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Toward a systematic study of kaonic nuclei with High-Momentum AMD

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Kaonic nuclei are the nuclear system involving anti-kaons ($\bar{K}$ mesons), which are expected to possess several interesting properties that we have never seen in normal nuclei, such as formation of dense state. Recently, a signal of a three-body system of kaonic nuclei, so-called "$K^- pp$", was found in a J-PARC hadron-hall experiment. Accordingly, experimentalists are going to search for heavier kaonic nuclei to reveal their properties. Corresponding to the experimental challenge, we will carry out a systematic study of kaonic nuclei theoretically. For this purpose, we have started our study based on a newly developed method, High-momentum Antisymmetrized Molecular Dynamics method (HM-AMD). The earlier studies with a coupled-channel Complex Scaling Method suggested that the two ingredients, 1. Channel coupling and 2. Resonances, should be respected in the theoretical study of kaonic nuclei. The HM-AMD method is expected to be easily improved to cover these necessary ingredients. In this article, we explain the improvement of the HM-AMD method for the study of kaonic nuclei.

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1. Introduction

Kaonic nuclei are the nuclear system involving anti-kaons ($\bar{K}$ mesons, $\bar{K} = (K^-, \bar{K}^0)$; $s\bar{q}$). The $\bar{K}N$ attraction is known to be so strong to generate a quasi-bound state of isospin-0 $\bar{K}N$ system with about 20 MeV binding energy, which corresponds to the excited hyperon $\Lambda(1405)$ [1]. Due to such a strong $\bar{K}N$ attraction, kaonic nuclei have been expected to have unique natures such as formation of dense state. The density of their inner region might exceed the normal nuclear density ($\rho_0 = 0.16$ fm^{-3}) against the NN short-range repulsion [2].

In kaonic nuclear study, theorists and experimentalists have devoted lots of effort to investigate the simplest kaonic nucleus, so-called "K^-pp", which is a three-body system composed of two protons and a K^- meson. The K^-pp has been studied with various theoretical approaches and its quasi-bound state has been searched experimentally for a long time. In 2020, J-PARC E15 collaboration found a signal of a K^-pp quasi-bound state via ^{3}He$(K^-, \Lambda pn)$ reaction. According to their analysis of the experimental data, the binding energy of K^-pp is $42 \pm 3^{+3}_{-4}$ MeV and the decay width is $100 \pm 7^{+19}_{-9}$ MeV [3]. After the success of their K^-pp experiment, they planned to conduct a systematic study of light kaonic nuclei (more than 3-body system) with the same approach as that of J-PARC E15 experiment which uses the K^- beam and analyzes the invariant mass of emitted particles in the reaction. Actually, the experiment to search for quasi-bound states of 4-body system ($\bar{K}NNN$) is now going on. (J-PARC E80 experiment)

So far, we have investigated the K^-pp in detail with a coupled-channel Complex Scaling Method [4] as explained in the next section. Corresponding to the current experimental plan, we will also investigate kaonic nuclei beyond the 3-body system. For such a systematic study, we will employ a new method which has been recently developed based on antisymmetrized molecular dynamics (AMD) method [5], which is called "High-momentum AMD (HM-AMD)" [6]. In this article, we will explain the framework of the HM-AMD method and its further improvement for the kaonic nuclear study.

2. Coupled-channel complex scaling method for the K^-pp system

As mentioned in Introduction, the $\bar{K}N$ interaction brings strong attraction to nucleons, which is the most unique and important in the physics of kaonic nuclei. So, we should treat this interaction as carefully as possible. It is well known that $\bar{K}N$ interaction causes the coupling of the $\bar{K}N$ pair to the $\pi\Lambda$ / $\pi\Sigma$ pairs. The diagonal $\bar{K}N$ interaction ($\bar{K}N$-$\bar{K}N$) is rather attractive, but the additional attraction to the $\bar{K}N$ pair is provided via the $\bar{K}N$-πY couplings ($Y = \Lambda, \Sigma$). Due to such couplings, kaonic nuclei should contain the $\pi YN...N$ components as well as the $\bar{K}NN...N$ component. In other words, we need to treat the kaonic nuclear system as a coupled-channel system of $\bar{K}NN...N$ and $\pi YN...N$ channels.

When such channel couplings are taken into account in theoretical calculations, kaonic nuclear systems are no longer the $\bar{K}NN...N$ bound states. The $\pi\Sigma$ and $\pi\Lambda$ thresholds are energetically lower than the $\bar{K}N$ threshold by 103 MeV and 180 MeV, respectively. So, the binding energy of kaonic nuclear systems needs to exceed 180 MeV so that they become a purely bound state. Otherwise, they could decay into $\pi YN...N$ states. As for the K^-pp, most of theoretical studies reported that

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its binding energy does not exceed 100 MeV. In other words, the K^-pp is considered to be not a bound state but a resonant state, which is a quasi-bound state of $\bar{K}NN$ decaying into πYN states.

