

Anisotropic quark star and the energy conditions

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Abstract. One of the compact objects that are attractive for investigating their energy conditions is quark stars. The existence of radial and tangential pressure differences in quark stars can cause anisotropic effects on the stars. We focus on and examine the energy conditions of quark stars using the Einstein Field Equation Solution. The energy stability of an anisotropic quark star can be determined by evaluating the profile of the pressure and energy density of the star using the anisotropic EOS as an input. The used equation is the extended MIT Bag Model, which involves the constant B and interaction parameter a_4 and the corresponding parameters of anisotropic part. It is known that parameter a_4 affects the mass distribution of quark stars to be more anisotropic. This anisotropic pressure also affects the energy condition profile of the star. We find the energy state of an anisotropic quark star satisfies the energy state of an ideal fluid.

1. Introduction

For a compact object like a quark star, the properties such as mass and radius depend on the EOS of matter model and the gravity theory used. General relativity is usually used to limit theoretical possibilities of the gravity theory. However, the quark star pressure is commonly assumed as an isotopically perfect ideal fluid. On the other hand, other studies have shown that solid bodies have an anisotropic distribution of pressure, such as quark stars. Therefore, to have more realistic description, we need to use anisotropic EOS model. Anisotropy is modeled in two forms of expression for pressure. Radial pressure is written as linear EOS, while complex expression that depends on radial coordinates is tangential pressure [5]. Neutron stars are stars that result from supernova explosions. When the red supergiant collapses are more violent, it takes a denser solid object, namely a neutron star, to maintain its shape. However, when a supernova for a massive star is denser than a neutron star and when the collapse does not cause the pressure inside the star to be no more intense to form a black hole, a sea of quarks is formed. Thus, a quark star can be considered as intermediate compactness between a neutron star and a black hole. Vergara et al. [1] recently describe the interaction of quarks in anisotropic quark stars using anisotropic EOS known as the modified MIT bag Model which involves the bag constant B term, interaction term with parameter a_4 , and other correction terms including the logarithmic and anisotropic pressure terms. Meanwhile, in reference [2], Setiawan et al. investigated the energy conditions in anisotropic neutron stars using several anisotropic models. Energy conditions analyses are needed to check how realistic the EOS used to describe stars. The quark star matter model proposed by Vergara et al. [1], it is not yet been extensively studied previously. In this proceeding, we investigated the energy condition of quark stars based on the EOS proposed in Ref. [1] using the standard Einstein Field Equation Solution.



2. EOS Equation

The EOS equation for quark stars is the extended MIT bag proposed in Ref. [1], can be expressed as

$$P = \frac{1(\epsilon - 4B)}{3} - \frac{m_s^2}{3\pi} \sqrt{\frac{\epsilon - B}{a_4}} + \frac{m_s^4}{12\pi^2} \left[1 - \frac{1}{a_4} + 3 \ln \left(\frac{8\pi}{3m_s^2} \sqrt{\frac{\epsilon - B}{a_4}} \right) \right] \quad (1)$$

Where P is the radial pressure, ϵ is the energy density, a_4 is the parameter from the QCD correction on the pressure, m_s is the quark strange mass whose value is 100 MeV and B is the bag constant whose values set from $52 \text{ MeV}/fm^3$ to $92 \text{ MeV}/fm^3$.

$$P_{\perp} = P_C + \frac{1(\epsilon - 4B_{\perp})}{3} - \frac{m_s^2}{3\pi} \sqrt{\frac{\epsilon - B_{\perp}}{a_4^{\perp}}} + \frac{m_s^4}{12\pi^2} \left[1 - \frac{1}{a_4} + 3 \ln \left(\frac{8\pi}{3m_s^2} \sqrt{\frac{\epsilon - B_{\perp}}{a_4^{\perp}}} \right) \right] \\ - \frac{1(\epsilon_C - 4B_{\perp})}{3} + \frac{m_s^2}{3\pi} \sqrt{\frac{\epsilon_C - B_{\perp}}{a_4^{\perp}}} - \frac{m_s^4}{12\pi^2} \left[1 - \frac{1}{a_4} + 3 \ln \left(\frac{8\pi}{3m_s^2} \sqrt{\frac{\epsilon_C - B_{\perp}}{a_4^{\perp}}} \right) \right] \quad (2)$$

