QUARKS: CURRENTS AND CONSTITUENTS

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ABSTRACT

In this thesis an attempt is made to clarify the connection between two physically different $SU(6)_W$ algebras. The one $SU(6)_W$ is an approximate symmetry group, and leads to the idea that hadrons are composed of just a few "constituent" quarks. The other $SU(6)_W$ is an algebra of physically observable operators, integrals over various components of vector, axial, and tensor currents. These currents behave, algebraically, as if they were simple bilinear combinations of "current" quark fields.

We propose that these two physically different algebras are related by a unitary transformation. This transformation is necessarily very different from the identity. We identify several properties of this transformation, and then go on to construct it explicitly in the free quark model, where it yields an exactly conserved $SU(6)_W$ symmetry of constituent quarks.

We then show how this transformation may be constructed in models with interacting quarks. In general, the algebraic structure of the transformation depends upon the dynamical details of the interaction. We discuss the effect of interactions in more detail, distinguishing two cases. In one case the structure of the transformation in the interacting model need not have any relation to that of the free quark model. In the other case, the algebraic structure of the two transformations are the <u>same</u>. We cannot distinguish these two cases at present. However, as found in the last section, the

algebraic structure of the transformed currents in nature seems to be roughly that given by the free quark model transformation. The mechanism by which this occurs is obscure at present, and we can make no clearcut distinction between the two cases.

As we have indicated, the last section is devoted to the application of the algebraic structure of the free quark model transformation to the matrix elements of physical currents. We are thus led to many successful approximate relations among matrix elements, not the least of which is the recovery of the famous ratio μ_T (proton)/μ_T (neutron) = $-3/2$.

To Penny

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I. INTRODUCTION*

Two Algebras Named $SU(6)_W$

The name $SU(6)_W$ first appeared in particle physics during the year 1965. Since that time, it has been used to denote the algebra of two physically different sets of operators.

One of these $SU(6)_W$'s, which we shall hereafter call $SU(6)_{W,\mathrm{strong}}$, was discovered by H. J. Lipkin and S. Meshkov.[1] These authors sought to explain the approximate spin independence of the strong interactions by postulating that the strong interaction Hamiltonian, H_{st}, approximately commutes with a set of 35 operators W_i^{α}. These operators are taken to have the same commutation relations, charge conjugation, and parity as the quark model expressions

$$W_i \sim \int d^3x \;\; q^+(x) \;\; \frac{\lambda i}{2}\, q(x) \qquad \text{(I-1a)}$$

$$W_i^{\,1} \sim \int d^3x \;\; q^+(x) \;\; \frac{\beta\sigma^1}{2} \; \frac{\lambda i}{2}\, q(x) \qquad \text{(I-1b)}$$

$$W_i^{\,2} \sim \int d^3x \;\; q^+(x) \;\; \frac{\beta\sigma^2}{2} \; \frac{\lambda i}{2}\, q(x) \qquad \text{(I-1c)}$$

$$W_i^{\,3} \sim \int d^3x \;\; q^+(x) \;\; \frac{\sigma^3}{2} \; \frac{\lambda i}{2}\, q(x) \qquad \text{(I-1d)}$$

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* This chapter is essentially a review, for the purpose of making the distinction between the two $SU(6)_W$'s as clear as possible. It contains nothing which cannot be found in references 5 and 11.

where λ_i is the usual 3x3 matrix representation of SU(3), and i runs from 0 to 8. The β and σ^i are 4x4 Dirac matrices. Note that W_o is excluded from $SU(6)_W$; its inclusion simply enlarges the algebra slightly to $U(6)_W$. One must be careful about interpreting expressions (I-1a)-(I-1d). They are not equations. The tilde here means simply "has the same algebra as." As we shall see later, the W_i^α cannot even be written as integrals over local operators; nevertheless, their algebra is the same. It is important not to become confused here. The expressions on the right-hand sides of (I-1a) - (I-1d) are often used in quark models to denote an $SU(6)_W$ algebra of local currents. We do not wish to imply any such thing here. We only write these expressions as a shorthand for the assumed algebraic structure of the W_i^α .

Now it should be immediately clear -- from the hadron mass spectrum, for example -- that $SU(6)_{W,strong}$ symmetry, insofar as it exists, must be badly broken. Nevertheless, the general hope is that there is some ideal limit, not "too" far removed from reality, in which the symmetry becomes exact. For example, we might hope that H_{st} can be broken up into the form $H_o + \alpha H_{int}$, where $[W_i^\alpha, H_o] = 0$ so that only H_{int} breaks the symmetry. If the matrix elements of H_o are large in comparison to those of H_{int}, then we have broken symmetry, which becomes exact in the "ideal limit" $\alpha \to 0$.

If we simply assume that such an ideal limit exists for $SU(6)_{W,strong}$, and proceed to test the predictions of this hypothetical symmetry, we find quite good agreement with experiment in many cases.

In making these predictions, the lowest lying negative parity mesons (the pseudoscalar and vector octets and singlets) are assumed to belong to the <u>35</u> and <u>1</u> representations, and the lowest lying positive parity baryons (the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decimet) to the <u>56</u>.

Perhaps the most interesting feature of $SU(6)_{W,strong}$ is the fact that the W-charge* of a particle seems to be independent of its momentum in the $\hat{z}$ direction. Although this property is <u>suggested</u> by the quark model expressions (I-1a)-(I-1d), (which are, moreover, only invariant under boosts in the $\hat{z}$ direction in the ideal limit where they commute <u>exactly</u> with H_{st}), there is really no compelling reason to believe that the postulated W_i^{α} have the same Lorentz properties as these expressions. That the W_i^{α} actually do commute with boosts in the $\hat{z}$ direction, $[W_i^{\alpha}, \Lambda^3] = 0$, seems to be demonstrated by the validity of the Johnson-Treiman relations[2] for a wide range of particle momenta. This peculiar feature makes symmetry predictions particularly easy to evaluate for collinear processes. The fact that these predictions hold equally well for states with any momentum in the $\hat{z}$ direction has earned $SU(6)_{W,strong}$ the name of "relativistic spin symmetry."

In summary, there seems to be a fair amount of purely empirical evidence supporting the existence of a badly broken $SU(6)_{W,strong}$ symmetry. At present, we know very little about the

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* Note that we shall use the term "charge" in a generic sense. Thus, we shall refer to any set of integrated operators forming a closed algebra as "charges" regardless of whether or not they can be expressed as the spatial integral of the timelike component of a 4-vector density.

charges which generate this symmetry. We know definitely only their commutation relations, charge conjugation, and parity. We suspect that the charges are invariant under boosts in the $\hat{z}$ direction (that is, in the ideal limit in which the symmetry is exact). Further than this, we cannot go at the moment.

The second $SU(6)_W$ was introduced in the same year by R. Dashen and M. Gell-Mann.(3) These authors considered extensions of the well-known chiral SU(3) x SU(3) algebra of the vector and axial vector charges, F_i and F_i^5, respectively. These charges are the space integrals of the 0^{th} components of the vector and axial vector current densities $\mathcal{F}_i^{\mu}(x)$ and $\mathcal{F}_i^{\mu 5}(x)$. Both densities are directly measurable in weak and electromagnetic processes, and hence are well-defined physical operators. Dashen and Gell-Mann proposed adding a new density, that of a world scalar, to the system and commuting operators at equal times until the algebra is complete. If the equal time commutation relations are those of the quark model, the algebra closes on a U(12) of 144 operators. Among this set of operators one finds an anti-symmetric tensor current, $\mathcal{F}_i^{\mu\nu}(x)$, which behaves like the quark model expression $\bar{q}\,(x)\,\sigma^{\mu\nu}\,\frac{\lambda i}{2}\,q(x)$. Although nothing seems to be directly coupled to this current in nature, unlike $\mathcal{F}_i^{\mu}(x)$ and $\mathcal{F}_i^{\mu 5}(x)$, it may nevertheless appear on the right-hand side of commutators of directly observable operators, eg. in $[\partial_{\mu}\mathcal{F}_i^{\mu 5}(x),\ \mathcal{F}_i^{k}(y)]$. We may thus assume that it makes physical sense to talk about operators in which these tensor currents appear.

The tensor currents $\mathcal{F}_i^{\mu\nu}(x)$ are particularly interesting for us because they can be used in conjunction with the vector and axial vector currents to define an $SU(6)_W$ subgroup of U(12) whose operators have the same commutation relations, charge conjugation, and parity as the W_i^α of $SU(6)_{W,\text{strong}}$. Let us be careful not to identify the two sets of operators too quickly, however, and instead call the new algebra by the name $SU(6)_{W,\text{currents}}$. This algebra consists of 35 operators F_i^α which are defined as

$$F_i = \int d^3x \; \mathcal{F}_i^{\,o}(x) \tag{I-2a}$$

$$F_i^{\,1} = \frac{1}{2} \int d^3x \; \mathcal{F}_i^{\,23}(x) \tag{I-2b}$$

$$F_i^{\,2} = \frac{1}{2} \int d^3x \; \mathcal{F}_i^{\,31}(x) \tag{I-2c}$$

$$F_i^{\,3} = \frac{1}{2} \int d^3x \; \mathcal{F}_i^{\,35}(x) \tag{I-2d}$$

The $SU(6)_{W,\text{currents}}$ algebra of this set of operators was suggested by the quark model, where the local currents are bilinear in quark fields, $q^+(x)\,\Gamma\,\frac{\lambda i}{2}\,q(x)$. The similarity in algebraic structure between the W_i^α in (I-1a) - (I-1d) and the F_i^α above becomes clearly evident with this identification. It is vital to distinguish similarity in structure from equality, however. In various quark models we find that the above charges are equal to integrals over the bilinear expressions $q^+(x)\,\Gamma\,\frac{\lambda i}{2}\,q(x)$, whereas in these same models the W_i^α only have the same algebraic structure (indicated by the tilde). The W_i^α in these models are not even integrals over local operators.

In the case of the algebra (I-2a) - (I-2d) above, the operators may be completely defined, in principle, in a context other than that of the presumed symmetry itself. In particular, the Lorentz transformation properties are known, and we clearly see the property that $[F_i^\alpha, \Lambda^3] = 0$ if the F_i^α are conserved. In fact, the forms (I-2a)-(I-2d) show us how to sharpen our expression of this property: we need only note that the F_i^α are "good" operators, in the sense that their matrix elements don't vanish when taken between finite mass states with infinite momentum in the $\hat{z}$ direction. The "goodness" of the F_i^α automatically implies $[F_i^\alpha, \Lambda^3] = 0$ in the limit of conservation, and gives an unambiguous statement of what we mean when the F_i^α are not conserved. In a like manner, we can summarize the idea that the $SU(6)_{W,\text{strong}}$ charges are independent of momentum in the $\hat{z}$ direction by making the W_i^α "good" operators.

Equations (I-2a)-(I-2d) demonstrate the existence of a set of 35 operators F_i^α which are physically well defined, and which seem in all respects similar to the 35 operators W_i^α. The fundamental question is, are these two sets of operators really the same? There seem to be several good reasons for identifying the W_i^α with the F_i^α. In particular, it seems that the W_i, the generators of the ordinary SU(3) of strong interactions, are the same as the F_i, the integrals of the timelike components of the vector currents, by the success of CVC in weak interactions. Since the two sets of operators behave alike in algebraic respects, it would be quite natural to use the hint from CVC and set them <u>all</u> equal. There are, however, several formidable objections to such a procedure.

The first objection is that no sensible ideal limit (wherein the charges of $SU(6)_{W,\text{currents}}$ commute with H_{st}) is known to exist. $SU(6)_{W,\text{currents}}$ is not even a symmetry of the free quark model, where the F_i^{α} fail to commute with the quark kinetic energy term. The only limit in which $SU(6)_{W,\text{currents}}$ can become a symmetry is one where the quark mass tends to infinity -- i.e., the nonrelativistic limit -- but such a limit can be of no value for the manifestly relativistic processes which we wish to consider. One can argue that (as we find in the free quark model) the commutator $[F_i^{\alpha}, H_{st}]$ has a zero expectation value when taken between collinear states moving in the $\hat{z}$ direction. However, if one takes this idea seriously, one faces the difficulty of how to propagate the symmetry through the non-collinear intermediate states which presumably make large contributions to many low-energy strong interaction processes where $SU(6)_{W,\text{strong}}$ is known to work fairly well (for higher energy processes, one might, however, argue that the cutoff in transverse momentum at 300 MeV/c would be effective in suppressing the non-collinear intermediate states).

Another possibility is to couple the quarks to fields which transform according to non-trivial $SU(6)_{W,\text{currents}}$ representations, hoping to arrange the interaction in such a way that the interacting Hamiltonian is $SU(6)_{W,\text{currents}}$ invariant. Such an approach, somewhat in the spirit of the non-linear chiral Lagrangians, has actually been proposed by P. Chang and F. Gürsey.[(4)] As we shall find, however, the identification of $SU(6)_{W,\text{currents}}$ as an approximate symmetry group leads to results which are quite unsatisfactory. In the present approach, we shall assume that the generators of $SU(6)_{W,\text{currents}}$ are far from being conserved.

There is direct objection to the use of $SU(6)_{W,\text{currents}}$ as an approximate symmetry group. R. Dashen and M. Gell-Mann[5] have attempted to classify the $\frac{1}{2}^{+}$ octet and $\frac{3}{2}^{+}$ decimet baryons in a $\underline{56}$ of $SU(6)_{W,\text{currents}}$ in the infinite momentum frame. They have shown that such a classification would imply that the anomalous magnetic moments of all $\frac{1}{2}^{+}$ octet baryons must vanish, along with the octet-decimet magnetic transition amplitudes. Since these results are far from being true in nature, one arrives at the necessity of describing the physical baryons as complex mixtures of many $SU(6)_{W,\text{currents}}$ irreducible representations. This impurity of physical states under the transformations generated by $SU(6)_{W,\text{currents}}$ (or its subgroup $SU(3) \times SU(3)_{,\text{currents}}$) is, in fact, the raison d'être of the many mixing schemes which have been proposed to obtain information about the matrix elements of currents between states of infinite momentum.[6] While these different schemes vary in detail, they all seem to agree on the need for very appreciable mixing between a variety of irreducible representations.

