Fermi National Accelerator Laboratory

FERMILAB-Pub-95/395-E
E665

Determination of the Gluon Distribution Function of the Nucleon Using Energy-Energy Angular Pattern in Deep-Inelastic Muon-Deuteron Scattering

M.R. Adams et al.
The E665 Collaboration

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

December 1995

Determination of the gluon distribution function of the nucleon using energy-energy angular pattern in deep-inelastic muon-deuteron scattering

(E665 Collaboration)

Abstract

We have used the energy-energy angular pattern of hadrons in inelastic muon-deuteron scattering to study perturbative QCD effects and to extract the gluon distribution function $\eta G(\eta)$ of the nucleon, where η is the fractional momentum carried by the gluon. The data were taken with the E665 spectrometer using the Fermilab Tevatron muon beam with a mean beam energy of 490 GeV. We present $\eta G(\eta)$ for $0.005 < \eta < 0.05$ and at an average Q^2 of 8 GeV^2 using this new technique. We find that $\eta G(\eta)$ in this region can be described by $\eta G(\eta) \propto \eta^{\lambda}$ with $\lambda = -0.87 \pm 0.09(stat.) \pm^{0.32}_{0.37}(sys.)$. We compare our results to expectations from various parametrizations of the parton distribution function and also to results from HERA.

M.R. Adams [6], S. Aïd [10,a], P.L. Anthony [11,b], M.D. Baker [11], J.Bartlett [4], A. A. Bhatti [13,c], H.M. Braun [14], W. Busza [11], T.J. Carroll [12], J.M. Conrad [5,d], G. Coutrakon [4,e], R. Davisson [14], I. Derado [12], S.K. Dhawan [15], W. Dougherty [14], T. Dreyer [1], K. Dziunikowska [9], V. Eckardt [12], U. Ecker [15,f], M. Erdmann [1,g], A. Eskreys [8], J. Figiel [8], H.J. Gebauer [12], D.F. Geesaman [2], R. Gilman [2,h], M. C. Green [2,i], J. Haas [1], C. Halliwell [6], J. Hanlon [4], D. Hantke [12,j], V.W. Hughes [16], H.E. Jackson [2], G. Jancso [7,12], D.M. Jansen [15,k], S. Kaufman [2], R.D. Kennedy [3], T. Kirk [4,l], H.G.E. Kobrak [3], S. Krzywdzinski [4], S. Kunori [10], J.J. Lord [14], H.J. Lubatti [14], D. McLeod [6], S. Magill [6,m], P. Malecki [8], A. Manz [12], H. Melanson [4], D. G. Michael [5,n], W. Mohr [1], H.E. Montgomery [4], J.G. Morfin [4], R.B. Nickerson [5,o], S. O'Day [10,p], K. Olkiewicz [8], L. Osborne [11], V. Papavassiliou [16,q], B. Pawlik [8], F.M. Pipkin [5,*], E.J. Ramberg [10,p], A. Röser [15,r], J.J. Ryan [11], C.W. Salgado [4,s], A. Salvarani [3,t], H. Schellman [13], M. Schmitt [5,u], N. Schmitz [12], K.P. Schüler [16,v], H.J. Seyerlein [12], A. Skuja [10], G.A. Snow [10], S. Söldner-Rembold [12,w], P.H. Steinberg [10,*], H.E. Stier [1,*], P. Stopa [8], R.A. Swanson [3], R. Talaga [10,m], S. Tentindo-Repond [2,x], H.-J. Trost [2,y], H. Venkataramania [16,z], M. Wilhelm [1,aa], J. Wilkes [14], Richard Wilson [5], W. Wittek [12], S.A. Wolbers [4], T. Zhao [14]

(E665 Collaboration)

[1] *Albert-Ludwigs-Universität Freiburg i. Br., Germany*

[2] *Argonne National Laboratory, Argonne, IL USA*

[3] *University of California, San Diego, CA USA*

[4] *Fermi National Accelerator Laboratory, Batavia, IL USA*

[5] *Harvard University, Cambridge, MA USA*

[6] *University of Illinois, Chicago, IL USA*

[7] *KFKI Research Institute for Particle and Nuclear Physics, H-1525 Budapest, Hungary*

[8] *Institute for Nuclear Physics, Krakow, Poland*

[9] *Institute for Nuclear Physics, Academy of Mining and Metallurgy, Krakow, Poland*

[10] *University of Maryland, College Park, MD USA*

[11] *Massachusetts Institute of Technology, Cambridge, MA USA*

[12] *Max-Planck-Institut für Physik, Munich, Germany*

[13] *Northwestern University, Evanston, IL USA*

[14] *University of Washington, Seattle, WA USA*

[15] *University of Wuppertal, Wuppertal, Germany*

[16] *Yale University, New Haven, CT USA*

Present addresses:
[a] University of Hamburg, D-22603 Hamburg, Germany.
[b] SLAC, Stanford, CA 94309, USA.
[c] The Rockefeller University, New York NY 10021, USA.
[d] Columbia University, New York, NY 10027.
[e] Loma Linda University Medical Center, Loma Linda CA 92350, USA.
[f] Jenfelderstr. 147, D-22045 Hamburg, Germany.
[g] Heidelberg University, D-69120, Heidelberg Germany.
[h] Rutgers University, Piscataway, NJ 08855, USA.
[i] LeCroy Research Systems, Spring Valley, NY, USA.
[j] 1097 GSF - Forschungszentrum für Umwelt und Gesundheit GmbH, 85764 Oberschleißheim, Germany.
[k] Max-Planck Institut für Kernphysik, 69029 Heidelberg, Germany.
[l] Brookhaven National Laboratory, Upton, NY 11973, USA.
[m] Argonne National Laboratory, Argonne, IL 60439, USA.
[n] California Institute of Technology, Pasadena, CA 91125, USA.
[o] Oxford University, Oxford OX1 3RH, UK.
[p] Fermi National Accelerator Laboratory, Batavia, IL 60510, USA.
[q] New Mexico State University, Las Cruces NM 88003, USA.
[r] Klinikum Barmen, Abt. Radiologie, D-42283 Wuppertal, Germany.
[s] CEBAF, Newport News, VA 23606, USA.
[t] A.T. & T., Bell Labs, 2000 North Naperville Road, Naperville, IL USA.
[u] University of Wisconsin, Madison, WI 53706, USA.
[v]DESY, D-22603 Hamburg, Germany.
[w] Albert-Ludwigs-Universität Freiburg, 79104 Freiburg, Germany.
[x] Northern Illinois University, Dekalb, IL 60115, USA.
[y] Texas A&M University, College Station, 77843, USA.
[z] Northwestern University, Evanston, IL 60208, USA.
[aa] Hoffmann-LaRoche, CH-4002 Basel, Switzerland.
∗ Deceased

1 Introduction

While the quark content of nucleons has been measured very precisely in deep-inelastic scattering [1], the gluon content is much less well known, especially at low values of the momentum fraction η. At small η the gluon is believed to be the dominant parton, but because it carries no electromagnetic charge, it contributes to deep-inelastic scattering only through higher order processes.

Both conventional leading log QCD calculations [2] and the improved BFKL [3] calculations which also resum $\alpha_s \log(1/\eta)$ contributions predict a singular rise in the gluon momentum distribution $\eta G(\eta)$ as η goes to zero. The BFKL prediction yields $\eta G(\eta) \propto \eta^{-\frac{1}{2}}$. In contrast conventional Regge expectations are that $\eta\mathrm{G}(\eta)$ should go as $\eta^{-0.08}$ [4].

In this paper we have used properties of final state hadrons produced in muon-nucleon deep inelastic scattering to extract $\eta G(\eta)$ for small values of η, applying the QCD formalism developed by Peccei and Rückl [5]. According to QCD, properties of final state hadrons are affected by parton distribution functions, by α_s and by fragmentation. In figure 1 we show the Feynman diagrams considered by Peccei and Rückl [5] in their calculation of the properties of hadrons in the final state produced in muon-nucleon deep inelastic scattering. At the energies considered, the deep-inelastic muon-nucleon scattering process is dominated by single virtual photon (γ^*) exchange. We shall denote by l, q and P, the 4-momentum of the initial lepton, the virtual photon and the target nucleon(N) respectively, and shall consider the scaling variables

$$x = \frac{q^2}{-2P\cdot q}, y_{Bj} = \frac{P\cdot q}{P\cdot l}, z_i = \frac{P\cdot p_i}{P\cdot q} \tag{1}$$

where p_i is the 4-momentum of an outgoing hadron and q^2 is the space-like four-momentum transfer squared. The diagrams c) and d) produce two forward going jets in the final state, where 'forward' is defined by the direction of the virtual photon in the overall hadronic center-of-mass system. Because of this, events produced by hard QCD radiation processes are characterized by a broader angular spread of hadrons than those produced by simply knocking out a quark from the nucleon. Note that the gluons in the nucleon take part in the interaction via diagram d) - the photon-gluon fusion diagram - and this is why properties of the final state hadrons are affected by $\eta G(\eta)$. The presence of such processes in particular results in larger values for $< sin^2\theta >$ and $< sin^2\chi >$ whose definitions are given below:

For each event we define:

- Width of angular energy flow, which is,

$$sin^2\theta = \sum_{i=1}^{N_{trk}} z_i^{res.} sin^2\theta_i \tag{2}$$

- Width of the energy-energy angular pattern:

$$sin^2\chi = \sum_{i=1}^{N_{trk}} \sum_{j=1}^{N_{trk}} z_i^{res.} z_j^{res.} sin^2\chi_{ij} \tag{3}$$

where N_{trk} is the number of charged tracks in the event, θ_i is the angle between track i and γ^*, χ_{ij} is the angle between tracks i and j and $z_i^{res.} = z_i/(\sum_{j=1}^{N_{trk}} z_j)$ i.e. $z_i^{res.}$ is obtained by rescaling z_i (motivation for using $z_i^{res.}$ is given below). In evaluating the sums only particles with θ between 0 and $\pi/2$ are considered. All the quantities are calculated in the γ^* N center-of-mass frame, assuming the target nucleon N in the deuteron to be at rest in the laboratory system. $< sin^2\theta >$ and $< sin^2\chi >$ denote the averages over events of $sin^2\theta$ and $sin^2\chi$ respectively.

