Classical and quantum correlation of quantum fluctuation

Y Nambu

Department of Physics, Graduate School of Science, Nagoya University, Chikusa, Nagoya 464-8602, Japan

E-mail: nambu@gravity.phys.nagoya-u.ac.jp

Abstract. We have investigated the classical and quantum correlations of a quantum field in the inflationary universe using a particle detector model. We have found that the entanglement between two comoving detectors becomes zero after their physical separation exceeds the Hubble horizon. We have also found that the quantum discord, which is defined as the quantum part of total correlation, approaches zero on the super horizon scale.

1. Introduction

According to the inflationary scenario of cosmology, all structure in the Universe can be traced back to primordial quantum fluctuations during an accelerated expanding phase of the very early universe. Short wavelength quantum fluctuations generated during inflation are considered to lose quantum nature when their wavelength exceed the Hubble horizon length. Then, the statistical property of generated fluctuations can be represented by classical distribution functions. This is the assumption of the quantum to classical transition of quantum fluctuations generated by the inflation. As the structure in the present Universe is classical objects, we must explain or understand how this transition occurred and the quantum fluctuations changed to classical fluctuations.

We have investigated this problem from the viewpoint of entanglement [1, 2]. In our previous study, we have defined spatially separated two regions in the inflationary universe and investigated the bipartite entanglement between these regions. We have found that the entanglement between these two regions becomes zero after their physical separation exceeds the Hubble horizon. This behaviour of the bipartite entanglement confirms our expectation that the long wavelength quantum fluctuations during inflation behave as classical fluctuations and can become seed fluctuations for the structure formation in the Universe. Our previous analysis concerning the entanglement of quantum fluctuations in the inflationary universe relies on the separability criterion for continuous bipartite systems [3, 4], of which dynamical variables are continuous. The applicability of this criterion is limited to systems with Gaussian states: The wave function or the density matrix of the system is represented in a form of Gaussian functions. Thus, we cannot say anything about the entanglement for the system with non-Gaussian state such as excited states and thermal states. Furthermore, from a viewpoint of observation or measurement, information on quantum fluctuations can be extracted via interaction between quantum fields and measurement devices. Hence, it is more natural to consider a setup that the entanglement of quantum field is probed using detectors.

In this paper, we have considered particle detectors [5, 6, 7] with two internal energy levels, which are assumed to interact with a scalar field. By preparing spatially separated two equivalent detectors interacting with the scalar field, we can extract the information on entanglement of the scalar field by evaluating the entanglement between these two detectors. As a pair of such detectors is two qubits system, we have the necessary and sufficient condition for entanglement of this system [8, 9].

2. Two detectors system

We have considered a system with two equivalent detectors interacting with the massless scalar field in an expanding universe [10, 11]. The detectors have two energy level state $|\uparrow\rangle, |\downarrow\rangle$ and their energy difference is given by Ω. The interaction Hamiltonian is assumed to be:

$$V = g(t) \sum_{j=A,B} \left(\sigma^+_{(j)} + \sigma^-_{(j)}\right)\phi(\boldsymbol{x}_j(t)), \quad \sigma^+ = |1\rangle\langle 0|, \quad \sigma^- = |0\rangle\langle 1|. \tag{1}$$

Two detectors are placed at $\boldsymbol{x}_{A,B}(t)$ and $(t, \boldsymbol{x}_{A,B}(t))$, to represent their world lines. We have assumed that the detectors are comoving with respect to cosmic expansion. Strength of the coupling $g(t)$ is controlled in accordance with the Gaussian window function $g(t) = g_0 \exp\Big(-(t-t_0)^2/2\sigma^2\Big)$.

The initial state of the total system is assumed to be $|\Psi_0\rangle = |\downarrow\downarrow\rangle|0\rangle$. Then the final state $(t \to +\infty)$ of the detector after interaction becomes:

$$\rho^{AB} = \begin{pmatrix} X_4 & 0 & 0 & X \\ 0 & E & E_{AB} & 0 \\ 0 & E_{AB} & E & 0 \\ X^* & 0 & 0 & 1-2E-X_4 \end{pmatrix}. \tag{2}$$

The matrix elements are expressed by integral of the Wightman function of the scalar field.

