Leptogenesis and Low Energy CP Violation

G. C. Branco[1], T. Morozumi[2], B. M. Nobre[1], and M. N. Rebelo[1]

[1] Departamento de Física Instituto Superior Técnico, Av. Rovisco Pais P-1049-001, Lisboa, Portugal
[2] Graduate School of Science, Hiroshima University 1-3-1 Kagamiyama, Higashi Hiroshima - 739-8526, Japan

Abstract. We discuss the possibility of relating the size and sign of the observed baryon asymmetry of the universe to CP violation observable at low energies, in a framework where the observed baryon asymmetry is produced by leptogenesis through out of equilibrium decay of heavy Majorana neutrinos. We have shown that although in general such a connection cannot be established, there are specific frameworks where a link does exist. Furthermore, we identify the CP violating phases relevant to leptogenesis and those relevant for low energy CP violation and build weak basis invariant conditions for CP conservation.

1 Introduction

In the Standard Model (SM) neutrinos are massless and there is no CP violation in the leptonic sector. However at present there is strong evidence for neutrino oscillations reported by the SuperKamiokande experiment [1] and recently confirmed by the results of the Sudbury Neutrino Observatory (SNO) [2], both pointing towards nonzero neutrino masses. The most straightforward way of extending the SM in order to incorporate neutrino masses is to add one right-handed neutrino field per generation, singlet under the $SU(3)_c \times SU(2) \times U(1)$ gauge symmetry, in analogy with the quark sector. In Grand Unified Theories (GUTs), such as $SO(10)$ GUTs [3], these right-handed neutrino states appear in irreducible representations, together with quarks and leptons. Although this might look like a trivial extension of the SM it is far reaching in its consequences, giving rise to entirely new phenomena in the leptonic sector, due to the fact that neutrinos are neutral particles. In fact, if lepton number conservation is not imposed, a Majorana mass term for the neutral right-handed gauge singlets must be included in the Lagrangean, together with the usual Dirac mass term, leading to the seesaw mechanism [4] which accounts, in an elegant way, for the smallness of neutrino masses. Furthermore mixing and CP violation in the leptonic sector naturally arise once right-handed neutrino singlets are included [1]. CP violation in the leptonic sector can have profound cosmological implications

[1] It was shown long ago that mixing and CP violation in the leptonic sector can also occur with strictly massless neutrinos in a model where, in addition to right-handed neutrinos, an equal number of gauge singlet leptons are included [5]

leading to the generation of the observed Baryon number Asymmetry of the Universe (BAU) via Leptogenesis. In this framework, the starting point is a CP asymmetry generated through out-of-equilibrium L-violating decays of the heavy Majorana neutrinos [6] leading to a lepton asymmetry L≠0 while B=0 is still maintained. Subsequently, sphaleron processes [7], which are (B+L) violating and (B-L) conserving restore (B+L)=0 thus creating a nonvanishing B. This mechanism is, at present, one of the most appealing scenarios for Baryogenesis. Leptogenesis has been studied in detail by several groups [8] and it has been shown that the observed BAU of $n_B/s \sim 10^{-10}$ can be obtained in the above scenario without any fine-tuning of parameters. In our work [9] we address the question of whether it is possible to establish a connection between CP breaking necessary to generate leptogenesis and CP violation at low energies. More specifically, assuming that baryogenesis is achieved through leptogenesis, can one infer the strength of CP violation at low energies from the size and sign of the observed BAU? We start by studying the various sources of CP violation in the minimal seesaw model (i.e.,no left-handed Majorana mass terms) identifying both the CP violating phases and the weak-basis (WB) invariants which are associated to leptogenesis and those relevant for CP violation at low energies. We proceed by showing that this connection is not possible in general, but we present special scenarios where the connection can be established. Several authors have addressed this question under different assumptions [10].