where P_{\perp} is the tangential pressure, P_C and ϵ_C being the radial pressure and the energy density, respectively, at the center of the star, B_{\perp} and a_4^{\perp} are the contributing parameters on the tangential component of the pressure. The values of B_{\perp} and a_4^{\perp} are set in the same range of values B and a_4 .

3. TOV Equation

The TOV equation is an equation that can be used to calculate the mass and radius of a star. This equation is obtained by solving the Einstein Field Equation and the Energy Momentum Tensor. The general expression for an anisotropic quark star (in $c=1$ unit) is given by

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \quad (3)$$

$$\frac{dP}{dr} = - \frac{(\epsilon + P)(m + 4\pi r^2 P)}{r^2 (1 - \frac{2Gm}{r})} - \frac{2}{r} (P - P_{\perp}) \quad (4)$$

where r is the radial coordinate, $\epsilon(r)$ and $P(r)$ are the density and pressure, respectively of the star at radius r . The quantity $m(r)$ is the total mass of the star within r .

4. Energy Conditions

In this work, we use the TOV equations to calculate the profiles of the matter quantities. Then, the results are compared with the corresponding quantities required by ideal fluid energy conditions. If these energy conditions are satisfied by the obtained EOS profiles of the anisotropic quark stars, then we obtain a stable anisotropic quark star. Meanwhile, the energy conditions should be met by an ideal fluid EOS profile are [6-10]

1. $P \geq 0$, $P_{\perp} \geq 0$, and $\epsilon \geq 0$, means that inside the star, radial pressure, tangential pressure, and energy density have to be positive.
2. $P' = \frac{dP}{dr} \leq 0$ and $\epsilon' = \frac{d\epsilon}{dr} \leq 0$, means that inside the star, the gradient of radial pressure and energy density have to be negative.
3. $V_S^{P^2} = \frac{dP}{dr} \leq 1$ and $V_S^{P_{\perp}^2} = \frac{dP_{\perp}}{dr} \leq 1$ means that inside a star, the speed of sound for both radial and tangential pressures is always lower than the speed of light.
4. $\epsilon \geq P_{\perp}$ and $\epsilon \geq P$ means that inside a star, the energy density has to be higher than radial and tangential pressure.
5. The strong energy conditions are met for $\epsilon + P + 2P_{\perp} \geq 0$ and $\epsilon + P_{\perp} \geq 0$.

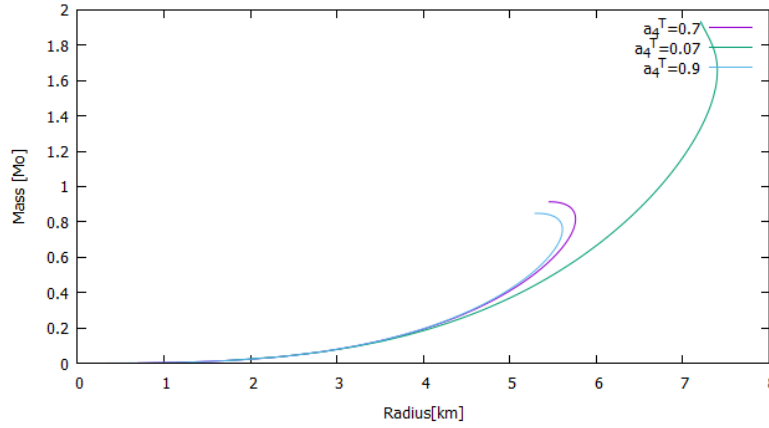


Figure 1. The mass-radius relation of anisotropic quarks with $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