Peccei and Rückl [5] use the variable z_i instead of the variable $z_i^{res.}$ in the definitions of $sin^2\theta$ and $sin^2\chi$. For a perfect detector z_i would be equal to $z_i^{res.}$. In this analysis the acceptance for forward going particles was less than 100% mainly because we did not include neutrals in our analysis. We have found that using $z_i^{res.}$ rather than z_i reduces event-to-event fluctuations. The effect of using $z_i^{res.}$ instead of z_i is included in the Monte Carlo simulation.

In order to extract $\eta G(\eta)$ in the region of small η which in our case is $0.005 < \eta < 0.05$ we make use of the fact that the values of $< sin^2\theta >$ and $< sin^2\chi >$ depend on the value of $\eta G(\eta)$ in this kinematical region. For the determination of $\eta G(\eta)$ we have chosen to use the variable $< sin^2\chi >$ rather than $< sin^2\theta >$, because as can be seen from equations (2,3), the virtual photon momentum vector does not enter directly in the calculation of $< sin^2\chi >$; $< sin^2\chi >$ is independent of the primordial transverse momentum k_T of the partons in the nucleon. For the same reason $< sin^2\chi >$ is also less affected by the QED μ-bremsstrahlung process, which leads to an experimental smearing of the virtual-photon direction. Although $< sin^2\theta >$ is not used in the determination of $\eta G(\eta)$, Monte Carlo - data comparisons are also shown for this variable in some cases.

In the experimental determination of $\eta G(\eta)$ we compare the value of $< sin^2\chi >$ as measured for the data with that for Monte Carlo events generated using different gluon distribution functions. This gives us a map connecting $< sin^2\chi >$ and the parameters in the expression for $\eta G(\eta)$. The Monte Carlo program uses the physics of reference [5] to simulate the interaction of the virtual photon with the nucleon at the parton level. The details of the Monte Carlo program are discussed in section 5.

The variable η is related but not equal to the variable x. η stands for the fractional momentum of the nucleon carried by the gluon while x reflects the kinematics of the photon-quark vertex. Since in photon-gluon fusion the photon interacts with a quark with momentum less than the parent gluon momentum, $\eta \geq x$. The relationship between x and η can be expressed more quantitatively using a Monte Carlo simulation. A Monte Carlo simulation of the photon-gluon fusion process with reasonable parton distribution functions and a reasonable value for the QCD Λ parameter shows that for $0.005 < x < 0.015$ $(0.015 < x < 0.05)$, on average, η is larger than x by 0.007(0.013).

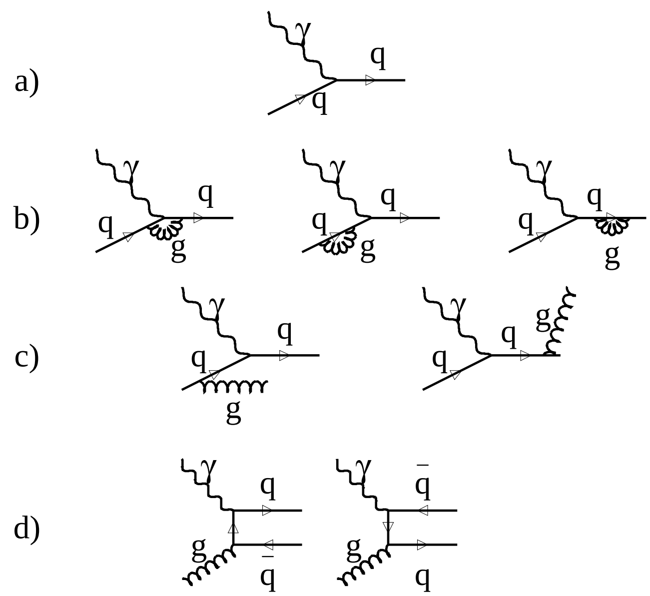

Figure 1: Feynman diagrams considered in the calculation performed by Peccei and Rückl. Diagrams c) and d) lead to a broader angular spread of hadrons in the final state because of the presence of more than one jet in the forward direction.

Both the energy flow and the energy-energy pattern are infrared safe quantities, i.e. there are no divergences associated with soft and collinear partons [5]. Therefore quantities based on the energy flow and the energy-energy pattern such as $< sin^2\theta >$ and $< sin^2\chi >$ should also be infrared safe. Both these quantities, apart from being affected by hard QCD processes, also get significant contributions from the fragmentation process.

At the center-of-mass energies(W) considered in this analysis the energy-energy angular pattern function is quite sensitive to fragmentation while also being sensitive to parton distribution functions and α_s. Since the QCD Λ parameter and the parton distribution functions for $x > 0.05$ are fairly well known [6-13], we will determine the transverse momentum due to fragmentation($\sigma_{frag.P_T}$) such that the width of the energy-energy angular pattern function in the Monte Carlo simulation agrees with the data for $x > 0.05$. The Monte Carlo simulation is based on the LUND event generators(LEPTO4.3 [14],JETSET4.3 [15]), in which Λ is set to its currently best value and in which standard sets of parton distribution functions are used. In order to extract $\eta G(\eta)$ for $\eta < 0.05$ using the angular width of the energy-energy angular pattern we take α_s, the quark distribution functions and $\eta G(\eta)$ for $\eta > 0.05$ as known quantities and we take $\sigma_{frag.P_T}$ as determined from the data at $x > 0.05$.

2 Theoretical calculation

Peccei and Rückl [5] using perturbative QCD have calculated angular widths for the energy flow and the energy-energy angular pattern functions at the parton level. The angular widths of these functions will clearly depend on parton distribution functions and α_s. They have also estimated the size of contributions to these quantities from the fragmentation process. They find that at our energies this contribution is sizable. The QCD matrix elements calculation in the Monte Carlo program(described below) used in the analysis is also based on ref. [5].

Peccei and Rückl [5] give an explicit formula for the normalized width of the energy-energy angular pattern function ($< sin^2\chi >$) for neutrino deep inelastic scattering using perturbative QCD to order $O(\alpha_s)$. We have modified their formula to describe muon deep inelastic scattering. The formula is:

$$< sin^2\chi >= \frac{\int d\Omega_1 d\Omega_2 sin^2\chi(\frac{d\sigma}{d\Omega_1 d\Omega_2 dxdy})}{2\pi \int_0^1 dcos\theta(\frac{d\sigma}{dcos\theta dxdy})} \tag{4}$$

where χ is the angle between two detectors that subtend solid angles of $d\Omega_1$ and $d\Omega_2$ respectively, $\frac{d\sigma}{d\Omega_1 d\Omega_2 dxdy}$ is the energy-energy angular pattern function and $\frac{d\sigma}{dcos\theta dxdy}$ is the azimuthally averaged energy flow function.

$$(\frac{d\sigma}{dcos\theta dxdy})_{\theta\neq 0} =$$

$$\begin{aligned}
\left(\frac{8\pi\alpha_{em}^2}{q^2 y}\right)\frac{2\alpha_s(q^2)}{3\pi}\int_0^1 d\eta\{[F_q(\xi;q^2)+(1-y)^2 F_{\overline{q}}(\xi;q^2)] \times [(1+x_p^2)A(\eta,cos\theta)-3x_p^2 B(\eta,cos\theta)] \\
+[F_{\overline{q}}(\xi,q^2)+(1-y)^2 F_q(\xi;q^2)][(1-x_p)^2 B(\eta,cos\theta)] \\
+[(1-y)(F_q(\xi;q^2)+F_{\overline{q}}(\xi;q^2))][4x_p(1-x_p)B(\eta,cos\theta)] \\
+\frac{3}{8}[(1+(1-y)^2)F_g(\xi;q^2)] \times [(1-x_p)(x_p^2+(1-x_p)^2(A(\eta,cos\theta)-2B(\eta,cos\theta))] \\
+\frac{3}{8}[(1-y)F_g(\xi;q^2)][16x_p(1-x_p)^2 B(\eta,cos\theta)]\}
\end{aligned}$$

where $x_p = x/\xi$, $\xi = 1-\eta(1-x)$,

$$A(\eta,cos\theta) = \frac{2}{(1-cos\theta)[2-\eta(1+cos\theta)]} \tag{5}$$

$$B(\eta,cos\theta) = \frac{2(1-\eta)(1+cos\theta)}{[2-\eta(1+cos\theta)]^3} \tag{6}$$

$F_q(\xi;q^2) = \sum_i \frac{1}{2}e_i^2 q_i(\xi;q^2)$, $F_{\overline{q}}(\xi;q^2) = \sum_i \frac{1}{2}e_i^2 \overline{q_i}(\xi;q^2)$, $F_g(\xi;q^2) = \frac{1}{2}\sum_i \frac{1}{2}e_i^2 G(\xi;q^2)$, where the sums over i are understood to be sums over the active flavors n_f. $q_i(\xi,q^2)$ is the parton distribution function of quark i, $G(\xi;q^2)$ is the gluon distribution function of the target and e_i is the charge of quark i.

