CP Violation in Kaons: $\varepsilon'_K/\varepsilon_K$ vs $K \to 3\pi$

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The measurement of $\varepsilon'_K/\varepsilon_K$ has left open several theoretical problems, the most important of which is an accurate calculation of the relevant hadronic matrix elements. Here we study the possibility of relating ε'_K to CP violation in charged $K \to 3\pi$ which can provide information on the reliability of such calculations. We discuss the rôle of final state interactions and the main sources of uncertainties. We find that the measurement of Δg_C in $K \to 3\pi$ is a crucial consistency test for the Standard Model calculations of ε'_K. Valuable information can be deduced also from other observables.

1 Status of ε'_K and its theoretical calculations

The experimental measurement of $\varepsilon'_K/\varepsilon_K$ by KTeV [1] and by NA48 [2] has proved to be a fundamental test for the understanding of CP violation in the Standard Model (SM). The world average of the experimental results [1,2,3,4] is

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) = (16.7 \pm 1.6) \cdot 10^{-4}. \tag{1}$$

The theoretical prediction for this observable has a long history and shown to be rather difficult. One needs several steps in order to arrive to a final prediction.

1.1 Short-distance contributions

The flavor changing phenomenon is born at a scale of the order of the gauge bosons mass, $\mathcal{O}(100~\mathrm{GeV})$, which is much bigger than the Kaon mass. Due to this difference the gluonic corrections are amplified by large logarithms. These gluonic corrections are summed up using the Operator Product Expansion (OPE) and renormalization group equations. The running of the corresponding Wilson coefficients has been performed up to two loops and is a well established result [5,6]. The Lagrangian for $\Delta S = 1$ processes in the three flavor theory is

$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} = \tilde{C} \sum_{i=1}^{10} C_i(\nu)\, Q_i(\nu) \; ; \qquad \tilde{C} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \; . \tag{2}$$

The renormalization scale ν separates the short-distance part contained in the Wilson coefficients $C_i(\nu)$ and the long-distance part encoded in the operators $Q_i(\nu)$. The final result is independent on this scale. The operators which are found to give the main contributions to direct CP violation are the hadronic penguin $Q_6(\nu)$ and the electroweak penguin $Q_8(\nu)$.

1.2 Long-distance

The left and most difficult problem is the calculation of the relevant hadronic matrix elements. On the other hand Chiral Perturbation Theory (CHPT) is the effective field theory of the SM [7,8,9], which describes the interaction of the low-lying pseudo-Goldston bosons and external

sources. At lowest order in the chiral expansion, $e^2 p^0$ and $e^0 p^2$, the effective realization of the Lagrangian in (2) is

$$\mathcal{L}^{(2)}_{|\Delta S|=1} = \frac{3}{5} \widetilde{C} F_0^6 e^2 G_E \, \text{tr} \left(\Delta_{32} u^\dagger Q u \right) + \frac{3}{5} \widetilde{C} F_0^4 \left[G_8 \, \text{tr} \left(\Delta_{32} u_\mu u^\mu \right) + G'_8 \text{tr} \left(\Delta_{32} \chi_+ \right) \right.$$
$$\left. + \; G_{27} \, t^{ij,kl} \, \text{tr} \left(\Delta_{ij} u_\mu \right) \, \text{tr} \left(\Delta_{kl} u^\mu \right) \right] + \text{h.c.} \tag{3}$$

with F_0 the chiral limit value of the pion decay constant $f_\pi = (92.4 \pm 0.4)$ MeV see ref. [10] for definitions. The real parts of the couplings G_8 and G_{27} can be measured from the CP conserving observables while their imaginary parts are responsible for CP violation. The theoretical determination of these couplings is the challenge. The main couplings which come in the estimation of ε'_K are Im G_8 and Im $(e^2 G_E)$. The calculation of the $K \to \pi\pi$ amplitudes has been done at NLO in CHPT [11,12,13] including isospin breaking [14]. In ε'_K there is a cancellation between the contributions proportional to these two couplings which is very much reduced when all the relevant effects are taken into account.

1.3 The unknown couplings

At large N_c, all the contributions to Im G_8 and Im $(e^2 G_E)$ are factorizable and the scheme dependences are not under control. The unfactorizable topologies are not included at this order and they bring in unrelated dynamics with its new scale and scheme dependences, so that it is difficult to give an uncertainty to the large N_c result for Im G_8 and Im $(e^2 G_E)$.

