

# EFFICIENT COMPUTATION OF TWO-DIMENSIONAL COHERENT SYNCHROTRON RADIATION WITH NEURAL NETWORKS\*

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## Abstract

The design and tuning of accelerators are both complicated processes involving many physical effects. Of these, the modeling of coherent synchrotron radiation has long been one of the most complicated and time consuming. This is especially true when modeling two and three-dimensional CSR, which is often neglected in state-of-the-art accelerator modeling due to its time consuming nature. We present a neural network designed to model 2D CSR, demonstrating both faithful accuracy to the physics and a dramatic speedup over even the fastest existing codes. We study its performance in the context of the last bunch compressor of the FACET-II facility, where the intense short pulse demands at least a 2D treatment, and find that we can reproduce the results of more standard tracking codes in a fraction of the time.

## INTRODUCTION

Coherent synchrotron radiation (CSR) is a deleterious effect found in linear accelerators that occurs during bunch compression [1, 2]. Bunch compression is achieved by chirping the time-energy distribution of the bunch then sending it through a series of magnets that modifies a particle's path length as a function of its energy. In those magnets, the bunch emits synchrotron radiation due to the bending, and that synchrotron radiation can catch up with the bunch downstream and modulate its energy in the dispersive magnets, leading to emittance growth. This is often one of the key brightness reducing effects in high brightness electron linacs.

The modelling of CSR is inherently complicated, since it is a collective effect that propagates with the bunch. To minimize computation time, a one-dimensional line charge approximation is used in most state-of-the-art codes [1, 3]. Such a model does a good job of predicting the behavior of most scenarios, however it is strictly valid only in the limit of a thin beam. The exact condition for the 1D model to apply is called the Derbenev criterion: for a bunch of transverse size  $\sigma_x$ , length  $\sigma_z$ , and a magnet with bending radius  $\rho$ , it reads  $\sigma_x \ll R^{1/3} \sigma_z^{2/3}$  [4]. Though historically this limit

has not been important, recent proposals are approaching regimes where 2D and 3D effects cannot be ignored [5, 6].

Although now many codes have implemented a 1D CSR model and some even 2D or 3D models, computational times are still prohibitive for large-scale design studies or accelerator control room use. 1D codes can easily increase total start to end simulation time by an order of magnitude, and the existing 2D/3D codes are so slow that they are generally run only in standalone simulations to isolate specific physics. Edelen *et al* proposed to speed up 1D CSR simulations using neural networks, which can learn the physics of 1D CSR and encode it in a dramatically less expensive calculation [7]. Here, we propose to do the same for the 2D CSR effect with a network architecture modified to tailor to the specific requirements of the 2D problem. We will show an example application to a dipole magnet inspired by the FACET-II facility [6].

## PROBLEM FORMULATION

2D CSR is calculated using the language of wakefields. In particular, the bunch wake functions which are proportional to the longitudinal and transverse kicks to a given particle are given by a convolution of the beam distribution function with the wake potentials [8–10]:

$$W_x(x, z) = \int dx' \int_{f(x-x')} dz' \psi_x(x-x', \frac{z-z'}{2R}) \rho_b(x', z')$$

$$W_z(x, z) = \int dx' \int_{f(x-x')} dz' \psi_z(x-x', \frac{z-z'}{2R}) \rho_b(x', z')$$

where  $x$  and  $z$  are the transverse (in the bend plane) and longitudinal coordinates,  $\rho_b$  is the beam distribution function,  $R$  is the bending radius in the magnet, and  $\psi_{x,z}$  are the single particle wake potentials defined in terms of the beam energy, bending radius, and distance into the magnet. Although convolutions can be implemented relatively efficiently, these are complicated by the fact that depending on the location of the emitter and observer electrons, the boundaries of the longitudinal integration change. Thus, although relatively fast codes have been written to account for 2D effects, they still present a dramatic slowdown to start to end simulations.