The expression for $\int d\Omega_1 d\Omega_2 sin^2\chi(\frac{d\sigma}{d\Omega_1 d\Omega_2 dxdy})$ for forward events, i.e. $0 \leq \theta_1 \leq \frac{1}{2}\pi$, $0 \leq \theta_2 \leq \frac{1}{2}\pi$ is analogous to that for $(\frac{d\sigma}{dcos\theta dxdy})_{\theta\neq 0}$ except that now the functions A and B are replaced by

$$< A_2 >= \frac{4}{(2-\eta)^2}\left\{\frac{2(1+(1-\eta)^2)}{\eta(2-\eta)}ln\frac{1+(1-\eta)^2}{2(1-\eta)} - \frac{\eta(2-\eta)}{1+(1-\eta)^2}\right\} \tag{7}$$

and

$$\begin{aligned} < B_2 >= & \frac{4[1+(1-\eta)+(1-\eta)^2]}{\eta^2(2-\eta)^2} \\ & \times\left\{\frac{\eta(2-\eta)}{1+(1-\eta)^2} - \frac{4(1-\eta)}{\eta(2-\eta)}ln\frac{1+(1-\eta)^2}{2(1-\eta)}\right\} \end{aligned} \tag{8}$$

The dependence on $G(\xi;q^2)$ is visible in the expression for $(\frac{d\sigma}{dcos\theta dxdy})_{\theta\neq 0}$.

In equation (4) the numerator and the denominator get different contributions from quarks and gluons and as a result the quantity $< sin^2\chi >$ can be sensitive to the gluon distribution function. As an example, if the only $O(\alpha_s)$ process that was present was the photon-gluon fusion process then in the numerator of equation (4) we would have only the contribution from gluons and no contribution from quarks while in the denominator we would have contributions from both quarks and gluons; at the parton level only QCD processes give non-zero values of χ.

3 Results on the gluon distribution function from other experiments

The gluon distribution function has been studied in direct photon production in hadron-hadron collisions [16-19]. However these measurements do not go below an η of 0.04. $\eta G(\eta)$ has also been studied by analysing the nucleon structure functions using perturbative QCD [20]. In both of the above mentioned analyses the gluon distribution function that is deduced is correlated with the QCD Λ parameter [20, 21]. This is because both the gluon distribution function and the strong coupling constant affect cross sections for direct photon emission and also for deep inelastic scattering of leptons off nucleons. $\eta G(\eta)$ has also been studied by looking at inelastic J/Ψ production in deep inelastic scattering [22, 23]. Once again these measurements do not go below an η of 0.04 and the results for $\eta G(\eta)$ are again correlated to the strong coupling constant. Furthermore, the interpretation of results on muoproduction of J/Ψ results requires an arbitrary normalization factor [22]. More recently there have been measurements from HERA of $\eta G(\eta)$ down to η of $\sim 10^{-3}$ [24-26].

4 Experimental procedure

The experiment E665 [27] was performed in the NM beam line at Fermilab and used a beam of 490 GeV muons. The beam energy had a 10% r.m.s. spread but was measured event by event to 0.5%. data taken during the 1987-88 run with a 1.15 m long deuterium target were considered in this analysis. Charged particles reconstructed in the tracking system and fitted to the primary vertex were used. The following event kinematic cuts were applied to define the event sample:

$$y_{Bj} = \nu / E_{Beam} < 0.8$$
$$Q^2 > 3.0 \text{ GeV}^2$$
$$\nu > 50 \text{ GeV}$$
$$x > 0.005$$
$$\theta_{scat} > 4 \text{ mrad}$$
$$W^2 > 300 \text{ GeV}^2$$

where $Q^2 = -q^2$,ν was the laboratory energy of the γ^*, θ_{scat} was the scattering angle of the muon in the laboratory frame, E_{Beam} was the energy of the incident muon and W was the overall hadronic center-of-mass energy. The minimum cut for Q^2 was chosen to be 3 GeV^2 in order to avoid the non-perturbative QCD region $Q^2 \sim \Lambda^2$. The y_{Bj} requirement was imposed in order to reduce the contribution from events with QED bremsstrahlung. A high minimum value of W^2 was chosen because non-perturbative effects are expected to go down as W increases [5]. We also required exactly one beam track to be present in the event. Most of the interesting quantities used in the analysis were calculated in the γ^* N center of mass frame with the virtual photon moving along the +Z direction. Only particles going forward (with a positive momentum component along the +Z axis) in the virtual photon-nucleon center-of-mass system were considered. In addition, events with less than 2 charged particles in the forward hemisphere were not included in the analysis. The following particle kinematic cuts were applied:

- particle momentum in the lab. frame $>$ 10 GeV/c
- χ^2 fit probability for the geometrical reconstruction of the particle track > 0.001
- $|z'| < 30$ mrad
- $|y'| < 70$ mrad

where y' and z' were the slopes in the laboratory frame of the particle track at the interaction vertex in horizontal and vertical directions respectively. The slope cuts were made in order to ensure that the tracks were in the region of uniform acceptance of the spectrometer. An event was kept if at least two particles passed the particle kinematic cuts. The data were divided into two samples: the low-x sample with $0.05 > x > 0.005$ and the high-x sample with $x > 0.05$. We used the high-x sample to determine the contribution of fragmentation to energy flow and energy-energy angular pattern while we used the low-x region to study $\eta G(\eta)$ in a poorly explored region of η. These cuts resulted in 10362 (1216) accepted events for the low-x (high-x) sample for the deuterium target. The average value of x for the low-x sample was 0.0166 ± 0.0001. The average values of Q^2, ν and W^2 for the low-x sample were (8.04 ± 0.05) GeV^2, (268.0 ± 0.7) GeV and (495.8 ± 1.3) GeV^2 respectively. The corresponding values for the high-x sample were (47.9 ± 0.9) GeV^2, (262.3 ± 1.9) GeV and (445.1 ± 3.2) GeV^2 respectively.

In practice, for calculating $< sin^2\chi >$ ($< sin^2\theta >$) we required $0.03 < sin^2\chi < 0.3$ ($0.04 < sin^2\theta < 0.4$); this was done in order to be independent of tails and resolution effects in the distributions of $sin^2\chi$ ($sin^2\theta$).

5 Monte Carlo simulation

The present analysis is based on comparisons of the properties of experimental events with those generated in Monte Carlo simulations. The simulation took into account the primary interaction using the LUND Monte Carlo program [14, 15], the radiative effects and the recording and reconstruction of the events in the detector.

We used the LUND 4.3 Monte Carlo program in a standard fashion except for the following changes:

- An event with 3 partons in the final state can sometimes look like an event with 2 partons in the final state if one of the partons is either collinear with another parton or has very low energy. In the Monte Carlo program used in the analysis [14, 15] the parameters PARL(11) and PARL(12) determine whether an event will be simulated as a 3 parton event or as a 2 parton event. For clarity we will use the symbol m_a for PARL(11) and the symbol x_{jet} for PARL(12). m_a is the average "transverse hadron mass" used for matrix element cuts, while x_{jet} is the minimum required value of $x_i = 2 \cdot E_i/W$ for parton i, where E_i is the energy of the parton in the center-of-mass of the hadronic system. The default values of m_a and x_{jet} are 1 GeV and 0.05

respectively. For a photon-gluon fusion event $\sqrt{8} \cdot m_a$ is the minimum value of the energy of the target remnant in the frame in which the quark and the anti-quark are moving back-to-back and the target remnant is moving perpendicular to the quarks. We used $m_a = 0.65$ GeV and $x_{jet} = 0.015$ and studied the sensitivity of our result to changes in these parameters.

- The matrix element calculation that is used in the Monte Carlo program to simulate the perturbative QCD processes is first order in α_s and according to ref. [28] the calculation is considered to be of next-to-leading order (NLO); the leading order (LO) process being the simple parton model with no QCD. In simulating hard QCD processes we used the next-to-leading order (NLO) Λ and NLO parton distribution functions. So in order to be consistent we used the NLO expression for α_s. When using different parton distribution functions we have always tried to use the NLO parametrization in the $\overline{MS}$ scheme. For arguments why this is the right thing to do see ref. [28]. In leading order, the QCD parton framework reproduces original parton model results, with scale-dependent parton distributions.

  We used the Λ value for number of quark flavors (n_f) equal to 3 if the condition $Q^2 > 4 \cdot m_c^2$ was not satisfied and the Λ value for $n_f = 4$ if the condition $Q^2 > 4 \cdot m_c^2$ was satisfied; where m_c was the mass of the charm quark. The value of m_c in the LUND Monte Carlo program [15] is 1.6 GeV.

  The value of Λ for $n_f = 4$ assumed in the analysis is (260 ± 50) MeV [6]. This implies Λ is (312 ± 50) MeV for $n_f = 3$.

- As our main set of parton distribution functions we have used that by Glück, Reya and Vogt (GRV-HO) [2].The GRV-HO parton distribution functions agree with the measurements of the structure function $F_2(x, Q^2)$ from the SLAC, EMC, BCDMS, NMC and HERA experiments [29-33] . For $Q^2 > 3$ GeV^2, the GRV-HO parton distribution functions give also a good description of the $F_2(x, Q^2)$ data from the

E665 experiment [34] even though these data were not used in the fit to determine the free parameters of the GRV-HO parton distribution functions.

- Simulation of soft gluons was not included in the Monte Carlo. The effect of soft gluons was studied by varying the QCD cutoff parameters.

$\eta G(\eta)$ was parametrized as:

$$
\begin{aligned}
\eta G(\eta) &= (\eta G_{pdfn}(\eta) \text{ evaluated at } \eta = 0.05) \cdot \left(\frac{\eta}{0.05}\right)^{\lambda} \quad \text{for } 0.005 < \eta < 0.05 \\
\eta G(\eta) &= \eta G_{pdfn}(\eta) \text{ for } \eta > 0.05
\end{aligned} \tag{9}
$$

where *pdfn* is the name of the parton distribution function parametrization of $\eta G(\eta)$. The η^{λ} behavior at small η is theoretically motivated by the BFKL [3] calculation. We generated Monte Carlo events for different values of λ.