There are also calculations which take into account NLO large-Nc corrections and also lattice computations. The most recent results are summarized in Fig. 1. Here the horizontal band represents the region allowed by the experimental value of ε'_K. The rectangle on the right is the result from [15,16]. The rectangle on the left is the result from [17,18]. The vertical band shows the lattice findings on Im $(e^2 G_E)$ [19,20]. The leading order large-N_c result is marked with a small circle.

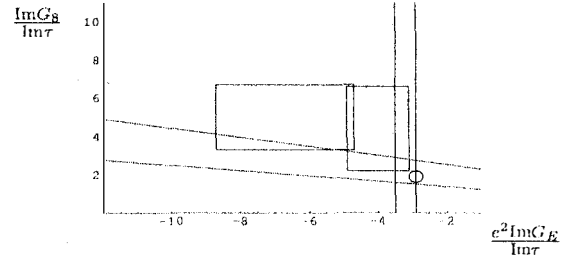Figure 1: Status of the most recent determination of the chiral weak couplings confronting ε'_K. The horizontal band is the allowed by the experimental result on ε'_K. The theoretical results discussed in section 1 are depicted.

2 CP violation observables in charged $K \to 3\pi$

CP violating observables in $K \to 3\pi$ are both experimentally and theoretically very promising and have attracted a lot of recent effort (see ref. [10] for a complete discussion of other works [21,22,23,24]).

In [10], we discussed CP-violating asymmetries in the decay of the charged Kaon into three pions; namely, asymmetries in the slope g defined as

$$\frac{|A_{K^+ \to 3\pi}(s_1, s_2, s_3)|^2}{|A_{K^+ \to 3\pi}(s_0, s_0, s_0)|^2} = 1 + g\,y + h\,y^2 + k\,x^2 + \mathcal{O}(yx^2, y^3) \tag{4}$$

and some asymmetries in the integrated $K^+ \to 3\pi$ decay rates. Above, we used the Dalitz variables $x \equiv \frac{s_1 - s_2}{m_{\pi^+}^2}$ and $y \equiv \frac{s_3 - s_0}{m_{\pi^+}^2}$ with $s_i \equiv (k - p_i)^2$, $3s_0 \equiv m_K^2 + m_{\pi(1)}^2 + m_{\pi(2)}^2 + m_{\pi(3)}^2$. The CP-violating asymmetries in the slope g are defined as

$$\Delta g_C \equiv \frac{g[K^+ \to \pi^+\pi^+\pi^-] - g[K^- \to \pi^-\pi^-\pi^+]}{g[K^+ \to \pi^+\pi^+\pi^-] + g[K^- \to \pi^-\pi^-\pi^+]}$$

$$\text{and} \quad \Delta g_N \equiv \frac{g[K^+ \to \pi^0\pi^0\pi^+] - g[K^- \to \pi^0\pi^0\pi^-]}{g[K^+ \to \pi^0\pi^0\pi^+] + g[K^- \to \pi^0\pi^0\pi^-]}. \tag{5}$$

The CP-violating asymmetries in the decay rates, $\Delta\Gamma_{C(N)}$, are defined analogously.

Recently, two experiments, namely, NA48 at CERN and KLOE at Frascati, have announced the possibility of measuring the asymmetries Δg_C and Δg_N with a sensitivity of the order of 10^{-4}, i.e., two orders of magnitude better than at present[25], see for instance[26] and[27]. It is therefore mandatory to have these predictions at NLO in CHPT as they are provided in[10].

2.1 CP-Violating Predictions at Leading Order

The slopes g and decay rates Γ start at order p^2 in CHPT. We have used the LO result to make some checks. We have checked that the effect of Re $(e^2 G_E)$ is very small also for the Δg and $\Delta\Gamma$ asymmetries. We have also checked that the asymmetries $\Delta g_{C(N)}$ are very poorly sensitive to Im $(e^2 G_E)$. This fact makes an accurate enough measurement of these asymmetries very interesting to check if Im G_8 can be as large as predicted in[15,18,28]. It also makes these CP-violating asymmetries complementary to the direct CP-violating parameter ε'_K. The same poor dependence on Im$(e^2 G_E)$ is observed in the LO result for the total widths asymmetries[10]. However, in order to make a more precise analysis, it is convenient to go at NLO in CHPT.

