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# Field signature for apparently superluminal particle motion

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**Abstract.** In the context of Stueckelberg's covariant symplectic mechanics, Horwitz and Aharonovich [1] have proposed a simple mechanism by which a particle traveling below light speed almost everywhere may exhibit a transit time that suggests superluminal motion. This mechanism, which requires precise measurement of the particle velocity, involves a subtle perturbation affecting the particle's recorded time coordinate caused by virtual pair processes. The Stueckelberg framework is particularly well suited to such problems, because it permits pair creation/annihilation at the classical level. In this paper, we study a trajectory of the type proposed by Horwitz and Aharonovich, and derive the Maxwell 4-vector potential associated with the motion. We show that the resulting fields carry a signature associated with the apparent superluminal motion, providing an independent test for the mechanism that does not require direct observation of the trajectory, except at the detector.

## 1. Introduction

The interpretation of antiparticles as negative energy particles propagating backward in time was proposed by Stueckelberg [2] in the context of his covariant Hamiltonian theory of interacting spacetime events  $x^\mu(\tau)$  evolving as functions of a Poincaré-invariant parameter  $\tau$ . His goal was to represent a particle/antiparticle process by a single worldline whose time coordinate advances and retreats with respect to the laboratory clock as its instantaneous energy changes sign under interaction with gauge fields. In order to obtain a well-posed electrodynamic theory, Sa'ad, Horwitz, and Arshansky [3] generalized Stueckelberg's formalism by introducing five  $\tau$ -dependent gauge fields. The resulting pre-Maxwell theory differs from conventional electrodynamics, but reduces to Maxwell theory at  $\tau$  equilibrium. An overview of pre-Maxwell electrodynamics can be found in [4].

In the context of Stueckelberg electrodynamics, Horwitz and Aharonovich [1] have proposed a simple mechanism by which a particle traveling below light speed almost everywhere may exhibit a transit time that suggests superluminal motion. This mechanism is analogous to a mountain trail with a switchback, on which a hiker may walk for 2 hours covering 12 km, but arrive at a point only 8 km from the starting point, with an apparent average speed of 4 km/hr instead of 6 km/hr. Exchanging time and space, an event may briefly reverse its time direction without affecting its space motion. In this case, the particle will continue its progress in space coordinates from the source to the detector, but the laboratory clock will report a difference



between time-of-start and time-of-arrival that is less than expected. In this paper, we study the Coulomb field expected from motion of this type and indicate a small deviation from the field expected for continuous linear motion.

## 2. Pre-Maxwell electrodynamics

In pre-Maxwell theory, interactions take place between events in spacetime rather than between worldlines. Each event occurring at  $\tau$  induces a current density in spacetime that disperses for large  $\tau$  for free particles (and hence asymptotically). Conversely, the current density induces an electromagnetic field that acts on the events. The *five* gauge fields are  $\tau$ -dependent and invariant under local gauge transformations that also depend on  $\tau$ . The combined theory can be derived from a unique scalar action and is integrable. It is convenient to write  $x^5 = \tau$  and adopt the index convention

$$\lambda, \mu, \nu = 0, 1, 2, 3 \qquad \alpha, \beta, \gamma = 0, 1, 2, 3, 5 . \quad (1)$$

The fields act on the events through the Lorentz force given by

$$M \dot{x}^\mu = e_0 f^\mu{}_\alpha(x, \tau) \dot{x}^\alpha \qquad \frac{d}{d\tau} \left( -\frac{1}{2} M \dot{x}^2 \right) = e_0 f_{5\alpha} \dot{x}^\alpha \quad (2)$$

where the gauge invariant fields are formed from the potentials

$$f_{\alpha\beta}(x, \tau) = \partial_\alpha a_\beta(x, \tau) - \partial_\beta a_\alpha(x, \tau) . \quad (3)$$