We have studied how our result depends on the choice of parton distribution functions by also using the Morfin-Tung [35] (MT) B_2 parametrization in the $\overline{MS}$ scheme (see section 8).

For studying the effect of the parametrization we have tried the following alternative:

$$
\begin{aligned}
\eta G(\eta) &= (\eta G_{pdfn}(\eta) \text{ evaluated at } \eta = 0.05) + (\eta - 0.05)(\frac{d(\eta G_{pdfn}(\eta))}{d\eta} \text{ evaluated at } \eta = 0.05) + \\
&\quad \frac{1}{2}(\eta - 0.05)^2 \cdot C_0 \cdot 1000 \text{ for } 0.005 < \eta < 0.05 \\
\eta G(\eta) &= \eta G_{pdfn}(\eta) \text{ for } \eta > 0.05
\end{aligned} \tag{10}
$$

where C_0 is a parameter used to generate different gluon distribution functions.

Although this parametrization is not physically motivated it was chosen to see how the final result depends on the choice of parametrization.

6 Determination of $\sigma_{frag.P_T}$

In a first step the Gaussian width $\sigma_{frag.P_T}$ of the p_x and p_y transverse momentum distributions for primary hadrons relative to the fragmenting jet was determined using the high-x data. We generated Monte Carlo events for different values of $\sigma_{frag.P_T}$ and then compared the value of $< sin^2\chi >$ seen in the data with that expected from the Monte Carlo sample. In figure 2 we show $< sin^2\chi >$ versus $\sigma_{frag.P_T}$ for the Monte Carlo sample - $< sin^2\chi >$ was calculated by requiring $0.03 < sin^2\chi < 0.3$ in order to be independent of tails and resolution effects in the distribution of $sin^2\chi$. The measured value of $sin^2\chi$ from the data, shown as the horizontal line bounded by two dash-dotted horizontal lines from above and below in figure 2, is 0.1158 ± 0.0022, that of $< sin^2\theta >$ is 0.1192 ± 0.0022. We extract $\sigma_{frag.P_T} = (366 \pm 20)$ MeV. In figure 3 we compare the measured distributions of $sin^2\chi$ and $sin^2\theta$ with those from the Monte Carlo sample, in which $\sigma_{frag.P_T}$ was set to 366 MeV. There is good agreement between data and Monte Carlo in the region for which the means $< sin^2\theta >$ and $< sin^2\chi >$ were calculated. The χ^2 per degree of freedom for the ratio of data over Monte Carlo distributions shown in figure 3 for $< sin^2\chi >$ when fitted to a constant is 1.047 for 17 degrees of freedom. For $\sigma_{frag.P_T} = 350$ MeV χ^2 per degree of freedom is 1.655 while for $\sigma_{frag.P_T} = 380$ MeV it is 1.466. The default value for $\sigma_{frag.P_T}$ in the Lund Monte Carlo program is 440 MeV. The difference is partly due to our QCD cutoff parameters being lower than the default values in the Monte Carlo program (see section 5).

7 Determination of the gluon distribution function at low x.

We then used the value of $\sigma_{frag.P_T}$ determined using the high-x data to investigate perturbative QCD effects in the low-x data. Here we are assuming that $\sigma_{frag.P_T}$ is independent of x. We generated Monte Carlo events for different values of λ in eq. (9) and then compared

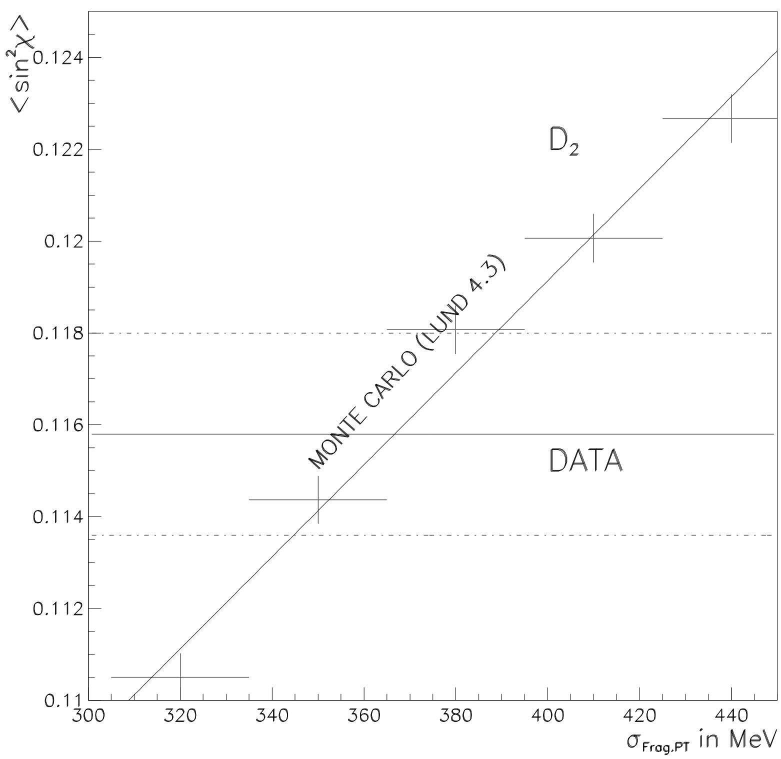

Figure 2: $< sin^2\chi >$ versus $\sigma_{frag.P_T}$ for Monte Carlo events (crosses) and the measured value of $< sin^2\chi >$, for $x > 0.05$. The tilted line in the figure is a straight line fit to the Monte Carlo points.

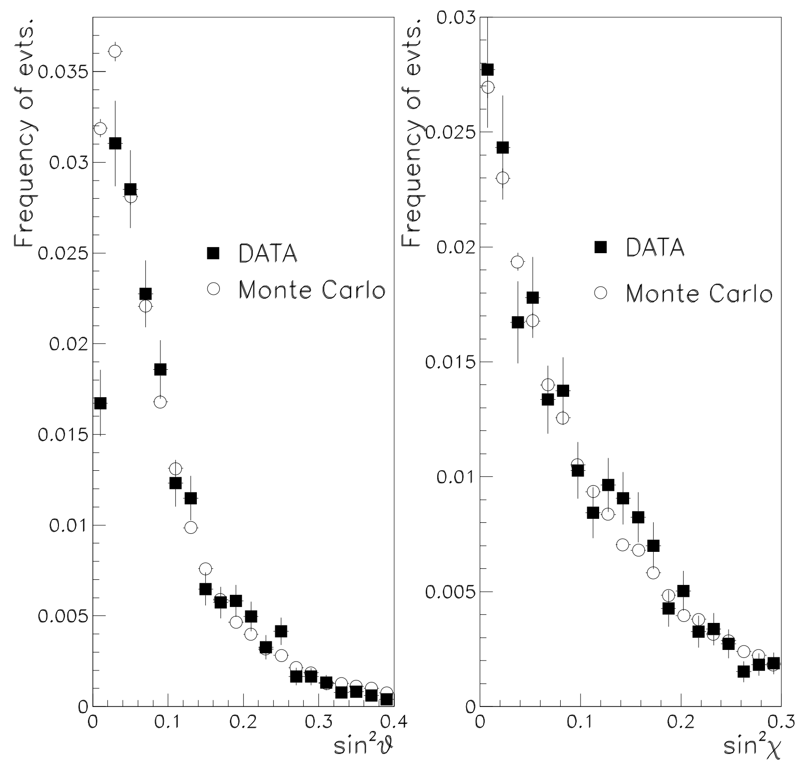

Figure 3: $sin^2\theta$ and $sin^2\chi$ distributions for the data and for the Monte Carlo sample (normalized to the same number of events), generated with $\sigma_{frag.P_T} = 366$ MeV, for $x > 0.05$

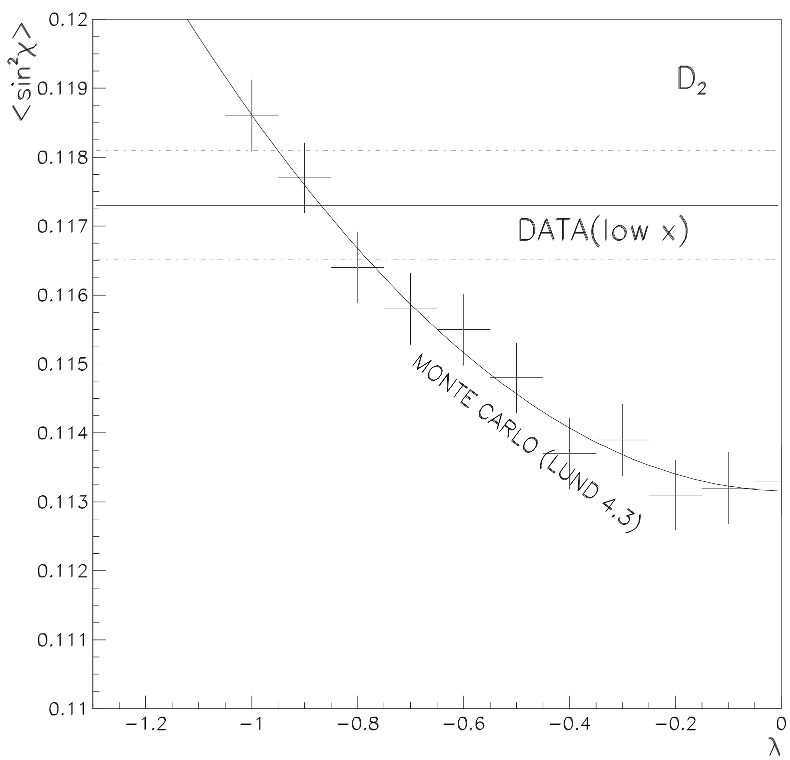

Figure 4: $< sin^2\chi >$ versus λ for Monte Carlo events and the measured value of $< sin^2\chi >$, for $0.005 < x < 0.05$. The curve in the figure is a fit to the Monte Carlo (crosses) points.