We can thus contrast the $SU(6)_{W,\text{currents}}$, whose charges are far from being conserved, and under which physical states appear to be complicated mixtures of irreducible representations, to the $SU(6)_{W,\text{strong}}$, whose empirical success presents the quite different aspect of nearly conserved charges with physical states lying in simple irreducible representations.

In more picturesque terms, we can say that the image of a baryon we develop using the $SU(6)_{W,strong}$ is one of just three "constituent" quarks sitting in an S state, whereas the $SU(6)_{W,currents}$ baryon is a complex object with many "current" quarks and quark pairs, all moving in states with various orbital angular momenta. This picture is reinforced by the results on deep inelastic electron scattering from SLAC. The infinite numbers of "partons" required to explain the observed neutron and proton structure functions are presumably nothing other than these current quarks and current quark pairs along with some kind of neutral glue.

Thus, we are forced to conclude that the two $SU(6)_W$'s are quite different. We might expect, however, that there is some relation between them. Their close similarity should not be dismissed as purely accidental. Since we can't set them equal, let us suppose instead that they are related by some unitary transformation, which we shall call V.[21]

Since the quantum numbers of either $SU(6)_W$ do not uniquely define a state, it is not <u>necessary</u> that a unitary V exists. V will exist only if the structure of representations of either $SU(6)_W$ on the physical states is the <u>same</u>. For example, if we find a <u>70</u> of $SU(6)_{W,strong}$ in the spectrum, there must be a corresponding <u>70</u> of $SU(6)_{W,currents}$ in the Hilbert space of physical states. We postulate this to be the case, writing

$$W_i^{\alpha} = V F_i^{\alpha} V^{-1} \qquad \text{(I-3)}$$

where V is unitary.

Some of the properties of V are immediately evident: It must have parity $P = +$, charge conjugation $C = +$ (since these properties are identical for both sets of charges, and we wish V to be continuously connected to the unit transformation). It must be invariant under spatial rotations about the $\hat{z}$ axis, $[V, J^3] = 0$, and it must be an SU(3) singlet, $[V, F_i] = 0$, at least to a very good approximation, in order for CVC generalized to SU(3) to remain valid. Other properties will be discussed in the next chapter.

Before going on, a few words about the classification of states would seem appropriate. Simple classification of states in $SU(6)_W$ multiplets is not enough to uniquely specify a state. It has been found many other quantum number are required. One of these quantum numbers can be taken to be the spin in the $\hat{z}$ direction (i.e., the helicity for collinear processes), and the group denoted $SU(6)_W \times O(2)$ (for either $SU(6)_W$). A more useful classification, however, is derived by noting that W_o^3 and F_o^3 act like quark spin operators for "constituent" and "current" quarks respectively. Since both operators commute with J^3, it makes sense to define "quark orbital angular momenta" by $L^3(W) = J^3 - W_o^3$ and $L^3(F) = J^3 - F_o^3$. Although these identifications are suggested by the quark model, the procedure is perfectly general. We may now classify states in terms of their $SU(6)_W$ quantum numbers, and "angular momenta." For example, the $SU(6)_{W,\text{strong}}$ classification for the low-lying baryons would be $\underline{56}$, $L^3(W) = 0$. The next highest set of baryons seems to be a $\underline{70}$, $L^3(W) = -1,0,1$.

Such a classification can be used to make the naïve quark picture of hadrons more precise. Let us suppose that $[V, F_o^3] \neq 0$, so that F_o^3 and W_o^3 are different operators. Then in general, where the $SU(6)_{W,\text{strong}}$ picture indicates, say, three quarks with a simple spin structure, the $SU(6)_{W,\text{currents}}$ classification of the same state will indicate a large mixture of different spins. Correspondingly, the values of $L^3(W)$ will be simple, while those of $L^3(F)$ will indicate a mixture of "orbital angular momenta." $L^3(F)$ must be complicated in order that $L^3(F) + F_o^3$ add up to the same J^3 as $L^3(W) + W_o^3$ (remember $[J^3, V] = 0$, so that the value of J^3 is the same for both classifications).

The transformation V thus expresses the general idea of the phenomenological mixing schemes in a compact way. It allows one to describe the hadrons as simple objects, "containing" just three quarks where strong interactions are involved, and at the same time giving them the necessary complicated structure where current matrix elements are concerned.

The usefulness and structure of such a transformation have been demonstrated phenomenologically by F. Buccella, H. Kleinert, C. A. Savoy et al.[7] in the infinite momentum frame. These authors have succeeded in fitting many coupling constants by means of this approach. The present work will be more theoretical in nature, and will concentrate more on showing how such a transformation arises physically. Our result will be seen to have transformation properties similar to that of Buccella, Kleinert, and Savoy.

The major problem is to actually say something about V. Assuming that such an operator actually exists in nature, what can we find out about its properties? In the next chapter we shall define V in an unambiguous way and make a list of what we know about V from physics. We will show how such a V can be explicitly constructed in the free quark model (to give an exact $SU(6)_{W,strong}$). We then go on to show how V may be constructed in any model based on interacting quarks.

A mystery then arises. The problem is that the algebraic structure of transformed charges and currents in nature seems to be roughly that of the free quark model. The evidence for this is that blind use of the free quark model transformation properties yields many successful relations between physical matrix elements.

We propose a solution to this mystery, arguing that the free quark algebraic structure may persist in interacting models if the spectrum of the interacting model shows an $SU(6)_{W,strong}$ symmetry of the appropriate kind. The two miracles, that of an $SU(6)_{W,strong}$ symmetry apparent in the spectrum and that of a free quark-like algebraic structure may be related, leaving only one miracle to explain.

Since the hadron spectrum does show an approximate $SU(6)_{W,strong}$ symmetry, it is therefore possible to suppose that relations between physical matrix elements derived via the algebraic structure of the free quark transformation V_{free} will be accurate to the order of $SU(6)_{W,strong}$ symmetry breaking in the spectrum, that is, to 20% or 30%.

This expectation is verified in the last chapter where many successful relations among the matrix elements of the axial vector charge and the electromagnetic current are found. Nevertheless, one should not regard this solution as anything more than a possibility. The actual mechanism by which the structure of V is determined is obscure, and it would be premature to try to guess the mechanism from the present work. In this respect we have only just begun to approach the problem, and there is a great deal yet to be learned.

II. PROPERTIES OF THE TRANSFORMATION

Thus far, we have not stated precisely what job this transformation V performs. Ambiguities occur because $SU(6)_{W,\text{strong}}$ is not an exact symmetry. If $SU(6)_{W,\text{strong}}$ were exact, there would be no difficulty: There would exist a well-defined set of generators W_i^{α} which could be related to the F_i^{α}. However, the generators of an approximate symmetry are not particularly well defined: We can always add other small operators, operators which may either better or worsen the appearance of symmetry without changing the "approximately conserved" nature of the generators. Thus, when there is no exact $SU(6)_{W,\text{strong}}$ we need yet another unitary transformation: a unitary transformation which takes us from the actual energy eigenstates to states transforming irreducibly under $SU(6)_{W,\text{strong}}$. This transformation, too, is poorly defined. However, the combined transformation, the one which takes us from energy eigenstates to irreducible representations of $SU(6)_{W,\text{strong}}$ and thence to irreducible representations of $SU(6)_{W,\text{currents}}$ is well defined, since the generators of $SU(6)_{W,\text{currents}}$ are integrals over (in principle) observable operators. This is the transformation which we shall study. Note that it reduces to our naïve idea of a relation between $SU(6)_{W,\text{strong}}$ and $SU(6)_{W,\text{currents}}$ when $SU(6)_{W,\text{strong}}$ is exactly conserved, since in that case energy eigenstates coincide with irreducible representations of $SU(6)_{W,\text{strong}}$.

Before proceeding, we need some sort of notation for labeling states. Let us label a state as the ρ member of the R representation of an $SU(6)_W$. Which $SU(6)_W$ is meant will be denoted by a subscript "strong" or "currents." Let us denote any other quantum numbers which may be needed to uniquely specify a state by "α". The operators connected with the "α" are assumed to commute with the generators of the appropriate $SU(6)_W$, and hence are included within the subscript. This is unnecessary only if the operators in question commute with V, which will generally not be the case. Note that V does not change momentum, so that momentum is always a good quantum number, although energy may not be. (This unpleasant tie to a particular Lorentz frame will be dissolved later, when we switch to charges integrated over a light-like plane. For now, we feel that there is some pedagogic value in remaining fixed in a particular frame.)

Using this notation, we designate a state as $|\vec{p}\,(R, \rho, \alpha)_{\text{currents}}\rangle$ when we wish to work with states which are irreducible representations of $SU(6)_W$, currents, and as $|\vec{p}\,(R, \rho, \alpha)_{\text{strong}}\rangle$ when irreducible representations of $SU(6)_{W,\text{strong}}$ are more convenient. In many cases we shall not need the full list of quantum numbers, and so shall write R, or some other letter, in place of the list. Thus, we shall often use the notation $|R_{\text{currents}}\rangle$ or $|R_{\text{strong}}\rangle$ to denote a state which transforms irreducibly under $SU(6)_{W,\text{currents}}$ or $SU(6)_{W,\text{strong}}$, respectively. When $SU(6)_{W,\text{strong}}$ is an exact symmetry, V specifies the relationship between the two labeling schemes, by definition. In general, a state labeled by one scheme will have a projection upon many states labeled by the other scheme.

Finally, we shall designate "physical states" -- that is, states which are eigenstates of the Hamiltonian -- by the traditional Poincaré group quantum numbers: momentum $\vec{p}$, energy E, spin j, and the spin component along $\hat{z}$, m. Furthermore, we may have to designate the channel we are interested in by other conserved quantum numbers such as charge, isotopic spin, and strangeness (which are conserved by the hadronic part of the Hamiltonian). We shall use the symbol Q to collectively designate these quantum numbers (note that Q is a subset of ρ, since both $SU(6)_W$'s include SU(3)). Thus, we designate a "physical state" by $|\vec{p}, E\, j\, m; Q >$ or, for brevity, simply by $|E >$ when we do not need to specify the other indices in more detail.

Armed with these notational devices, we can expand a physical state in terms of states which transform irreducibly under $SU(6)_{W,\ \mathrm{currents}}$:

$$|\vec{p},\ Ejm;\ Q\rangle = \sum_{R,\rho,\alpha} \langle \vec{p}\ (R,\rho,\alpha)_{\mathrm{currents}}\ |\vec{p},\ Ejm;\ Q\rangle |\ \vec{p}\ (R,\rho,\alpha)_{\mathrm{currents}}\rangle$$

or simply

$$|E\ \rangle = \sum_{R} \langle R_{\mathrm{currents}}\ |E\rangle\ \ |R_{\mathrm{currents}}\rangle \qquad \text{(II-1)}$$

the specific values of the indices being understood. In operator form this becomes

$$|E\ (R_{\mathrm{currents}})\rangle = V\ \ |R_{\mathrm{currents}}\rangle \qquad \text{(II-2)}$$

where

$$V = \sum_{R} |E\ (R_{\mathrm{currents}})\rangle\ \langle R_{\mathrm{currents}}| \qquad \text{(II-3)}$$

V is not fully defined until we specify an association $(R\ \rho\ \alpha) \leftrightarrow (E\ j\ m;\ Q)$. In principle, we can choose any association we wish without changing the physical content of the transformation, but in practice we shall find a "natural" one.

The transformation V in eq. (II-3) relates physical states to states which transform irreducibly under $SU(6)_{W,\text{currents}}$. As such, it is the appropriate generalization of the transformation between $SU(6)_{W,\text{currents}}$ and $SU(6)_{W,\text{strong}}$ discussed in the introduction.

In the following work we shall retain eq. (I-3), although it is now more a definition of W_i^{α} than of V. In this way we avoid the complication of considering some transformation which takes us from physical states to states which transform irreducibly under a poorly defined $SU(6)_{W,\text{strong}}$. The $SU(6)_{W,\text{strong}}$ classification of a given state is thus completely determined by the association $(R, \rho, \alpha) \leftrightarrow (E\ j\ m;\ Q)$. From this point of view, broken $SU(6)_{W,\text{strong}}$ is a very subordinate kind of symmetry -- as it should be since its operators are not physically well determined. The actual algebraic framework of the theory derives from $SU(6)_{W,\text{currents}}$, whose operators <u>are</u> well determined. The reason that we even consider $SU(6)_{W,\text{strong}}$ as a separate entity is due to a dynamical accident: physical states do seem to lie in approximately degenerate $SU(6)_{W,\text{strong}}$ multiplets. The $SU(6)_{W,\text{currents}}$ algebra, however, is far from being a symmetry, so we need some transformation V to take us over to the physical states. Once among the physical states, the approximate degeneracy of multiplets implies that we are only a small step away from some $SU(6)_{W,\text{strong}}$. In view of this situation, we have <u>defined</u> the irreducible representations of $SU(6)_{W,\text{strong}}$ to coincide with these multiplets. Moreover, this is how we establish our "natural" association of physical states with $SU(6)_{W,\text{currents}}$ states: We first group a set of states $\left\{ |\vec{p}\ \ Ejm;\ Q\rangle \right\}$ into a multiplet $\left\{ |\vec{p}\ (R,\rho,\alpha)_{\text{strong}}\rangle \right\}$ (an empirical process), then we associate $(R,\rho,\alpha)_{\text{strong}} \leftrightarrow (R,\rho,\alpha)_{\text{currents}}$. This process is unique to the extent that we can find all the states in a given multiplet, but is only possible in practice because of the approximate degeneracy of multiplets. It should be clear from the definitions that $[W_i^{\alpha}, H_{st}] = 0$ when the multiplets are exactly degenerate, and $[W_i^{\alpha}, H_{st}] \approx 0$ when the splitting is small.