The fields are induced by event currents through the pre-Maxwell equations

$$\partial_\beta f^{\alpha\beta}(x, \tau) = \frac{e_0}{\lambda} j^\alpha(x, \tau) = e j^\alpha(x, \tau) \qquad \epsilon^{\alpha\beta\gamma\delta\epsilon} \partial_\alpha f_{\beta\gamma}(x, \tau) = 0 \quad (4)$$

in which the parameter  $\lambda$  acts as a coherence length. The classical current for an event  $\xi^\mu(\tau)$  is

$$j^\alpha(x, \tau) = e \dot{\xi}^\alpha(\tau) \delta^4(x - \xi(\tau)) \quad (5)$$

where  $\xi^5 = 1$ . Under the boundary conditions

$$a^5(x, \tau) \xrightarrow{\tau \rightarrow \pm\infty} 0 \qquad j^5(x, \tau) \xrightarrow{\tau \rightarrow \pm\infty} 0 \quad (6)$$

the standard Maxwell theory is extracted as the equilibrium limit of (4) by integration over the worldline

$$\left. \begin{array}{l} \partial_\beta f^{\alpha\beta}(x, \tau) = e j^\alpha(x, \tau) \\ \partial_{[\alpha} f_{\beta\gamma]} = 0 \end{array} \right\} \xrightarrow{\int d\tau} \left\{ \begin{array}{l} \partial_\nu F^{\mu\nu}(x) = e J^\mu(x) \\ \partial_{[\mu} F_{\nu\rho]} = 0 \end{array} \right. \quad (7)$$

where

$$F^{\mu\nu}(x) = \int d\tau f^{\mu\nu}(x, \tau) \qquad A^\mu(x) = \int d\tau a^\mu(x, \tau) \qquad J^\mu(x) = \int d\tau j^\mu(x, \tau) . \quad (8)$$

This integration has been called concatenation [5] and links the event current  $j^\mu(x, \tau)$  with the divergenceless Maxwell particle current  $J^\mu(x)$  defined on the entire worldline. The field equations lead to a wave equation

$$\partial_\alpha \partial^\alpha a^\beta(x, \tau) = (\partial_\mu \partial^\mu - \partial_\tau^2) a^\beta(x, \tau) = -e j^\beta(x, \tau) \quad (9)$$

and the principal part Green's function [6] is

$$G(x, \tau) = -\frac{1}{2\pi} \delta(x^2) \delta(\tau) - \frac{1}{2\pi^2} \frac{\partial}{\partial x^2} \frac{\theta(x^2 - \tau^2)}{\sqrt{x^2 - \tau^2}} = D(x) \delta(\tau) - G_{correlation}(x, \tau). \quad (10)$$

The first term has support on the lightcone at instantaneous  $\tau$ , and recovers the standard Maxwell Green's function under concatenation. The second term has spacelike support ( $x^2 > \tau^2 \geq 0$ ) and vanishes under concatenation, so it may contribute to correlations but not to Maxwell potentials. Terms of this type have been studied in [7].

### 3. Piecewise linear trajectories

We define a particle trajectory as a set of piecewise linear event segments

$$\xi(\tau) = \sum_{i=1}^n \xi_i(\tau) \Theta_i(\tau) = \sum_{i=1}^n (u_i \tau + q_i) \Theta_i(\tau) \quad (11)$$

where  $u_i$  and  $q_i$  are constant 4-vectors and  $\Theta_i(\tau)$  is some combination of  $\theta$  functions that defines the support of segment  $i$ . The velocities satisfy

$$u_i = \dot{\xi}_i(\tau) = \frac{d\xi_i}{d\tau} = \frac{1}{\sqrt{1 - \mathbf{v}^2}} (1, \mathbf{v}) \quad u_i^2 = -1. \quad (12)$$

