ARE THE PSEUDOSCALARS LIGHT BECAUSE OF HADRON SELF ENERGIES

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Abstract: We discuss unitarity effects to the quark model S-wave $q\bar{q}$ states.

Because of a spin counting factor unitarity effects should be much larger for the pseudoscalars (P) than for the vector mesons (V) and could account for the V-P mass difference and the η-η' mixing.

Contrary to QCD annihilation graphs the signs of the unitarity effects are in the right direction compared to experiment.

Résumé: Nous discutons les effets d'unitarité pour les états d'onde S des modèles des quarks.

En raison d'un facteur de spin, les effets d'unitarité sont beaucoup plus grands pour les mésons pseudoscalaires (P) que pour les mésons vectoriels (V) et on peut expliquer la différence de masse V-P et le mixing η-η'.

Contrairement aux graphes d'annihilation en gluons, les corrections dues à l'unitarité vont dans la bonne direction par rapport à l'expérience.

It is a widely recognized fact that unitarity plays an important role in strong interaction physics. The fact that all hadrons couple strongly to each others implies that the meson states, in particular the pseudoscalars (P) and the vector mesons (V), depend on the presence of nearby two body channels (PP, PV etc.). In other words the physical meson masses and mixing angles get contributions from self energy loop diagrams (fig. 1).

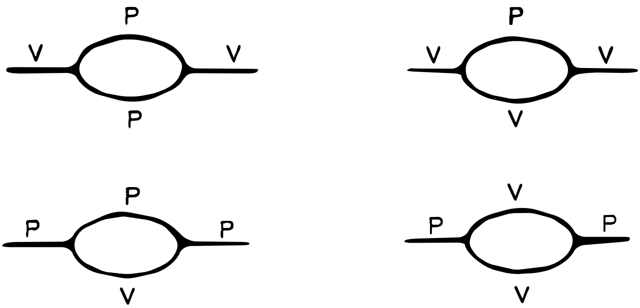

Fig. 1 Meson self energy diagrams for V → PP → V V → PV → V, P → PV → P and P → VV → P.

Many years ago Feynman[1] emphasized the importance of such hadronic self energy diagrams, although he stated that "no calculation of such virtual strong interactions in any problem has ever been successful". The difficulty is of course that of regularization or the choice of cut-off in calculating the real part of the loop diagram:

$$\mathrm{Re}\Pi_{ab}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} \frac{-\mathrm{Im}\Pi_{ab}(s')}{s - s'}\, ds' \qquad (1)$$

Thereby, in particular, the over all mass shift will be uncertain, but assuming the same cut for different thresholds related by flavour symmetry there are many predictions (mass differences, mixing angles) which are relatively unsensitive to the choice of cut off. Below we outline a scheme, discussed in more detail in Ref.[2], by which one can phenomenologically estimate them.

The imaginary part $\mathrm{Im}\Pi_{ab}$ is through unitarity constrained by experimental widths. ($-\mathrm{Im}\Pi_{aa} \simeq m\Gamma_a$). For the $V \to PP \to V$ and $V \to PV \to V$ loops we have:

$$-\mathrm{Im}\Pi^V_{ab} = \gamma^2_{V,PP} \sum_{cd} c^{(-)}_{a,cd}\, c^{(-)}_{b,cd} \frac{k^3_{cm}}{\sqrt{s}} F^2(s) + \gamma^2_{V,PV} \sum_{cd} c^{(+)}_{a,cd}\, c^{(+)}_{b,cd} \frac{k^3_{cm}}{\sqrt{s}} F^2(s) \;, \tag{2}$$

where a-d are flavour indices, $\gamma_{V,PP}$ and $\gamma_{V,PV}$ are over all constants (assuming exact $SU(6)_W$: $\gamma_{V,PV} = 2\gamma_{V,PP}$), k_{cm} the c.m. momentum of the threshold cd and $c^{(\pm)}_{a,cd}$ are flavour symmetry Clebsch Gordan constants (see below, eq. (4)) and F(s) is the hadronic form factor giving the cut off. In a composite model, like the nonrelativistic quark model, F corresponds to the overlap of three $q\bar{q}$ wave functions. For a harmonic oscillator potential F would be of the form $\exp\{-(k_{cm}/k_{cut\ off})^2\}$