To move towards the language of neural networks, we can describe the 2D CSR problem as one of image generation. In particular, given an image (the 2D beam distribution on a grid) and the parameters of the bend magnet (distance into the magnet  $s$ , bending radius  $R$ , beam energy  $\gamma$ ), we would like to predict two images: the transverse and longitudinal wakes on the same grid as the beam distribution. This suggests that an appropriate network architecture would be one that resembles a convolutional autoencoder. In convolutional autoencoders, an input image is first represented in

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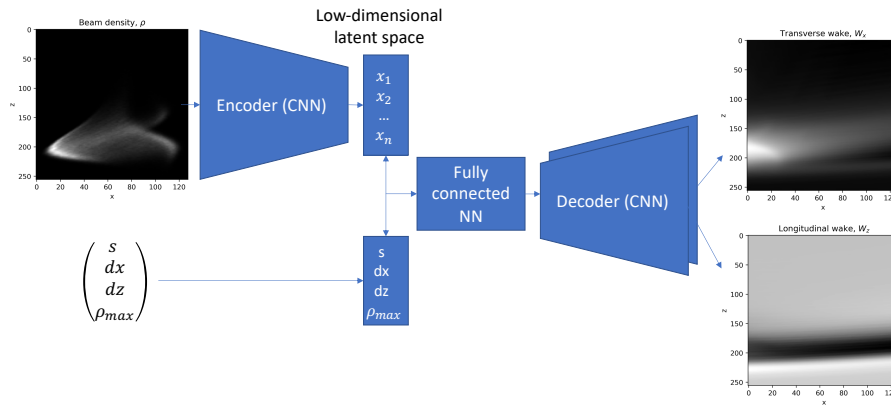


Figure 1: Schematic depiction of the concept behind the 2D CSR neural network. An input image representing the beam density is first reduced through a set of convolutional layers to a low-dimensional latent space representation which is concatenated with the scalar inputs. After further processing in a fully connected network, a set of deconvolutional layers expands the scalars back up to two images representing the wakes.

a lower dimensional format using convolutional layers in a stage called the encoder. One can think of this encoder as performing more general principal component analysis (PCA) tailored to the specific problem at hand. Given this low-dimensional, latent space representation of the image, one can construct another set of layers called the decoder that can reconstruct the input image, potentially modified in a desired way. This method has been applied to many problems, such as image denoising.

In our case this might look something like Fig. 1. We will focus our attention on a particular bend magnet and beam energy such that the bend radius and beam energy are implicit parameters of the model. In that case our inputs are a 2D beam density image on a grid, the distance into the bend magnet  $s$ , the grid step sizes  $dx$  and  $dz$ , and the peak charge density  $\rho_{max}$ . These inputs are shown on the left of Fig. 1. We can distinguish then between our image input and our scalar inputs. In order to combine these inputs in a way the network will understand, we first reduce the dimensionality of the beam image using an encoder section composed of convolutional layers. The low-dimensional latent space vector is then concatenated with the scalar inputs, sent into a fully connected neural network for intermediate processing, then finally into a decoder section. The decoder consists of a set of deconvolutional layers which outputs two new images: one representing the transverse wake and one representing the longitudinal wake.

## APPLICATION TO FACET-II LIKE DIPOLE

We now make these concepts concrete by applying them to the case of a dipole magnet inspired by FACET-II. We consider a beam energy of 30 GeV and magnet length of 20 m. We have obtained an input beam distribution from start-to-end simulations of FACET-II dumped just before the final compressor and tracked through the first three dipole magnets up to the entrance of the fourth, where the beam is shortest and therefore CSR effects are most relevant. The

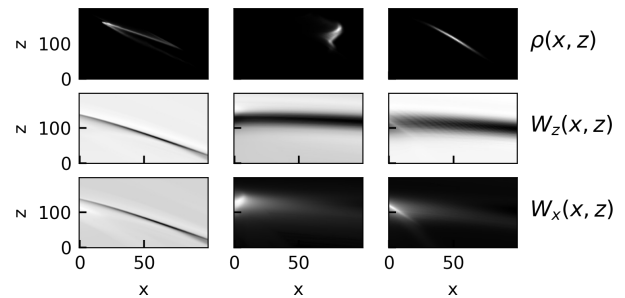


Figure 2: Example images of the beam density (top), longitudinal wake (middle) and transverse wake (bottom) from the dataset.

2D CSR calculation in the fourth dipole is performed using a Julia-based code that runs on a GPU [11]. As a first pass to generating training data for the neural network, we scale this input beam in  $x$  and  $z$  by factors between 0.1 and 2. By computing the CSR wakes at 20 steps in the magnet, we obtain from each simulation 20 training datasets. In total we generated 2000 such datasets, relegating 1990 for training and 10 for validation. A sample of the datasets is shown in Fig. 2, where we plot the beam charge density (top row), longitudinal wake (middle), and transverse wake (bottom) for three of the examples. We see that the dataset consists of a diverse set of input images despite the simplistic approach to generating training data.