2 Framework

Let us consider a minimal extension of the SM which consists of adding to the standard spectrum one right-handed neutrino per generation. After spontaneous gauge symmetry breaking the leptonic mass terms can be written as

$$\begin{aligned}\mathcal{L}_m &= -[\overline{\nu_L^0} m_D \nu_R^0 + \frac{1}{2}\nu_R^{0T} C M_R \nu_R^0 + \overline{l_L^0} m_l l_R^0] + h.c. = \\ &= -[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0] + h.c. \end{aligned} \tag{1}$$

where m_D, M_R and m_l denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix and the charged lepton mass matrix, respectively, and $n_L = (\nu_L^0, (\nu_R^0)^c)$. Obviously in this minimal extension of the SM a term of the form $\frac{1}{2}\nu_L^{0T} C m_L \nu_L^0$ does not appear in the Lagrangean and the matrix $\mathcal{M}$ is given by:

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} \tag{2}$$

with a zero entry on the (11) block. For notation simplicity, we have dropped the subscript in m_D and M_R. It has been shown that in the most general case, when $\mathcal{M}$ includes m_L, the number of CP violating phases in $\mathcal{M}$ is given by [11]:

$$N_{CP} = nn' + \frac{n(n-1)}{2} \tag{3}$$

where n is the number of ν_L fields and n' the number of ν_R fields. In our case $n = n'$. Without m_L the number of independent CP violating phases was computed in Ref.[12] to be:

$$n_{CP} = n^2 - n \tag{4}$$

For defineteness, we shall consider the case of three generations (three light neutrinos). In this case the number of physical parameters contained in $\mathcal{L}_m$ is a total of fifteen real parameters and six CP violating phases as can be easily seen by going to the weak-basis (WB) where m_l and M are chosen to be diagonal and real matrices. Of course there is no loss of generality in the choice of a weak-basis. In this WB, there will be six real parameters in m_l and M, on the other hand m is a general three-by-three matrix and can be written as the product of a unitary times a Hermitian matrix:

$$m = UH = P_\xi \hat{U}_\rho P_\tau P_\beta^\dagger \ \hat{H}_\sigma \ P_\beta \tag{5}$$

with $P_\xi = \mathrm{diag.}\,(\exp(i\xi_1), \exp(i\xi_2), \exp(i\xi_3))$, $P_\tau = \mathrm{diag.}(1, \exp(i\tau_1), \exp(i\tau_2))$ and $P_\beta = \mathrm{diag.}(1, \exp(i\beta_1), \exp(i\beta_2))$. In the second equality we have factored out of U and H as many phases as possible leaving $\hat{U}_\rho$ and $\hat{H}_\sigma$ with only one phase each. Since P_ξ can be rotated away by a WB transformation corresponding to a simultaneous phase redefinition of the left-handed charged lepton fields, and the left-handed neutrino fields, the matrix m is left with six independent phases and nine real parameters. We shall denote the six independent phases as ρ in $\hat{U}_\rho$, α_1, α_2 in the product $(P_\tau P_\beta^\dagger)$, σ in $\hat{H}_\sigma$ and β_1, β_2 in P_β.

3 CP violating phases relevant for leptogenesis

Leptogenesis gives rise to the BAU through out-of-equilibrium decay of heavy Majorana neutrinos in the symmetric phase (i.e., before spontaneous gauge symmetry breakdown). The computation of the lepton number asymmetry, in this extension of the SM, resulting from the decay of a heavy Majorana neutrino N^j into charged leptons $l_i^{\pm}$ (i= e, μ , τ) can be done both in the symmetric phase [13] and in the broken phase [12], [9]. We define the lepton family number asymmetry as $\Delta A^j{}_i = N^j{}_i - \overline{N}^j{}_i$. The lepton number asymmetry from j th heavy Majorana particle is then given by:

$$A^j = \frac{\sum_i \Delta A^j{}_i}{\sum_i \left(N^j{}_i + \overline{N^j}_i\right)} \tag{6}$$

with the sum in i running over the three flavours $i = \mathrm{e}\ \nu\ \tau$. In this framework the calculations lead to:

$$A^j = \frac{g^2}{{M_W}^2} \sum_{k \neq j} \left[\mathrm{Im}\left((m^\dagger m)_{jk} (m^\dagger m)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{(m^\dagger m)_{jj}},$$
$$= \sum_{k \neq j} \left[\mathrm{Im}\left(({y_D}^\dagger y_D)_{jk} ({y_D}^\dagger y_D)_{jk} \right) \frac{1}{8\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{({y_D}^\dagger y_D)_{jj}}, \quad (7)$$

The second equality results from the substitution $m_{ij} = y_{Dij} \frac{v}{\sqrt{2}}$, with y_{Dij} denoting the coefficients of the neutrino Yukawa couplings and v the Higgs vacuum expectation value. The variable x_k is defined as $x_k = \frac{{M_k}^2}{{M_j}^2}$ and $I(x_k) = \sqrt{x_k} \left(1 + (1 + x_k) \log(\frac{x_k}{1+x_k}) \right)$. From Eq.(7) it can be seen that the lepton number asymmetry is only sensitive to the CP violating phases appearing in $m^\dagger m$. With the choice of phases of Eq.(5) leptogenesis is only sensitive to σ, β_1 and β_2.