The trajectory is associated with an instantaneous event current

$$j(x, \tau) = e \sum_{i=1}^n u_i \Theta_i(\tau) \delta^4(x - q_i - u_i \tau) \quad (13)$$

with delta function support. To handle the delta functions the instantaneous current is spread along the worldline through

$$\begin{aligned} j_\varphi(x, \tau) &= e \int ds \varphi(\tau - s) j(x, s) \\ &= e \sum_{i=1}^n \int ds \varphi(\tau - s) u_i \Theta_i(s) \delta^3(\mathbf{x} - \mathbf{q}_i - \mathbf{u}_i s) \delta(t - q_i^0 - u_i^0 s) \\ &= e \sum_{i=1}^n \int ds \varphi(\tau - s) \frac{u_i \Theta_i(s)}{|u_i^0|} \delta^3(\mathbf{x} - \mathbf{q}_i - \mathbf{u}_i s) \delta\left(\frac{t - q_i^0}{u_i^0} - s\right) \\ &= e \sum_{i=1}^n \varphi\left(\tau - \frac{t - q_i^0}{u_i^0}\right) \frac{u_i}{|u_i^0|} \Theta_i\left(\frac{t - q_i^0}{u_i^0}\right) \delta^3\left(\mathbf{x} - \mathbf{q}_i - \mathbf{u}_i \frac{t - q_i^0}{u_i^0}\right) \end{aligned} \quad (14)$$

where

$$\varphi(\tau) = \frac{1}{2\lambda} e^{-|\tau|/2\lambda} \quad \int_{-\infty}^{\infty} d\tau \varphi(\tau) = 1. \quad (15)$$

The Maxwell current can be found by integrating this event current over  $\tau$ , summing the event contributions along worldline and extracting the equilibrium current

$$\begin{aligned} J(x) &= e \int d\tau \sum_{i=1}^n \varphi\left(\tau - \frac{t - q_i^0}{u_i^0}\right) \frac{u_i}{|u_i^0|} \Theta_i\left(\frac{t - q_i^0}{u_i^0}\right) \delta^3(\mathbf{x} - \mathbf{q}_i - \mathbf{v}_i(t - q_i^0)) \\ &= e \sum_{i=1}^n \frac{u_i}{|u_i^0|} \Theta_i\left(\frac{t - q_i^0}{u_i^0}\right) \delta^3(\mathbf{x} - \mathbf{q}_i - \mathbf{v}_i(t - q_i^0)). \end{aligned} \quad (16)$$

The Liénard-Wiechert potential is found from the principal part Green's function

$$G(x - x', \tau - \tau') = -\delta \left[ (x - x')^2 \right] \delta(\tau - \tau') \quad (17)$$

and the event current, so that

$$\begin{aligned} a_\varphi(x, \tau) &= -\frac{e}{2\pi} \int d^4x' d\tau' \delta \left[ (x - x')^2 \right] \delta(\tau - \tau') j_\varphi(x', \tau') \\ &= -\frac{e}{2\pi} \int d^4x' \delta \left[ (x - x')^2 \right] j_\varphi(x', \tau) . \end{aligned} \quad (18)$$

Inserting (14) and performing the  $x'$  integration leads to

$$a_\varphi(x, \tau) = -\frac{e}{2\pi} \sum_{i=1}^n \int d\tau' \varphi(\tau - \tau') u_i \Theta_i(\tau') \delta \left[ (x - q_i - u_i \tau')^2 \right] . \quad (19)$$

Using the identity

$$\int d\tau f(\tau) \delta[g(\tau)] = \frac{f(s)}{|g'(s)|} \quad (20)$$

we may perform the integral over  $\tau'$  to find

$$a_\varphi(x, \tau) = -\frac{e}{4\pi} \sum_{i=1}^n \frac{u_i \Theta_i(s_i) \varphi(\tau - s_i)}{|u_i \cdot (x - q_i - u_i s_i)|} \quad (21)$$

where  $s_i$  is determined by the fixed observation point  $x$  and the *a priori* trajectory  $\xi_i(\tau)$  through

$$(x - \xi_i(s_i))^2 = (x - q_i - u_i s_i)^2 = 0 \quad x^0 - q_i^0 - u_i^0 s_i^0 > 0 . \quad (22)$$

Using  $u_i^2 = -1$  we find the retarded times as

$$s_i = -u_i \cdot (x - q_i) \mp \sqrt{(u_i \cdot (x - q_i))^2 + (x - q_i)^2} . \quad (23)$$

