

Acceleration of the Universe in Type-0 Non-Critical Strings

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Abstract. I review cosmology within the framework of type-0 non-critical strings [1]. The instabilities of the tachyonic backgrounds, due to the absence of space-time supersymmetry, are treated in this framework as a necessary ingredient to ensure cosmological flow. The model involves D3 brane worlds, whose initial quantum fluctuations induce the non criticality. I argue that this model is compatible with the current astrophysical observations pointing towards acceleration of the Universe. A crucial rôle for the correct “phenomenology” of the model, in particular the order-one value of the deceleration parameter, is played by the relative magnitude of the flux of the five form of the type-0 string as compared to the size of the volume of five of the extra dimensions, transverse to the direction of the flux-field.

Recently there is some preliminary experimental evidence from type Ia supernovae data [2], which supports the fact that our Universe *accelerates* at present: distant supernovae (redshifts $z \sim 1$) data indicate a slower rate of expansion, as compared with that inferred from data pertaining to nearby supernovae. Distant supernovae look dimmer than they should be, if the expansion rate of the Universe would be constant. If the data have been interpreted right this means that 70% of the present energy density of the Universe consists of an unknown substance (“*dark energy*”), which, for the fit of [2,3], has been taken to be the standard cosmological constant. This last statement, in turn, would imply that our Universe would be eternally accelerating (de Sitter), according to standard cosmology [4]. Such evidence is still far from being confirmed, but it is reinforced by combining these data with Cosmic Microwave background (CMB) data (first acoustic peak) implying a spatially $\Omega_{\text{total}} = 1.0 \pm 0.1$ flat Universe [3].

An important phenomenological parameter, which is of particular interest to astrophysicists is the *deceleration parameter* q of the Universe [4], which is defined as:

$$q = - \frac{(d^2 a_E / dt_E^2) a_E}{(da_E / dt_E)^2} \quad (1)$$

where $a_E(t)$ is the Robertson-Walker scale factor of the Universe, and the subscript E denotes quantities computed in the so-called Einstein frame, that is where the gravity action has the canonical Einstein form as far as the scalar curvature term is concerned. This distinction is relevant in string-inspired effective theories with four-dimensional Brans-Dicke type scalars, such as dilatons, which will be dealing with here.

It should be mentioned that in standard Robertson-Walker cosmologies with matter the deceleration parameter can be expressed in terms of the matter and vacuum (cosmological constant) energy densities, Ω_M and Ω_Λ respectively, as follows: $q = \frac{1}{2}\Omega_M - \Omega_\Lambda$. For the best fit Universe [2] one can then infer a present-era deceleration parameter $q_0 = -0.55 < 0$, indicating that the Universe accelerates today with an order one deceleration parameter.

If the data have been interpreted right, then there are three possible explanations [4]:

(i) Einstein's General Relativity is incorrect, and hence Friedman's cosmological solution as well. This is unlikely, given the success of General Relativity and of the Standard Cosmological Model in explaining a plethora of other issues.

(ii) the 'observed' dark energy and the acceleration of the Universe are due to an 'honest' cosmological *constant* Λ in Einstein-Friedman-Robertson-Walker cosmological model. This is the case of the best fit Universe which matches the supernova and CMB data. In that case one is facing the problem of eternal acceleration, for the following reason: let $\rho_M \propto a^{-3}$ the matter density in the Universe, with $a(t)$ the Robertson-Walker scale factor, and t the cosmological observer (co-moving) frame time. The vacuum energy density, due to Λ , is assumed to be constant in time, $\rho_\Lambda = \text{const}$. Hence in conventional Friedmann cosmologies one has:

$$\Omega_\Lambda/\Omega_M = \rho_\Lambda/\rho_M \propto a(t)^3, \quad (2)$$

and hence eventually the vacuum energy density component will dominate over matter. From Friedman's equations then, one observes that the Universe will eventually enter a de-Sitter phase, in which $a(t) \sim e^{\sqrt{\frac{8\pi G_N}{3}}\Lambda t}$, where G_N is the gravitational (Newton's) constant. This implies eternal expansion and acceleration, and most importantly the presence of a *cosmological horizon*

$$\delta = a(t) \int_{t_0}^{\infty} \frac{cdt}{a(t)} < \infty \quad (3)$$

It is this last feature in de Sitter Universes that presents problems in defining proper *asymptotic states*, and thus a consistent scattering matrix for field theory in such backgrounds [5]. The analogy of such global horizons with microscopic or macroscopic black hole horizons in this respect is evident, the important physical difference, however, being that in the cosmological de Sitter case the observer lives "inside" the horizon, in contrast to the black hole case.