4 Weak-basis invariants and CP violation

The most general CP transformation of the leptonic fermion fields, still in a WB, which leaves the gauge interaction invariant is of the form

$$\mathrm{CP} l_L (\mathrm{CP})^\dagger = U' \gamma^0 \mathrm{C} \overline{l_L}^T \quad \mathrm{CP} l_R (\mathrm{CP})^\dagger = V' \gamma^0 \mathrm{C} \overline{l_R}^T$$
$$\mathrm{CP} \nu_L (\mathrm{CP})^\dagger = U' \gamma^0 \mathrm{C} \overline{\nu_L}^T \quad \mathrm{CP} \nu_R (\mathrm{CP})^\dagger = W' \gamma^0 \mathrm{C} \overline{\nu_R}^T \quad (8)$$

where U', V', W' are unitary matrices acting in flavour space and where for notation simplicity we have dropped here the superscript 0 in the fermion fields. Invariance of the mass terms under the above CP transformation, requires that the following relations have to be satisfied:

$$W'^T M W' = -M^* \quad (9)$$
$$U'^\dagger m W' = m^* \quad (10)$$
$$U'^\dagger m_l V' = {m_l}^* \quad (11)$$

From Eqs. (10), (9), one obtains:

$$W'^\dagger h W' = h^*$$
$$W'^\dagger H W' = H^* \quad (12)$$

with $h = m^\dagger m$, $H = M^\dagger M$. It can be then readily derived, from Eqs. (9) and (12), that CP invariance requires:

$$I_1 \equiv \mathrm{ImTr}[h H M^* h^* M] = 0$$
$$I_2 \equiv \mathrm{ImTr}[h H^2 M^* h^* M] = 0$$
$$I_3 \equiv \mathrm{ImTr}[h H^2 M^* h^* M H] = 0 \quad (13)$$

By construction, these WB invariants are only sensitive to the CP violating phases which appear in leptogenesis. This is due to the fact that m always appears in the combination $m^\dagger m$. WB invariant conditions are particularly useful because they can be evaluated and analysed in any conveniently chosen WB (see Ref. [9] for further discussion of these conditions). Since there are six independent CP violating phases, one may wonder whether one can construct other three independent WB invariants, apart from I_i, which would describe CP violation in the leptonic sector. This is indeed possible, a simple choice are the WB invariants $\bar{I}_i (i = 1, 2, 3)$, obtained from I_i, through the substitution of h by $\bar{h} = m^\dagger h_l m$, where $h_l = m_l {m_l}^\dagger$. For example one has:

$$\bar{I}_1 = \mathrm{ImTr}(m^\dagger h_l m H M^* m^T {h_l}^* m^* M) \tag{14}$$

and similarly for $\bar{I}_2, \bar{I}_3$. As it was the case for I_i, CP invariance requires that $\bar{I}_i = 0$.

5 CP violating phases relevant at low energies

The neutrino mass matrix $\mathcal{M}$ is diagonalized by the transformation:

$$V^T \mathcal{M}^* V = \mathcal{D} \tag{15}$$

where $\mathcal{D} = \mathrm{diag.}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3})$, with m_{ν_i} and M_{ν_i} denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write V and $\mathcal{D}$ in the following form:

$$V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}; \quad \mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}. \tag{16}$$

From Eq. (15) one obtains to an excellent approximation:

$$S^\dagger = -K^\dagger m M^{-1} \tag{17}$$

$$-K^\dagger m \frac{1}{M} m^T K^* = d \tag{18}$$

Eq.(18) is the usual seesaw formula. In this approximation K is a unitary matrix corresponding to the three-by-three Maki-Nakagawa-Sakata matrix [14]. The neutrino weak-eigenstates are related to the mass eigenstates by:

$$\nu^0_{i\,L} = V_{i\alpha} \nu_{\alpha L} = (K, R) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \quad \begin{pmatrix} i = 1, 2, 3 \\ \alpha = 1, 2, ...6 \end{pmatrix} \tag{19}$$

and thus the leptonic charged current interactions are given by:

$$-\frac{g}{\sqrt{2}} \left(\overline{l_{iL}} \gamma_\mu K_{ij} \nu_{j\,L} + \overline{l_{iL}} \gamma_\mu R_{ij} N_{j\,L} \right) W^\mu + h.c. \tag{20}$$