For the pseudoscalars the nearest thresholds correspond to $P \to PV \to P$ and $P \to VV \to P$. With the same effective Lagrangians as used in deriving eq. (2) one has:

$$-\mathrm{Im}\Pi^P_{ab} = (3\,\frac{s}{m^2_v})\, \gamma^2_{V,PP} \sum_{cd} c^{(-)}_{a,cd}\, c^{(-)}_{b,cd} \frac{k^3_{cm}}{\sqrt{s}} F^2(s) + (3)\, \gamma^2_{V,PV} \sum_{cd} c^{(+)}_{a,cd}\, c^{(+)}_{b,cd} \frac{k^3_{cm}}{\sqrt{s}} F^2(s) \;. \tag{3}$$

Note in particular that the expressions are very similar to those of eq. (2) apart from a spin counting factor 3 (coming from the fact that $P \to PV$ is isotropic, where $V \to PV$ is not) and a factor s/m^2_V in $P \to PV$ (coming from the fact that the vector meson is longitudinally polarized). Thus with everything else equal we expect the self energy effects to be three or more (with $\frac{s}{m^2_V} > 1$) times larger for the pseudoscalars than for the vector mesons.

The input coupling constants are assumed to have the simplest possible flavour symmetry structure, i.e. the constants $c^{(\pm)}_{a,bc}$ are given by the quark line rules

$$c^{(\pm)}_{a,bc} = \mathrm{Tr}\{\Delta_a\Delta_b^\dagger\Delta_c^\dagger \pm \Delta_a\Delta_c^\dagger\Delta_d^\dagger\}/2, \qquad (4)$$

where the sign is determined by the C quantum numbers (- for F-type, + for D-type coupling, and where Δ are usual N x N matrices given by the quark content of the input states. Each loop diagram of Fig. 1 can then be decomposed in terms of quark line graphs as shown in Fig. 2 below.

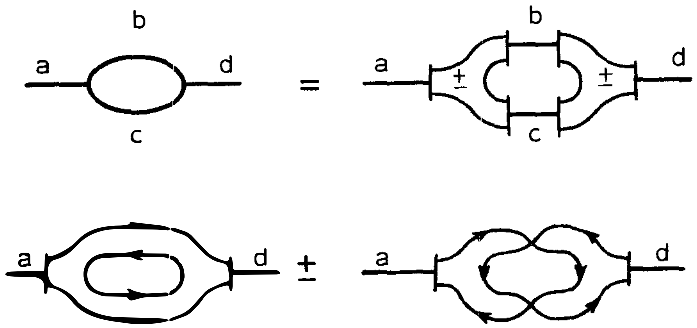

Fig. 2 The self energy diagram for a definite intermediate state e.g. $K\bar{K}$ decomposed into two quark line graphs.

An important point is that the signs of the diagonal elements of $\mathrm{Re}\Pi_{aa}$ are in general negative, i.e. the unitarity effects shift the masses down. This sign is basically the same sign, which in elementary quantum mechanics makes two energy levels "repel" each others when they are disturbed by a perturbation, - the lower state (E_1) is "pushed down" by the higher state (E_2), ($\Delta E = H_{12}^2/(E_1-E_2) < 0$). In the same way the two body continuum of states (e.g. $K\bar{K}$ above the ϕ mass) "push" the resonance (ϕ) mass down due to the virtual transitions ($\phi \to K\bar{K} \to \phi$).

A similar argument[3,4] can be applied to the sign of the gluon annihilation graphs, which contribute only to the flavour singlet states (P_1,V_1). Thus the gluon annihilation graphs make the singlet states lighter than the octet states. This is opposite to what is observed experimentally. If one disregards from this sign argument, reasonable fits to the experimental masses have been obtained by many authors some of which are listed in

Ref.[5].

Unitarity predicts the right sign if one assumes that the nearest thresholds, $P \to PV \to P$ or $V \to PP \to V$ are the most important. This can easily be seen since the first thresholds are antisymmetric in flavour (F-coupled) and therefore do not couple to the singlet states. Thus they shift only the octet states down in mass.

The scheme outlined by eqs. (1) - (4) above fixes the unitarity effects from each threshold (e.g. $\rho \to \pi\pi \to \rho$, $\rho \to K\bar{K} \to \rho$, $\phi \to K\bar{K} \to \omega$, $\eta \to K\bar{K}^{*} \to \eta'$). For more details see Ref.[2].