Such eternal-acceleration Universes are bad news for critical string theory [5], due to the fact that strings are by definition theories of on-shell S -matrix and hence, as such, can only accommodate backgrounds consistent with the existence of the latter.

(iii) the 'observed effects' are due to the existence of a *quintessence* field φ , which has not yet relaxed in its absolute minimum (ground state), given that

the relaxation time is longer than the age of the Universe. Thus we are still in a non-equilibrium situation, relaxing to equilibrium gradually. In this drastic explanation, the vacuum energy density, due to the potential of the field φ will be time-dependent. In fact the data point towards a $1/t^2$ relaxation, with $t \geq 10^{60}$ in Planck units, where the latter number represents the age of the observed Universe.

It is this third possibility that we have attempted to adopt in a proper non-critical string theory framework [6,7] in ref. [1]. Non-critical strings can be viewed as non-equilibrium systems in string theory [8]. The advantage of this non-equilibrium situation lies on the possibility of an eventual exit from the de Sitter phase, which would allow proper definition of a field-theory scattering matrix, thus avoiding the problem of eternal horizons mentioned previously. Exit from de Sitter inflationary phases cannot be accommodated (at least to date) within the context of critical strings [8,9]. This is mainly due to the fact that such a possibility requires time-dependent backgrounds in string theory, which are not well understood within the conformal σ -model setting. On the other hand, there is sufficient evidence that such a ‘graceful exit possibility’ from the inflationary phase can be realized in non-critical strings, with a time-like signature of the Liouville field, which thus plays the rôle of a Robertson-Walker comoving-frame time [8,9]. The evidence came first from toy two-dimensional specific models [10], and was extended in [1] to four-dimensional models based on the so-called type-0 non-supersymmetric strings [11].

The latter string theory has four-dimensional brane worlds, whose fluctuations have been argued in [1] to lead to super criticality of the underlying string theory, necessitating Liouville dressing with a Liouville mode of time-like signature. In general, Liouville strings become critical strings after such a dressing procedure, but in one target-space dimension higher. However in our approach [8,1], instead of increasing the initial number of target space dimensions ($d = 10$ for type-0 strings), we have identified the world-sheet zero mode of the Liouville field with the (existing) target time. In this way, the time may be thought of as being responsible of re-adjusting itself (in a non-linear way), once the fluctuations in the brane worlds occur, so as to restore the disturbed conformal invariance of the underlying world sheet theory.

One of the most important results of [1] is the appearance of a time-dependent central charge deficit in the underlying conformal theory, acting as a vacuum energy density in the respective target-space lagrangian. This is crucial for a ‘graceful exit’ from the inflationary de Sitter phase, and the absence of eternal acceleration. In fact the asymptotic (in time) theory is that of a flat (Minkowski) target-space σ -model with a linear dilaton [7] in the string frame. In the Einstein frame, i.e. in a redefined metric background in which the Einstein curvature term in the target-space effective action has the canonical normalization, the universe is linearly expanding, which is the limiting case in which the horizon (3) diverges logarithmically; hence such

a theory can admit properly defined asymptotic states and S -matrix amplitudes. The linear dilaton background has been shown [7] to be a consistent background for string theory, despite being time dependent, in the sense of satisfying factorizability (for certain discrete values of the asymptotic central charge though), modular invariance and unitarity.

Another important aspect of our solution is the fact that the extra bulk dimensions are compactified in such a way that one is significantly larger than the others, thereby leading to effective five-dimensional brane world scenarios. This is a consequence of an appropriately chosen five-form flux background. This feature is one of the most important ones of the type-0 stringy cosmologies, which are known to be characterized by the existence of non-trivial flux form fields coming from the Ramond sector of the brane worlds [11,1].