So much for the general structure. In order to actually learn something about V we must construct it somehow. We shall attempt to do this in the next two chapters.

To aid this construction, we can derive some constraints from nature, requirements which can be imposed upon major portions of V. Several properties were mentioned in the introduction. Beside these, a further property follows directly from definition (II-2) and the fact that the F_i^{α} are "good" operators. This requires that V must be constructed only from "good" operators itself or, at least, that it can only take good operators into good operators. For the present, we shall assume the stronger form, viz. that V contains only "good" operators. We shall see how this must be modified in Chapter IV.

As a result of the "goodness" requirement, we expect that V contains no world scalar densities, no transverse components of vector densities, and no spatial or longitudinal components of tensor density operators. V should be expressible as a spatial integral (to preserve translation invariance) over operators having one and only one time or z index among their Lorentz indices. All other indices must be transverse, x or y.

An immediate consequence of these restrictions is that V cannot be invariant under the complete O(3) of spatial rotations. We must thus define a new V (i.e., a new $SU(6)_{W,strong}$) for each direction in space. This should come as no surprise: by defining a "collinear group" like $SU(6)_W$, we have picked out a privileged direction in space, and it is only natural that operators connected with the group should depend upon that direction.

Let us now summarize our knowledge about the transformation V:

(a) V transforms states lying in irreducible representations of $SU(6)_{W,\,currents}$ into states with definite energy and spin

$$|E\,(R_{currents})\rangle \;=\; V\;|R_{currents}\rangle\ .$$

(b) V transforms the F_i^{α} in such a way that the $VF_i^{\alpha}V^{-1}$ are conserved in some sensible limit not "too" far removed from reality. In other words, physical states fall into nearly degenerate $SU(6)_{W,strong}$ multiplets.

(c) V contains only "good" operators or, at least, takes "good" operators only into "good" operators. This ensures that finite mass states at infinite momentum are mixed only among themselves.

(d) V is an SU(3) singlet, $[F_i, V] = 0$, in the limit where all physical processes are SU(3) invariant.

(e) V has P = +, C = + and is invariant under O(2), $[J^3, V] = 0$.

(f) V is unitary.

In order to get further insight into the structure of V, it seems necessary to resort to explicitly constructing it in a simple model.

III. EXPLICIT CONSTRUCTION OF V IN THE FREE QUARK MODEL

In the free quark model the fundamental operator of the theory is $q(x)$, a local relativistic field obeying the equal time anticommutation relations,

$$\left\{ q_\alpha(x),\ q_\beta^+(y) \right\}_{et} = \delta_{\alpha\beta}\, \delta^3(x - y) \qquad \text{(III-1)}$$

$$\left\{ q_\alpha(x),\ q_\beta(y) \right\}_{et} = 0, \text{ etc.}$$

Local current densities can be constructed from bilinear combinations of these fields, being of the form $q^+(x)\,\Gamma\,\frac{\lambda i}{2}\,q(x)$, where Γ is a 4x4 Dirac matrix. In terms of these densities, the generators F_i^{α} of $SU(6)_{W,\text{currents}}$ are definable in this model. Note that these operators exist <u>only</u> in the model, and that there is no reason to believe that the corresponding operators in nature can be written in such a bilinear form. However, since the F_i^{α} in nearly any quark model are the same as the F_i^{α} in the free quark model (so long as whatever gluons which may be present are $SU(6)_{W,\text{currents}}$ singlets), we shall adopt the form below without special subscripts in the work that follows. Nevertheless, one must keep in mind that such F_i^{α} are model operators, and that the generators (I-2a) - (I-2d) of the physical $SU(6)_{W,\text{currents}}$ need not have the same form.

With these reservations we can define

Model

$$F_i = \int d^3x \; q^+(x) \frac{\lambda i}{2} q(x) \qquad \text{(III-2a)}$$

$$F_i^{\,1} = \frac{1}{2} \int d^3x \; q^+(x) \; \beta\sigma^1 \; \frac{\lambda_i}{2} \; q(x) \qquad \text{(III-2b)}$$

$$F_i^{\,2} = \frac{1}{2} \int d^3x \; q^+(x) \; \beta\sigma^2 \; \frac{\lambda_i}{2} \; q(x) \qquad \text{(III-2c)}$$

$$F_i^{\,3} = \frac{1}{2} \int d^3x \; q^+(x) \; \sigma^3 \; \frac{\lambda_i}{2} \; q(x) \qquad \text{(III-2d)}$$

The $SU(6)_{W,\text{strong}}$ generators appear in this model as

$$W_{i,\text{ free}}^{\alpha} = V_{\text{free}} \, F_i^{\alpha} \; V_{\text{free}}^{-1} \qquad \text{(III-3)}$$

Finally, the Hamiltonian of the theory is

$$H_{\text{free}} = \int d^3x \; q^+(x) \left\{ -i\vec{\alpha} \cdot \vec{\partial} + m\,\beta \right\} q(x) \qquad \text{(III-4)}$$

It is easy to check that this model contains all the difficulties with $SU(6)_{W,\text{currents}}$ previously mentioned. $[F_i^{\alpha}, H_{\text{free}}] \neq 0$ in any limit except the non-relativistic one where $m \to \infty$, with the kinetic energy becoming a negligibly small portion of the total energy.

In the following, we shall find it convenient to work in a representation where the $W_{i,\text{free}}^{\alpha}$ have a simple form. We thus define the "W-representation" whose operators $O_W = V^{-1}OV$. In this representation, the operators $(W_{i,\text{free}}^{\alpha})_W$ have the forms (III-2a) - (III-2d), while $(F_i^{\alpha})_W = V_{\text{free}}^{-1} \; F_i^{\alpha} \; V_{\text{free}}$ and hence contain whatever complexities V_{free} may contain. The advantage of this representation is that we can tell at a glance whether or not the $W_{i,\text{free}}^{\alpha}$ commute with

H_{free}: if $(H_{free})_W$ contains any other Dirac matrices than 1, β, α^3, $\beta\alpha^3$, then the $(W^{\alpha}_{i,free})_W$ will not commute with it (that is, so long as $(H_{free})_W$ is bilinear in quark fields. Consideration of more complicated forms is not necessary at present).

Let us write the unitary operator V_{free} as

$$V_{free} = \exp(i\, Y_{free}) \qquad \text{(III-5)}$$

where Y_{free} is an Hermitian operator. Then we can readily check that if we choose

$$Y_{free} = \frac{1}{2} \int d^3x \;\; q^+(x) \arctan\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) q(x) \qquad \text{(III-6)}$$

then

$$\begin{aligned}(H_{free})_W &= V^{-1}_{free} \;\; H_{free} \;\; V_{free} \\ &= \int d^3x \; q^+(x) \left\{ -i\alpha^3\partial^3 + \beta\sqrt{m^2 + (\vec{\gamma}_\perp \cdot \vec{\partial}_\perp)^2} \right\} q(x)\end{aligned} \qquad \text{(III-7)}$$

which manifestly commutes with $(W^{\alpha}_{i,free})_W$, since $(\vec{\gamma}_\perp \cdot \vec{\partial}_\perp)^2 = -\vec{\partial}_\perp \cdot \vec{\partial}_\perp = -\partial_\perp^{\,2}$. In equations (III-6) - (III-7), the operator functions of $\vec{\gamma}_\perp \cdot \vec{\partial}_\perp$ are defined by their power series expansions; that is, in terms of the string of operators of decreasing physical dimension ℓ : $\bar{q}(x)\, q(x)$ $\ell = -3$, $\bar{q}(x)\, \partial_\perp^{\,2}\, q(x)$ $\ell = -5$, $\bar{q}(x)\, \partial_\perp^{\,2}\, \partial_\perp^{\,2}\, q(x)$ $\ell = -7$, etc. Note that V_{free} becomes the identity transformation only in the limit $m \to \infty$, as expected.

Thus, at least within this model, we can define an exactly conserved set of operators forming an $SU(6)_{W,strong}$. The transformation V_{free} explicitly possesses the properties (a) - (f) of Chapter II. One can easily check that the $W_{i,free}^{\alpha}$, written below in terms of the local quark fields q(x), commute with H_{free}:

$$W_{i,free} = F_i \qquad \text{(III-8a)}$$

$$W_{i,free}^{1} = F_i^{1} + \int d^3x\, q^+(x) \frac{1}{\varkappa} \left\{ \frac{1}{1+\varkappa} \frac{\vec{\gamma}_{\perp}\cdot \vec{\partial}_{\perp}}{m} - i \right\} \gamma^5 \frac{\partial^1}{m} \frac{\lambda_i}{2} q(x) \qquad \text{(III-8b)}$$

$$W_{i,free}^{2} = F_i^{2} + \int d^3x\, q^+(x) \frac{1}{\varkappa} \left\{ \frac{1}{1+\varkappa} \frac{\vec{\gamma}_{\perp}\cdot \vec{\partial}_{\perp}}{m} - i \right\} \gamma^5 \frac{\partial^2}{m} \frac{\lambda_i}{2} q(x) \qquad \text{(III-8c)}$$

$$W_{i,free}^{3} = F_i^{3} + \int d^3x\, q^+(x) \frac{1}{\varkappa} \left\{ \frac{1}{1+\varkappa} \frac{\vec{\gamma}_{\perp}\cdot \vec{\partial}_{\perp}}{m} - i \right\} \sigma^3 \frac{\vec{\gamma}_{\perp}\cdot \vec{\partial}_{\perp}}{m} \frac{\lambda_i}{2} q(x) \qquad \text{(III-8d)}$$

where the operator $\varkappa$ is defined as

$$\varkappa = \left[1 + \left(\frac{\vec{\gamma}_{\perp}\cdot \vec{\partial}_{\perp}}{m} \right)^2 \right]^{\frac{1}{2}} \qquad \text{(III-9)}$$

hence $\varkappa$ contains only operators of even dimension, $\ell = 0, -2, -4, \ldots$ The additional terms in (III-8b) - (III-8d) vanish if $m \to \infty$, or if the quarks have no transverse motion; however, these terms must be present for the W_i^{α} to be generally conserved.

These transformed operators look quite complex as written above. Is there any way to see why such complexity is necessary? In fact, there is. Let us write the q(x) in terms of creation and annihilation operators. Using a non-relativistic box normalization, we write

$$q(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k},r} \left\{ a_{\vec{k}}^{(r)} \; u^{(r)}(\vec{k}) \; e^{-i\,k\cdot x} + b_{\vec{k}}^{+(r)} \; v^{(r)}(\vec{k}) \; e^{i\,k\cdot x} \right\} \tag{III-10}$$

where the notation is standard, except that r runs over both Dirac and SU(3) indices. In terms of these $a_{\vec{k}}^{(r)}$ and $b_{\vec{k}}^{(r)}$ operators, the $W_{i,free}^{\alpha}$ take on a simple form:

$$W_{i,free} = \sum_{\vec{k};r,s} \left\{ a_{\vec{k}}^{+(r)} \; a_{\vec{k}}^{(s)} \left[u^{+(r)}(0) \; \frac{\lambda i}{2} \; u^{(s)}(0) \right] - b_{-\vec{k}}^{+(s)} \; b_{-\vec{k}}^{(r)} \left[v^{+(r)}(0) \; \frac{\lambda i}{2} \; v^{(s)}(0) \right] \right\} \tag{III-11a}$$

$$W_{i,free}^{1} = \sum_{\vec{k};r,s} \left\{ a_{\vec{k}}^{+(r)} \; a_{\vec{k}}^{(s)} \left[u^{+(r)}(0) \; (R^{-1} \sigma^{1} \frac{\lambda^{i}}{2} R) \; u^{(s)}(0) \right] + b_{-\vec{k}}^{+(s)} \; b_{-\vec{k}}^{(r)} \left[v^{+(r)}(0) \; (R^{-1} \sigma^{1} \frac{\lambda i}{2} R) \; v^{(s)}(0) \right] \right\} \tag{III-11b}$$

$$W_{i,free}^{2} = \sum_{\vec{k};r,s} \left\{ a_{\vec{k}}^{+(r)} \; a_{\vec{k}}^{(s)} \left[u^{+(r)}(0)(R^{-1} \sigma^{2} \frac{\lambda^{i}}{2} R) \; u^{(s)}(0) \right] + b_{-\vec{k}}^{+(s)} \; b_{-\vec{k}}^{(r)} \left[v^{+(r)}(0)(R^{-1} \sigma^{2} \frac{\lambda i}{2} R) \; v^{(s)}(0) \right] \right\} \tag{III-11c}$$

$$W_{i,free}^{3} = \sum_{\vec{k};r,s} \left\{ a_{\vec{k}}^{+(r)} \; a_{\vec{k}}^{(s)} \left[u^{+(r)}(0)(R^{-1} \sigma^{3} \frac{\lambda^{i}}{2} R) \; u^{(s)}(0) \right] - b_{-\vec{k}}^{+(s)} \; b_{-\vec{k}}^{(r)} \left[v^{+(r)}(0)(R^{-1} \sigma^{3} \frac{\lambda i}{2} R) \; v^{(s)}(0) \right] \right\} \tag{III-11d}$$

where the $u^{(r)}(0)$ and $v^{(r)}(0)$ are rest spinors. The matrix R represents a rotation of the Dirac indices, given by the 4x4 matrix

$$R = \sqrt{\frac{(\omega + k^o)(\omega + m)}{2\,\omega\,(k^o + m)}} \left\{ 1 - \frac{i\,k^3\,(\vec{k} \times \vec{\sigma})^3}{(\omega + k^o)(\omega + m)} \right\} \qquad \text{(III-12)}$$

here, ω is the "transverse energy," $\sqrt{|\vec{k}_\perp|^2 + m^2}$, and k^o is the ordinary energy, $\sqrt{|\vec{k}\,|^2 + m^2}$. The significance of R is readily determined by means of a simple example. First, note that R is unity for $k^3 = 0$, i.e., for a quark moving in a direction transverse to $\hat{z}$. A single quark state moving in this way would be created by $a_{k_\perp}^{+(1,s)}$, for example. Under $SU(6)_{W,strong}$ it would be classified as a $\underline{6}$, spin up, strange quark. Now consider what happens if we boost this state and examine it in a frame of reference where $k^3 \neq 0$. What we see is no longer a spin up state, if $\vec{k}_\perp \neq 0$. In general, there is a rotation of the spin, a Wigner Rotation,(8) which takes the boosted state into a mixture of spin up and spin down components. Now if R were unity, it is clear that the $SU(6)_{W,strong}$ classification of the state would change; however, R is just the inverse of the Wigner rotation in this case. Thus, the mixture of spin components produced by the boost is precisely the eigenstate of $W^{\alpha}_{i,free}$ for that momentum k^3. This is the detailed mechanism by which the W_i^{α} remain invariant under boosts: as already stated, the $SU(6)_{W,strong}$ classification of a state is independent of its momentum in the $\hat{z}$ direction. Note in particular that R does not approach unity as $k^3 \to \infty$: thus, we can already see that a state with infinite momentum, classified in an irreducible representation of $SU(6)_{W,\text{strong}}$, will contain a mixture of representations of $SU(6)_{W,\text{currents}}$ (this is clear from the fact that $F_o^{\ 3}$ simply counts the number of spin up and spin down quarks, regardless of their transverse momenta -- but an eigenstate of $W_o^{\ 3}$ contains a mixture of up and down quarks, as specified by R).