Therefore, the K^-pp should be treated as a resonant state of the $\bar{K}NN - \pi\Sigma N - \pi\Lambda N$ coupled-channel system. In the past studies, we developed the coupled-channel Complex Scaling Method (ccCSM), in which both ingredients, channel coupling and resonances, are simultaneously treated [4]. The ccCSM is based on the complex scaling method (CSM), which has succeeded in many studies of resonant states of ordinary nuclei. In the CSM, we carry out the complex rotation (scaling) of coordinates $\{\boldsymbol{r}_i\}$ and their conjugate momenta $\{\boldsymbol{p}_i\}$: $\boldsymbol{r}_i \to \boldsymbol{r}_i \exp(i\theta)$ and $\boldsymbol{p}_i \to \boldsymbol{p}_i \exp(-i\theta)$, where the scaling angle θ is a real value. With this scaling, wave functions of resonant states, which are diverged in the asymptotic region, are transformed to be L^2-integrable and they can be represented with the superposition of Gaussian functions like bound-state wave functions. In actual calculations, the Hamiltonian H is transformed with the complex rotation and then the complex-scaled Hamiltonian H_θ is diagonalized with Gaussian basis functions. The complex-eigenvalues which are independent of the scaling angle θ correspond to those of resonant states, while those of continuum states vary with the θ value. Thus, we can identify the resonant states among obtained many eigenstates. It is noted that in the CSM the resonant states are correctly obtained by solving Schrödinger equation under Siegert boundary condition. In addition, as mentioned above, it is the greatest advantage of the CSM that we can obtain resonant states completely in the similar way to bound states, by diagonalization of (complex-scaled) Hamiltonian with Gaussian basis functions.

In the past study of the K^-pp, we have extended the CSM to the coupled-channel case. The K^-pp wave function is described as follows:

$$|\Phi_{K^-pp}\rangle = \sum_{MB_1B_2} \left\{ \sum_n C^{(\theta)}_{MB_1B_2,n} G_{MB_1B_2,n}(\boldsymbol{r}_{B_1B_2}, \boldsymbol{r}_{M-(B_1B_2)}) \right\} |MB_1B_2; S_{B_1B_2} = 0\rangle, \quad (1)$$

in which the channel index MB_1B_2 runs among $\bar{K}NN$, $\pi\Sigma N$ and $\pi\Lambda N$. The spatial wave function in each channel is expanded with the correlated Gaussian functions $\{G_{MB_1B_2,n}(\boldsymbol{r}_{B_1B_2}, \boldsymbol{r}_{M-(B_1B_2)})\}$ to describe the spatial correlation between particles accurately. The coefficients $\{C^{(\theta)}_{MB_1B_2,n}\}$ are determined with the diagonalization of the complex-scaled Hamiltonian H_θ. In our calculation, the quantum numbers of K^-pp is assumed to be $J^\pi = 0^-$ with the spin-singlet baryons ($S_{B_1B_2} = 0$) and the total isospin $T = 1/2$. More details are explained in the papers [4]. By diagonalizing the complex-scaled Hamiltonian with the coupled-channel basis functions, we have successfully obtained the K^-pp resonance pole in the complex energy plane with a good separation from continuum states of $\bar{K}NN$, $\pi\Sigma N$ and $\pi\Lambda N$ as well as $\Lambda(1405)N$ as shown in Fig. 1. As a result of the studies with ccCSM [4], the K^-pp is obtained to be a resonant state with $30 - 50$ MeV binding energy and $40 - 60$ MeV decay width.

Through our studies of K^-pp, we have learned that two ingredients, **1. Channel coupling and 2. Resonant states** should be appropriately treated in the study of kaonic nuclei, in order to respect the $\bar{K}N$ interaction which plays a key role in this physics.