The reader might object to our use of type-0 backgrounds due to the existence of tachyonic backgrounds. Although at tree level it has been demonstrated that the above-described flux forms can stabilize such backgrounds by shifting away the tachyonic mass poles, however, recently this feature has been questioned at string loop level. Nevertheless, in the context of our cosmological model, such quantum instabilities are expected probably as a result of the *non-equilibrium* nature of our relaxing background, in which the time dependent dilaton field plays the rôle of the quintessence field. Indeed, as demonstrated in [1] the asymptotic in time value of the tachyon background is zero, and hence such a field disappears eventually from the spectrum, which is consistent with the asymptotic equilibrium nature of the ground state.

The effective ten-dimensional target space action of the type-0 String, upon which we base our low-energy analysis, reads to $\mathcal{O}(\alpha')$ in the Regge slope α' [11]:

$$S = \int d^{10}x \sqrt{-G} \left[e^{-2\Phi} \left(R + 4(\partial_M \Phi)^2 - \frac{1}{4}(\partial_M T)^2 - \frac{1}{4}m^2 T^2 - \frac{1}{12}H_{MNP}^2 \right) - \frac{1}{4}(1 + T + \frac{T^2}{2})|\mathcal{F}_{MNP\Sigma T}|^2 \right] \quad (4)$$

where capital Greek letters denote ten-dimensional indices, Φ is the dilaton, H_{MNP} denotes the field strength of the antisymmetric tensor field, which we shall ignore in the present work, and T is a tachyon field of mass $m^2 < 0$. In our analysis we have ignored higher than quadratic order terms in the tachyon potential. The quantity $\mathcal{F}_{MNP\Sigma T}$ denotes the appropriate five-form of type-0 string theory, with non trivial flux, which couples to the tachyon field in the Ramond-Ramond (RR) sector via the function $f(T) = 1 + T + \frac{1}{2}T^2$.

From (4) one sees easily the important rôle of the five-form \mathcal{F} in stabilizing the ground state. Due to its special coupling with the quadratic T^2 term in the Ramond-Ramond (RR) sector of the theory, it yields an effective mass term for the tachyon which is positive, despite the originally negative m^2 contribution [11]. As mentioned previously, such a stability has recently been

questioned in the context of string loop corrections, but as we have mentioned previously this is rather a desirable feature of the approach, in view of the claimed cosmological instabilities.

As argued in [1] *fluctuations of the brane worlds* involved in the construction of type-0 string theory result in *supercriticality* of the underlying σ -model, with inevitable consequence the addition of the following term to the action (4)

$$-\int d^{10}x \sqrt{-G} e^{-2\Phi} Q(t)^2 \quad (5)$$

where $Q(t)$ is the central-charge deficit of the non-equilibrium non conformal σ -model theory. The time here is identified with the (world-sheet zero mode of) the Liouville field, and the t dependence of the central charge deficit is in accordance with the concept of a Zamolodchikov C -function [12], a “running central charge” of a non-conformal theory, interpolating between two conformal (fixed point) theories. The sign of $Q(t)^2$ is positive if one assumes supercriticality of the string [7,6,10,1], which is the case of the model of [1]. It is important to remark that in general, $Q^2(t)$ depends on the σ -model backgrounds fields, being the analogue of Zamolodchikov’s C -function [12]. As explained in [1], the explicit time dependence of $Q(t)$ reflects the existence of relevant operators in the problem, other than the background fields considered in (4) which are treated collectively in the present context. Such operators have been argued to represent *initial* quantum fluctuations of the brane world. A plausible scenario, for instance, would be that the initial disturbance that takes the system out of equilibrium is due to an impulse on the D3 brane worlds coming from either a scattering off it of a macroscopic number of closed string bulk states or another brane in scenarios where the bulk space is uncompactified (e.g. ekpyrotic universes *etc.* [13]). For times long after the event, memory of the details of this process is kept in the temporal evolution of $Q^2(t)$, which is determined self-consistently by means of the Liouville equations, as we shall see below.