We see by this example whence the complexity of (III-8b) - (III-8d) arises: it is from the need for invariance of the $SU(6)_{W,\text{strong}}$ charges under boosts in the $\hat{z}$ direction. Also note that equations (III-11a) - (III-11d) do not contain a^+b^+ - type cross terms: i.e., the $W_{i,\text{free}}^{\alpha}$ annihilate the vacuum, as required.

Let us now analyze the structure of V_{free} in more detail. The first striking property we observe is that it is non-local in the transverse directions. This property can be readily demonstrated by examining the behavior of the quark field q(x) when it is transformed by V_{free}:

$$V_{\text{free}}\, q(x)\, V^{-1}_{\text{free}} = \frac{(\varkappa + 1) - i\,\dfrac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}}{\sqrt{2\varkappa\,(\varkappa + 1)}}\, q(x) \qquad \text{(III-13a)}$$

$$= \int d^3y\;\; K_{\text{free}}\,(x-y)\; q(y) \qquad \text{(III-13b)}$$

where the kernel $K_{\text{free}}(x)$ is given by

$$K_{\text{free}}(x) = \frac{\delta(x^3)}{(2\pi)^2} \int d^2p_\perp\; e^{i\,\vec{p}_\perp \cdot\, \vec{x}_\perp} \left\{ \frac{\omega + m + \vec{p}_\perp \cdot \vec{\gamma}_\perp}{\sqrt{2\omega(\omega + m)}} \right\} \qquad \text{(III-14)}$$

in which ω is again the "transverse energy." Since the integrand of (III-14) approaches unity only for $|p_\perp| \ll m$, we expect $K_{\text{free}}(x)$ to receive contributions from $|x_\perp| \lesssim \frac{1}{m}$, i.e., from a distance comparable to the Compton wavelength of a quark.

In order to further analyze the structure of V_{free}, we introduce the quark spin $\Sigma^i{}_{,free}$ and orbital angular momentum $L^i{}_{,free}$ operators of this model. The sum of these two operators is the total angular momentum, $J^i = L^i{}_{,free} + \Sigma^i{}_{,free}$. Note that $L^3{}_{,free}$ and $\Sigma^3{}_{,free}$ correspond to the previously introduced operators $L^3(F)$ and $F_o{}^3$, respectively. $L^i{}_{,free}$ and $\Sigma^i{}_{,free}$ are defined in the usual way:

$$L^i{}_{,free} = i\,\epsilon_{ijk} \int d^3x\; x^k q^+(x)\, \partial^j\, q(x) \qquad \text{(III-15a)}$$

$$\Sigma^i{}_{,free} = \int d^3x\; q^+(x)\, \frac{\sigma^i}{2}\, q(x) \qquad \text{(III-15b)}$$

We may now employ these operators, and the $F_i{}^\alpha$, to discuss the properties of Y_{free} under the group $SU(6)_{W,currents} \times O(2)$. We see that Y_{free} is the uncharged, $\Delta J^3 = 0$, $\Delta\,\Sigma^3{}_{,free} = \pm 1$ member of a $\underline{35}$ (transforming like an ω, helicity + and -), or the corresponding member of a $(3,\overline{3}) + (\overline{3},3)$ under the chiral $SU(3) \times SU(3)_{currents}$ subgroup. While only changing $L^3{}_{,free}$ by ± 1, the presence of the power series in $\vec{\gamma}_\perp \cdot \vec{\partial}_\perp/m$ implicit in the arctan $\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right)$ implies a very complex tensor structure, involving strings of operators with $\Delta\, j = 0,1,2,\ldots$ Hence Y_{free} can effect the mixing of essentially all total angular momenta, and also leads to states of different quark spin $\Sigma^3{}_{,free}$. The cutoff in these expansions, hence the maximum range of angular momenta mixed, is regulated by the rate of convergence of the expansions in $\vec{\gamma}_\perp \cdot \vec{\partial}_\perp/m$. Inasmuch as there is reason to think that convergence may be very slow, we can expect very complete mixing. (Such slow convergence is suggested by the identification of $|p_\perp|$ in this model with the transverse momentum cutoff of ca. 300MeV/c observed in strong interactions. Taking $m \approx 300$MeV also, as suggested by magnetic moment calculations, we find $|p_\perp|/m \approx 1$. Thus in coordinate space, we expect $\langle \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m} \rangle \approx 1$.)

Thus, we see that V_{free} in this model possesses many of the qualitative features expected for the actual transformation V (i.e., it possesses features which show up in the phenomenological mixing schemes for states at infinite momentum). In particular, V_{free} can connect states which lie in nonexotic representations (eg, positive parity baryons lying in a 56, $L^3_{free}(F) = 0$) to states lying in exotic-containing representations (such as 700, $L^3_{free}(F) = \pm 1$). Note that, insofar as V_{free} is an SU(3) singlet*, it will take SU(3) 1's, 8's, and 10's only into 1's, 8's and 10's, respectively. These latter SU(3) multiplets, however, lie in $SU(6)_W$ representations (like 700) which may contain exotic SU(3) multiplets (27's, say). V_{free} can also lead

*If the analysis by M. Gell-Mann, R. J. Oakes, and B. Renner (9) is correct, and the bare mass of the strange quark is actually much larger than that of the non-strange quarks, V_{free} may contain an enormous SU(3) violation (assuming that the rule for constructing V_{free} in this case is to replace m by the bare mass of each quark it acts on). Since this seems to conflict with generalized CVC, it may rule out V_{free} as an approximation to V. It could nevertheless happen that V has an algebraic structure similar to that of a V_{free} with equal quark masses, although the mechanism is presently unknown.

directly to states with the "exotic" quantum numbers $J^{PC} = 0^{--}$, $(\text{odd})^{-+}$, $(\text{even})^{+-}$ by creating $q\bar{q}$ pairs.

The structure of Y_{free} in equation (III-6) is very similar to that of the Foldy-Wouthusen transformation. In fact, they are identical, except for one important omission: Y_{free} is a function of $\vec{\gamma}_{\perp} \cdot \vec{\partial}_{\perp}$, not $\vec{\gamma} \cdot \vec{\partial}$. It is the additional term $\gamma^3 \partial^3$ (which makes the F-W transformation invariant under the full rotation group O(3)) that distinguishes these two transformations. This difference is crucial, since $\gamma^3 \partial^3$ is <u>not</u> a "good" operator: Its inclusion would lead to an $SU(6)_{W,strong}$ whose charges depend upon the momentum in the $\hat{z}$ direction, directly contradicting the experimental evidence.

Thus, we have found that, so far as the free quark model is concerned, we can construct an <u>exact</u> $SU(6)_{W,strong}$ which explicitly satisfies all our requirements. We have constructed a set of non-local operators $W^{\alpha}_{i,free}$ which exactly annihilate the vacuum. The problem which now arises is the relevance of the model to physics. Before going on, however, it may be worthwhile to say a few words about this kind of non-local transformation in general.

Since we have made our non-local $SU(6)_{W,strong}$ an exact symmetry, it is no longer restricted to purely collinear processes. As is obvious, however, the $\hat{z}$ direction still plays a very special role in theory, and we can expect that when states do not have momenta parallel to $\hat{z}$, their charges will have some special properties. In fact, since the $W^{\alpha}_{i,free}$ do not commute with boosts perpendicular to $\hat{z}$, we see that the $SU(6)_{W,strong}$ charges, while exactly conserved in this model, depend upon the value of the transverse momentum. Only for states with momenta parallel to $\hat{z}$ are the charges independent of momentum. Thus, between collinear states the $SU(6)_{W,strong}$ will appear as a conventional symmetry where states have fixed charges. For non-collinear states, the $SU(6)_{W,strong}$ symmetry may still hold, but the charges will depend upon the energy and angle, resulting in predictions which may be less obvious than those of the conventional type of symmetry. Moreover, these results should be correspondingly more difficult to see when large symmetry breaking combines with rapidly changing differential cross sections.

The non-local transformation V_{free} presented here is only one member of a wide class of such transformations, all of which lead to charges depending upon momentum. For example, the close resemblance of V_{free} to the Foldy-Wouthusen transformation, V_{FW}, may induce us to investigate the properties of some of the non-local F-W transformed operators. We would find, for example, that the model contains a set of non-local operators $V_{FW} \int d^3x\, q^+(x) \frac{\sigma^k}{2} \frac{\lambda^i}{2} q(x) V^{-1}_{FW}$ which generate an SU(6) (The idea that the F-W transformation may be useful for generating such a symmetry group has been independently suggested[10] by several authors). In this case, although V_{FW} possesses full O(3) invariance, it does not commute with boosts in any direction, so that the charges are always momentum dependent. We have here a static SU(6) which is a true invariance of the theory, its operators being exactly conserved, but which only appears as a conventional symmetry between states at rest. Use of these types of non-local transformations may thus provide us with a bridge which will allow us to circumvent the difficulties of combining internal and spacetime symmetries in a consistent and useful manner.

There has never been any real reason to believe that the symmetries observed in strong interaction physics are generated by the integrals of local current densities: all that we have ever been able to observe in the case of $SU(6)_W$, for example, is the algebra of the integrated charges. Hopefully, the present discussion provides some evidence that these charges may, in fact, be integrals of non-local operators.

IV. CONSTRUCTION OF V IN INTERACTING QUARK MODELS

In the previous chapter we saw how to construct a transformation V_{free} in the free quark model. This transformation takes us from a set of local operators F_i^{α} which do not commute with H_{free} to a set of nonlocal operators $W_{i,free}^{\alpha}$ which exactly commute with H_{free}. *The problem is to extend these free quark results to more* general models, models in which an exact $SU(6)_{W,strong}$ symmetry may not exist.

We shall approach this problem through the free quark model: That is, we shall assume that the Hilbert space of the interacting problem can be spanned by free particle states. Note that this is a highly non-trivial assumption: in particular, it is certainly <u>not</u> true in ordinary quantum field theories, as is demonstrated by Haag's theorem. However, as emphasized by H. Fritzsch and M. Gell-Mann,[11] experience with the behavior of matrix elements in the Bjorken limit of deep inelastic scattering suggests that the strong interactions are "softer" than the conventional, barely renormalizable, theories we know of at present. It therefore may make sense to follow a naïve approach to the problem.

If the Hilbert space of the interacting model can be spanned by free particle states (that is, states of free quarks plus free gluons of some sort), then we can expand any given eigenstate of the full Hamiltonian into a sum over a large number (perhaps infinite) of free particle states. The matrix which is formed from the projection coefficients is unitary, and can be considered to be an operator acting upon states within the Hilbert space. Calling this operator $\mathcal{U}$, we see that when it acts upon a free particle state it transforms this state into the "corresponding" eigenstate of the full Hamiltonian,

$$\mathcal{U} \; |E_{free}\rangle \; = \; |E\,(E_{free})\rangle \tag{IV-1}$$

What we mean by the "corresponding" eigenstate has to be clarified. For most purposes it will be sufficient to say that an eigenstate $|E_{free}\rangle$ of the free Hamiltonian "corresponds" to an eigenstate $|E(E_{free})\rangle$ of the full Hamiltonian if $|E\,(E_{free})\rangle$ develops out of $|E_{free}\rangle$ as the interaction is adiabatically switched on. That is, if we write the full Hamiltonian $H_{st} = H_{free} + \lambda H_{interaction}$, then the eigenvalue $E(\lambda)$ of the state in question is a continuous function of λ such that $E(0) = E_{free}$ and $E(1) = E$. $\mathcal{U}$ will therefore be uniquely defined so long as we are not troubled by level crossing or some such pathology. Note that we shall always quantize in a large but finite box, so that the spectrum of H_{free} is discrete. In this way we shall avoid any counting difficulties due to comparing discrete spectra of bound states to continuous spectra of free states.

The operator $\mathcal{U}$ should be familiar from elementary quantum mechanics. It has a simple implicit definition,

$$\mathcal{U} = \lim_{t \to -\infty} e^{iH_{st}t} \, e^{-i\hat{H}_{free}\,t} \tag{IV-2}$$

where H_{st} is the full strong interaction Hamiltonian and $\hat{H}_{free}$ is a "renormalized" free Hamiltonian which is diagonal in the basis of free particle states, but whose eigenvalues are those of the corresponding eigenstates of H_{st}. That is, $\hat{H}_{free}\,|E_{free}\rangle = E(E_{free})|E_{free}\rangle$.