the value of $< sin^2\chi >$ seen in the data with that expected from the Monte Carlo sample. In figure 4 we show $< sin^2\chi >$ versus λ for the Monte Carlo sample. The measured value of $< sin^2\chi >$, shown as the horizontal solid line bounded by two horizontal dash-dotted lines in figure 4, is 0.1173 ± 0.0008. We extract $\lambda = -0.87 \pm 0.09$. In figure 5 we compare the measured distributions of $sin^2\chi$ and $sin^2\theta$ with those of the Monte Carlo in which λ was set to -0.87. There is good agreement between data and Monte Carlo in the region where the means $< sin^2\theta >$ and $< sin^2\chi >$ were calculated.The χ^2 per degree of freedom for the ratio of data over Monte Carlo distributions shown in figure 5 for $< sin^2\chi >$ when fitted to a constant comes out to be 1.073 for 17 degrees of freedom. For $\lambda = -0.8$ χ^2 per degree of freedom is 1.529 while for $\lambda = -1.0$ it is 1.314. The value of χ^2 per degree of freedom for $\lambda = 0$ is 2.419.

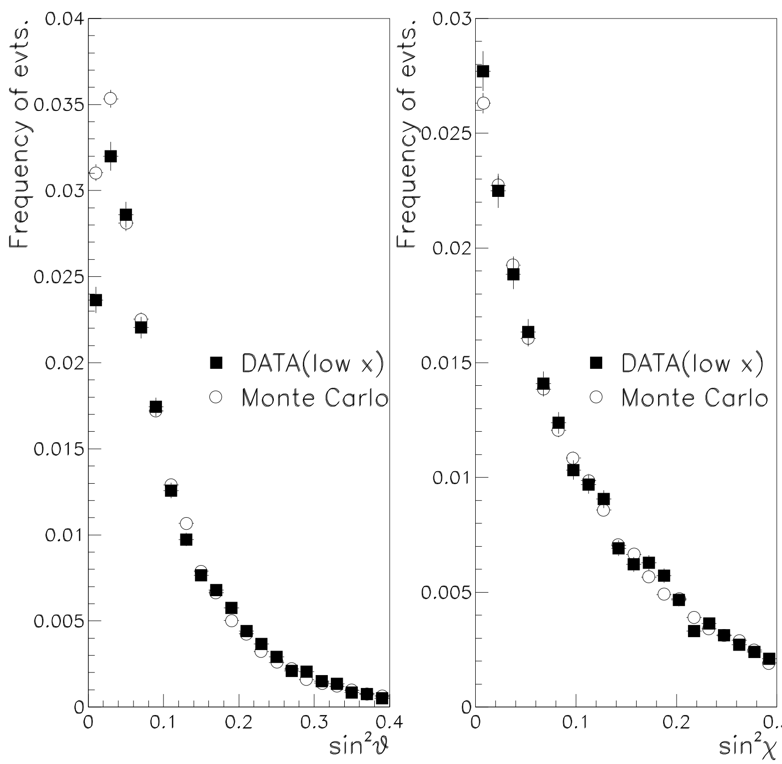

Figure 5: $sin^2\chi$ and $sin^2\theta$ distributions for the data and for the Monte Carlo sample (normalized to the same number of events), generated with $\lambda = -0.87$, for $0.005 < x < 0.05$.

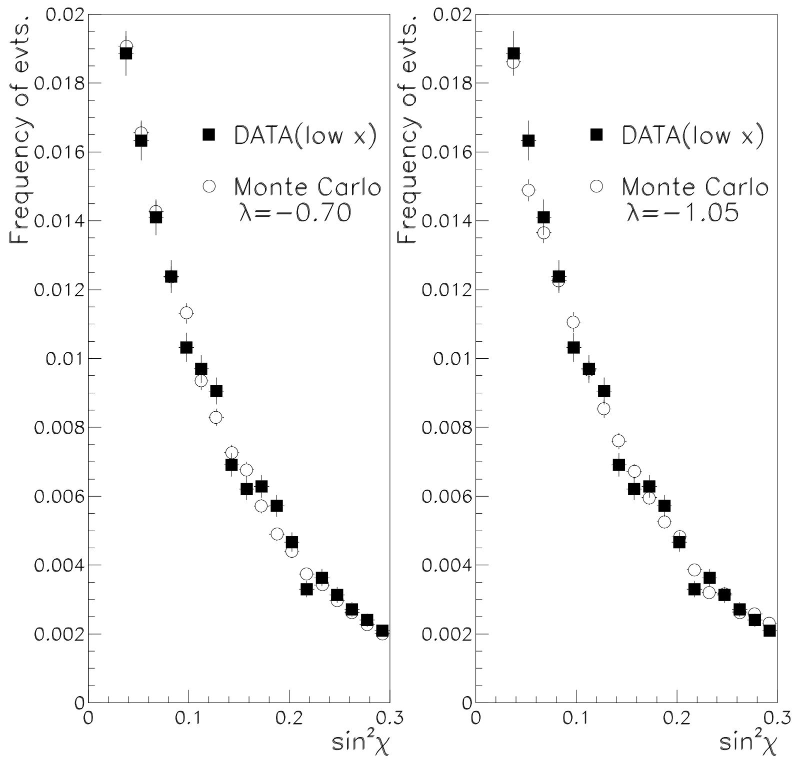

Figure 6: $sin^2\chi$ distributions for the data and for two Monte Carlo samples (normalized to the same number of events), generated with $\lambda = -0.70$ and $\lambda = -1.05$ respectively, for $0.005 < x < 0.05$.

In figure 6 we have compared $sin^2\chi$ distributions for data and Monte Carlo samples generated with $\lambda = -0.7$ and -1.05 respectively. We see that the distribution tends to be steeper in the Monte Carlo sample than in the data for $\lambda = -0.7$ and shallower for $\lambda = -1.05$.

8 Systematic errors and checks

There are several potential sources of systematic error. They come from uncertainties in fragmentation, the QCD Λ parameter, the gluon distribution function for $\eta > 0.05$, the quark distribution functions and the acceptance of the detector for hadrons. The respective contributions to the systematic error of λ are compiled in table 1.

Systematic error from fragmentation: As was stated earlier fragmentation of partons into hadrons is potentially an important source of systematic error. One of the ways we have looked at this is by extracting λ using different values of $\sigma_{frag.P_T}$. We can see by how much λ changes as we change $\sigma_{frag.P_T}$. If we raise $\sigma_{frag.P_T}$ by 20 MeV from its central value and use that to determine λ we find $\lambda = -0.60 \pm 0.11$. So a $+20$ MeV change in $\sigma_{frag.P_T}$ leads to $+0.27$ change in λ. Similarly a -20 MeV change in $\sigma_{frag.P_T}$ leads to a -0.33 change in λ. We assign a systematic error on λ due to fragmentation of $\pm^{0.27}_{0.33}$.

Systematic error from the QCD Λ parameter: Another source of systematic error on λ comes from the uncertainty in the QCD Λ parameter. If we repeat the analysis using a value for Λ that is 50 MeV larger than our standard value we get a value of $\sigma_{frag.P_T} = (363 \pm 20)$ MeV and the value of λ comes out to be -0.80 ± 0.10. However lowering Λ by 50 MeV gives $\sigma_{frag.P_T} = (368 \pm 20)$ MeV and $\lambda = -0.94 \pm 0.10$. Thus a ± 50 MeV change in Λ leads to a ± 0.07 change in λ. We assign a systematic error on λ due to uncertainty in Λ of ± 0.07 .

Systematic error from $\eta G(\eta)$ for $\eta > 0.05$: We have also examined the systematic error on λ due to the particular choice of the function $\eta G(\eta)$ for $\eta > 0.05$. If for parton distributions we use the Morfin-Tung [35] (MT) B_2 parametrization in the $\overline{MS}$ scheme we obtain $\sigma_{frag.P_T} = (370 \pm 20)$ MeV and $\lambda = -0.98 \pm 0.10$. We assign a systematic uncertainty of ± 0.11 due to this source.

Systematic error from quark distributions: Changing the quark distributions for $\eta < 0.05$ can also affect the result for λ. If the number of quarks in this kinematic region is reduced the perturbative QCD effects will be attributed to a larger extent to the photon-gluon fusion graph. The resulting increase in the fraction of photon-gluon fusion events will make the energy-energy angular pattern more sensitive to the gluon distribution function, thus leading to a lower absolute value of λ. A -5% change in the quark distributions leads to a change in the value of λ of $+0.01$. The reason we are considering a -5% change in the quark distributions is because for the kinematic region of interest in the analysis described here, the F_2 structure function of the nucleon is known to 5% or better and F_2 is a measure of the quark distributions in the nucleon. We assign a systematic error on λ of ± 0.01 due to this source.

Systematic error from acceptance: Lack of full knowledge of the acceptance of the detector can also influence the determination of λ. We have investigated this by randomly throwing away 10% of all the tracks in the data and repeating the analysis. First using the high-x data we deduce a value of $\sigma_{frag.P_T} = (357 \pm 22)$ MeV. Then using this information together with the low-x data results in a value of -0.98 ± 0.18 for λ. Since this value is in magnitude higher by 0.11 from our value determined using all tracks we assign a systematic error of ± 0.11 due to this source.