The ten-dimensional metric configuration we considered in [1] was:

$$G_{MN} = \begin{pmatrix} g_{\mu\nu}^{(4)} & 0 & 0 \\ 0 & e^{2\sigma_1} & 0 \\ 0 & 0 & e^{2\sigma_2} I_{5 \times 5} \end{pmatrix} \quad (6)$$

where lower-case Greek indices are four-dimensional space time indices, and $I_{5 \times 5}$ denotes the 5×5 unit matrix. We have chosen two different scales for internal space. The field σ_1 sets the scale of the fifth dimension, while σ_2 parametrize a flat five dimensional space. In the context of cosmological models, we are dealing with here, the fields $g_{\mu\nu}^{(4)}$, σ_i , $i = 1, 2$ are assumed to depend on the time t only.

As we demonstrated in [1], a consistent background choice for the flux form field will be that in which the flux is parallel to the fifth dimension

σ_1 . This implies actually that the internal space is crystallized (stabilized) in such a way that this dimension is much larger than the remaining five σ_2 .

Upon considering the fields to be time dependent only, i.e. considering spherically-symmetric homogeneous backgrounds, restricting ourselves to the compactification (6), and assuming a Robertson-Walker form of the four-dimensional metric, with scale factor $a(t)$, the generalized conformal invariance conditions and the Curci-Pafutti σ -model renormalizability constraint [14] imply a set of differential equations, which we solved numerically in [1]. The system can also be solved analytically in the phase of large times after the initial fluctuations, where the various fields obey linearized approximations.

The generic form of the Liouville equations, which express restoration of conformal invariance after Liouville dressing, reads [6,8,15,1]:

$$\ddot{g}^i + Q(t)\dot{g}^i = -\tilde{\beta}^i \quad (7)$$

where $\tilde{\beta}^i$ are the Weyl anomaly coefficient of the stringy σ -model on the background $\{g^i\}$. In the model of [1] the set of $\{g^i\}$ contains graviton, dilaton, tachyon, flux and moduli fields $\sigma_{1,2}$ whose vacuum expectation values control the size of the extra dimensions. As mentioned above, these equations should be supplemented by the Curci-Pafutti renormalizability condition [14].

As argued in [1] such equations correspond to solutions of equations of motion derived from a ten-dimensional effective action. This is an important and non-trivial consequence of the gradient flow property of the σ -model $\tilde{\beta}^i$ functions, according to which: $\tilde{\beta}^i = \mathcal{G}^{ij} \frac{\delta \mathcal{F}[g]}{\delta g^j}$ where the flow functional $\mathcal{F}[g]$ is essentially the target-space effective action, depending on the background configuration under consideration.

An equivalent set of equations (in fact at most linear combinations) come out from the corresponding four-dimensional action after dimensional reduction. Of course this reduction leads to the string or σ -model frame, in which there are dilaton exponential factors in front of the Einstein term in the action. We may turn to the Einstein frame, in which such factors are absent, and the Einstein term is canonically normalized. In this latter frame, the cosmological time is defined by $dt_E = e^{-\Phi + \frac{\sigma_1 + 5\sigma_2}{2}} dt$. The extra dimensions freeze out quickly [1] and their constant values can be absorbed in a redefinition of the Newton's constant [1]. This yields the Einstein-frame time in the form:

$$t_E = \int^t e^{-\Phi(z)} dz \quad (8)$$

In this late-time phase, the Einstein-frame dilaton varies logarithmically with the Einstein time t_E (8)

$$\Phi_E = \text{const} - \ln t_E, \quad (9)$$

while the Einstein-frame “vacuum” energy is related to the central charge deficit as [1]:

$$\Lambda_E = e^{2\Phi} Q^2(t) \quad (10)$$

in Planck M_P units. As discussed in [1], due to the non-equilibrium nature of the non-critical string Universe, which has not yet relaxed to its ground state, Λ_E should be considered rather as an effective potential, in much the same way as the potential of a (non-equilibrium) quintessence field, whose rôle is played here the dilaton Φ [9,1].