3. High-momentum AMD method

As mentioned in Introduction, we will utilize the High-Momentum AMD method (HM-AMD) [6] to investigate kaonic nuclei theoretically beyond the three-body system of K^-pp. Here, we

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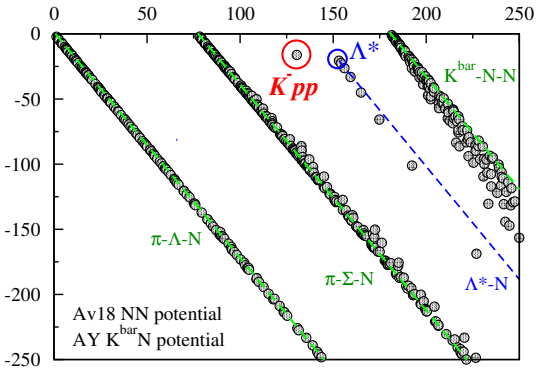


Figure 1: Eigenvalues distribution of $\bar{K}NN$-$\pi\Sigma N$-$\pi\Lambda N$ three-body system on the complex energy plane, which is calculated with the coupled-channel Complex Scaling Method [4]. The eigenvalue marked with a red (blue) circle corresponds to the "$K^- pp$" ($\Lambda(1405)$) resonance. Eigenvalues appearing along lines (green, blue) indicate $\bar{K}NN$, $\pi\Sigma N$, $\pi\Lambda N$ and $\Lambda(1405)N$ continuum states.

explain the framework of HM-AMD briefly.

In the standard AMD, each nucleon wave function is expressed with a Gaussian wave packet:

$$\phi_i(\boldsymbol{r}) \;=\; \left(\frac{2\pi}{\nu}\right)^{3/4} \exp\left[-\nu\,(\boldsymbol{r}-\boldsymbol{Z}_i)^2\right]\chi_i\tau_i, \tag{2}$$

where a real variable ν is a width parameter and a complex vector $\boldsymbol{Z}_i$ represents the centroid of Gaussian wave packet. The functions χ_i and τ_i indicate the spin and isospin (proton or neutron), respectively. The wave function of a nucleus is constructed with nucleon wave functions being antisymmetrized to be a Slater determinant, and then projected onto an eigenstate of parity and total angular momentum:

$$|\Phi\rangle \;=\; \det[|\phi_i(j)\rangle] \quad\Longrightarrow\quad |\Phi^{J\pi}_{MK}\rangle \;=\; \hat{P}^J_{MK}\hat{P}^\pi|\Phi\rangle, \tag{3}$$

where $\hat{P}^\pi$ is the parity projection operator with $\pi = +$ or $-$ and $\hat{P}^J_{MK}$ is the angular-momentum projection operator with the total angular momentum J. Each centroid of Gaussian wave packets $\{\boldsymbol{Z}_i\}$ is determined in an energy-variational way by means of so-called frictional cooling method. Details of the standard version of AMD are explained in Ref. [5].

In HM-AMD, we focus on the fact that the imaginary part of the complex vector $\boldsymbol{Z}_i$ indicates the momentum of a nucleon. Actually, it is easily confirmed that the expectation value of the momentum operator $\hat{\boldsymbol{p}} = -i\hbar\nabla$ calculated with a nucleon wave function ϕ_i is as follows:

$$\langle\phi_i|\hat{\boldsymbol{p}}|\phi_i\rangle \;=\; 2\hbar\sqrt{\nu}\,\mathrm{Im}\boldsymbol{Z}_i. \tag{4}$$

Therefore, using complex vectors $\{\boldsymbol{Z}_i\}$ with large imaginary values, we can introduce high-momentum components to the AMD wave function. In particular, by introducing large vectors $\{\boldsymbol{D}_a\}$ as an imaginary part of the centroid of nucleon wave functions in pairwise way such as

$$\boldsymbol{Z}_i + i\boldsymbol{D}_a \;\Longleftrightarrow\; \boldsymbol{Z}_j - i\boldsymbol{D}_a, \tag{5}$$

we can generate a nuclear wave function involving strong correlations between pairs of nucleons. We call such AMD wave functions involving high-momentum components as *HM-AMD wave function*, which are denoted as $\{\Phi^{(HM)}_a\}$.

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We generate various HM-AMD wave functions $\{\Phi_p^{(HM)}\}$ with many parameters sets of $\{\boldsymbol{Z}_i\}$ and $\{\boldsymbol{D}_a\}$. In the same way as the standard AMD, we carry out the parity- and total angular momentum projections for each HM-AMD wave function: $\Phi_n^{(HM),J\pi} = \hat{P}^J_{MK}\hat{P}^\pi\Phi_p^{(HM)}$. Using such the projected HM-AMD wave functions $\{\Phi_n^{(HM),J\pi}\}$ as basis functions, we diagonalize the Hamiltonian of the system (solving Schrödinger equation) to obtain eigen energies and eigen wave functions.