The operator $\mathcal{U}$ should not be at all mysterious. Although the implicit definition (IV-2) may be somewhat infamiliar, it actually expresses nothing more than the Rayleigh-Schroedinger perturbation series (when that series converges) in operator form. Thus, when perturbation expansions are valid, we can explicitly evaluate the matrix elements of $\mathcal{U}$ by this familiar technique. In principle, $\mathcal{U}$ exists even when perturbation expansions fail, but in this case it may be hard to compute.

Note that (IV-2) shows how $\mathcal{U}$ is related to the Møller wave matrix $\Omega^{(+)}$ of scattering theory. $\mathcal{U}$ and $\Omega^{(+)}$ are identical if there are no bound states in the theory, but if bound states are present $\mathcal{U}$ is still unitary, although $\Omega^{(+)}$ is not.

This operator $\mathcal{U}$ can now help us to determine the $SU(6)_{W,\text{currents}}$ *structure of a given interacting state, and thus allow* us to compute V in any given model. The procedure is simple: we use $\mathcal{U}$ to project the interacting state onto free states, and then use V_{free} to describe the $SU(6)_{W,\text{currents}}$ content of each free state. We can use precisely the operator V_{free} developed in Chapter III as long as whatever gluons that may be present in the model are singlets of $SU(6)_{W,\text{currents}}$. In this case the generators of $SU(6)_{W,\text{currents}}$ in the model will be the same as for the free quark model, and the gluon content of a given free state will not change its $SU(6)_{W,\text{currents}}$ structure. These conditions are automatically fulfilled in any model with a neutral gluon whose field commutes with the quark field.

Thus, we can easily write an explicit form for V, one which may be used to evaluate it, in principle, in any given interacting model.

Since $SU(6)_{W,strong}$ is exact in the free quark model, we can label states either according to Poincaré quantum numbers or $SU(6)_{W,strong}$ representation. Thus,

$$|E_{free}(R)\rangle = |R_{strong}\rangle = V_{free}\,|R_{currents}\rangle \tag{IV-3}$$

by definition of V_{free}. Recalling (IV-1),

$$|E\,(R)\rangle = \mathcal{U}\,|E_{free}(R)\rangle = \mathcal{U}\,V_{free}|R_{currents}\rangle \tag{IV-4}$$

Comparing this with (II -3), we see that

$$V = \mathcal{U}\,V_{free} \tag{IV-5}$$

and thus

$$W_i^{\alpha} = V\,F_i^{\alpha}\,V^{-1} = \mathcal{U}\,W_{i,free}^{\alpha}\,\mathcal{U}^{-1} \tag{IV-6}$$

The above definition (IV-5) of V will be employed for the remainder of this work. It is not the only possible form, however, due to our neglect of gluon degrees of freedom. In fact, (IV-5) could be multiplied on the right by any arbitrary function of the gluon fields without changing its $SU(6)_{W,currents}$ algebraic structure. Moreover, $\mathcal{U}$ is somewhat arbitrary also, in that we could have chosen some other way to associate interacting states and free states. It just happens that the form (IV-5) is most convenient for our purposes at present, so there is no reason to consider other possible forms.

Note that V in (IV-5) satisfies the requirements (a) - (f) of Chapter II. The only reservation we must make is that $\mathcal{U}$ need not be a good operator. However, it can only take good operators to good operators, since in any Lorentz invariant theory dressed operators must have the same Lorentz structure as bare operators. Thus, the definition (IV-5) satisfies our intuition about this property of V.

The structure of the $SU(6)_{W,strong}$ operators W_i^{α} in interacting models is also clarified by this form of V. As can be seen from (IV-6), W_i^{α} takes us from one physical state $|E\rangle$ (eigenstate of H_{st}) to another by referring back to the free eigenstate $|E_{free}\rangle$, from which the physical state developed as the interaction was turned on. Once back in the realm of free states, $W_{i,free}^{\alpha}$ is applied, and the resulting free state is finally related to the corresponding eigenstate $|E'\rangle$ of H_{st}. The changes from eigenstates of H_{st} to those of H_{free} are accomplished by the operator $\mathcal{U}$ in (IV-6).

The W_i^{α} are conserved when this process always leads from a given eigenstate of H_{st} to another with the <u>same</u> energy. Thus, if $[W_i^{\alpha}, H_{st}] = 0$, we expect to find an $SU(6)_{W,strong}$ multiplet structure among the eigenstates of H_{st} paralleling that of the eigenstates of H_{free}.

Exact conservation of W_i^{α} is an extremely strong condition, however. Nevertheless, we do see approximately degenerate $SU(6)_{W,strong}$ multiplets in nature, and it is often an interesting approximation to consider them <u>exactly</u> degenerate. Does this circumstance compel us to consider the case where the W_i^{α} are completely conserved? Fortunately, the answer is no.

One of the principal reasons that we consider an $SU(6)_{W,strong}$ approximate symmetry at all is because the observed particle and resonance states seem to fall into multiplets of states with different spin. Thus, the most we can be certain of is that $[W_i^{\alpha}, H_{st}] \approx 0$ <u>within the subspace of these states</u>. That is, there exists a particular set of states, which we shall call $|P(M)\rangle$, for which

$$\langle P(M)| \quad [W_i^{\alpha}, H_{st}] \quad |P(M')\rangle \approx 0 \qquad \text{(IV-7)}$$

The notation $|P(M)\rangle$ is meant to convey that these are what we naïvely call "single particle" states with various masses M. In the light of our present knowledge we have no right to extend (IV-7) to any of the other states in Hilbert space. In particular, we need not assume that the W_i^{α} are conserved when states containing several stable particles are involved (eg. a πp collision state). Thus, there is no reason to believe that the existence of approximately degenerate multiplets necessarily implies a general symmetry of H_{st}. Experimentally, the situation is unclear at the present time, and it is not known whether a rough $SU(6)_{W,strong}$ symmetry exists for collisions or not. There is even some question about whether there is an $SU(6)_{W,strong}$ <u>vertex</u> symmetry, as we shall see in Chapter VI.

How shall we interpret (IV-7) in these circumstances? On one hand, we can regard the existence of degenerate multiplets in the spectrum as being the result of a dynamical conspiracy among states with different spin and parity. In this case there need be no relationship between the $SU(6)_{W,strong}$ multiplet to which a physical state $|E\rangle$ belongs and the (free) $SU(6)_{W,strong}$ multiplet to which the corresponding state $|E_{free}\rangle$ belongs. On the other hand, we may find it more reasonable to attribute the observed degeneracies to a kind of "spectrum symmetry." That is, we presume that the states $|E\rangle$ in a given $SU(6)_{W,strong}$ multiplet correspond to states $|E_{free}\rangle$ which lie in the same multiplet of the free $SU(6)_{W,strong}$. In other words, we presume that the matrix elements of the operator $\mathcal{U}$ in, say, the basis of free states, $\langle E_{free}|\,\mathcal{U}\,|E'_{free}\rangle$, are such that they are large only between states $|E_{free}\rangle$ and $|E'_{free}\rangle$ which have the same classification under the free $SU(6)_{W,strong}$.

Putting these alternatives into mathematical language, we can interpret (IV-7) as being due to either:

(a) Detailed dynamical cancellations, in which case

$$\langle P(M)|\ [W_i^{\alpha},\mathcal{U}]\ \ |P(M')\rangle \neq 0 \qquad \text{(IV-8)}$$

or

(b) A "spectrum symmetry,"

$$\langle P(M)|\ [W_i^{\alpha},\mathcal{U}]\ \ |P(M')\rangle \approx 0 \qquad \text{(IV-8b)}$$

If nature has actually chosen alternative (a), then the structure of V is obscure. In order to relate matrix elements of currents taken between different members of an $SU(6)_{W,strong}$ multiplet, we must know $\mathcal{U}$. Algebraic predictions become a matter of the detailed dynamics of the strong interactions. This is the case, for example, if a real neutral vector gluon theory underlies the strong interactions. In this example, the inevitable spin-orbit interactions break up the free $SU(6)_{W,strong}$ multiplets. Even if $SU(6)_{W,strong}$ multiplets somehow reappear for large coupling strengths, they will probably not bear any relation to the original multiplets of the free fields.

Case (b) is more clearcut. Admittedly, there is no known theory in which (IV-8b) is true when couplings are large. However, there are equally few theories which show $SU(6)_{W,strong}$ multiplets in their spectra.

In these circumstances, we could simply assume that whatever the real forces underlying the strong interaction are, they yield an approximate $SU(6)_{W,strong}$ symmetry by means of alternative (b).

The advantage of this assumption is that it allows us to use V_{free} to compute the algebraic structure of physical currents. This proposition is easily proved. If we assume that (IV-8b) is true, (IV-6) tells us that

$$\langle P(M)|\; W_i^{\alpha}\; |P(M')\rangle \approx \langle P(M)|\; W_{i,free}^{\alpha}\; |P(M')\rangle \qquad \text{(IV-10)}$$

That is, the states $|P(M)\rangle$ have nearly the same classifications under the free $SU(6)_{W,strong}$ as under the $SU(6)_{W,strong}$ for physical states. However, we already know the algebraic structure of currents $\mathcal{F}_j^{\beta}(x)$ with respect to the free $SU(6)_{W,strong}$, since the $\mathcal{F}_j^{\beta}(x)$ are just bilinear operators like $q^+(x)\,\Gamma^{\beta}\,\frac{\lambda i}{2}\,q(x)$ in any quark model in which the glue (whatever it is) is an $SU(6)_{W,currents}$ singlet.

Knowing roughly the classification of both states and operators under the free $SU(6)_{W,strong}$, we can find many approximate relations between matrix elements.

In Chapter VI we shall find it convenient to consider the $SU(6)_{W,currents}$ algebraic structure of operators like $V^{-1}\mathcal{F}_j^{\beta}(x)\,V$ taken between states classified as irreducible representations of $SU(6)_{W,currents}$. That is, we consider

$$\langle R'_{\text{currents}}| \; [F_i^{\alpha}, \; V^{-1}\mathcal{F}_j^{\beta}(x) \; V] \; |R_{\text{currents}}\rangle \qquad \text{(IV-11a)}$$

rather than

$$\langle R'_{\text{strong}}| \; [W_i^{\alpha}, \mathcal{F}_j^{\beta}(x)] \; |R_{\text{strong}}\rangle \qquad \text{(IV-11b)}$$

These expressions are actually equal, since $|R_{\text{currents}}\rangle = V^{-1}|R_{\text{strong}}\rangle$. Thus, the assumption that the algebraic structure of $\mathcal{F}_j^{\beta}(x)$ with respect to $SU(6)_{W,\text{strong}}$ is given by $W_{i,\text{free}}^{\alpha}$ in (IV-11b) is equivalent to the assumption that the algebraic structure of $V^{-1} \mathcal{F}_j^{\beta}(x) \; V$ with respect to $SU(6)_{W,\text{currents}}$ in (IV-11a) is given by $V_{\text{free}}^{-1} \mathcal{F}_j^{\beta}(x) \; V_{\text{free}}$. This fact will be of great utility in the evaluation of matrix elements.

Although the symmetry may be realized by (b), so that W_i^{α} and $W_{i,\text{free}}^{\alpha}$ have the same matrix elements among the "single particle" states, note that this does not imply that the $\mathcal{F}_j^{\beta}(x)$ have the same matrix elements as in the free quark model, since we never assume that $[\mathcal{U}, \; \mathcal{F}_j^{\beta}(x)]$ or $[\mathcal{U}, \; V_{\text{free}}]$ even approximately vanish. The only thing that V has in common with V_{free} is its algebraic structure -- the values of its matrix elements may be much different.

In summary, we have attempted to supply a rationale for the fact that the algebraic structure of the V that is used by nature is roughly that of V_{free}. It may be that this is explained by the realization of $SU(6)_{W,\text{strong}}$ via mechanism (b). Nevertheless, it is possible that mechanism (a) is actually used, and some complicated (but obscure) cancellation allows V and V_{free} to be roughly the same algebraically. Since we are ignorant of the Hamiltonian which governs the strong interactions, this point cannot be readily settled. Perhaps the safest point of view is a purely pragmatic one: we shall simply apply the algebraic structure of V_{free} to physical currents, and note whether or not the results seem to approximate the physical world.

Before evaluating the consequences of V_{free}, we shall introduce the language of light-like charges and moments. This language avoids some of the clumsiness of the infinite-momentum frame, and will permit a rapid and elegant evaluation of the consequences of the free quark algebraic structure for the matrix elements of physical currents.

V. LIGHT-LIKE CHARGES AND MOMENTS

The first step in performing any calculations with nonconserved operators is to work exclusively with charges and moments integrated over a null surface $x^0 + x^3 = \text{const.}$, or a "light-like plane." Such charges have many advantages over equivalent formulations at infinite momentum, as has been emphasized by H. Leutwyler.(12) Their principal advantage is that they annihilate the vacuum, whether or not they are conserved. This allows such charges the possibility of having finite dimensional representations, and may even invalidate Coleman's Theorem in their case.

As a result, these light-like charges are certainly to be preferred over other formulations for discussing the representations of algebras of nonconserved operators, and our first care will be to re-express the transformation obtained in the free quark model in terms of them.