3. Entanglement and correlations of detectors

As a measure of the entanglement between two detectors, we have considered the negativity [12], defined via a partial transpose operation to the density matrix given in Eq. (2) with respect to detector B's degrees of freedom:

$$\mathcal{N} = |X| - E. \tag{3}$$

The negativity gives the necessary and the sufficient condition of the entanglement for two qubits systems [8, 9], and two detectors are entangled when $\mathcal{N} > 0$ and separable when $\mathcal{N} < 0$.

To define classical part of correlation between two detectors (two qubits), we have performed a local projective measurement of detectors' state. The measurement operators for each detector are:

$$M^A_\pm = \frac{I \pm \boldsymbol{n}_A \cdot \boldsymbol{\sigma}}{2}, \quad \textit{and} \quad M^B_\pm = \frac{I \pm \boldsymbol{n}_B \cdot \boldsymbol{\sigma}}{2}, \qquad |\boldsymbol{n}_A| = |\boldsymbol{n}_B| = 1, \tag{4}$$

where $\pm$ denotes output of the measurement. $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrix and $\boldsymbol{n}_A, \boldsymbol{n}_B$ represent internal direction of measurement. The joint probability p_{jk} attaining measurement result j for detector A, and k for detector B $(j, k = \pm 1)$ is obtained as:

$$p_{jk} = \mathrm{Tr}\Big(M^A_j \otimes M^B_k \, \rho^{AB}\Big). \tag{5}$$

Using the joint probability obtained by the measurement, the classical mutual information I_C is defined by [13]:

$$I_C(p) = H(p_j) + H(p_k) - H(p_{jk}), \quad H(p_j) = -\sum_j p_j \log_2 p_j, \quad H(p_{jk}) = -\sum_{jk} p_{jk} \log_2 p_{jk}$$

where $H(p)$ is a Shannon entropy for a probability distribution p. The measure of classical correlation is defined via the maximization done over all possible projective measurements as:

$$\mathcal{C}(p) = \sup_{\{\boldsymbol{n}_A, \boldsymbol{n}_B\}} I_C(p) = \max\left[\frac{1}{\ln 2}\left(E^2 - X_4 + X_4 \ln\left(\frac{X_4}{E^2}\right)\right), \frac{2}{\ln 2}\Big(|E_{AB}| + |X|\Big)^2\right]. \tag{6}$$

On the other hand, the quantum mutual information I_Q of the bipartite state ρ^{AB} is defined independent of measurement procedure given as:

$$I_Q(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}), \qquad S(\rho) = -\mathrm{tr}(\rho \log_2 \rho), \tag{7}$$

where $S(\rho)$ is the von Newmann entropy for the state ρ. This quantity represents the total correlations of the bipartite system including both quantum and classical part of correlations. The quantum mutual information for the state in Eq. (2) is given by:

$$\begin{aligned} I_Q(\rho) = \frac{1}{\ln 2}&\Big[-2E\ln E + (E+E_{AB})\ln(E+E_{AB}) + (E-E_{AB})\ln(E-E_{AB})\Big] \\ &+ \frac{1}{\ln 2}\Big[E^2 - X_4 + |X|^2 - 2X_4 \ln E + (X_4 - |X|^2)\ln(X_4 - |X|^2)\Big] + O(g^6). \end{aligned} \tag{8}$$

The quantum discord [14, 15, 16] is introduced as the difference between the quantum mutual information and the classical mutual information given as:

$$\mathcal{Q}(\rho) = I_Q(\rho) - \mathcal{C}(p). \tag{9}$$

For arbitrary local projective measurements, it can be shown that $\mathcal{Q} \geq 0$. Thus, the quantum mutual information can be decomposed into a positive classical mutual information and a positive quantum discord, which represents the quantum part of the total correlation. The state with zero quantum discord can be said as a classical state. For pure state, we have $\mathcal{Q} = \mathcal{C}$ and the state with zero quantum discord has no classical correlations.