2.2 CP-Violating Predictions at Next-to-Leading Order

At NLO one needs the real parts of the amplitudes at order p^4 and the FSI at order p^6. The real part of the amplitudes for the octet and 27-plet components were recently computed in[29] and checked by us in[10]. The electroweak part and the relevant FSI were computed in[10]. We refer to these papers for the details.

To describe $K \to 3\pi$ at NLO, in addition to Re G_8, G_{27}, Re $(e^2 G_E)$, Im G_8 and Im $(e^2 G_E)$, we also need several other ingredients. Namely, for the real part we need the chiral logs and the counterterms. The relevant counterterm combinations were called $\widetilde{K}_i$ in[29]. The real part of the counterterms, Re $\widetilde{K}_i$, can be obtained from the fit of the $K \to 3\pi$ CP-conserving decays to data done in[29].

The imaginary part of the order p^4 counterterms, Im $\widetilde{K}_i$, is much more problematic. They cannot be obtained from data and there is no available calculation for them at NLO in $1/N_c$. A direct calculation of them at NLO in $1/N_c$ can be done using the appropriate hadronic Green functions a scheme to get them has been setted up recently in[30].

One can use several approaches to get the order of magnitude and/or the signs of Im$\widetilde{K}_i$. We will follow here a more naive approach that will be enough for our purpose of estimating the effect of the unknown counterterms. We can assume that the ratio of the real to the imaginary
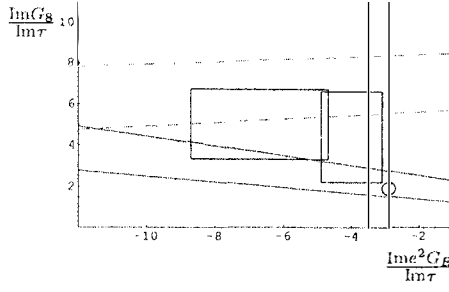
Figure 2: $\Delta g_C \sim -3.5 \cdot 10^{-5}$ (band between blue dashed lines) confronting the experimental value of ε'_K (band between red solid lines) and its theoretical estimates (see text).
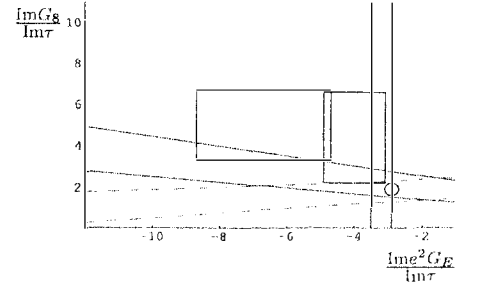
Figure 3: $\Delta g_C \sim -10^{-5}$ (band between blue dashed lines) confronting the experimental value of ε'_K (band between red solid lines) and its theoretical estimates (see text).

parts is dominated by the same strong dynamics at LO and NLO in CHPT, therefore

$$\frac{\mathrm{Im}\widetilde{K_i}}{\mathrm{Re}\widetilde{K_i}} \simeq \frac{\mathrm{Im}G_8}{\mathrm{Re}G_8} \simeq \frac{\mathrm{Im}G'_8}{\mathrm{Re}G'_8} \simeq (0.9 \pm 0.3)\,\mathrm{Im}\tau \,. \tag{6}$$

In particular, we set to zero those $\mathrm{Im}\,\widetilde{K_i}$ whose corresponding $\mathrm{Re}\,\widetilde{K_i}$ are set also to zero in the fit to CP-conserving amplitudes done in [29]. Of course, the relation above can only be applied to those $\widetilde{K_i}$ couplings with non-vanishing imaginary part. Octet dominance to order p^4 is a further assumption implicit in (6). The second equality in (6) is well satisfied by the model calculation in [15], which is the only full calculation at NLO in $1/N_c$ at present.

The values of $\mathrm{Im}\,\widetilde{K_i}$ obtained using (6) will allow us to check the counterterm dependence of the CP-violating asymmetries. They will also provide us a good estimate of the counterterm contribution to the CP-violating asymmetries that we are studying.