One can calculate a phenomenological (mass matrix)^2

$$M^2_{ab} = (M^2_{input})_{ab} + \Pi_{ab}(s), \qquad (5)$$

where M^2_{input} (but not Π) is diagonal in the ideal reference frame. The physical meson masses are the eigenvalues of M . The matrix which diagonalizes M^2 also determines the physical meson couplings, which are complicated (s dependent) linear combinations of the input coupling constants. Thus although the input states have a simple flavour symmetry structure the physical coupling constants will break the flavour symmetry in a complicated way through the loop diagrams and the input quark mass differences.

The self consistency requirement, that the input (intermediate meson) masses should be equal to the output masses allows us to use the experimental masses for the intermediate states (PP, PV etc.). Applying such a scheme to the vector mesons[6] one gets a very good agreement with experiment if one adjusts the overall relative weight γ^2 of the F and D coupled thresholds such that $m_\omega - m_\rho$ is small as in experiment. This fixes $\gamma_F^2 \simeq \gamma_D^2$ (an exchange degeneracy relation) and allows us to predict the ϕ-ω and ψ-ω mixing

$$\delta_{\phi\omega} = (+0.08 \pm 0.03)e^{-i(0.09 \pm 0.1)} \text{ radians or,}$$

$$\theta^{SU(3)}_{\phi\omega} = \theta_{ideal} + \delta_{\phi\omega} = [(39.5 \pm 1.5) - i(0.4 \pm 0.5)]^{o} \text{ and,}$$

$$|\delta_{\psi\omega}| \simeq |\delta_{\psi\phi}| \lesssim 0.006 \text{ radians.}$$

The ϕ-ω mixing agrees both in sign and magnitude with experimental estimates. The imaginary part[7] of the mixing is predicted to be small and agrees in sign with experiment[8] (arg $\delta^{exp}_{\phi\omega} = (-25 \pm 29)^{o}$, assuming $g_{\psi\omega}/g_{\gamma\phi}$ to be negative). The uncertainties in the theoretical estimates are due to different choices of the cut off. The ψ-ω and ψ-ϕ mixing is consistent with zero because of almost exact cancellation of F and D type thresholds

($D\bar{D}$, $D\bar{D}^*$).

As we already pointed out for the pseudoscalars the unitarity effects should be much larger due to the spin counting factor 3 and the factor s/m_V^2 in eq. (3). This means (using the same sign argument as above) that the pseudoscalars are shifted down much more than the vector mesons. The same sign also fixes the direction of the η-η' mixing (assuming the nearest thresholds (PV) to be dominant). If one assumes the cut off to be such that $m_{K^*}^2 - m_K^2$ comes out correct one gets (neglecting quark mass differences)

$$m_{\eta_1}^2 - m_{\eta_8}^2 \simeq m_{K^*}^2 - m_K^2 = 0.55\ \mathrm{GeV}^2 .$$

Thus disregarding, for a moment, from SU(3) breaking this would be $m_{\eta'}^2 - m_\eta^2$ in reasonable agreement with the experimental value 0.61 GeV^2. With usual quark mass difference in the input one predicts a pseudoscalar η-η' mixing angle[2)]

$$\theta_P^{SU(3)} = -19^\circ ,$$

which is a quite acceptable value compared to usual mass formulas values[9)] (-11° to -24°) or a recent experimental[10)] value from η'/η production ratios $(-18.2 \pm 1.4)^\circ$. Analogously the $\eta_1 \simeq \eta'$ and $\eta_8 \simeq \eta$ mixing in η_c is predicted to be

$$\delta_{\eta_c\eta_8} \simeq \frac{1}{\sqrt{2}}\,\delta_{\eta_c\eta_1} \simeq -5^\circ ,$$

giving a suppression factor of $\simeq$ 100 in η hadronic decays. Thus in this scheme the dominant OZI breaking in η_c decay is due to $\eta_c \to D\bar{D}^* \to$ hadrons, - not two gluon annihilation. Similarly the ψ-η_c mass difference is due to the fact that virtual $\eta_c \to D\bar{D}^* \to \eta_c$ transitions are much larger than the corresponding ones for ψ ($\psi \to D\bar{D} \to \psi$). Detailed χ^2 fits with application also to the 0^{++} mesons are in progress[11)].

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