In order to make the structure of these charges clear, we write down the form of the light-like charges of $SU(6)_{W,\,\text{currents}}$, since they are known in terms of integrals of local currents. Thus,

$$\hat{F}_i = \int d^4x\, \delta(x^+) \, [\mathcal{F}_i^{\,0}(x) + \mathcal{F}_i^{\,3}(x)] \,/\, \sqrt{2} \tag{V-1a}$$

$$\hat{F}_i^{\,1} = \tfrac{1}{2} \int d^4x\, \delta(x^+) \, [\mathcal{F}_i^{\,23}(x) + \mathcal{F}_i^{\,20}(x)] \,/\, \sqrt{2} \tag{V-1b}$$

$$\hat{F}_i^{\,2} = \tfrac{1}{2} \int d^4x\, \delta(x^+) \, [\mathcal{F}_i^{\,31}(x) + \mathcal{F}_i^{\,01}(x)] \,/\, \sqrt{2} \tag{V-1c}$$

$$\hat{F}_i^{\,3} = \tfrac{1}{2} \int d^4x\, \delta(x^+) \, [\mathcal{F}_i^{\,05}(x) + \mathcal{F}_i^{\,35}(x)] \,/\, \sqrt{2} \tag{V-1d}$$

where $x^+ = (x^0 + x^3) / \sqrt{2}$ and the caret over the charges denotes that they are integrated over the light-like plane $x^+ = 0$.

Let us briefly examine some of the properties of these charges. The most striking property is evidently their momentum transfer: when applied to a state of momentum p^μ the $\hat{F}_i^\alpha$ do not change $\vec{p}_\perp$, nor $p^0 + p^3$. They can alter only $p^0 - p^3$. Thus, the $\hat{F}_i^\alpha$ carry zero mass, $(\Delta p)^2 = 0$. Since there are no zero mass hadrons, this implies that the $\hat{F}_i^\alpha$ cannot produce anything out of the vacuum

$$\hat{F}_i^\alpha | 0 \rangle = 0 \qquad \text{(V-2)}$$

so long as they are decoupled from infinite momentum states with $p^0 - p^3 = 0$. Of course, if the $\hat{F}_i^\alpha$ act on a state at rest with mass m, they <u>can</u> lead to a state of different mass $m^* \neq m$, provided that the final state has momentum $p^3 = (m^2 - m^{*2})/2m$. Only if the $\hat{F}_i^\alpha$ are conserved, $[H - P^3, \hat{F}_i^\alpha] = 0$, will they not lead to states of different mass. What condition (V-2) <u>does</u> guarantee, however, is that the $\hat{F}_i^\alpha$ cannot produce any disconnected pairs. Thus, application of various $\hat{F}_i^\alpha$'s to a state an arbitrary number of times does not lead to the creation of an arbitrary number of pairs: there exists a possibility that we shall return to the original state after a finite number of steps, and thus obtain a finite dimensional representation of the algebra.

The Lorentz properties of the $\hat{F}_i^\alpha$ are not simple. It is clear that they commute with finite boosts along $\hat{z}$, $[\hat{F}_i^\alpha, \Lambda^3] = 0$, but they do not commute with transverse boosts. In particular, a state with given helicity and transverse momentum $\vec{p}_\perp \neq 0$ does not have the same $SU(6)_{W,\text{currents}}$ classification as a state with the same helicity but different transverse momentum. To remedy this defect, we must introduce special transverse boost operators which leave the $\hat{F}_i^{\alpha}$ invariant. Thus, defining

$$E^1 = \Lambda^1 + J^2 / p^0 + p^3 \tag{V-3a}$$

$$E^2 = \Lambda^2 - J^1 / p^0 + p^3 \tag{V-3b}$$

we can easily check that E^1 and E^2 commute with all $\hat{F}_i^{\alpha}$. States with transverse momenta $\vec{p}_{\perp}$ generated by means of the $\vec{E}_{\perp}$ can thus be classified in the same $SU(6)_{W,\text{currents}}$ representation, whatever $\vec{p}_{\perp}$ may be:

$$|R, \vec{p}_{\perp}\rangle = e^{-i\vec{p}_{\perp} \cdot \vec{E}_{\perp}} |R, 0_{\perp}\rangle \tag{V-4}$$

where R signifies the particular representation. The transverse boost-rotation in (V-4) can be decomposed into a pure Lorentz boost preceeded and followed by rotations. We find that an E-boosted state like that in (V-4) can be related to a certain mixture of helicity states with the same momenta.

As to other properties of the light-like charges (V-1a) - (V-1d), we merely note that they have the same charge conjugation C as their spacelike $x^0 = 0$ counterparts, but do not have definite parity. Instead of parity, we can define the operation $\mathcal{R} = e^{-i\pi J^2} P$ under which the $\hat{F}_i^{\alpha}$ <u>are</u> eigenvectors. Thus, $\mathcal{R}^{-1} \hat{F}_i \mathcal{R} = + \hat{F}_i$, $\mathcal{R}^{-1} \hat{F}_i^3 \mathcal{R} = -\hat{F}_i^3$, etc. Hence, instead of requiring that $\hat{V}$ have parity P = +, we allow the light-like version $\hat{V}$ to be a parity mixture, but require that it have $\mathfrak{R} = +$ instead.

We are interested in matrix elements of various moments of currents as well as their charges. That is, objects like

$$\langle A,\vec{p}\,' |\ \int d^4x\, \delta(x^+)\ \mathcal{F}_i^{\alpha}(x) e^{+i(\vec{p}\,'_\perp - \vec{p}_\perp)\cdot \vec{x}_\perp} | B,\vec{p}\rangle \qquad \text{(V-5)}$$

where the states always have the normalization $\langle A,\vec{p}\,' |\ B,\vec{p}\rangle = \delta_{AB}\ \delta_{\vec{p}\,',\vec{p}}$. If the transverse momenta of $|A,\vec{p}\,'\rangle$ and $|B,\vec{p}\rangle$ have been generated by E boosts, the matrix element (V-5) depends only upon the <u>difference</u> $(\vec{p}\,'_\perp - \vec{p}_\perp)$ of transverse momenta. (There is no dependence, of course, on the longitudinal momentum -- the difference $p'^3 - p^3$ is fixed by the requirement that $(p^0 + p^3)$ is the same for both states.) This property is easily demonstrated: If the transverse momenta are generated by E boosts, we can write (V-5) as

$$\langle A,\vec{p}\,'_\perp = 0 |\ \int d^4x\, \delta(x^+)\, \mathcal{F}_i^{\alpha}(x) e^{+i(\vec{p}\,'_\perp - \vec{p}_\perp)\cdot(\vec{x}_\perp + \vec{E}_\perp)} | B,\vec{p}_\perp = 0\rangle$$

$$= \langle A,\vec{p}\,'_\perp = 0 \,|\, \hat{F}_i^{\alpha}\ (\vec{p}\,'_\perp - \vec{p}_\perp)\ | B,\vec{p}_\perp = 0\rangle \qquad \text{(V-6)}$$

where we have defined

$$\hat{F}_i^{\alpha}\ (\vec{p}_\perp) \ = \ \int d^4x\, \delta\ (x^+)\, \mathcal{F}_i^{\alpha}(x) e^{+i\vec{p}_\perp\cdot\ (\vec{x}_\perp + \vec{E}_\perp)} \qquad \text{(V-7)}$$

The matrix element (V-6) obviously depends only upon $\vec{p}\,'_\perp - \vec{p}_\perp$, as required. The operators $\hat{F}_i^{\alpha}(\vec{p}_\perp)$ have many nice properties -- for example, they commute with the translation operator $\vec{P}_\perp$, so that moments of $\mathcal{F}_i^{\alpha}(x)$ can be expressed in terms of forward matrix elements. This is a useful property for $SU(6)_{W,\text{strong}}$ calculations, since we only know how to classify states with collinear momenta. Other properties of these operators will be discussed as needed.

In a similar way, we can define light-like $SU(6)_{W,\text{strong}}$ operators, $\hat{W}_i^{\alpha}$, which have the same property of annihilating the physical vacuum, $\hat{W}_i^{\alpha}|0\rangle = 0$. The relation between these $\hat{W}_i^{\alpha}$ operators and the $\hat{F}_i^{\alpha}$ is given by the light-like transformation $\hat{V}$

$$\hat{W}_i^{\alpha} = \hat{V}\,\hat{F}_i^{\alpha}\,\hat{V}^{-1} \tag{V-8}$$

where $\hat{V}$ is such that $[H-P^3, \hat{W}_i^{\alpha}] \approx 0$. The $\hat{W}_i^{\alpha}$, then, lead to states of nearly the same mass, while the $\hat{F}_i^{\alpha}$ can lead to states of quite different mass.

As before, we can construct the operators $\hat{W}_i^{\alpha}$, $\hat{F}_i^{\alpha}$, and $\hat{V}$ in the free quark model, obtaining essentially the same results as in Chapter III. This generalization is quite simple, and we only record the form of the result. Following J. B. Kogut and D. E. Soper,(13) we quantize on the $x^+ = 0$ plane, using the anticommutation relations

$$\left\{q_+^{\dagger}(x),\; q_+(y)\right\}_{x^+ = y^+} = 1/\sqrt{2}\; P_+\,\delta(x^- - y^-)\,\delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$$\left\{q_+(x),\; q_+(y)\right\}_{x^+ = y^+} = 0\text{ , etc.} \tag{V-9}$$

where the "independent fields" $q_+(x)$ are obtained from $q(x)$ by the projection operator $P_+ : q_+(x) = P_+ q(x) = \frac{1}{2}(1+\alpha^3)q(x)$. The light-like $SU(6)_{W,\text{currents}}$ generators for the quark model are

$$\hat{F}_i = \sqrt{2} \int d^4x\ \delta(x^+) q_+^\dagger\ (x)\ \frac{\lambda i}{2}\ q_+(x) \qquad \text{(V-10a)}$$

$$\hat{F}_i^{\ 1} = \sqrt{2} \int d^4x\ \delta(x^+) q_+^\dagger\ (x)\ \frac{\beta\sigma^1}{2}\ \frac{\lambda i}{2}\ q_+(x) \qquad \text{(V-10b)}$$

Model

$$\hat{F}_i^{\ 2} = \sqrt{2} \int d^4x\ \delta(x^+) q_+^\dagger\ (x)\ \frac{\beta\sigma^2}{2}\ \frac{\lambda i}{2}\ q_+(x) \qquad \text{(V-10c)}$$

$$\hat{F}_i^{\ 3} = \sqrt{2} \int d^4x\ \delta(x^+) q_+^\dagger\ (x)\ \frac{\sigma^3}{2}\ \frac{\lambda i}{2}\ q_+(x) \qquad \text{(V-10d)}$$

The "good" classification of the $SU(6)_{W,\text{currents}}$ operators is reflected by the fact that they contain only $q_+(x)$ fields. "Bad" operators contain one $q_+(x)$ and one $q_-(x)=P_-(x)q(x)=\frac{1}{2}(1-\alpha^3)q(x)$ "dependent" field (which does not have canonical commutation relations). Those operators containing only $q_-(x)$ fields (like $\mathcal{F}_i^{\ 0}(x) - \mathcal{F}_i^{\ 3}(x)$) are called "terrible."

The transformation $\hat{V}_{\text{free}}$ is a "good" operator. It is therefore well-defined, and has a simple algebraic structure. Writing $\hat{V}_{\text{free}} = \exp i\hat{Y}_{\text{free}}$, the old free quark result becomes

$$\hat{Y}_{\text{free}} = 1/\sqrt{2} \int d^4x\ \delta\ (x^+) q_+^\dagger\ (x)\ \arctan\ \left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) q_+(x) \qquad \text{(V-11)}$$

The algebraic structure of $\hat{Y}_{\text{free}}$ is the same as before: it belongs to a $(3,\overline{3}) + (\overline{3},3)$, $L^3(\hat{F}) = \pm 1$ of $SU(3) \times SU(3) \times O(2)_{\text{currents}}$, etc. As before, the $\hat{W}^{\ \alpha}_{i,\text{free}}$ are conserved, $[H_{\text{free}} - P^3,\ \hat{W}^{\ \alpha}_{i,\text{free}}] = 0$. Note, however, that for free quarks the light-like $SU(6)_{W,\text{currents}}$ is also conserved, $[H_{\text{free}} - P^3,\ \hat{F}_i^{\ \alpha}] = 0$. Several authors[14] have recently taken this to mean that a unique $\hat{V}_{\text{free}}$ cannot be defined on the light-like plane. It is true that the simple criterion by which V_{free} was derived can no longer be used; however, $\hat{V}_{free}$ probably does more than just ensure the existence of a conserved $SU(6)_{W,strong}$.

Experience with the way V_{free} acts on quark states in the infinite momentum frame suggests that the rotations performed by $\hat{V}_{free}$ are necessary to ensure that the multiquark states may have definite total spin. The definition (a) in Chapter II may still be useful -- only now we must emphasize spin rather than merely energy.

That the $\hat{F}_i^{\alpha}$ and the spin generators are not automatically compatible is the upshot of Dashen and Gell-Mann's[15] "angular condition" on the matrix elements of the $\hat{F}_i^{\alpha}$. The fact that Gell-Mann[16] has found that the angular condition forces mixing of $SU(6)_{W,currents}$ representations even in the free quark model seems to support this idea.

Much more work remains to be done on this subject. Can $\hat{V}$ be uniquely defined by means of some spin criterion? If the answer is yes, such a criterion may be more useful in the case of broken symmetry than the $\mathcal{U}V_{free}$ prescription. The understanding of this question may thus be of very great importance.

The above form (V-11) for $\hat{Y}_{free}$ is particularly significant in the light of a proposal by H. Fritzsch and M. Gell-Mann.[17] The large algebra of good light-like plane operators which they postulate include the operators of which $\hat{Y}_{free}$ is composed. Moreover, they hope to be able to construct the relevant components of the energy-momentum tensor within this large algebra, formulating a complete theory of strong interactions in terms of physical functions of quark and gluon fields on the light-like plane. Within the context of this theory

$\hat{Y}_{free}$ may be found to be a useful approximation to the more complicated $\hat{Y}$. In any event, this algebra may provide us with a way in which $\hat{Y}_{free}$ may be defined in terms of observable operators.

