On the Meissner-like effect of an extreme black hole

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Abstract

It is known that the Meissner-like effect is seen in the vacuum solutions of black-hole magnetosphere: no non-monopole component of magnetic flux penetrates the event horizon if the black hole is extreme. In this article, in order to see the effects of charge currents, we study the force-free magnetic field on the extreme Reissner-Nordström background. In this case, we should solve one elliptic differential equation called the Grad-Shafranov equation which has singular points called light surfaces. Due to the surfaces, it is difficult to solve the equation in the region from the event horizon to infinity. In order to see the Meissner effect, we consider the region near the event horizon and try to solve the equation by Taylor expansion about the event horizon. Moreover, we assume that the small rotational velocity of the magnetic field, and then, we construct a perturbative method to solve the Grad-Shafranov equation considering the effect of the inner light surface and study the behavior of the magnetic field near the event horizon.

1 Introduction

It is widely believe that there is a super massive black hole in the center of a galaxy. The black holes are expected as engines of active galactic nucleis(AGNs) and gamma ray bursts(GRBs). There are two kinds of energy source. One is the gravitational energy of accreting matter. The other is the rotational energy of an accretion disk or a black hole. In this article, we concentrate on the rotational energy of a black hole and consider the Blandford-Znajek(BZ) mechanism which is one of ways to extract the rotational energy [1]. The BZ mechanism is expected as a mechanism that supports jets in AGNs.

The efficiency of the BZ mechanism is getting larger if the magnetic field lines penetrating the event horizon increase. However, it is known that stationary axisymmetric magnetic fields in a vacuum are expelled from the event horizon of an extremely rotating black hole[2]. This effect is analogous to the Meissner effect in superconductors. The efficiency of the BZ mechanism would decrease by this Meissner effect of the black holes. Since there would be plasma around a rotating black hole in realistic astrophysical cases, it is important to study the effect of charge current for the Meissner effect of the black holes.

The realistic situation is complicated to treat, then we consider a simple toy model in this article; (i)we consider a system of electromagnetic fields and plasma which is stationary, axisymmetric and force-free, (ii)we use a static and spherical black hole spacetime with a degenerate horizon as a background geometry instead of a Kerr spacetime. It is known that a static and spherical symmetric black hole with a degenerate horizon shows the Meissner effect of the black hole[3]. Moreover, we assume that the small rotational velocity of the magnetic field, and then, we study the behavior of the magnetic field near the event horizon by using a pertubative method.

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2 Background geometry

We consider a static and spherical symmetric spacetime with the metric in the form

$$ds^2 = -\frac{\Delta}{r^2}dt^2 + \frac{r^2}{\Delta}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where $\Delta = (r - r_+)(r - r_-)$. The spacetime is known as the Reissner-Nordström(RN) spacetime. The radius of the event horizon is determined by $\Delta = 0$, therefore, there are two event horizons at $r = r_\pm$ in this spacetime. In the case of $r_+ = r_-$, these two horizons coincide, and such a case is called the extreme one.

3 Grad-Shafranov equation

In order to study the effect of charge currents for the Meissner effect of the black hole, we consider stationary and axisymmetric electromagnetic fields on the RN background [6]. Moreover, we ignore the rest mass of plasma, that is, we assume force-free. The formulation of the force-free electrodynamics on a black hole background was constructed by Macdonald and Thorne[4]. Our formulation is based on their work. In this system, we can get one quasi-linear elliptic equation for the magnetic flux, $\Psi(r, \theta)$, so called the Grad-Shafranov(GS) equation,

$$\partial_r^2\Psi + \frac{\sin\theta}{\Delta}\partial_\theta\left(\frac{\partial_\theta\Psi}{\sin\theta}\right) + \frac{U}{D} + \frac{W}{\Delta D} = 0, \tag{2}$$

where

$$U = \left(\partial_r D + \frac{r^2}{2}\sin^2\theta\frac{d\Omega_{\rm F}^2}{d\Psi}\partial_r\Psi\right)\partial_r\Psi, \quad W = \left(\partial_\theta D + \frac{r^2}{2}\sin^2\theta\frac{d\Omega_{\rm F}^2}{d\Psi}\partial_\theta\Psi\right)\partial_\theta\Psi + 8\pi^2 r^2\frac{dI^2}{d\Psi}$$

$$\text{and} \quad D = \frac{\Delta}{r^2} - r^2\Omega_{\rm F}^2\sin^2\theta, \tag{3}$$

$\Omega_{\rm F} = \Omega_{\rm F}(\Psi)$ is the angular velocity of the magnetic field line and $I = I(\Psi)$ is the charge current. In general, the GS equation for the black hole magnetosphere has three regular singular points; one is at the event horizon $\Delta = 0$ and the other two are at the light surfaces determined by $D = 0$. The speed of the magnetic field line becomes the speed of light at the light surfaces.