Combining the systematic errors due to fragmentation($\pm^{0.27}_{0.33}$), QCD Λ parameter (± 0.07), $\eta G(\eta)$ for $\eta > 0.05$ (± 0.11), quark distributions (± 0.01) and acceptance (± 0.11) in quadrature we obtain a combined systematic error of $\pm^{0.32}_{0.37}$.

In table 1 we have summarized the various components of the combined systematic error for the parameter λ as determined for the deuterium data.

In figure 7 we show our result after combining systematic and statistical errors in quadrature. For comparison we have also shown $\eta G(\eta)$ for the GRV-HO, MRS-$\overline{MS}$ [40] and CTEQ3-$\overline{MS}$ [41] parametrizations of parton distribution functions evaluated at $Q^2 =$

Source of systematic error	systematic error on λ
Fragmentation	$\pm^{0.27}_{0.33}$
QCD Λ parameter	± 0.07
$\eta G(\eta)$ for $\eta > 0.05$	± 0.11
Quark distributions	± 0.01
Acceptance	± 0.11
Combined systematic error	$\pm^{0.32}_{0.37}$

Table 1: Components of the systematic error on λ as determined for the deuterium data.

8 GeV^2 and $\eta G(\eta)$ obtained by the H1 collaboration at HERA using jets at higher W^2 [24]. For the x region shown in the figure $\eta G(\eta)$ for the GRV-HO distribution function at $Q^2 = 8$ GeV^2 can be described by $\lambda \approx -0.5$. From the figure we see that our result is within errors in agreement with the result from the H1 experiment [24]. Our result also supports the theoretical expectation based on the BFKL [3] calculation that for small η $\eta G(\eta)$ rises steeply as η is lowered.

8.1 Cross-checks

In addition to the systematic errors listed above we have also carried out some other checks to confirm our result.

Sensitivity to QED corrections: From figure 4 we see that changing λ from 0.00 to -0.87 corresponds to a change in the Monte Carlo prediction for $< sin^2\chi >$ of only 4%. Since this is such a small effect we repeated the analysis by using a Monte Carlo simulation in which QED radiative events were not present to see if the result would change. Using the high-x data we obtained $\sigma_{frag.P_T} = (372 \pm 20)$ MeV and then using this information

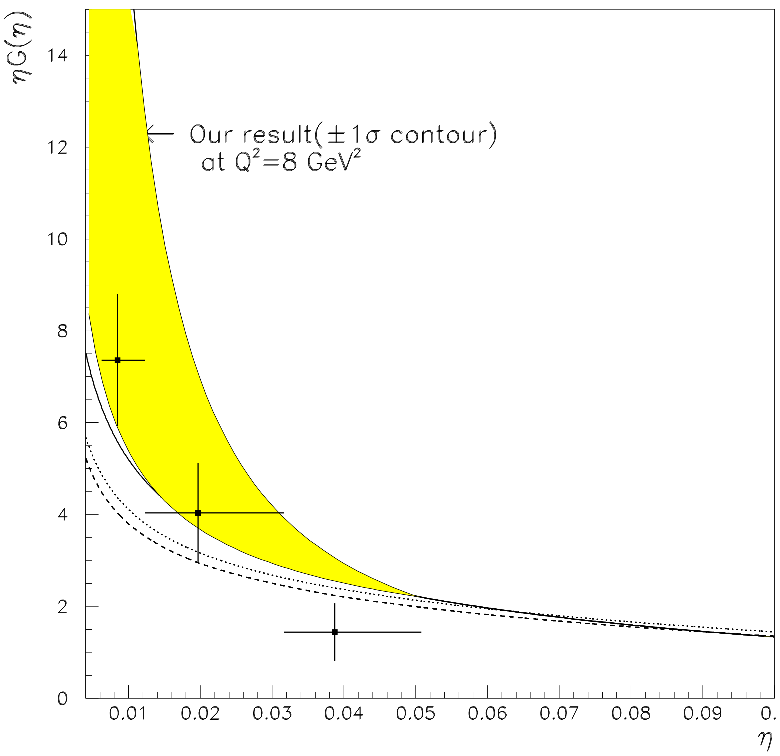

Figure 7: Gluon distribution function for the parametrization (9). The shaded area represents the experimental error after combining statistical and systematic errors in quadrature. The 3 points with error bars are those obtained by the H1 experiment using jets. $\eta G(\eta)$ for GRV-HO (solid line), MRS (dashed line) and CTEQ3-$\overline{MS}$ (dotted line) parametrizations of parton distribution functions are also shown.

together with the low-x data we obtained $\lambda = -0.85 \pm 0.10$ which is very close to the original value obtained for λ. We conclude that our result is insensitive to the details of QED corrections.

Sensitivity to QCD cutoff parameters: We have also looked at the sensitivity of our result to the QCD cutoff parameters m_a and x_{jet}. Raising m_a from 0.65 GeV to 0.75 GeV gives a slightly higher value of (368 ± 20) MeV for $\sigma_{frag.P_T}$. However the resulting value of λ is -0.85 ± 0.08 which is quite close to the original value of -0.87 ± 0.09. Similarly raising x_{jet} from 0.015 to 0.025 gives a value of (365 ± 25) MeV for $\sigma_{frag.P_T}$, however λ comes out to be -0.90 ± 0.09. If for m_a and x_{jet} we used the default values we would obtain for $\sigma_{frag.P_T}$ a value of (382 ± 20) MeV and for λ a value of -0.85 ± 0.08.

Sensitivity to W^2: We have also examined the effect of raising the minimum W^2 cut from 300 GeV^2 to 400 GeV^2. Fragmentation effects are expected to decrease with increasing W^2. However raising the minimum value of W^2 also lowers the mean value of x and the number of events. Using the high-x data we determined the value of $\sigma_{frag.P_T}$ to be (370 ± 30) MeV. This is consistent with the value determined with the lower minimum W^2 cut. For $W^2 > 400$ GeV^2, for the data we measured $< sin^2\chi >= 0.1160 \pm 0.0009$ and the value of λ comes out to be $-0.65 \pm^{0.15}_{0.10}$.

Sensitivity to Q^2: If instead of raising the minimum W^2 cut we raise the minimum Q^2 cut from 3 GeV^2 to 4 GeV^2 we deduce for the data $< sin^2\chi >= 0.1175 \pm 0.0009$ which implies $\lambda = -0.95 \pm 0.10$. For 3 GeV$^2 < Q^2 < 4$ GeV^2 the value of $< sin^2\chi >$ for the data is 0.1163 ± 0.0018, which corresponds to $\lambda = -0.70 \pm 0.12$.

Sensitivity to the aperture for hadrons: Tightening the requirements on the horizontal and vertical slopes of tracks by demanding $|z'| < 25$ mrad and $|y'| < 60$ mrad gives a value of (359 ± 20) MeV for $\sigma_{frag.P_T}$ and for λ we obtain a value of -0.95 ± 0.09.

Sensitivity to exclusive ρ^0 meson production: There are special processes in the muon-nucleon interaction which are not simulated in the Monte Carlo program. An example is exclusive ρ^0 meson production [36]. We have investigated the sensitivity to the presence of this channel by dropping events where exactly two charged tracks pass the track and event selection cuts and the invariant mass of the charged track pair lies between 0.4 GeV and 1.2 GeV. For $x > 0.05$ for the data we measured $< sin^2\chi >= 0.1183 \pm 0.0027$ and this implies $\sigma_{frag.P_T} = (367 \pm 23)$ MeV which is in good agreement with the value measured without dropping exclusive ρ^0 meson candidates. For $0.005 < x < 0.06$ for data we measured $< sin^2\chi >= 0.1210 \pm 0.0009$ and the value of λ came out to be -0.95 ± 0.08 using $\sigma_{frag.P_T}$ $= 366$ MeV.

Sensitivity to the order of parton distribution functions: According to ref. [28] NLO parton distribution functions are the proper parton distribution functions to use. We have, however, also examined how our results would change if we were to use LO parton distribution functions. Using the GRV-LO parton distribution functions, the high-x data lead to $\sigma_{frag.P_T} = (360 \pm 22)$ MeV. With this value of $\sigma_{frag.P_T}$ the low-x data then yield $\lambda = -0.80 \pm 0.09$. We notice that the result for λ changes only little as we go from NLO to LO parton distribution functions.

Sensitivity to the small $sin^2\chi$ region: In calculating $< sin^2\chi >$ we have so far required $0.03 < sin^2\chi < 0.3$. If instead we require $0.0 < sin^2\chi < 0.3$ we obtain $\sigma_{frag.P_T} =$ (366 ± 16) MeV and $\lambda = -0.85 \pm 0.07$. Both these values are very close to the values obtained by requiring $0.03 < sin^2\chi < 0.3$.

Sensitivity to different versions of the LUND Monte Carlo program: In this analysis we used versions 4.3 of LEPTO [14] and JETSET [15] Monte Carlo programs to simulate deep-inelastic lepton scattering. We have also investigated how our results would change if we were to use a different version of the JETSET program. In version 6.3 of JETSET fragmentation of partons is done quite differently compared to version 4.3 of JETSET. If we use version 4.3 of LEPTO [14] and version 6.3 of JETSET [37] we obtain a value of (387 ± 22) MeV for $\sigma_{frag.P_T}$ but for λ the result comes out to be -0.82 ± 0.10.