In the next chapter $\hat{Y}_{free}$ will be used to transform various good light-like charges and moments, extracting the algebraic structure from the model and applying it to physical operators.

VI. APPLICATION TO CURRENT MATRIX ELEMENTS

We have now come to the point where some calculations can be made. The form of the transformation between $SU(6)_{W,\text{currents}}$ and $SU(6)_{W,\text{strong}}$ for the free quark model has been discussed in Chapter III. In Chapter IV we discussed the form of the transformation in interacting models. We saw that in at least one case, that of an $SU(6)_{W,\text{strong}}$ multiplet structure realized by a "spectrum symmetry," we may expect the algebraic structure of the free quark model to persist even in interacting models. Whether this actually occurs in nature, we do not know. It is, nevertheless, interesting to compute the consequences of the idea that $\hat{V}$ may have the same algebraic structure as $\hat{V}_{\text{free}}$. We shall see that we get reasonable results from this assumption.

Using the formalism of light-like charges and moments, we are interested in matrix elements like $\langle A|\hat{F}_i^{\alpha}(\vec{p}_{\perp})|B\rangle$ where $|A\rangle$ and $|B\rangle$ are physical "single particle" states (resonances) with zero transverse momentum and finite longitudinal momentum. We assume that these states have simple transformation properties under $SU(6)_{W,\text{strong}}$. For actual computation however, it will be convenient to use the $SU(3) \times SU(3),_{\text{currents}}$ subgroup of $SU(6)_{W,\text{currents}}$, since the transformation properties of operators are more easily discussed in these terms.

Using this trick, we can throw all of the complexity of the mixing onto the operators. Thus, the matrix element

$$\langle A,_{\text{strong}}|\hat{F}_i^{\alpha}(\vec{p}_\perp)|B,_{\text{strong}}\rangle$$

$$= \langle A,_{\text{currents}}|\hat{V}^{-1}\hat{F}_i^{\alpha}(\vec{p}_\perp)\hat{V}|B,_{\text{currents}}\rangle \qquad \text{(VI-1)}$$

The problem now reduces to one of the structure of $\hat{V}^{-1}\hat{F}_i^{\alpha}(\vec{p}_\perp)\hat{V}$ under $SU(3) \times SU(3) \times O(2)_{,\text{currents}}$. We shall abstract this structure (but not the numerical values of matrix elements, of course!) from the free quark model.

We shall consider the matrix elements of the electromagnetic current, $\hat{F}_{em}(\vec{p}_\perp)$, and the axial vector current, $\hat{F}_i^{3}(\vec{p}_\perp)$, since many of the matrix elements of these currents have been measured.

The truly remarkable thing about the transformation properties derived from the free quark model is their simplicity: since V_{free} is bilinear in quark fields, bilinear operators are transformed only into bilinear operators. Thus, the resulting transformed current can contain the irreducible representations (1,8) + (8,1) and $(3,\bar{3}) + (\bar{3},3)$ and nothing else. This property of rapid termination is unique to the free quark model, and unless we have some special reason to think that the free quark algebra should be preserved, we would not expect it to show up in an interacting model.

In general, the $\mathcal{U}$ in V will introduce products of these simple irreducible representations, spoiling the termination property and invalidating the free quark results. However, due to the argument made in Chapter IV, it is conceivable that these extra terms do not contribute very greatly to matrix elements of physical currents between hadron states. The results below are all based upon this assumption. We must, of course, expect some deviations of the predictions from the experimental values, since the validity of the $\hat{V}_{free}$ structure can only be approximate.

A. The Axial Vector Current.

The axial vector charge, $\hat{F}_i^3 = \hat{F}_i^3(\vec{p}_\perp = 0)$, yields the first interesting results. Referring to the Appendix, where the form of $\hat{V}^{-1}\hat{F}_i^3\hat{V}$ is explicitly written out for the free quark model, we see that the structure of the transformed operator is:

$$\hat{V}^{-1}\hat{F}_i^3\hat{V} \sim (1,8) + (8,1)L^3(\hat{F}) = 0 \qquad \text{(VI-2)}$$

$$+ (3,\overline{3}) + (\overline{3},3)L^3(\hat{F}) = \pm 1$$

The first term transforms like $(\gamma^5 \frac{\lambda i}{2})$, and the second term like $(\beta\vec{\sigma}_\perp \cdot \vec{\partial}_\perp \frac{\lambda i}{2})$. The first term transforms like the original charge $\hat{F}_i^3$ (Although there is no reason whatsoever to think that it is $\hat{F}_i^3$! It is not equal to $\hat{F}_i^3$ even in the free quark model.). The second term is new, and can lead from $L^3(\hat{W}) = 0$ representations (like the baryon 56, $L^3(\hat{W}) = 0$) to higher ones (like 70, $L^3(\hat{W}) = \pm 1$, or 56, $L^3(\hat{W}) = \pm 1$). This is the kind of behavior we expect of the physical $\hat{F}_i^3$ -- such behavior is actually seen in, for example, the Adler-Weisberger sum rule, where we find many resonances contributing to the empirical sum over states.

More detailed results are obtained by sandwiching the transformed charge (VI-2) between wellknown states, like the baryons, classified as $\underline{56}$, $L^3(\hat{W}) = 0$ under $SU(6)_{W,strong}$. We see that only the first, $L^3(\hat{F}) = 0$ term can contribute to the matrix element (Remember that $\hat{V}^{-1}\,\hat{F}_i^{\,3}\,\hat{V}$ is taken between $L^3(\hat{F}) = 0$ states, which correspond to the $L^3(\hat{W}) = 0$ classification of the physical states. This jumping back and forth between $L^3(\hat{F})$ and $L^3(\hat{W})$ may seem confusing at first, but, by enabling us to put $\hat{V}$ on the operator, it actually results in a great simplification.). Since the first term of (VI-2) has precisely the same $SU(3) \times SU(3)$, $_{currents}$ structure as $\hat{F}_i^{\,3}$ itself, we see that we simply get back all the old $SU(6)_W$ results for these matrix elements -- with one important proviso. The difference is that the first term of (VI-2) is not a generator of $SU(3) \times SU(3)$, $_{currents}$. The values of its matrix elements are not determined by the symmetry; there is always some reduced matrix element η, which is in general different from 1 (we have defined η so that $\eta = 1$ if there is no transformation). Thus, we find the traditional $SU(6)_W$ results modified by factors:

$$\frac{G_A}{G_V} = -\eta\,\frac{5}{3} \qquad G^* = -\eta\,\frac{4}{3} \qquad (D/F)_{axial} = \frac{3}{2} \qquad \text{(VI-3)}$$

The D/F ratio is the same as before, since η cancels out. Whether or not $\eta = 1/\sqrt{2}$ is a question of dynamics which the present work cannot decide (It is nevertheless interesting, although probably not too significant, that the free quark model gives $\eta = \left\langle \frac{1}{\sqrt{1+p_\perp^{\,2}/m^2}} \right\rangle$ which one can argue should be about $1/\sqrt{2}$.).

The structure (VI-2) which we have proposed for the transformed axial vector charge has been applied to the decays of

$L^3(\hat{W}) = -1,0,+1$ mesons by F. J. Gilman and M. Kugler,[18] who use PCAC to relate the axial vector charge to pion decay amplitudes. These authors have also introduced the simplifying assumption that the (1,8) + (8,1) $L^3(\hat{F}) = 0$ term in (VI-2) is the generator $\hat{F}_i^3$ times a constant. Although this assumption is not true even in the free quark model, Gilman and Kugler have found it useful for some phenomenological purposes. However, as a consequence of this drastic restriction, Gilman and Kugler find that the Roper N'(1470) cannot decay into a nucleon and pion. This decay is fully allowed by our structure in (VI-2), since the first term is not a generator.

Gilman and Kugler nevertheless find generally satisfactory results for the decays of $L^3(\hat{W}) = -1,0, +1$ mesons to $L^3(\hat{W}) = 0$ mesons. In particular, they find that the decay $B \to \omega\pi$ is purely transverse and that $g_{B\omega}/g_{A_2\rho} = \sqrt{2}$ for the $J^3 = 1$ (transverse) part. Both of these results seem to be in good agreement with recent experiments. If we were to be more general, however, and admit a (1,8) + (8,1) $L^3(\hat{F}) = 0$ term in the transformed charge which is not a generator, certain results of theirs would be altered. The longitudinal decay $B \to \omega\pi$ would be allowed, for example. In general we would have two reduced matrix elements to characterize the decays rather than one.

It is something of a mystery that this second reduced matrix element should vanish, but perhaps further investigation will clarify the issue.

The use of PCAC raises an interesting problem since, as Gilman and Kugler have found, when PCAC is combined with the transformation properties (VI-2) one sometimes finds results which conflict with the predictions of $SU(6)_{W,\text{strong}}$ vertex symmetry. The $B \to \omega\pi$ decay is a good example of this conflict, since $SU(6)_{W,\text{strong}}$ symmetry forbids the transverse decay, whereas the structure of the transformed axial charge, via PCAC, allows it. $SU(6)_{W,\text{strong}}$ is thus not even a good <u>vertex</u> symmetry when non-zero quark orbital angular momenta are involved. $SU(6)_{W,\text{strong}}$ must be restricted solely to the classification of states, and some new prescription adopted for the discussion of strong interaction vertices.[19]

The algebraic structure of the transformed $\hat{F}_i^{\,3}$ thus seems to give adequate results for forward matrix elements. The area where the old results first broke down, however, was in the first moments of the electromagnetic current. We must therefore turn to these to these to see that the proposed algebraic structure really does make the corrections we intended it to make.

B. Moments of the Electric Current

The matrix elements of the electromagnetic charge, $\hat{F}_{em} = Q$, are, of course, trivial since $[\hat{V}, \hat{F}_{em}] = 0$. The moments are more interesting, however. In particular, the first moment, $\frac{\partial}{\partial k_x} \hat{F}_{em}(k_x) \big|_{k_x = 0}$ is nothing less than the anomalous magnetic moment operator. For spin-$\frac{1}{2}$ particles, the anomalous magnetic moment μ_A, is given by

$$\frac{\mu_A}{2M} = \frac{\partial}{\partial k_x} \langle A,_{\text{strong}};\text{rest};-\tfrac{1}{2}|\hat{F}_{em}(k_x)|A,_{\text{strong}};\text{rest};+\tfrac{1}{2}\rangle\big|_{k_x = 0} \tag{VI-4}$$

This identification can be readily checked by expanding out the matrix element in terms of the traditional invariants, being careful to remember the spin rotations induced by the E boosts. One then sees that only the Pauli form factor $F_2(0)$ is projected out.

We can also differentiate $\hat{F}_{em}(k_x)$ directly, finding

$$\frac{\mu_A}{2M} = +i \langle A, {}_{strong}; rest; -\tfrac{1}{2} | \left\{ \int d^3x \, \delta(x^+) x \mathfrak{F}^+_{em}(x) + QE^1 \right\} | A, {}_{strong}; rest + \tfrac{1}{2} \rangle \tag{VI-5}$$

The second term is just kinematic: Q is the net charge of the state $|A, {}_{strong}\rangle$, while $E^1 = \frac{J^2 + \Lambda^1}{M}$. But $\langle rest | E^1 | rest \rangle = \frac{1}{M} \langle rest | J^2 | rest \rangle$. Λ^1 has negative parity, and therefore has no diagonal matrix elements between states at rest. Since $\langle rest, -\tfrac{1}{2} | J^2 | rest, +\tfrac{1}{2} \rangle = i/2$, we find

$$\frac{\mu_A}{2M} = +i \langle A, {}_{strong}; rest; -\tfrac{1}{2} | \int d^4x \, \delta(x^+) x \mathfrak{F}^+_{em}(x) | A, {}_{strong}; rest; +\tfrac{1}{2} \rangle - \frac{Q}{2M} \tag{VI-6}$$

The second term is just the Dirac Moment! Note that this identification is a bit more subtle than it seems, since E^1 and $\int d^4x \, \delta(x^+) x \mathfrak{F}^+_{em}(x)$ individually can change $\vec{p}_\perp$, and their forward matrix elements may not be well defined. However, if the evaluations are done in terms of symmetric wave packets, rather than plane wave states, no ambiguities arise and the matrix elements are perfectly well defined. The upshot of this argument is that we can write

$$\begin{aligned} \frac{\mu_T}{2M} &= +i \langle A, {}_{strong}; rest; -\tfrac{1}{2} | \int d^4x \, \delta(x^+) x \mathfrak{F}^+_{em}(x) | A, {}_{strong}; rest; +\tfrac{1}{2} \rangle \\ &= +i \langle A, {}_{currents}; rest; -\tfrac{1}{2} | \hat{V}^{-1} \int d^4x \, \delta(x^+) x \mathfrak{F}^+_{em}(x) \hat{V} | A, {}_{currents}; rest; +\tfrac{1}{2} \rangle \end{aligned} \tag{VI-7}$$

where μ_T is the total magnetic moment of the particle. The algebraic structure of this operator is readily determined (see Appendix):

$$\hat{V}^{-1} \int d^4x\ \delta(x^+) x \mathcal{F}^+_{em}(x) \hat{V} \sim (1,8) + (8,1),\ L^3(\hat{F}) = \pm 1 \\ + (3,\overline{3}) + (\overline{3},3),\ L^3(\hat{F}) = 0,\ \pm 2 \qquad \text{(VI-8)}$$

where the $(3,\overline{3}) + (\overline{3},3)$, $L^3(\hat{F}) = 0$ term transforms like ($\gamma^1 \frac{\lambda i}{2}$). With the assignment of the nucleon, spin-up to $(6,3)L^3(\hat{W}) = 0$ and the nucleon, spin-down to $(3,6)L^3(\hat{W}) = 0$, we find that the $(1,8) + (8,1)$ parts give no contribution, the $(3,\overline{3}) + (\overline{3},3)$, $L^3(\hat{F}) = 0$ term alone connecting the two states. It is easy to verify that this yields

$$\frac{\mu_T\ (\text{proton})}{\mu_T\ (\text{neutron})} = -\frac{3}{2} \qquad \text{(VI-9)}$$

We have thus recovered this famous ratio. The fact that one obtains the result $\mu_A = 0$ when the transformation $\hat{V} \to 1$ is, alone, a striking proof of how badly such a transformation is needed, since an $SU(6)_{W,\text{currents}}$ "symmetry" would predict that $\langle \text{rest};-\frac{1}{2}|E^1|\ \text{rest};+\frac{1}{2}\rangle = 0$ -- in clear contradiction to the Lorentz algebra! One must conclude that states belonging to irreducible representations of $SU(6)_{W,\text{currents}}$ cannot even have definite spin!