It is well known that there are important two correlations in nuclei due to nuclear forces. One is the short-range correlation induced by repulsive core in nuclear forces, and the other is the tensor correlation induced by the tensor force. Both correlations involve high-momentum components between paired nucleons. The HM-AMD wave function is expected to treat both correlations, since it contains the high-momentum components explicitly. Actually, HM-AMD method can give sufficiently good result of the triton (^{3}H) with a realistic nuclear potential which contains repulsive core and tensor force. The obtained binding energy of ^{3}H is 7.70 MeV, which is comparable or better than other precise calculations with the same potential: 7.76 MeV obtained by Green function Monte Carlo method and 7.68 MeV obtained by Tensor-optimized AMD. (Ref. [6] and references therein.)

4. Merging High-momentum AMD with coupled-channel CSM

We expect that HM-AMD is a suitable approach for our purpose. Since we will make a systematic study of kaonic nuclei, the method to be employed should be able to treat kaonic nuclear systems precisely with arbitrary particle numbers and quantum numbers of spin and parity. Such demands are considered to be satisfied with the HM-AMD method: 1. This approach is expected to carry out the precise calculation with bare interactions. As mentioned in the previous section, it is confirmed that HM-AMD gives the binding energy of triton with the same accuracy as other precise calculations, when the same realistic NN potential is used. 2. The HM-AMD wave function is composed of Gaussian wave packets, completely in the same way as the standard AMD. Although many AMD wave functions are superposed in HM-AMD method, various matrix elements can still easily be calculated. 3. We can generate kaonic nuclear states with possible quantum numbers as we hope by the spin-parity projection. Especially the angular momentum projection is carried out by rotating the wave function in terms of Euler angles $\Omega = (\alpha, \beta, \gamma)$ as follows:

$$|\Phi^J_{MK}\rangle \;=\; \hat{P}^J_{MK}|\Phi\rangle \;=\; \int d\Omega\, D^{J*}_{MK}(\Omega)\,\hat{R}(\Omega)\,|\Phi\rangle, \tag{6}$$

where $D^J_{MK}(\Omega)$ is Wigner's D function and $\hat{R}(\Omega)$ is the rotation operator acting on the spin wave function as well as the spatial wave function. This projection is realized numerically. It is noted that this projection method enables us to avoid complicated calculations involving Clebsch-Gordan coefficients to construct a wave function with a specific total angular momentum. In addition, the isospin projection is also possible in this method [2].

Therefore, HM-AMD is an ideal approach for our purpose. However, we need to improve this method to treat the channel coupling and the resonance which are the key ingredients in the kaonic nuclear study. As explained in the section 2, these demands have been realized in the coupled-channel Complex Scaling Method (ccCSM). So, we try to merge HM-AMD with ccCSM.

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In principle, the merge of the two method is expected to be possible, because of the features of HM-AMD method. The channel coupling is realized in ccCSM via the diagonalization of Hamiltonian matrix whose elements are calculated with basis functions involving channels. (See Eq. (1).) As explained in the previous section, HM-AMD also obtains binding energies and wave functions by means of the diagonalization of the Hamiltonian matrix. So, by expanding the basis functions in a single channel to those in multi channels, the coupled-channel calculation will be realized in HM-AMD as well. In addition, the building block of HM-AMD function is Gaussian wave packets as shown in Eq. (2), which are essentially Gaussian functions. As mentioned in the section 2, Gaussian functions match well with the complex scaling method (CSM) to obtain resonant states. Therefore, it is expected that CSM will function well on the HM-AMD method.

Thus, HM-AMD could be merged with ccCSM so that it can treat the channel coupling and resonances correctly. With the combination of both methods, we will be able to realize the systematic study of kaonic nuclei.

5. Summary and future prospect

Kaonic nuclei have been considered to possess interesting properties such as the possibility to form a dense state beyond the normal nuclear density, due to the strong $\bar{K}N$ attraction. For a systematic study of kaonic nuclei, we believe that the newly developed method, High-momentum antisymmetrized molecular dynamics (HM-AMD) is a suitable method, because it can easily generate various spin-parity (as well as isospin) states without bothersome calculation using Clebsch-Gordan coefficients. Additionally, HM-AMD relies on the diagonalization method and the building block of its wave function is Gaussian function, similarly to the coupled-channel complex scaling method (ccCSM) which was applied to the K^-pp study. Therefore, HM-AMD is expected to be extended to the multi-channel calculation and to implement the CSM to it. In other words, it will be merged with the ccCSM that is an adequate treatment of channel coupling and resonant states, which are the key ingredients for kaonic nuclear study.

As for the future prospect, firstly we will apply the HM-AMD+ccCSM to the two-body system of the isospin-0 $\bar{K}N$-$\pi\Sigma$ coupled-channel system which associates the hyperon resonance $\Lambda(1405)$. After we check the method functions correctly in the two-body system, we will proceed to kaonic nuclei beyond the three-body system.

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