The choice of sign is associated with the retarded and advanced times of the event evolution, and depends upon the direction of the time evolution. Consider the "static" event for which  $\mathbf{u} = 0$  and  $q = 0$ . For the forward time trajectory  $u = (1, \mathbf{0})$

$$s_\pm = t \mp \sqrt{(-t)^2 + \mathbf{x}^2 - t^2} = t \mp R . \quad (24)$$

At the observation point  $(t, \mathbf{x})$  the event observed must have been produced by the event current at a time earlier than  $t$ , and so we choose the upper sign and retarded time  $s_-$ . The event produced at time  $s_-$  is located at  $x = (t - R, \mathbf{0})$  and as expected is at future lightlike separation from the observation point. But for the "static" event evolving backward in time with  $u = (-1, \mathbf{0})$

$$s_\pm = -t \mp \sqrt{(-t)^2 + \mathbf{x}^2 - t^2} = -t \mp R \quad (25)$$

and we must choose the lower sign. Of the two times  $s_\pm$  it is clear that the event evolving toward negative times will reach the point  $s_+$  before reaching the point  $s_-$  and so this is the retarded time for that trajectory. At time  $s_+$  the event is located at

$$x = u s_+ = (-1, \mathbf{0}) (-t + R) = (t - R, \mathbf{0}) . \quad (26)$$

Adding the lightlike vector  $(R, \mathbf{x})$  takes this to the observation point  $(t, \mathbf{x})$  as for the forward evolving event. The choice of the upper sign can also be understood as choosing the  $T$ -reversed picture of the forward time evolution, respecting the discrete Lorentz symmetry. We summarize these cases as

$$s_i = -u_i \cdot (x - q_i) - \varepsilon (u_i^0) \sqrt{(u_i \cdot (x - q_i))^2 + (x - q_i)^2} \quad (27)$$

and it follows that

$$|u_i \cdot (x - q_i - u_i s_i)| = \sqrt{(u_i \cdot (x - q_i))^2 + (x - q_i)^2} \quad (28)$$

Finally, the Liénard-Wiechert potential takes the form

$$a_\varphi(x, \tau) = -\frac{e}{4\pi} \sum_{i=1}^n \frac{u_i \Theta_i(s_i) \varphi(\tau - s_i)}{\sqrt{(u_i \cdot (x - q_i))^2 + (x - q_i)^2}} \quad (29)$$

which under concatenation provides the Maxwell potential in the form

$$A(x) = -\frac{e}{4\pi} \sum_{i=1}^n \frac{u_i \Theta_i(s_i)}{\sqrt{(u_i \cdot (x - q_i))^2 + (x - q_i)^2}} . \quad (30)$$

#### 4. Field from pair annihilation

We first consider a trajectory on which an event moves uniformly forward in time until  $\tau = 0$  and then moves uniformly backward in time. In the laboratory, this will be seen as pair annihilation. The event trajectory is described by

$$\zeta(\tau) = \begin{cases} (u^0, \mathbf{u}) \tau & , \tau < 0 \\ (-u^0, \mathbf{u}) \tau & , \tau > 0 \end{cases} \quad (31)$$

from which

$$u_1 = (u^0, \mathbf{u}) \quad u_2 = (-u^0, \mathbf{u}) \quad q_1 = q_2 = 0 \quad (32)$$

$$\Theta_1 = [1 - \theta(\tau)] = \theta(-\tau) \quad \Theta_2 = \theta(\tau) \quad (33)$$

This trajectory is shown in Figure 1.