We can also compute μ^*, the M1 transition moment for $p \to \Delta^+$, assigning the Δ^+, spin $+\frac{1}{2}$ to $(6,3)$, $L^3(W) = 0$. As before, we can verify that

$$\mu^* = 2 \frac{\partial}{\partial k_x} \langle \Delta^+_{\text{strong}}, \tfrac{1}{2}, \text{rest}\ |\hat{F}_{em}(k_x)|\ p_{\text{strong}}, -\tfrac{1}{2}, p_3 = \frac{m_\Delta^2 - m_p^2}{2m_\Delta} \rangle |_{k_x = 0} \qquad \text{(VI-10)}$$

Again, only the $(3,\overline{3}) + (\overline{3},3)$, $L^3(\hat{F}) = 0$ term contributes, yielding the traditional

$$\mu^* = \frac{2\sqrt{2}}{3}\ \mu_T(\text{proton}) \qquad \text{(VI-11)}$$

which is within about 30% of the measured value. Finally, we can work out the E2 transition moment for $p \rightarrow \Delta^+$,

$$E2 = \tfrac{1}{2}\frac{\partial}{\partial k_x}\Bigg\{\frac{-1}{\sqrt{2}}\langle\Delta^+_{strong},\tfrac{1}{2},\text{rest}\ |\hat{F}_{em}(k_x)|\,p_{strong},-\tfrac{1}{2},p_3=\frac{m_\Delta^{\ 2}-m_p^{\ 2}}{2m_\Delta}\rangle$$

$$+\tfrac{1}{2}\sqrt{\tfrac{2}{3}}\langle\Delta^+_{strong},\tfrac{3}{2},\text{rest}\ |\hat{F}_{em}(k_x)|\ p_{strong},\tfrac{1}{2},p_3=\frac{m_\Delta^2-m_p^{\ 2}}{2m_\Delta}\ \rangle\Bigg\}_{k_x = 0}$$

(VI-12)

With the assignment of Δ^+, spin $+\frac{3}{2}$, to $(10,1)L^3(\hat{W}) = 0$, this yields $E2 = 0$, in good agreement with experiment. This moment has a very special importance for our work, since neither $(1,8) + (8,1)$ terms nor $(3,\overline{3}) + (\overline{3},3)$ can give any contributions to E2. Products of these representations, however, can contribute, so that the vanishing of E2 provides a test for the absence of such terms. Experimentally,[20] $E2/M1 = .02 \pm .02$, which seems to indicate that any terms transforming like products of currents (i.e., terms not bilinear in quark fields) are absent, or at least contribute very little to $\Delta L^3(\hat{F}) = 0$ transitions.

We can now go on to higher moments, finding, for example, charge radii of spin $\frac{1}{2}$ particles,

$$R^2 = \frac{d}{dq^2} \; G_E(q^2)\Big|_{q^2 = 0}$$

$$= -\tfrac{1}{2} \langle A, {}_{\text{strong}}; +\tfrac{1}{2}, \text{rest}| \int d^4x\delta(x^+)x^{\;2}\mathcal{F}^+_{em}(x)|A, {}_{\text{strong}}; +\tfrac{1}{2}, \text{rest}\rangle + Q/8M^2$$

(VI-13)

where $G_E(q^2)$ is the Electric Sachs form factor. From the Appendix we find that the transformed operator has the structure,

$$\hat{V}^{-1} \int d^4x \; \delta(x^+)x^{\;2}\mathcal{F}^+_{em}(x)\hat{V} \sim (1,8) + (8,1) \quad L^3(\hat{F}) = 0$$

$$+ (3,\overline{3}) + (\overline{3},3) \quad L^3(\hat{F}) = \pm 1 \qquad \text{(VI-14)}$$

where the first term has pieces transforming like $(\frac{\lambda_{em}}{2})$ and $(L^3(F)\;\sigma^3\;\frac{\lambda_{em}}{2})$. This second piece can be shown to vanish by means of $\mathcal{R}$ parity and a spin flip operator $\exp\,(i\pi\;\hat{F}_o{}^2)$. Note that $(L^3(F)\;\;\sigma^3\;\frac{\lambda_{em}}{2})$ is even under $\mathcal{R}$, but odd under $\exp\,(i\pi\;\hat{F}_o{}^2)$.

We thus conclude that the charge radii are proportional to the charge for the entire 56 (i.e., pure F coupling for the nucleon 8). In particular, this means R^2(neutron) = 0, which is to be compared with the experimental value of .027 ± .001 fm^2, or -1/5 of the proton charge radius. Although this violation of 20% is acceptable when we remember the approximations that led to the prediction, its size is still a bit surprising in comparison with the accuracy of our other results. Perhaps this is a warning that as we increase the complexity by going to higher moments we may also be increasing the sensitivity of the predictions to terms not present in the free quark model.

The evaluation of these higher moments becomes extremely laborious, as can be seen from the transformation properties of the charge radius in the Appendix. Furthermore, the exp $ik_x(x+E^1)$ operator alone complicates the extraction of form factors.

What we need is a cleaner method for deriving these higher moments, so that we can get a better idea about exactly what is going wrong (if anything). The most fruitful approach to this problem seems to be the application of Dashen and Gell-Mann's angular condition,[15] which relates higher moments to lower moments. Perhaps in this way the problem of the higher moments can be clarified. This work remains to be done.

VII. CONCLUSION

The results of the last chapter make it apparent that the algebraic structure of the transformation between $SU(6)_{W,\,currents}$ and $SU(6)_{W,\,strong}$ in the free quark model may be close to that of the transformation in the real world.

Although we expect that the actual transformation $\hat{V}$ will yield more than just operators belonging to $(3,\bar{3}) + (\bar{3},3)$ and $(1,8) + (8,1)$ when $\hat{V}$ is used to transform charges and moments like $\hat{F}_j^{\,\beta}(k\ )$, such terms seem to be nearly absent, at least for matrix elements of $\hat{F}_j^{\,\beta}(k\ )$ between baryon states in the $\underline{56}$, $L^3(\hat{W}) = 0$. The mechanism which produces the similarity of the algebraic properties of $\hat{V}$ and $\hat{V}_{free}$ is presently unknown. The problem was discussed in Chapter IV, but the issue has yet to be completely clarified.

We have found what seems like a reasonable definition of the transformation between the two $SU(6)_W$'s, one which can be applied (in principle, at least) to study a wide range of models. Such a study would be one avenue of learning more about the structure of this transformation.

Another means of learning more about the structure of the transformation would be to investigate the impact of, say, $\hat{V}_{free}$ on the problem of saturating the local $SU(3) \times SU(3)$ at infinite momentum.

This work remains to be done. In the meantime, however, we have made some progress in understanding how to use the $SU(6)_{W,\,currents}$ algebra to predict relations between matrix elements. An extremely simple (although somewhat unrealistic) example of the transformation between the two $SU(6)_W$'s has been found. Hopefully, the existence of this concrete example of the relation between the two $SU(6)_W$'s will make the wide difference between the "current quark" and "constituent quark" points of view clear.

Lastly, we have found some rational basis for understanding where the "naive quark model" results come from -- we have recovered all the successes of these schemes, and tempered the failures.

APPENDIX

Transformation of Light-like Charges and Moments in the Free Quark Model

A. The Axial Vector Charge

$$\hat{V}^{-1}_{free} \hat{F}^3_i V_{free} =$$

$$\sqrt{2} \int d^4x \, \delta(x^+) \, q_+^\dagger(x) \frac{1}{\varkappa} \left(1 - i \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m} \right) \frac{\sigma^3}{2} \frac{\lambda_i}{2} q_+(x)$$

where

$$\varkappa = \sqrt{1 + \left(\frac{\gamma_\perp \cdot \partial_\perp}{m} \right)^2}$$

B. The Electromagnetic Current

Magnetic Moment

$$\hat{V}^{-1}_{free} \int d^4x \, \delta(x^+) \, x \, \mathcal{F}^+_{em}(x) \, \hat{V}_{free} = \int d^4x \, \delta(x^+) \, x \, \mathcal{F}^+_{em}(x)$$

$$- i \sqrt{2} \int d^4x \, \delta(x^+) \, q_+^\dagger(x) \frac{1}{2\varkappa^2} \left\{ \left(1 - \frac{1}{1+\varkappa} \frac{\partial^2 \partial^2}{m^2} \right) \frac{\gamma^1}{m} + \right.$$

$$\left. \frac{1}{1+\varkappa} \frac{\partial^1 \partial^2}{m^2} \frac{\gamma^2}{m} + \frac{\varkappa}{1+\varkappa} \frac{\sigma^3 \partial^2}{m^2} \right\} \frac{\lambda_{em}}{2} q_+(x)$$

Charge Radius and Quadrupole Moment

$$\hat{V}_{free}^{-1} \int d^4x\, \delta(x^+)\, x^i x^j \mathcal{F}_{em}^+(x)\, V_{free} = \int d^4x\, \delta(x^+)\, x^i x^j \mathcal{F}_{em}^+(x)$$

$$- \sqrt{2} \int d^4x\, \delta(x^+)\, x^i q_+^\dagger(x) \left\{ \frac{\frac{\partial^j}{m^2}\left[\frac{i}{\varkappa}\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) - 1\right] + \frac{\gamma^j}{m}\left[i\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) + (\varkappa+1)\right]}{2\varkappa(\varkappa+1)} \right\} \frac{\lambda_{em}}{2} q_+(x)$$

$$- \sqrt{2} \int d^4x\, \delta(x^+)\, x^j q_+^\dagger(x) \left\{ \frac{\frac{\partial^i}{m^2}\left[\frac{i}{\varkappa}\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) - 1\right] + \frac{\gamma^i}{m}\left[i\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) + (\varkappa+1)\right]}{2\varkappa(\varkappa+1)} \right\} \frac{\lambda_{em}}{2} q_+(x)$$

$$+ 2\, \delta_{ij} \int d^4x\, \delta(x^+)\, q_+^\dagger(x)\, \frac{\left[1 - \frac{i}{\varkappa}\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right)\right]}{2\varkappa(\varkappa+1)}\, \frac{\lambda_{em}}{2}\, q_+(x)$$

$$+ \sqrt{2} \int d^4x\, \delta(x^+)\, q_+^\dagger(x) \frac{\partial^i \partial^j}{16 m^4 \varkappa^5} \frac{1}{(\varkappa+1)^2} \Big[(12\varkappa^2 - 2\varkappa - 1)(\varkappa+1)$$
$$- (12\varkappa^2 + 10\varkappa + 1)\left(i\,\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right)\Big] \frac{\lambda_{em}}{2}\, q_+(x)$$

$$- i\sqrt{2} \int d^4x\, \delta(x^+)\, q_+^\dagger(x) \frac{\gamma^i \partial^j + \gamma^j \partial^i}{m^3 \varkappa [2\varkappa(\varkappa+1)]^2} \left[i\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) + (\varkappa+1)\right] \frac{\lambda_{em}}{2}\, q_+(x)$$

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19. M. Gell-Mann has made a suggestion on how to resolve this problem. PCAC and vector dominance allow us to evaluate the consequences of $\hat{V}_{free}$ for certain strong interaction vertices (those involving pions or vector mesons). Strong interaction vertices in general can be expanded in terms of invariants, each of which has definite $SU(6)_{W,strong}$ properties, after the manner of Sakita and Wali (B. Sakita and K. C. Wali, Phys. Rev. 139, B1355, (1965)). In this scheme mesons and baryons are described as objects with appropriate sets of SU(12) indices. Vertices are described by means of invariant terms consisting of the various possible contractions of these indices with the external momenta and with each other.

We can see what the structure of $\hat{V}_{free}$ implies for these invariants in processes where $\hat{V}_{free}$ can be applied. The next step of generalization would be to assume this structure applies for all strong interaction processes, whether $\hat{V}_{free}$ can be applied or not.

It seems likely that this procedure will result in the Rosner-Colglazier $^3P_0$ prescription for analyzing strong-interaction vertices (E. W. Colglazier and J. L. Rosner, Nuclear Physics B27, 349 (1971). See also W. P. Petersen and J. L. Rosner, Phys. Rev. D6, 820 (1972)). This hypothesis has yet to be verified, however.

This scheme has the great virtue that it allows us to use the ideas gained from the properties of currents to suggest the structure of purely hadronic vertices.

20. R. Walker has kindly supplied the following data for the $\gamma p \to n^0\pi^+$ transition (resonance parameters are taken to be $M = 1233$ MeV, $\Gamma = 120$ MeV)

$$M_{1+} = 2.49 \pm .04$$

$$E_{1+} = .038 \pm .03$$

(errors are approximate).

21. The idea that a unitary transformation V is responsible for the mixing of $SU(6)_{W,currents}$ representations at infinite momentum is an old one. It appears in attempts to find representations of the current algebra at infinite momentum (Dashen and Gell-Mann,[5] Buccella, Klinert, et al.[7]) and in many other research efforts of the last seven years. The existence of such a transformation is also implicit in the phenomenological mixing schemes.[6]