If we use the most recent version of the LUND Monte Carlo program with version 6.1 for LEPTO [38] and with version 7.3 for JETSET [39] we obtain for $\sigma_{frag.P_T}$ a value of (312 ± 20) MeV and for λ we get -1.10 ± 0.12. The cutoff values we used were $m_{cut} = 0.65$ GeV and $y_{cut} = 0.005$ where m_{cut} is the minimum value of the invariant mass of any parton pair (including the target remnant) and y_{cut} is the minimum value of m_{ij}^2/W^2, m_{ij} being the invariant mass of the parton i - parton j system.

Sensitivity to changing the low-x region: In our analysis we used x=0.05 to divide the data into high-x and low-x regions. If instead we call the $0.005 < x < 0.06$ region as the low-x region we obtain $\lambda = -0.86 \pm 0.09$.

Sensitivity to how $\eta G(\eta)$ is parametrized: If we parametrize the gluon distribution function in the low-x region using equation (10) instead of equation (9) and extract the value of C_0 from the data we deduce $C_0 = 4.7 \pm 0.8$. In figure 8 we compare the gluon distributions extracted using the two different parametrizations at $Q^2 = 8$ GeV^2. We see that the two results are consistent with each other for $\eta > 0.01$. For $\eta < 0.01$ the result depends on the choice of the parametrization of the gluon distribution function. The contours shown have been determined using statistical errors only.

In order to determine which of the two solutions is closer to reality we determined the

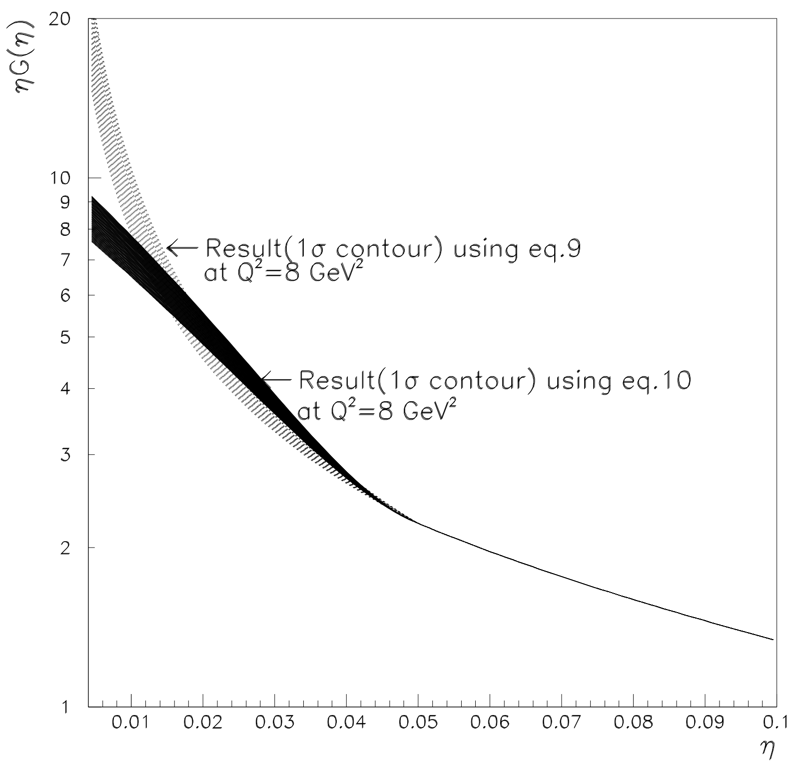

Figure 8: Gluon distribution function extracted using two different parametrizations of $\eta G(\eta)$ for $\eta < 0.05$. Here the 1σ contour reflects only statistical errors

parameters λ and C_0 for $0.005 < x < 0.01$. We picked this region because this is where the two solutions differ the most. For this kinematic region we have for the data $< sin^2\chi >=$ 0.1180 ± 0.0014. Using the procedure outlined earlier we get $\lambda = -0.85 \pm 0.10$ and $C_0 =$ 6.0 ± 1.1. The two C_0 values differ by 1.6 standard deviations while the two λ values differ by 0.2 standard deviations respectively. Equation (9) thus appears to give a better description of the data than equation (10).

Result from the hydrogen target: During the 1987-88 run we also took some data on a hydrogen target. If we analyze this data in the same manner that we analyzed the data taken on the deuterium target we obtain for the high-x data a value of $< sin^2\chi >=$ 0.1145 ± 0.0044 which implies $\sigma_{frag.P_T} = (350 \pm 44)$ MeV. For the low-x data we measure $< sin^2\chi >= 0.1168 \pm 0.0017$. This value of $< sin^2\chi >$ together with a value of 350 MeV for $\sigma_{frag.P_T}$ suggests a value of $-1.05 \pm 0.15(stat.)$ for λ. The systematic error on λ however is quite large. The 44 MeV uncertainty in $\sigma_{frag.P_T}$ alone leads to a systematic error in λ of $\pm^{0.7}_{0.3}$. Within the errors this result is consistent with the one obtained using the deuterium data.

Q^2 dependence of $\sigma_{frag.P_T}$: As was indicated in Section 4, the average Q^2 for the low-x and the high-x regions are quite different. In order to investigate the Q^2 dependence of $\sigma_{frag.P_T}$ we first divided the high-x region into three bins, the first one with $Q^2 < 30$ GeV^2, the second one with $30 < Q^2 < 45$ GeV^2 and the third one with $Q^2 > 45$ GeV^2. Then, using the technique described in Section 6, we determined $\sigma_{frag.P_T}$ for all the three regions. The values for $\sigma_{frag.P_T}$ for the three regions came out to be (310 ± 40) MeV, (412 ± 35) MeV and (370 ± 40) MeV respectively. The average values of Q^2 for these three regions were 23.77, 36.94 and 74.13 GeV^2 respectively. Within the errors the results are consistent with the Q^2 dependence of $\sigma_{frag.P_T}$ being flat.

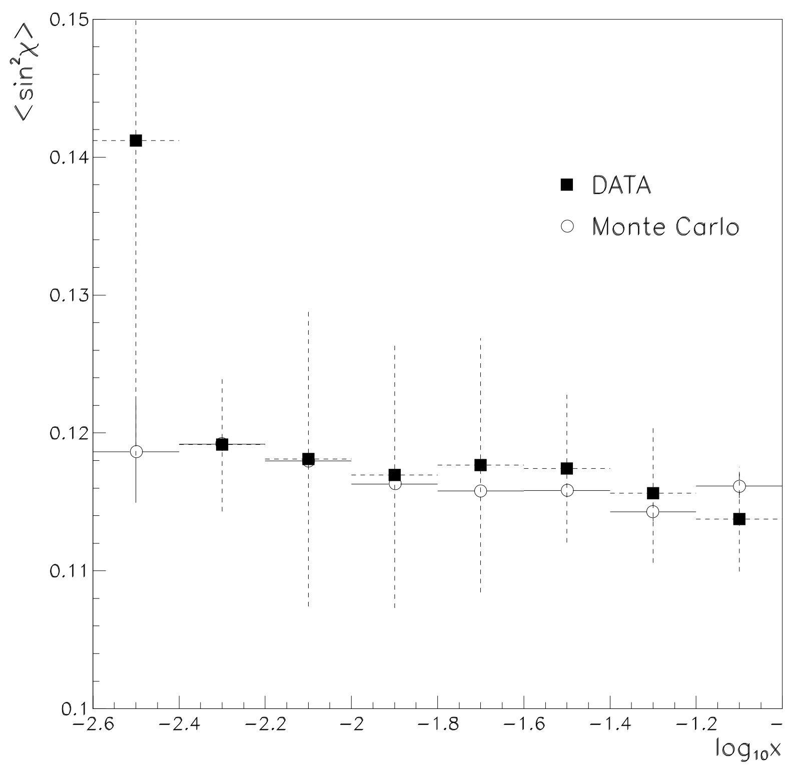

Figure 9: A comparison, for different x bins, of the values of $< sin^2\chi >$ for data events with those for Monte Carlo events generated using $\lambda = -0.87$ and $\sigma_{frag.P_T} = 366$ MeV.

x-dependence of $< sin^2\chi >$: We have also examined how well our result for the gluon distribution function describes the x-dependence of $< sin^2\chi >$ in the data. We did this by comparing the values of $< sin^2\chi >$ for data events with those for Monte Carlo events generated using $\lambda = -0.87$ and $\sigma_{frag.P_T} = 366$ MeV. This is illustrated in figure 9 and in table 2. We see that within the errors Monte Carlo events generated using $\lambda = -0.87$ and $\sigma_{frag.P_T} = 366$ MeV give a good description of the x-dependence of $< sin^2\chi >$ seen in the data.

In table 3 we have summarized the study of the systematics.

$log_{10}x$ bin	$< sin^2\chi >$ for data events	$< sin^2\chi >$ for Monte Carlo events
-2.6 to -2.4	0.14121 ± 0.02286	0.1186 ± 0.0037
-2.4 to -2.2	0.11915 ± 0.00484	0.1192 ± 0.0005
-2.2 to -2.0	0.11810 ± 0.01067	0.1180 ± 0.0003
-2.0 to -1.8	0.11696 ± 0.00965	0.1163 ± 0.0003
-1.8 to -1.6	0.11766 ± 0.00922	0.1158 ± 0.0004
-1.6 to -1.4	0.11741 ± 0.00535	0.1158 ± 0.0005
-1.4 to -1.2	0.11562 ± 0.00503	0.1143 ± 0.0007
-1.2 to -1.0	0.11375 ± 0.00379	0.1162 ± 0.0009

Table 2: A comparison, for different x bins, of the values of $< sin^2\chi >$ for data events with those for Monte Carlo events generated using $\lambda = -0.87$ and $\sigma_{frag.P_T} = 366$ MeV.