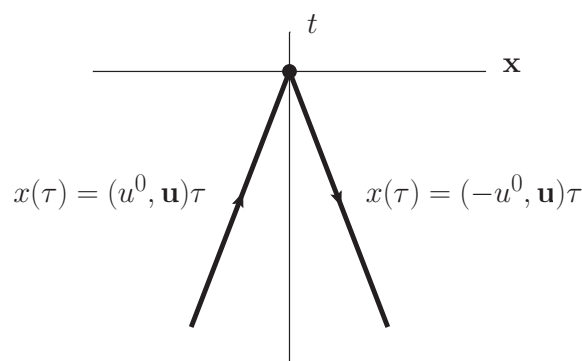


Figure 1: Pair annihilation event

The event current takes the form

$$j(x, \tau) = e (u^0, \mathbf{u}) [1 - \theta(\tau)] \delta^4(x - (u^0, \mathbf{u}) \tau) + (-u^0, \mathbf{u}) \theta(\tau) \delta^4(x - (-u^0, \mathbf{u}) \tau) \quad (34)$$

and the spread current is

$$j_\varphi(x, \tau) = e \sum_{i=1}^n \varphi\left(\tau - \frac{t}{u_i^0}\right) \frac{u_i}{|u_i^0|} \Theta_i\left(\frac{t}{u_i^0}\right) \delta^3\left(\mathbf{x} - \mathbf{u}_i \frac{t}{u_i^0}\right) \quad (35)$$

$$= e \left[ \varphi\left(\tau - \frac{t}{u^0}\right) (1, \mathbf{v}) \delta^3(\mathbf{x} - \mathbf{v}t) + \varphi\left(\tau + \frac{t}{u^0}\right) (-1, \mathbf{v}) \delta^3(\mathbf{x} + \mathbf{v}t) \right] \theta(-t) \quad (36)$$

In this form, the first term describes a particle located at  $\mathbf{x} = \mathbf{v}t$  and the second term describes an antiparticle located at  $\mathbf{x} = -\mathbf{v}t$ , until they mutually annihilate at  $t = 0$ . Similarly, the concatenated current is

$$J(x, \tau) = e [(1, \mathbf{v}) \delta^3(\mathbf{x} - \mathbf{v}t) + (-1, \mathbf{v}) \delta^3(\mathbf{x} + \mathbf{v}t)] \theta(-t) \quad (37)$$

The retarded times are

$$\begin{aligned} s_1 &= -(u^0, \mathbf{u}) \cdot x - \sqrt{((u^0, \mathbf{u}) \cdot x)^2 + x^2} \\ &= u^0 t - \mathbf{u} \cdot \mathbf{x} - R \sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} - u^0\right)^2 - 1\right]} \end{aligned} \quad (38)$$

$$\begin{aligned} s_2 &= -(-u^0, \mathbf{u}) \cdot x + \sqrt{((-u^0, \mathbf{u}) \cdot x)^2 + x^2} \\ &= -u^0 t - \mathbf{u} \cdot \mathbf{x} + R \sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]} \end{aligned} \quad (39)$$

and the potential takes the form

$$a^\beta(x, \tau) = -\frac{e}{4\pi R} \left[ \frac{(u^0, \mathbf{u}) \varphi(\tau - s_1) [1 - \theta(s_1)]}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} - u^0\right)^2 - 1\right]}} + \frac{(-u^0, \mathbf{u}) \varphi(\tau - s_2) \theta(s_2)}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]}} \right] \quad (40)$$

Under concatenation, the Maxwell potential is found to be

$$A(x) = -\frac{e}{4\pi R} \left[ \frac{(u^0, \mathbf{u}) \theta(-s_1)}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} - u^0\right)^2 - 1\right]}} + \frac{(-u^0, \mathbf{u}) \theta(s_2)}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]}} \right] \quad (41)$$