From Eqs.(19), (20) it follows that K and R give the charged current couplings of charged leptons to the light neutrinos ν_j and to the heavy neutrinos N_j, respectively. In this approximation, with K a unitary matrix, it is clear that we can rotate away three phases on the left by a redefinition of the physical charged leptonic fields so that K is left with three physical CP violating phases, one of Dirac type, δ, and two of Majorana type, which have an interesting geometrical interpretation in terms of unitarity triangles [15]. These are the three CP violating phases relevant at low energies. On the other hand from Eq.(15), taking into account the zero entry in $\mathcal{M}$, one derives the following exact relation:

$$R = mT^* D^{-1} \tag{21}$$

In the WB where the right-handed Majorana neutrino mass is diagonal it follows to an excellent approximation that:

$$R = mD^{-1} \quad \text{or else} \quad R_{ij} D_j = m_{ij} \tag{22}$$

leading to:

$$A^j = \frac{g^2}{{M_W}^2} \sum_{k \neq j} \left[(M_k)^2 \mathrm{Im}\left((R^\dagger R)_{jk} (R^\dagger R)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{(R^\dagger R)_{jj}}. \tag{23}$$

From the first equality in Eq.(22) and Eq.(5) we see, once again, that only the phases σ, β_1 and β_2 are relevant for leptogenesis. Furthermore the first equality in Eq.(22) implies that the two CP violating phases β_1 and β_2 in m (see Eq.(5)) appear in Eq.(20) as Majorana type phases since P_β commutes with D^{-1} and, as a result, these phases can be shifted to the physical heavy neutrino masses.

6 Relating CP violation in leptogenesis with CP violation at low energies

In this section we address the question of whether it is possible to infer about the size of CP violation at low energies from the size and sign of the observed BAU. For definiteness, let us consider the parametrization of m given before, where the six phases are ρ, α_1, α_2, σ, β_1, β_2. We have already seen that leptogenesis is controlled by the phases σ, β_1, and β_2. On the other hand the phases relevant at low energies are those appearing in K and resulting from the diagonalization of the effective left-handed neutrino mass matrix given by:

$$m_{ef} = -m \frac{1}{D} m^T \tag{24}$$

The strength of CP violation at low energies, observable for example through neutrino oscillations, can be obtained from the following low-energy WB invariant:

$$Tr[h_{ef}, h_l]^3 = 6i \Delta_{21} \Delta_{32} \Delta_{31} \mathrm{Im}\{(h_{ef})_{12} (h_{ef})_{23} (h_{ef})_{31}\} \tag{25}$$

where $h_{ef} = m_{ef} m_{ef}{}^{\dagger}$, $h_l = m_l m_l{}^{\dagger}$ and $\Delta_{21} = (m_\mu{}^2 - m_e{}^2)$ with analogous expressions for Δ_{31}, Δ_{32}. CP violation in neutrino oscillations [16] is only affected by the phase δ. The important point is that the phase δ is, in general, a function of all the six phases ρ, α_1, α_2, σ, β_1, β_2 as can be seen from Eq. (18). Since leptogenesis only depends on σ, β_1 and β_2, it is clear that, in general, one cannot directly relate the size of CP violation responsible for leptogenesis with the strength of CP violation at low energies. Yet it can be seen by computing the invariant $Tr[h_{ef}, h_l]^3$ that in a model where the leptonic mass matrices are constrained (e. g. by flavour symmetries) so that only one of the phases (for example σ) is non-vanishing, one can establish a direct connection between the size of the observed BAU and the strength of CP violation at low energies observable, for example, in neutrino oscillations. Of special interest are specific GUT inpired scenarios such as the case of m given by:

$$m = dU_R \tag{26}$$

where d is diagonal and U_R is a generic unitary matrix, in a WB where m_l and M are both real and diagonal. This case has been discussed with all generality in Branco et al. in Ref. [10]. Another interesting possibility are models with spontaneous CP violation at a high energy scale such as the one presented in Ref. [9].

7 Concluding remarks

This talk is based on a more detailed work [9] where we studied the possible sources of CP violation in the minimal seesaw model and addressed the question of whether it is possible to establish a connection between CP violation responsible for leptogenesis and CP violation observable at low energies. It was shown that, in general, such a connection does not exist but there are special interesting scenarios where it may be established.