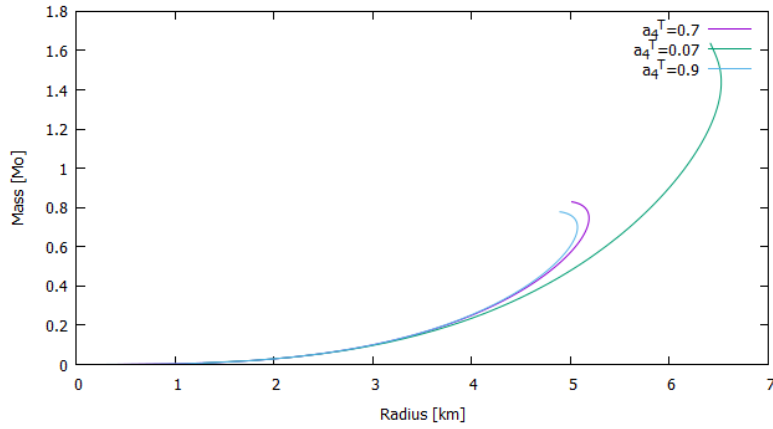


Figure 2. The mass-radius relation of anisotropic quarks with $B = B_{\perp} = 92 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

6. $-1 \leq V_S^{P^2} - V_S^{P^2} \leq 1$ where $-1 \leq V_S^{P^2} - V_S^{P^2} \leq 0$ and $0 \leq V_S^{P^2} - V_S^{P^2} - 1$ means that the acceptable range of difference between the speed of sound in the radial and tangential directions is in the range between -1 and 1.

5. Results and Discussion

The mass-radius relation for spherically symmetric solutions of anisotropic quark stars is calculated for these cases where the difference in the hydrostatic equation between the tangential and radial pressures is not zero. The anisotropy at the level of the tangential component of the pressure is confirmed by equations 1 and 2 due to spherical symmetry, then the radial constituents a_4 and B are fixed, while the tangential constituents a_4^{\perp} are varied and the bag constant B_{\perp} is fixed. The first solution for the anisotropic quark stars is shown in figure 1, when the value is set to $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ while a_4^{\perp} is varied. It is obtained that the more a smaller a_4 value, it will increase the higher the mass and radius of the anisotropic quark star, where the star mass reaches $\approx 2M_{\odot}$. In Figure 2, by increasing the value of the bag constant both in the radial and tangential directions to $B = B_{\perp} = 92 \text{ MeV}/\text{fm}^3$, the results do not change much compared to the value of $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$. Based on Figure 1 and 2,

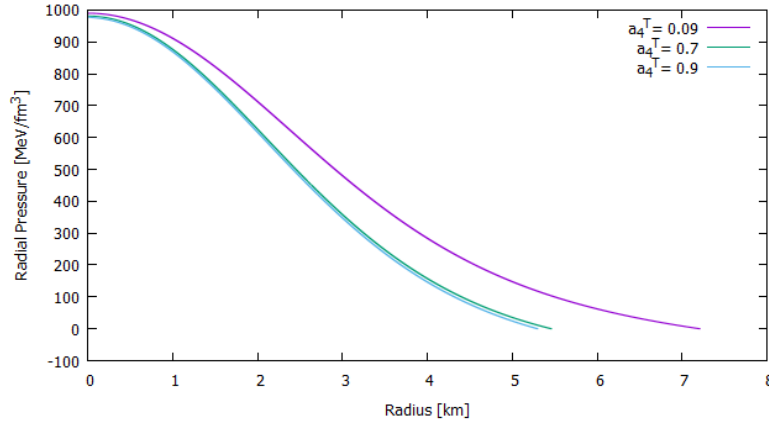


Figure 3. The radial pressure-radius relation of anisotropic quarks with $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