The final results for the slope asymmetries are

$$\frac{\Delta g_C}{10^{-2}} \simeq \left[(0.66 \pm 0.13)\,\mathrm{Im}G_8 + (4.3 \pm 1.6)\,\mathrm{Im}\widetilde{K_2} - (18.1 \pm 2.2)\,\mathrm{Im}\widetilde{K_3} - (0.07 \pm 0.02)\,\mathrm{Im}(e^2 G_E)\right],$$

$$\frac{\Delta g_N}{10^{-2}} \simeq - \left[(0.04 \pm 0.08)\,\mathrm{Im}G_8 + (3.7 \pm 1.1)\,\mathrm{Im}\widetilde{K_2} + (26.3 \pm 3.6)\,\mathrm{Im}\widetilde{K_3}\right.$$
$$\left. + (0.05 \pm 0.02)\,\mathrm{Im}(e^2 G_E)\right]\,. \tag{7}$$

Similar numerical formulas can be deduced also for the width asymmetry and can be found in ref. [10]. From these formulas one deduces that Δg_C is a quite clean observable to measure $\mathrm{Im}\,G_8$ and so cross-check the result of ε'_K. Using the inputs of ref. [16] for $\mathrm{Im}G_8$ and $\mathrm{Im}(e^2\,G_E)$ and the estimation of the counterterms contributions of ref. [10] one finds

$$\Delta g_C = -(2.4 \pm 1.2) \cdot 10^{-5}\,; \qquad \Delta g_N = (1.1 \pm 0.7) \cdot 10^{-5}\,. \tag{8}$$

3   Conclusions

The impact of the eventual measurement of Δg_C is depicted in Fig. 2 and in Fig. 3. A value of $10^5 \cdot \Delta g_C < -4$ would be a sign of new physics. For $-2 > 10^5 \cdot \Delta g_C > -4$ the high values of

$\text{Im}G_8$ would be confirmed, although the measure would not be in straight agreement with the measurement of ε'_K. A value of $10^5 \cdot \Delta g_C \sim -1$ would fit perfectly the SM value of ε'_K. An experimental precision of the order of $(1 \sim 1.5) \cdot 10^{-5}$ would be required.

The asymmetry Δg_N and the asymmetries in the total widths would give also important informations on the counterterms magnitude but not so much on $\text{Im} \, G_8$ and $\text{Im} \, (e^2 G_E)$. An example is provided in Fig. 4. In this figure we use

$$\frac{\text{Im}\widetilde{K}_2}{\text{Re}\widetilde{K}_2} \simeq \omega \frac{\text{Im}\widetilde{K}_3}{\text{Re}\widetilde{K}_3} \simeq k \frac{\text{Im}G_8}{\text{Re}G_8} \tag{9}$$

and we plot $\Delta g_N \cdot 10^5$ versus k for several values of the parameter ω. An upper limit on Δg_N of 10^{-5} would already put a valuable constraint on k, namely $k < 3$. The total width asymmetries are even more difficult to predict [10].

The measurement of all these charged $K \to 3\pi$ CP asymmetries would certainly be an experimental achievement and would provide extremely precious information for the understanding of the mechanism of CP-violation.

Acknowledgments

Misprints in reference [10]

We report here some misprints[a] found in our original paper[10]. The signs of the rhs of Eqs. B.12 and B.15 must be flipped except for A_c. In Eq. B.29 the counterterm Z_8^r must be proportional to $9(m_K^2 - 2m_\pi^2)$ and not to $9(m_K^2 - m_\pi^2)$.
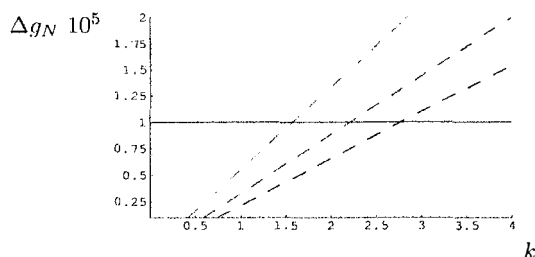

Figure 4: Constraints on the counterterms coming from an eventual upper limit of Δg_N of 10^{-5} (horizontal line). The three dashed lines correspond respectively from left to right to $\omega = 1/2$, 1, 2.

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