We notice that the first term is nonzero for  $s_1 < 0$  which holds in a region of spacetime for which

$$-(u^0, \mathbf{u}) \cdot x - \sqrt{((u^0, \mathbf{u}) \cdot x)^2 + x^2} < 0 \quad (42)$$

$$[(u^0, \mathbf{u}) \cdot x]^2 < ((u^0, \mathbf{u}) \cdot x)^2 + x^2 \quad (43)$$

$$0 < x^2 \quad (44)$$

and the second term is nonzero for  $s_2 > 0$  which requires

$$-(-u^0, \mathbf{u}) \cdot x + \sqrt{((-u^0, \mathbf{u}) \cdot x)^2 + x^2} > 0 \quad (45)$$

$$((u^0, \mathbf{u}) \cdot x)^2 + x^2 > [(u^0, \mathbf{u}) \cdot x]^2 \quad (46)$$

$$x^2 > 0 \quad (47)$$

so that

$$\theta(-s_1) = \theta(s_2) = \theta(R - t) \quad (48)$$

The field (41) thus describes the combined field of two opposite charges that when observed from a fixed distance  $R$  are seen to mutually annihilate at  $t = R$ . Put another way, the field is observable only at spacelike separation from the annihilation event  $\xi = 0$ . At any point in the future of the annihilation event, there is no observed field. We notice that if  $\mathbf{u} \cdot \mathbf{x} = 0$  the component  $A^0$  vanishes identically, describing a combined charge distribution built from a pair of opposite charges viewed symmetrically from the observation point  $x$ .

## 5. Switchback in time

We now consider an event evolving according to

$$\xi(\tau) = \begin{cases} (u^0\tau, \mathbf{u}\tau) & , \tau < 0 \\ (-u^0\tau, \mathbf{u}\tau) & , 0 \leq \tau \leq T \\ (u^0(\tau - 2T), \mathbf{u}\tau) = (u^0\tau, \mathbf{u}\tau) + q_3 & , \tau > T \end{cases} \quad (49)$$

where  $q_3 = (-2u^0T, \mathbf{0})$ . This trajectory is shown in Figure 2 — its laboratory phenomenology is discussed in section 6.

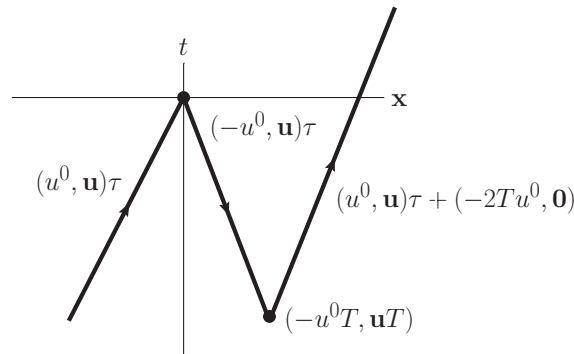


Figure 2: Switchback in time

As described in the Horwitz-Aharonovich model, this event proceeds moves linearly and continuously in space, but experiences a brief switchback in its time coordinate. For example, if the particle is observed at  $\tau = -\tau_0 < 0$  and observed again at  $\tau = \tau_0 > T$ , the particle is seen to travel the distance  $2|\mathbf{u}|\tau_0$  during an elapsed time  $2u^0\tau_0 - 2T$  with speed

$$v = \frac{|\mathbf{u}|}{u^0 \left(1 - \frac{T}{\tau_0}\right)} < \frac{|\mathbf{u}|}{u^0} . \quad (50)$$

The four-velocity for this trajectory is

$$\dot{\xi}(\tau) = \begin{cases} (u^0, \mathbf{u}) & , \tau < 0 \\ (-u^0, \mathbf{u}) & , 0 \leq \tau \leq T \\ (u^0, \mathbf{u}) & , \tau > T \end{cases} \quad (51)$$

and current is

$$j(x, \tau) = e \begin{cases} (u^0, \mathbf{u}) \delta^3(\mathbf{x} - \mathbf{u}\tau) \delta(t - u^0\tau) & , \tau < 0 \\ (-u^0, \mathbf{u}) \delta^3(\mathbf{x} - \mathbf{u}\tau) \delta(t + u^0\tau) & , 0 \leq \tau \leq T \\ (u^0, \mathbf{u}) \delta^3(\mathbf{x} - \mathbf{u}\tau) \delta(t - u^0(\tau - 2T)) & , \tau > T \end{cases} . \quad (52)$$

which can be put into the form (11) as

$$j(x, \tau) = e (u^0, \mathbf{u}) \delta^3(\mathbf{x} - \mathbf{u}\tau) \delta(t - u^0\tau) \theta(-\tau) \\ + e (-u^0, \mathbf{u}) \delta^3(\mathbf{x} - \mathbf{u}\tau) \delta(t + u^0\tau) [\theta(\tau) - \theta(\tau - T)] \\ + e (u^0, \mathbf{u}) \delta^3(\mathbf{x} - \mathbf{u}\tau) \delta(t - u^0(\tau - 2T)) \theta(\tau - T) . \quad (53)$$