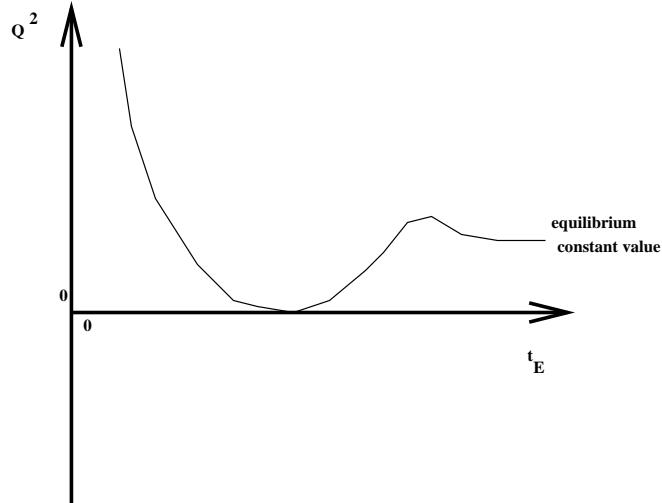


Fig. 1. The evolution of the central charge deficit Q^2 in the Einstein frame. Immediately after inflation, the deficit passes through a phase where it first vanishes, and then oscillates before relaxing to an equilibrium constant value asymptotically.

The results of the analysis of [1] are summarized in figs. 1 and 2. The numerical solution is supported by analytical considerations for the asymptotic field modes (late cosmological-frame times $t \rightarrow \infty$). Our solution demonstrates that the scale factor of the Universe, after the initial singularity, enters a short inflationary phase and then, in a smooth way, goes into a flat Minkowski spacetime with a linear dilaton for asymptotically long times $t \rightarrow \infty$ in the σ -model frame. Note that this is a consistent σ -model background [7] in the sense of satisfying modular invariance and factorization of S -matrix elements. This is actually one of the most important points of our work in [1], namely that there is a smooth exit from the de Sitter phase in such a way that one can *appropriately define* asymptotic on-shell states, and hence an S -matrix.

We observe from fig. 2 that, after its graceful exit from the inflationary phase, the type-0 non-critical string Universe passes first through a decelerating phase, which is then succeeded by an accelerating one, before the Universe relaxes asymptotically to its steady state (equilibrium) value.

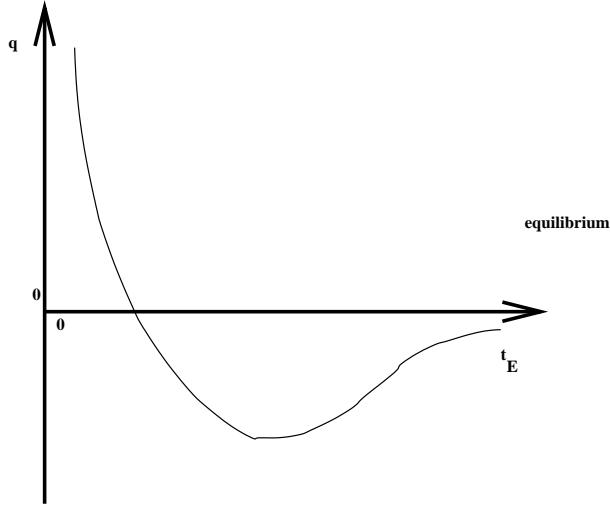


Fig. 2. The evolution of the decelerating parameter q of the type-0 string Universe in the Einstein frame.

From fig. 1 we also observe that, immediately after inflation, the central charge deficit Q^2 passes through a *metastable point* where it vanishes. This has to do with the fact that at this point the square root Q changes sign. After this point the central charge deficit oscillates (as a function of time) before it relaxes to its constant asymptotic value, q_0 , which should be taken as one of the values for which the conformal theory of [7] is valid. The oscillatory nature is consistent with the time-like signature of the Liouville mode as explained in [1].

This behaviour of the central charge deficit is important in determining the evolution of the energy densities of the type-0 string Universe, which are depicted in fig. 3. We should remark that in the type-0 string/brane scenario, ordinary matter may be assumed attached to the brane, and hence purely four dimensional. The latter is going to resist the deceleration of the universe, according to standard arguments, but it is not expected to change the order of magnitude of such quantities as the Hubble parameter and the vacuum energy, determined in [1] in the absence of matter. In this sense one may obtain the observed ‘coincidence situation’ of the present era, where the matter and ‘dark energy’ contributions are roughly of the same order of magnitude [4]. In our scenario it is the time dependence of both ‘dark energy’ and ‘matter contribution’, in conjunction with the value the time t_E has at present, roughly $t_E \sim 10^{60} M_P^{-1}$, that is held responsible for this coincidence situation.