Our final network structure is summarized in Fig. 3. It starts with the encoder section, which consists of 7 iterations of image dimension reduction and processing with  $3 \times 3$  and  $7 \times 7$  convolutional layers in order to make the model sensitive to both fine and coarse features of the input image. At the end of the encoder the output is reduced to 16 numbers using a 16 node dense layer, which are then concatenated with the 4 input scalars. This set of 20 numbers representing the inputs is then further processed in 3 dense layers before being reshaped to a small 2D image for input into the decoder. The

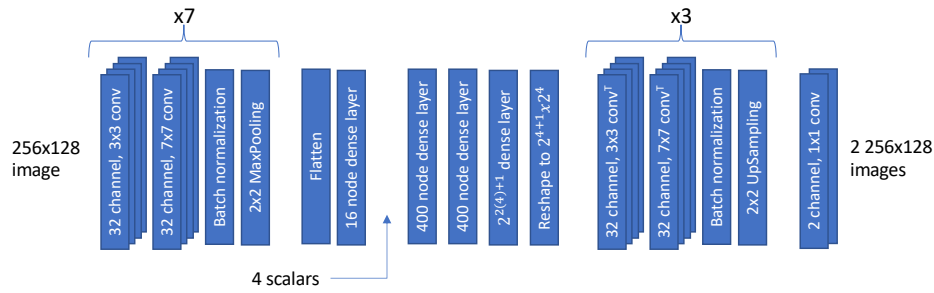


Figure 3: The final network architecture implemented. The specific layer choices are described in the text.

decoder is essentially an inverse of the encoder in structure, and at its output we use a two channel convolutional layer to generate the two output images. We trained the model using the Adam optimizer with a mean absolute error loss function. We employed learning rate reduction on plateaus to reach an optimal point in the training.

To test the model, we rewrote the previously mentioned Julia based 2D CSR code to have it call our model in the CSR tracking steps. This reduces the time taken to compute the 2D CSR wakes from 60+ seconds per step on a GPU using Julia to milliseconds on one CPU using the neural network. Figure 4 summarizes the results of tracking the same scaled inputs through the dipoles. The first four subplots on the left show the evolution of the geometric emittance of the beam for different starting scaling factors indicated by the  $s_x, s_z$ . The top right subplot shows the relative final emittance error as a function of the scale factor in  $x$  and the bottom right shows the average cumulative emittance error through the dipole. In general we observe errors around the 10% level at the dipole exit, which is quite good considering the relatively small quantity of training data employed thus far.

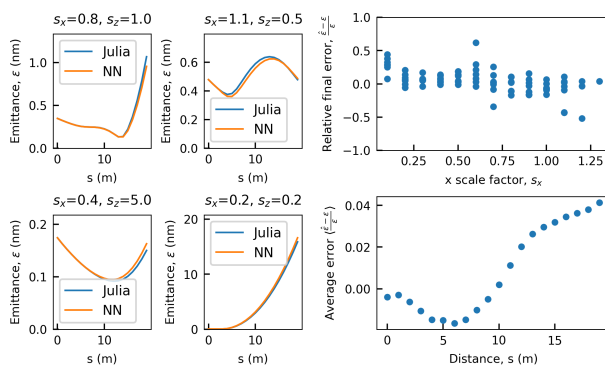


Figure 4: Summary of results tracking through the dipole using the CSR neural network compared to the Julia code. The left four subplots show example emittance evolutions through the dipole, the top right shows the relative final emittance error as a function of the  $x$  scaling factor, and the bottom right shows the average relative emittance error through the bend.

## CONCLUSIONS

We have presented preliminary results towards the development of a neural network for rapidly evaluating 2D CSR effects. In our initial tests we have found good agreement between simulations based on a full 2D CSR tracking code and our neural network. The neural network approach has a negligible impact on the simulation time, and represents a promising step towards being able to regularly take 2D CSR effects into account. The work in this paper isolates a single bend magnet operating at a single beam energy. Although this has its utility, for example in the accelerator control room, if this method is to become useful in design studies we must train it on more general datasets with more general inputs. More immediately, further work will focus on broadening the training dataset to include more exotic compression configurations for this magnet as well as additional hyperparameter tuning. Of course, in the long term another goal is to move to 3D CSR. Very little needs to change in terms of our network architecture to tackle that problem: simply adding a third output convolutional layer would be sufficient. The main challenge would be in efficiently and accurately generating training data.

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