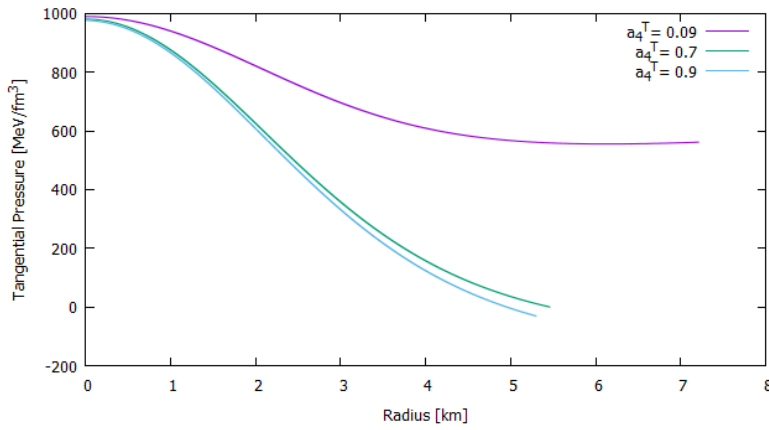


Figure 4. The tangential pressure-radius relation of anisotropic quarks with $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

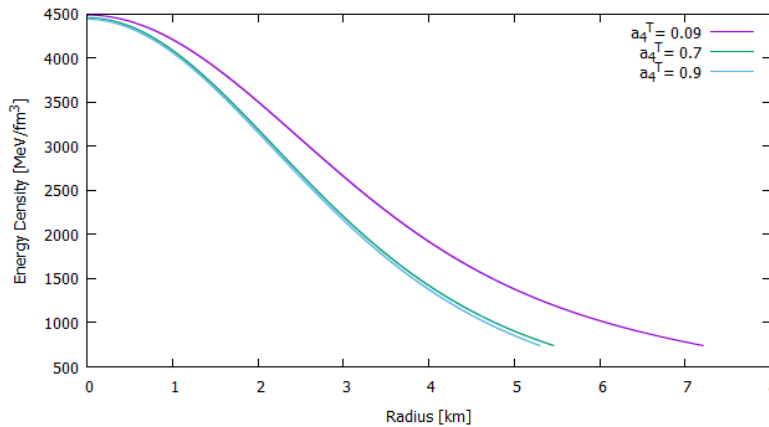


Figure 5. The energy density-radius relation of anisotropic quarks with $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

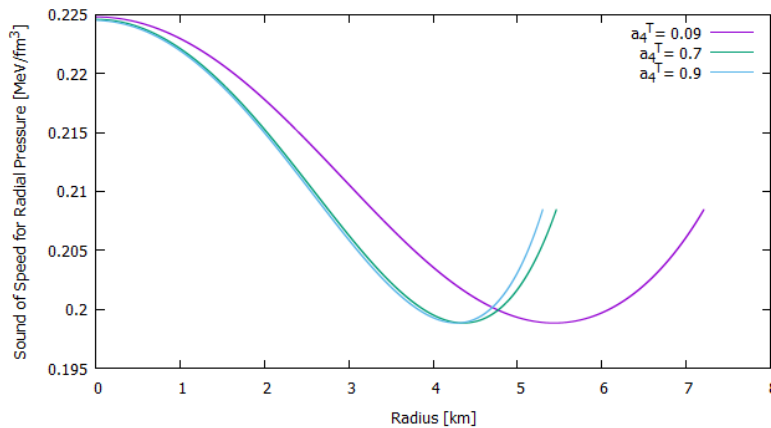


Figure 6. The sound of speed for radial pressure-radius relation of anisotropic quarks with $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