In our model the tracking (coincidence) of the matter energy density by that of the quintessence dilaton field is a feature only of the present era, which is a welcome feature phenomenologically. As the time t_E elapses further,

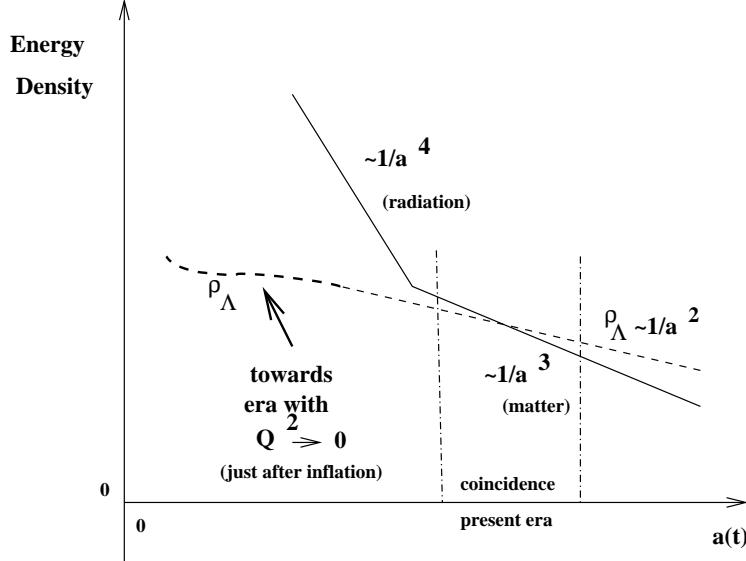


Fig. 3. The evolution of the energy densities of matter, radiation and of the quintessence field (dilaton) vs. the scale factor of the Universe in the Einstein frame. At early stages the energy density of the quintessence field decreases significantly, as compared with the rest, and the coincidence situation is lost. This is due to the behaviour of the central charge deficit of the model, shown in figure 1, which dives in to zero for a short period immediately after inflation.

the matter contribution will become subdominant, as scaling like a_E^{-3} . For very large times t_E in the far future, as we have seen above, the dominant contributions will be the ones due to the non-constant in time ‘dark energy component’ $\Lambda(t_E) \sim a_E^{-2}$, which asymptotes to zero, as the system reaches its equilibrium value. This makes a quantitative difference in scaling as compared with the standard Robertson-Walker scenario with a constant vacuum energy.

For large times, such as the present era, a straightforward computation yields [1] the Hubble parameter:

$$H(t_E) \simeq \frac{\gamma^2 t_E}{1 + \gamma^2 t_E^2} \quad (11)$$

where the constant $\gamma \propto V_5/|C_5|$, with C_5 the flux of the five form, which is taken along the fifth extra dimension, and V_5 the volume of the five extra dimensions, transverse to the flux direction.

The deceleration parameter (1), in the same regime of t_E , is:

$$q(t_E) \simeq -\frac{1}{\gamma^2 t_E^2} \quad (12)$$

Finally, the “vacuum energy” can be obtained from (10):

$$\Lambda(t_E) \simeq \frac{q_0^2 \gamma^2}{F_1^2(1 + \gamma^2 t_E^2)} \quad (13)$$

where q_0 is the constant asymptotic value of the central charge deficit Q^2 . In the model of [1] conformal invariance requires that $q_0/F_1 \sim \mathcal{O}(1)$.

If, therefore, one defines the *present era* by the time regime

$$\gamma \sim t_E^{-1} \quad (14)$$

in the Einstein frame, then from (12) it becomes clear that an order one negative value of q is obtained, in agreement with the preliminary astrophysical observations. The important point is that this is compatible with large enough times t_E (in string units) for $|C_5|/V_5 \gg 1$. This condition can be guaranteed *either* for small radii of the five of the extra dimensions *or* for a large value of the flux $|C_5|$ of the five-form of the type-0 string. Notice that the relatively large extra dimension, in the direction of the flux, decouples from this condition, thus allowing for the possibility of effective five-dimensional models with large uncompactified fifth dimension.

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