Using (27) we find the retarded times

$$s_1 = u^0 t - \mathbf{u} \cdot \mathbf{x} - \sqrt{[\mathbf{u} \cdot \mathbf{x} - u^0 t]^2 + R^2 - t^2} \quad (54)$$

$$s_2 = -u^0 t - \mathbf{u} \cdot \mathbf{x} + \sqrt{[\mathbf{u} \cdot \mathbf{x} + u^0 t]^2 + R^2 - t^2} \quad (55)$$

$$s_3 = u^0 (t + 2u^0 T) - \mathbf{u} \cdot \mathbf{x} - \sqrt{[\mathbf{u} \cdot \mathbf{x} - u^0 (t + 2u^0 T)]^2 + R^2 - (t + 2u^0 T)^2} \quad (56)$$

so that the potential is

$$a_{\varphi}(x, \tau) = -\frac{e}{4\pi R} \left\{ \frac{(u^0, \mathbf{u}) \varphi(\tau - s_1) \theta(-s_1)}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} - u^0\right)^2 - 1\right]}} \right. \\ \left. + \frac{(-u^0, \mathbf{u}) \varphi(\tau - s_2) [\theta(s_2) - \theta(s_2 - T)]}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]}} \right. \\ \left. + \frac{(u^0, \mathbf{u}) \varphi(\tau - s_3) \theta(s_3 - T)}{\sqrt{1 + \left(\frac{t + 2u^0 T}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t + 2u^0 T} - u^0\right)^2 - 1\right]}} \right\} . \quad (57)$$

Under concatenation, the Maxwell potential becomes

$$A(x) = -\frac{e}{4\pi R} \left\{ \frac{(u^0, \mathbf{u}) \theta(-s_1)}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} - u^0\right)^2 - 1\right]}} \right. \\ \left. + \frac{(-u^0, \mathbf{u}) [\theta(s_2) - \theta(s_2 - T)]}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]}} \right. \\ \left. + \frac{(u^0, \mathbf{u}) \theta(s_3 - T)}{\sqrt{1 + \left(\frac{t + 2u^0 T}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t + 2u^0 T} - u^0\right)^2 - 1\right]}} \right\} . \quad (58)$$

As seen in the pair annihilation problem, the functions  $\theta(-s_1)$  and  $\theta(s_2)$  become  $\theta(R - t)$  and we must similarly convert  $\theta(s_2 - T)$  and  $\theta(s_3 - T)$  to functions on spacetime. Using (55) the condition  $s_2 > T$  becomes

$$-u^0 t - \mathbf{u} \cdot \mathbf{x} + \sqrt{[\mathbf{u} \cdot \mathbf{x} + u^0 t]^2 + R^2 - t^2} > T \quad (59)$$

which can be put into the form

$$t^2 + 2Tu^0 t + T^2 + 2T\mathbf{u} \cdot \mathbf{x} - R^2 < 0 . \quad (60)$$

Since  $u_i^2 = -1$  this can be rewritten as

$$-(u_2T - x)^2 < 0, \quad (61)$$

which is equivalent to the requirement that  $u_2T - x$  be spacelike. For this term to be nonzero the observation point  $x$  must not be in the future of the event  $u_2T$ , the second turn-around point at which the trajectory returns to forward time evolution. Using (56) the condition  $s_3 > T$  is expressed as

