 + 4\eta)\exp(-\frac{3}{4\eta}t^2) + \frac{8}{9}\eta^2. \tag{9}$$

Now, in order to find the self-interacting potential for the scalar field, we consider the combination of components

$$T_r{}^r + T_t{}^t = \eta H \frac{d}{dt}\left(\frac{d\phi}{dt}\right)^2 + \eta\left(H' + 6H^2\right)\left(\frac{d\phi}{dt}\right)^2 - 2V(\phi) = 0, \tag{10}$$

from it, we isolate the potential $V(\phi)$. After the substitution of (8) into (10) one obtains

$$V(\phi) = \frac{1}{2}(9\eta H^2 - 1)\left(\frac{d\phi}{dt}\right)^2 = \frac{1}{2}\left(\frac{4\eta}{t^2} - 1\right)\phi_0 t^3 \exp\left(-\frac{3t^2}{4\eta}\right), \tag{11}$$

and it is worth mentioning that this coincides with the component $T_t{}^t$.

Now in this case it is possible to write the self-interaction potential just in terms of ϕ, that is possible by inverting the Eq.(9), i.e., taking the inverse function in order to get $t = t(\phi)$:

$$t = -\frac{2}{9}\sqrt{-3\eta}\sqrt{LambertW(\hat{\phi}) + 1}, \tag{12}$$

here $\hat{\phi} = (\frac{9}{8\eta^2}\phi - 1)e^{-1}$. Hence, the selft-interaction potential finally is written as

$$V(\phi) = \frac{\phi_0\sqrt{-3\eta}}{18\eta}\frac{(LambertW(\hat{\phi}) + 4)\sqrt{LambertW(\hat{\phi}) + 1}}{LambertW(\hat{\phi})}(8\eta^2 - 9\phi), \tag{13}$$

there are two points to discuss from the potential (13), first, in order to avoid complex values for the potential, η must be in the range $\eta \in (-\infty, 0)$, at second point is the LambertW‡ function; the principal branch is analytic at zero and it has real-values for the range $\phi \in [-e^{-1}, \infty)$, that is our case, so the potential is real and well behaved in the range $V(\phi) \in [0, \infty)$.

In the figure (1) we show the graph of $V(\phi)$ vs. ϕ, in order to show the behavior of the potential, actually, it is not so sensitive to the values of parameters, so that for any values of these, the graph will be similar.

‡ The LambertW function, satisfies $LambertW(x)e^{LambertW(x)=x}$, as the equation $ye^y = x$ has an infinite number of solutions y for each (non-zero) value of x, so has an infinity number of branches, but just one analytic at 0.

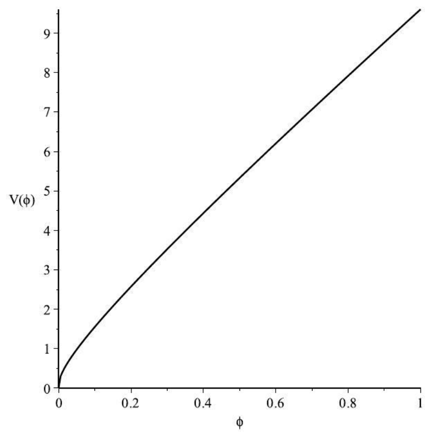

Figure 1. Self-interaction potential $V(\phi)$, for $\phi_0 = 1$ and $\eta = -1/4$.

3. Conclusions

The aim of the present work was to show the existence of solutions in theories with non-minimal derivative coupling– motivated by the self-tuning of this coupling and the behavior of stealth fields–, in order to study its possible cosmological implications. In particular, we studied the simplest case of a perfect fluid (dust) in an homogeneous and isotropic cosmology described by the FLRW spacetime. All the development shown the decoupling of the field equations to get the explicit integral of stealth field, and its self-interaction potential.

On the other hand, in the looking for stealth solutions, one can follow different philosophies, fixing the self-interaction potential, as in [12], and decoupling the equations to find the field, or as in [19], fixing the cosmology to find the field an its potential as is our case. When the cosmology was appointed, the most often is founding dissipative potentials and in order to get a potential just in terms of the scalar field one use the inverse function of the implicit variable to obtain it in terms of the scalar field.

Although the approach used in the present work helped us to found stealth solutions, it is not direct to obtain the self-tunning mechanism. Instead, there is an approach in which the stealth solutions and self-tuning mechanism are obtained directly [20, 21]. Unfortunately, the recent observations discard this particular coupling term [22], however, the study of all the approaches emerging from it could be interesting.

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