Acknowledgments

Contributed paper based on Ref. [9] The authors of Ref. [9] thank the CERN Theory Division for hospitality during its preparation and also T. Endoh, R. González Felipe, T. Onogi and A. Purwanto for useful discussions. The work of TM was supported by a fellowship for Japanese Scholar and Researcher abroad from the Ministry of Education, Science and Culture of Japan. The work of BMN was supported by Fundação para a Ciência e a Tecnologia (FCT) (Portugal) through fellowship SFRH/BD/995/2000; GCB, BMN and MNR received partial support during the preparation of Ref. [9] from FCT through Project CERN/P/FIS/40134/2000, Project POCTI/36288/FIS/2000 and Project CERN/C/FIS/40139/2000. The participation of MNR at the RTN Meeting "Across the Present Energy Frontier: Probing the Origin of

Mass", Corfu2001, where this talk was presented, was partially supported by the EC under contract HPRN-CT-2000-00148 and by the Project CERN/P/FIS/40134/2000. Finally we specially thank the Organizers of Corfu2001 for the stimulating Meeting.

References

1. Y. Fukuda *et al.*, Super-Kamiokande Collaboration, Phys. Rev. Lett. **81** (1998) 1158; Phys. Rev. Lett. **81** (1998) 4279(E); Phys. Lett. B **433** (1998) 9; Phys. Lett. B **436** (1998) 33; Phys. Lett. B **467** (1999) 185; Phys. Rev. Lett. **82** (1999) 1810; Phys. Rev. Lett. **82** (1999) 2430; Phys. Rev. Lett. **82** (1999) 2644; Phys. Rev. Lett. **85** (2000) 3999; Phys. Rev. Lett. **86** (2001) 5651; Phys. Rev. Lett. **86** (2001) 5656.
2. Q.R. Ahmad *et al.*, SNO Collaboration, Phys. Rev. Lett. **87** (2001) 071301.
3. For a recent review on grand unified theories see *e.g.* R.N. Mohapatra, *Lectures at ICTP Summer School in Particle Physics, Trieste, Italy, 7 June - 9 July 1999*, hep-ph/9911272, and references therein.
4. M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, KEK report 79-18 (1979), p. 95; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912.
5. J. Bernabéu, A. Santamaria, J. Vidal, A. Mendez and J. W. Valle, Phys. Lett. B **187** (1987) 303; P. Langacker and D. London, Phys. Rev. D **38** (1988) 886; G. C. Branco, M. N. Rebelo and J. W. Valle, Phys. Lett. B **225** (1989) 385.
6. M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.
7. V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.
8. For a review and references, see W. Buchmüller, M. Plümacher, Int. J. Mod. Phys. A **15** (2000) 5047.
9. For a more detailed discussion, see G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, Nucl. Phys. B **617** (2001) 475.
10. R. N. Mohapatra and X. Zhang, Phys. Rev. D **46** (1992) 5331; C. W. Chiang, Phys. Rev. D **63** (2001) 076009; A. S. Joshipura, E. A. Paschos and W. Rodejohann, JHEP **0108** (2001) 029; W. Buchmüller and D. Wyler, Phys. Lett. B **521** (2001) 291; J. R. Ellis, J. Hisano, S. Lola and M. Raidal, Nucl. Phys. B **621** (2002) 208; W. Rodejohann and K. R. Balaji, arXiv:hep-ph/0201052. T. Endoh, T. Morozumi and A. Purwanto, arXiv:hep-ph/0201309; G. C. Branco, R. González Felipe, F. R. Joaquim and M. N. Rebelo, arXiv:hep-ph/0202030.
11. G. C. Branco, L. Lavoura and M. N. Rebelo, Phys. Lett. B **180** (1986) 264.
12. T. Endoh, T. Morozumi, T. Onogi and A. Purwanto, Phys. Rev. D **64** (2001) 013006 [Erratum-ibid. D **64** (2001) 059904].
13. L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169; M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B **345** (1995) 248 [Erratum-ibid. B **382** (1995) 447]; M. Plümacher, Z. Phys. C **74** (1997) 549.
14. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
15. J. A. Aguilar-Saavedra and G. C. Branco, Phys. Rev. D **62** (2000) 096009.

16. A. De Rújula, M. B. Gavela and P. Hernandez, Nucl. Phys. B **547** (1999) 21; M. C. Gonzalez-Garcia, Y. Grossman, A. Gusso and Y. Nir, Phys. Rev. D **64** (2001) 096006.