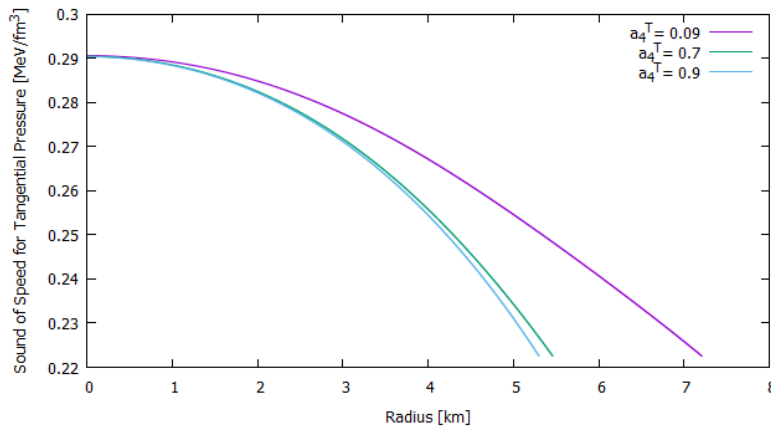


Figure 7. The sound of the speed of tangential pressure - radius relation of anisotropic quarks with $B = B_{\perp} = 52 \text{ MeV}/\text{fm}^3$ and $a_4 = 0.7$ by varying the value of a_4^{\perp}

we can see that parameter a_4^{\perp} controls the maximum mass and radius values of the star. This can be seen from the significant change in the mass-radius relation between the quark stars, where parameter a_4^{\perp} affects the mass distribution on the surface of the star. This indicates that on the surface of anisotropic quark stars, the mass distribution becomes more anisotropic and this observed effect is controlled by the a_4^{\perp} parameter. Meanwhile, varying the B constant does not result in significant change in the observed mass-radius relation of anisotropic quark stars. Figure 3 shows the result that inside the star, the radial pressure is positive for each increase in the radius of the star. However, the pressure in the radial direction decreases as the radius of the star increases. This shows that at the center of the star, the radial pressure is maximum, while at the star's surface it is minimum. Figure 4 shows a similar result in the radial pressure, that inside the star, the tangential pressure is also positive for each increase in the radius of the star. The image also shows that the tangential pressure decreases as the radius of the star increases. The tangential pressure is maximum at the center, while the minimum is at the surface. Figure 5 shows that inside a star, the energy density is also positive for each increase in the radius of the star. It can be seen that at the center, the energy density of the star is maximum and decreases as the radius of the star increases. Based on these results, we can see that in the

center, the density of quark stars is greater than at the star's surface. It can be seen that the smaller the value of a_4^\perp , the greater the pressure on the star on the surface. This shows that the parameter a_4^\perp affects the pressure and density of the star on the star's surface. Figures 6 and 7 show the results for $V_S^{P^2} - V_S^{P^\perp} \leq 1$ (in the range of values 0.1-0.3). This result is still acceptable for the relativistic limit where in stars, both the radial and tangential speed of sound are always less than the speed of light $V_S^{P^2} \leq V_S = 1$ and $V_S^{P^\perp} \leq V_S = 1$. This result is still within the QCD limit where the maximum sound speed limit is QCD $V_S = 0.3$. Based on the energy condition profile, we can see that all quark star energy conditions correspond to ideal fluid conditions. These results also prove that the Einstein Field Equation can be used to check the energy conditions of anisotropic quark stars which results in stable quark star conditions.

6. Conclusion

In this work, we calculated the pressure profiles both in the radial $P(r)$ and tangential P_\perp directions, and the energy density $\epsilon(r)$ using the TOV and EOS equations of anisotropic quark stars to determine the energy conditions required filled with anisotropic quark stars. Based on the results obtained, it can be concluded that the parameter a_4^\perp controls the mass distribution of anisotropic quark stars on the surface because on the surface of anisotropic quark stars, the mass distribution becomes more anisotropic. And also, the distribution of pressure in both radial and tangential directions and the energy density from the center to the surface of the star is getting smaller. This is because it is denser at the center than at the surface of the star. In addition, the parameter a_4^\perp also controls the distribution of pressure and energy density, where the distribution becomes uneven on the star's surface. The speed of sound in both the radial and tangential directions still meets the realistic limit, and also the QCD limit was ≤ 1 for the realistic limit (not exceeding the speed of light) and ≤ 0.3 for the QCD limit. Hence, we conclude that within the model parameters that we used, the profiles obey the energy conditions. It means that the corresponding quark stars are stable.

7. Acknowledgments

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