Due to those singular points, it is difficult to obtain a smooth solution of the GS equation in the global region from the event horizon to the infinity. Our purpose is to see the effect of charge current for the Meissner effect, therefore, we consider a region near the event horizon. Moreover, we assume a small rotating magnetic field. In this case, the inner light surface locates near the event horizon, thus, we can solve the GS equation by Taylor expansion about the event horizon in the region including the inner light surface. The outer light surface is not considered in our approach, but our study would be sufficient to see the Meissner effect of the black hole.

4 Perturbative method

In this section, we focus on the extreme case $r_+ = r_- := r_{\rm H}$ and, for simplicity, assume that the rotational velocity of the magnetic field $\Omega_{\rm F}$ is constant.

4.1 Basic equations

First, we introduce following dimensionless quantities

$$r =: r_{\rm H}y, \quad \Psi =: r_{\rm H}\psi, \quad \varepsilon := r_{\rm H}\Omega_{\rm F},$$

$$I(\Psi) =: \varepsilon\mathcal{I}(\psi) \quad \text{and} \quad 8\pi^2\frac{dI^2}{d\Psi} =: \varepsilon^2 r_{\rm H}^{-1}\mathcal{S}(\psi). \tag{4}$$

[6] Note that we only use the RN spacetime as a background instead of a Kerr spacetime. A RN black hole is a solution of charged black hole in Einstein-Maxwell system, but we treat in the way that the electromagnetic field considered in this article is not related to the Maxwell feild created by the source of the RN black hole. Perturbations of Einstein-Maxwell system without charge currents of the RN black hole was studied by Bičák and Dvořák[3]. They showed that the Meissner effect of the black hole appears in this case.

The regularity conditions for the electromagnetic field at the event horizon lead to

$$\mathcal{I} + \frac{1}{4\pi}\sin\theta\partial_\theta\psi = 0 \ \text{ at } \ y=1, \tag{5}$$

$$\frac{d\mathcal{I}}{d\psi}\partial_y\psi + \frac{1}{4\pi}\sin\theta\partial_\theta(\partial_y\psi) = 0 \ \text{ at } \ y=1. \tag{6}$$

From these conditions, we can obtain the GS equation at the event horizon as

$$\partial_y^2\psi + \frac{1}{2}L_\theta\psi + 2\partial_y\psi + \frac{1}{2}\partial_y^2\left(\frac{\mathcal{W}}{\mathcal{D}}\right) = 0 \ \text{ at } \ y=1, \tag{7}$$

where

$$\mathcal{D} = y\left[(y-1)^2 - \varepsilon^2 y^4\sin^2\theta\right] \quad \text{and} \quad \mathcal{W} = -\varepsilon^2 y^5\left[\sin 2\theta\partial_\theta\psi - \mathcal{S}(\psi)\right]. \tag{8}$$

Moreover, the inner light surface regularity condition can be written as

$$\partial_\theta\psi - \frac{\mathcal{S}}{\sin 2\theta} - \varepsilon y\tan\theta\sin\theta(1-\varepsilon y^2\sin\theta)\partial_y\psi = 0 \ \text{ at } \ y = y_{\mathrm{LS}_-}, \tag{9}$$

where y_{LS_-} is the location of the inner light surface.

Here we assume the ψ can be written in the form of Taylor series around $y=1$ as

$$\psi(y,\theta) = \sum_{n=0}^{\infty}\psi^{(n)}(\theta)(y-1)^n. \tag{10}$$

If we fix the functional form of $\mathcal{I}(\psi)$, the coefficients $\psi^{(n)}$ for $n \le 2$ are determined by Eqs. (5), (6), (7) and (9). In order to get $\psi^{(n)}$ of $n \ge 3$, we use the derivative of the GS equation with respect to y. The functional form of $\mathcal{I}(\psi)$ must be determined so that the regularity condition (9) at the inner light surface is satisfied [5][6][7][8].

4.2 Perturbative analysis

Hereafter, assuming $0 < \varepsilon << 1$, we construct a perturbative solution for ψ near the event horizon. We assume

$$\psi^{(n)} = \sum_{n=0}^{\infty}\psi_N^{(n)}\varepsilon^N \quad \text{and} \quad \mathcal{I}(\psi) = \sum_{n=0}^{\infty}\mathcal{I}_{N+1}(\psi)\varepsilon^N. \tag{11}$$

Note that $\mathcal{I}_1$ which is the lowest order of $\mathcal{I}$ is the first order of the charge current because we assume $I = \varepsilon\mathcal{I}$. Because of $y_{\mathrm{LS}_-} - 1 = \mathcal{O}(\varepsilon)$, the inner light surface regularity condition can be written in the form of Taylor series around $y=1$. Thus, we can obtain the regularity conditions in each order of ε and construct a perturbative solution for $\psi^{(n)}$ by solving Eqs. (5), (6), (7) and (9) in each order of ε. In this article, we construct a perturbative solution with corrections up to $\mathcal{O}(\varepsilon^2)$. Since $y_{\mathrm{LS}_-} - 1 = \mathcal{O}(\varepsilon^1)$, the magnetic flux up to $\mathcal{O}(\varepsilon^2)$ behaves near the inner light surface as

$$\psi(y,\theta) = \psi_0^{(0)}(\theta) + \varepsilon^1\psi_1^{(0)}(\theta) + \varepsilon^2\psi_2^{(0)}(\theta) + \left(\psi_0^{(1)}(\theta) + \varepsilon^1\psi_1^{(1)}(\theta)\right)(y-1) + \psi_0^{(2)}(\theta)(y-1)^2 + \mathcal{O}(\varepsilon^3). \tag{12}$$