$$u^0(t + 2u^0T) - \mathbf{u} \cdot \mathbf{x} - \sqrt{[\mathbf{u} \cdot \mathbf{x} - u^0(t + 2u^0T)]^2 + R^2} - (t + 2u^0T)^2 > T \quad (62)$$

equivalent to the requirement

$$-[(u_3T - q_3) - x]^2 > 0 \quad (63)$$

where again  $q_3 = (-2u^0T, \mathbf{0})$ . Since

$$u_3T - q_3 = u_2T \quad (64)$$

this requires that the observation point be in the future of the second turn-around point. It is convenient to write

$$\theta(s_3 - T) = \theta(-(x - u_2T)^2) = 1 - \theta((x - u_2T)^2) \quad (65)$$

which splits the potential into 3 pieces

$$A(x) = A_1(x) + A_2(x) + A_3(x) \quad (66)$$

where

$$A_1 = -\frac{e}{4\pi R} \frac{(u^0, \mathbf{u}) \theta(t + u^0T)}{\sqrt{1 + \left(\frac{t + 2u^0T}{R}\right)^2 \left[ \left(\frac{\mathbf{u} \cdot \mathbf{x}}{t + 2u^0T} - u^0\right)^2 - 1 \right]}} \quad (67)$$

appears most like the usual Liénard-Wiechert potential with support on the future of the second turn-around point, and

$$A_2 = -\frac{e}{4\pi R} \left[ \frac{(u^0, \mathbf{u})}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} - u^0\right)^2 - 1\right]}} + \frac{(-u^0, \mathbf{u})}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]}} \right] \theta(R - t) \quad (68)$$

$$A_3 = \frac{e}{4\pi R} \left[ \frac{(u^0, \mathbf{u})}{\sqrt{1 + \left(\frac{t + 2u^0T}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t + 2u^0T} - u^0\right)^2 - 1\right]}} + \frac{(-u^0, \mathbf{u})}{\sqrt{1 + \left(\frac{t}{R}\right)^2 \left[\left(\frac{\mathbf{u} \cdot \mathbf{x}}{t} + u^0\right)^2 - 1\right]}} \right] \theta(R - (t + u_2T)) \quad (69)$$

have support in the present (spacelike separation) of the “pull-back” in time.

## 6. Discussion

In the previous section we derived the Maxwell potential associated with a particle trajectory that includes a “pull-back” in time. Under the Stueckelberg-Feynman interpretation, the resulting zig-zag in the time coordinate will be observed in the laboratory as the following sequence:

- (i) Particle-1 moves on a linear trajectory with energy  $E = mu^0$  and velocity  $\mathbf{u}/u^0$ ,
- (ii) At clock time  $t = -u^0T$ , particle-1 reaches  $\mathbf{x} = -\mathbf{u}T$ , while a pair creation process at  $\mathbf{x} = \mathbf{u}T$  produces particle-2 and an antiparticle,
- (iii) At clock time  $t = 0$ , particle-1 and the antiparticle mutually annihilate,
- (iv) Particle-2 continues on a linear trajectory congruent with the original path of particle-1.

The observation of such a process and the measurement of the time “pull-back” would require many detectors along the particle trajectory. Given the good fortune to have placed detectors in (spacelike) proximity to the pair processes, one may also observe the unusual contributions (68) and (69) to the Maxwell potential — and hence the electromagnetic fields.

However, in addition to these signatures, the duration  $T$  of the “pull-back” in time will be embedded, through equation (67), in the form of the electromagnetic fields at any point along

the trajectory, specifically when the particle arrives at the detector. If the line of observation is transverse to the particle velocity, so that  $\mathbf{u} \cdot \mathbf{x} = 0$ , then at high energy, with  $(u^0)^2 \gg 1$ , one may have  $t \simeq 2u^0T$  causing the potential

$$A_1 \simeq -\frac{e}{4\pi R} \frac{(1, \mathbf{v})}{\sqrt{1 + \left(\frac{t + 2u^0T}{R}\right)^2}} \quad (70)$$

to deviate measurably from the expected value (found from equation (70) with  $T = 0$ ).

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