4. Behaviour of entanglement and correlations of scalar field

We have considered asymptotic estimation of the matrix elements E_{AB}, X, X_4 for $\Omega\sigma \gg 1, H\sigma \ll 1$, where H is the Hubble parameter and σ is duration time of interaction between detectors and the scalar field. For the massless minimal scalar field in de Sitter spacetime, for sufficiently large e-foldings $Ht_0 \gg 1$, the matrix elements of the density matrix are:

$$\begin{aligned} X &\approx -(2\pi g_0^2)\frac{e^{-(\Omega\sigma)^2}}{4\pi^2}(H\sigma)^2\Big[-\ln(Hr_p) + Ht_0\Big], \\ E_{AB} &\approx (2\pi g_0^2)\frac{e^{-(\Omega\sigma)^2}}{4\pi^2}(H\sigma)^2\left[\frac{1}{4\sin^2(H\Omega\sigma^2)} - \ln\sqrt{(Hr_p)^2 + 4\sin^2(H\Omega\sigma^2)} + Ht_0\right], \end{aligned}$$

and the negativity for the minimal scalar field is:

$$\mathcal{N} = \frac{g_0^2}{2\pi}e^{-\Omega^2\sigma^2}(H\sigma)^2\left[\ln\left(\frac{2\sin(H\Omega\sigma^2)}{Hr_p}\right) - \frac{1}{4\sin^2(H\Omega\sigma^2)}\right]. \tag{10}$$

The separability condition $\mathcal{N} < 0$ yields:

$$Hr_p \gtrsim 2\sin^2(H\Omega\sigma^2)\exp\left(-\frac{1}{4\sin^2 H\Omega\sigma^2}\right) \sim 1.0, \tag{11}$$

where the numerical value is obtained for $H\Omega\sigma^2 = 1$. Thus, we can find sufficiently large r_p at which detectors are separable for any value of detector's parameters Ω, σ. As r_p grows in time t_0, entangled two detectors $r_p < H^{-1}$ evolves to a separable state after their separation exceeds the Hubble horizon scale. This behaviour is consistent with our previous analysis of entanglement using the lattice model and the coarse-grained model of the scalar field [1, 2]: Bipartite entanglement of the scalar field between spatially separated regions in de Sitter spacetime disappears beyond the Hubble horizon scale.

The ratio of classical and total correlation of the minimal scalar field is:

$$\frac{\mathcal{C}}{I_Q} \approx 4E \sim g_0^2\, e^{-(\Omega\sigma)^2}(H\sigma)^2(Ht_0) = g_0^2\, e^{-(\Omega\sigma)^2}(H\sigma)^2 \ln\left(\frac{r_p}{r_0}\right).$$

For sufficiently large distance (large e-folding Ht_0) given by:

$$\ln\left(\frac{r_p}{r_0}\right) \sim \frac{e^{(\Omega\sigma)^2}}{g_0^2(H\sigma)^2} \gtrsim \frac{1}{g_0^2}\left(\frac{\Omega}{H}\right)^2, \tag{12}$$

$\mathcal{C} \approx I_Q$, and this means the quantum state of detectors approaches zero quantum discord state and can be regarded as "classical" in the sense that measurement procedure does not alter the state. Therefore, this behaviour of correlations supports that the long wavelength quantum fluctuations of the massless minimal scalar field in de Sitter spacetime can be treated as classical fluctuations.

5. Summary

We have investigated quantum and classical correlations of the quantum field in de Sitter spacetime using the detector model. Entanglement of the scalar field is swapped to that of two detectors interacting with the scalar field, and we can measure the entanglement of the quantum field by this setup of experiment. In de Sitter spacetime, the entanglement between detectors disappears on super horizon scale, and this behaviour is consistent with our previous analysis using the lattice model and the coarse-grained model of the scalar field [1, 2]. However, the behaviour of correlations shows different behaviour depending on type of scalar fields [7]. For the massless minimal scalar field, the ratio of classical correlation to the total correlation approaches unity for sufficiently large e-foldings. On the other hand, for the massless conformal scalar field, that ratio approaches a constant value smaller than unity, and the condition for classicality is not achieved. These results support the long wavelength quantum fluctuation of the minimal scalar field, which can be treated as classical fluctuations and can become seed fluctuations for the structure in the our Universe.

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