The results are as follows:

$$\psi_0^{(0)} = C_0^{(0)}(1-\cos\theta), \quad \psi_1^{(0)} = 0, \quad \psi_2^{(0)} = \left[C_2^{(0)}\cos\theta(3\cos^2\theta - 7) + \frac{C'C_0^{(0)}}{4}(\cos^2\theta - 5)\right]\sin^2\theta,$$
$$\psi_0^{(1)} = C'C_0^{(0)}\sin^2\theta, \quad \psi_1^{(1)} = 0 \ \text{ and } \ \psi_0^{(2)} = \frac{1}{2}\left[3\left(C_0^{(0)}C'^2 - 10C_2^{(0)}\right)\cos\theta + C'C_0^{(0)}\right]\sin^2\theta, \tag{13}$$

where $C_0^{(0)}$, $C_2^{(0)}$ and C' are integration constants. Moreover the small charge current are obtained as

$$\mathcal{I}_1 = -\frac{C_0^{(0)}}{4\pi}\hat{X}\left(2-\hat{X}\right), \quad \mathcal{I}_2 = 0 \quad \text{and}$$
$$\mathcal{I}_3 = \frac{1}{8\pi^2}\hat{X}^2\left(2-\hat{X}\right)^2\left[C_0^{(0)}C'\left(1-\hat{X}\right) + 2C_2^{(0)}\left(2 - 18\hat{X} + 9\hat{X}^2\right)\right], \tag{14}$$

where $\hat{X} = \psi_0^{(0)}/C_0^{(0)}$. The zeroth order of $\psi^{(n)}$ represent a vacuum solution because $\Omega_{\rm F}$ and I become zero in the limit $\varepsilon \to 0$. Therefore, $\psi_0^{(0)}$ represents a monopole solution in the vacuum. It means the Meisnner effect appear in the vacuum case if we accept the fact that there is no magnetic monopole charge in the nature. As a result, our perturbative solutions show magnetic fields with the small corrections to the monopole due to the existence of the charge currents. Here we should note that non-trivial solutions are obtained by our pertubative analysis, only if the configuration of the zeroth order solution $\psi_0^{(0)}$ is monopole, namely, $C_0^{(0)} \neq 0$. If we require $\psi_0^{(0)} = 0$, by integrating Eq. (5) of $\mathcal{O}(\varepsilon^1)$, we can obtain

$$\psi_1^{(0)} = C_1^{(0)} \left[\frac{\sin^2\theta}{(1+\cos\theta)^2} \right]^{-2\pi \frac{d\mathcal{I}_1}{d\psi}(0)}, \tag{15}$$

where $C_1^{(0)}$ is an integration constant. If we impose no-magnetic monopole charge, $\psi(\pi) = 0$, the integration constant should be $C_1^{(0)} = 0$, therefore, we get the trivial solution $\psi_1^{(0)} = 0$. Moreover, we get the same result for all order. Thus, we can obtain the only trivial solution $\psi_N^{(0)} = 0$ in the case $\psi_0^{(0)} = 0$.

5 Summary and Discussion

We constructed an approximate solution of the GS equation considering the inner light surface regularity condition in the region near the event horizon in the case of slow rotating magnetic field by using a perturbative analysis. Non-trivial solutions obtained by our perturbative analysis in the form of deformed monopole fields. However, we can not conclude that the Meissner effect of the black hole appears even if the current exists because our perturbative analysis dose not include all perturbative solutions of vacuum configurations. For example, we know an exact solution of GS equation as

$$\psi = C(1-\cos\theta) \quad \text{with} \quad \mathcal{I} = \frac{\psi}{4\pi}\left(2 - \frac{\psi}{C}\right), \tag{16}$$

where C is an integration constant, and the perturvative solution of (16) can not be obtained by our perturbative analysis with $\psi_0^{(0)} = 0$. Thus, there is a possibility that perturbative solutions with vanishing $\psi_0^{(0)}$. The origin of this possibility may come from the fact that the assumption for the charge current (11) is too strong. Unfortunately, the persent perturbation analysis would be hard to do without this assumption. According to Komissarov and McKinney[9], there was no sign of the Meissner effect in highly conductive magnetoshperes. Our results are not opposed to their results. In order to argue the disappearance of the Meissner effect, we should construct a magnetic configuration with only non-monopole components. This is a future work.

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