9 Summary

In conclusion we have measured $\eta G(\eta)$ of the nucleon for $0.005 < \eta < 0.05$ at an average Q^2 of 8 GeV^2. We have used a new technique that uses hadrons produced in deep-inelastic lepton scattering. We used the data at high x, where the gluon distribution function is relatively well known, to determine the fragmentation parameter $\sigma_{frag.P_T}$; we then used this information to determine the gluon distribution function at small values of η. We find that it can be described by $\eta G(\eta) \propto \eta^\lambda$ with $\lambda = -0.87 \pm 0.09(stat.) \pm^{0.32}_{0.37} (sys.)$.

Our result for $\eta G(\eta)$, within errors, agrees with recent results from HERA and also with the GRV-HO parametrization of the gluon distribution function.

Postscript. Recently there have been attempts in deep inelastic lepton scattering to calculate the contribution of multiple soft gluon emission when a parton is produced very close to the direction of the virtual photon. This has been done by summing the perturbation

Change	$\sigma_{frag.P_T}$ in MeV	λ
Ignore QED radiative events in the Monte Carlo	372 ± 20	-0.85 ± 0.10
Raise m_a to 0.75 GeV	368 ± 20	-0.85 ± 0.08
Raise x_{jet} to 0.025	365 ± 25	-0.90 ± 0.09
Use default values for m_a and x_{jet}	382 ± 20	-0.85 ± 0.08
Raise minimum W^2 cut to 400 GeV^2	370 ± 30	$-0.65\pm^{0.15}_{0.10}$
Raise minimum Q^2 cut to 4 GeV	366 ± 20	-0.95 ± 0.10
Lower maximum Q^2 cut to 4 GeV	366 ± 20	-0.70 ± 0.12
Require track slopes $\|z'\| < 25$ mr, $\|y'\| < 60$ mr	359 ± 20	-0.95 ± 0.09
Exclude exclusive ρ^0 candidate events	367 ± 23	-0.95 ± 0.08
Use GRV-LO parton distribution functions	360 ± 22	-0.80 ± 0.09
Require $0.0 < sin^2\chi < 0.3$	366 ± 16	-0.85 ± 0.07
Use LEPTO4.3 and JETSET6.3	387 ± 22	-0.82 ± 0.10
Use LEPTO6.1 and JETSET7.3	312 ± 20	-1.10 ± 0.12
Change the low-x region to $0.005 < x < 0.06$	366 ± 20	-0.86 ± 0.09
Sensitivity to gluon dbn. fun. parametrization	366 ± 20	See fig.7
Use hydrogen data	350 ± 44	-1.05 ± 0.15

Table 3: A summary of the cross-checks.The errors quoted in columns 2 and 3 are statistical errors.

series to all orders [42]. In ref. [42] a variable similar to our $sin^2\theta$ has been calculated using such a technique. At present there do not exist complete calculations for $sin^2\chi$. Since in our analysis we do not consider the small $sin^2\chi$ region this contribution is not expected to be large. In addition, in Section 8.1 we have shown that our result does not change even when we include the small $sin^2\chi$ region. This suggests that multiple gluon emission will probably not have much of an effect on the result of this analysis.

Acknowledements. We wish to thank R.Rückl for his critical comments and valuable suggestions.

This work was performed at the Fermi National Accelerator Laboratory, which is operated by Universities Research Association, Inc., under contract DE-AC02-76CHO3000 with the U.S. Department of Energy. The work of the University of California, San Diego was supported in part by the National Science Foundation, contract numbers PHY82-05900, PHY85-11584, and PHY88-10221; the University of Illinois at Chicago by NSF contract PHY88-11164; and the University of Washington by NSF contract numbers PHY83-13347 and PHY86-13003. The University of Washington was also supported by the U.S. Department of Energy. The work of Argonne National Laboratory was supported by the Department of Energy, Nuclear Physics Division, under Contract No. W-31-109-ENG-38. The Department of Energy, High Energy Physics Division, supported the work of Harvard University, the University of Maryland, the Massachussetts Institute of Technology under Contract No. DE-AC02-76ER03069, Northwestern University under Contract No. DE-FG02-91ER40684, and Yale University. Work at Northwestern University was also funded by the A.P. Sloan Foundation. The Albert-Ludwigs-Universität Freiburg and the University of Wuppertal were supported in part by the Bundesministerium für Forschung und Technologie. The work of the Institute for Nuclear Physics, Krakow, was supported in part by the Polish Committee for Scientific Research under grant No. 2P03B23008. The work of the KFKI, Institute for Particle and Nuclear Physics, Budapest, was supported by the

Hungarian Science Foundation under grant No. OTKA T 16609.

References

[1] See for example D. Espriu, Proceedings of the XXII International Meeting on Fundamental Physics, The Standard Model, Jaca (Huesca), Spain, February 7-11, 1994, edited by J.A. Villar and A. Morales

[2] M. Glück, E. Reya, A. Vogt, Z. Phys. C53 (1992) 127

[3] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45 (1977) 199 ; Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822

[4] A. Donnachie and P.V. Landshoff, Nucl. Phys. B244 (1984) 322; Z. Phys. C61 (1994) 139

[5] R.D. Peccei and R. Rückl, Nucl. Phys. B162 (1980) 125

[6] Particle Data Group, Phys. Rev. D45,Part 2 (1992)

[7] W.J. Stirling, in Proceedings of the 1987 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 27-31 July 1987, ed. by W.Bartel and R. Ruckl (North-Holland, Amsterdam, 1988), p.715

[8] CCFRR Collaboration, P.Z.Quintas *et al.*, Phys. Rev. Lett. 71 (1993) 1307

[9] K.Bazizi and S.J.Wimpenny, UCR/DIS/91-02

[10] M.Virchaux and A. Milsztajn, Phys. Lett. B274 (1992) 221

[11] T.Hebbeker, Proc. 1991 Joint Int. Lepton Photon Symposium at High Energies and European Physical Society Conference on High Energy, Geneva, 25 July - 2 August 1991, 73

[12] UA2 Collaboration, J. Alitti *et al.*, Phys. Lett. B 263 (1991) 563

[13] A.D. Martin, talk presented at the Workshop on Small-x and Diffractive Physics at the Tevatron, 16 - 18 September 1992, Fermi National Accelerator Laboratory, Batavia, Illinois

[14] G. Ingelman, CERN program library long writeups W5035,W5045

[15] T. Sjöstrand, Comput. Phys. Commun. 27 (1982) 243; 28 (1983) 229

[16] UA6 Collaboration, G. Ballocchi *et al.*, Phys. Lett. B317 (1993) 250

[17] E706 Collaboration, G. Alverson *et al.*, Phys. Rev. Lett. 68 (1992) 2584

[18] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D48 (1993) 2998

[19] UA2 Collaboration, J. Alitti *et al.*, Phys. Lett. B299 (1993) 174

[20] NMC Collaboration, M. Arneodo *et al.*, Phys. Lett. B309 (1993) 222

[21] P. Aurenche *et al.*: Phys. Rev. D39 (1989) 3275

[22] EMC Collaboration, J. Ashman *et al.*, Z. Phys. C56 (1992) 21

[23] NMC Collaboration, D. Allasia *et al.*, Phys. Lett. B258 (1991) 493

[24] H1 Collaboration, S. Aid *et al.*, Nucl. Phys. B449 (1995) 3

[25] H1 Collaboration, S. Aid *et al.*, Phys. Lett. B354 (1995) 494

[26] ZEUS Collaboration, M. Derrick *et al.*, Phys. Lett. B345 (1995) 576

[27] E665 Collaboration, M.R. Adams *et al.*, Nucl. Instrum. Methods A291 (1990) 533

[28] Wu_Ki Tung, Proceedings of the Workshop on Hadron Structure Functions and Parton Distributions, April 26 - April 28, 1990, Fermi National Accelerator Laboratory, edited by D.F. Geesaman, J. Morfin, C. Sazama, W.K. Tung, p.18

[29] M. Glück, E. Reya, A. Vogt, Phys. Lett. B306 (1993) 391-394

[30] NMC Collaboration, P. Amaudruz *et al.*, Phys. Lett. B295 (1992) 159

[31] NMC Collaboration, E. Rondio, XXVI Intern. Conf. on High energy physics (Dallas, TX, August 1992)

[32] BCDMS Collaboration, A.C. Benvenuti *et al.*, Phys. Lett. B223 (1989) 485

[33] H1 Collaboration, T. Ahmed *et al.*, Nucl. Phys. B439 (1995) 471-502

[34] E665 Collaboration, Ashutosh V. Kotwal, Ph.D. Thesis, Harvard University, Cambridge, USA, May 1995

[35] J.G. Morfin, Wu_Ki Tung, Z.Phys. C52 (1991) 13

[36] E665 Collaboration, M.R. Adams *et al.*, Phys. Rev. Lett. 74, 1525 (1995)

[37] T. Sjöstrand, Comput. Phys. Commun. 39 (1986) 347; T.Sjöstrand, LU TP 86-22

[38] G. Ingelman, LEPTO 6.1 - The Lund Monte Carlo for deep inelastic lepton-nucleon scattering, Proceedings of the workshop on Physics at HERA, October 29-30, 1991, Hamburg, Germany, edited by W. Buchmuller and G. Ingelman, p. 1366

[39] T. Sjöstrand, A Manual to The Lund Monte Carlo for Jet Fragmentation and e^+e^- Physics, JETSET version 7.3, July 1990

[40] A.D. Martin, R.G. Roberts and W.J. Stirling, University of Durham preprint DTP/94/78 (1994)

[41] H.L. Lai, J. Botts, J. Huston, J.G. Morfin, J.F. Owens, J. Qiu, W.K. Tung and H. Weerts; Preprint MSU-HEP/41024, CTEQ 404

[42] R. Meng, F.I. Olness, D.E. Soper, paper under preparation, 1995