



Studying Coulomb correction at EIC and EicC

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ABSTRACT

We study the gauge link contribution to the dipole type transverse momentum dependent distributions of coherent photons, which is conventionally referred to as the Coulomb correction. We further propose to search for the evidence of the Coulomb correction in the Bethe-Heitler process in eA collisions at EIC and EicC.

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1. Introduction

The study of QED processes in a strong Coulomb field has a long history pioneered by the Bethe and Maximon's seminal work [1], in which the Furry-Sommerfeld-Manue wave functions were used to calculate Coulomb corrections (CC) to the pair production and bremsstrahlung cross section. For a comprehensive review on this topic, we refer readers to the reference [2]. It later received renewed interest in the heavy ion physics community around the time when physics operation began at Relativistic Heavy Ion Collider (RHIC). A lot of efforts have been made to compute pure electromagnetic lepton pair production in Ultra-Peripheral heavy ion Collisions (UPC) to all orders in $Z\alpha$ where Z is the nuclear charge number. The summation of multiple photon re-scattering can be achieved by either making the systematical Eikonal approximation formulated in the impact parameter space [3–6] or solving the Dirac equation in the presence of a strong Coulomb field [7–10]. The agreement between these more modern methods and the original Bethe-Maximon's results was confirmed in Ref. [11].

The total cross section of lepton pair production in UPCs is predicated to be reduced by the Coulomb correction. However, there is no clear evidence of the Coulomb correction observed in heavy ion collisions so far [12,13]. The fact that experimental data is well described by the lowest order QED calculation [14–22] leaves no much room for any higher order QED effect. On the other hand, as the experimental observation of the Coulomb correction crucially depends on the overall normalization of the total cross section that suffers from various uncertainties (reliable Coulomb dissociation estimations, luminosity of heavy ion beams, and faithful reproduction of experimental momenta cuts, etc.), no definitive conclusion can be drawn at this stage.

In this work, we study the Coulomb correction to the Bethe-Heitler (BH) process in eA collisions. The deviation from the single photon exchange can be experimentally checked by comparing with the cross section of the BH process in ep collisions. The precise determination of the absolute normalization is thus not required for searching the evidence of the Coulomb correction in this process. The distribution of the total transverse momentum of the scattered electron and the emitted photon is found to be very sensitive to the Coulomb correction. It should be feasible to test our predictions at the future Electron Ion Collider (EIC) in US and the Electron Ion Collider in China (EicC). We notice that this subject has been addressed in some earlier publications [23–25]. The present work differs from the previous studies in two aspects: 1) The problem is re-formulated in the framework of transverse momentum dependent factorization [26], based on which the multiple photon re-scattering effect is naturally incorporated into the gauge link. In addition, the Sudakov effect arising from soft photon radiation can be easily taken into account in our calculation. 2) The Coulomb correction to the linear polarization of coherent photons [29,30] is included. We investigate how the polarization dependent observable is affected by the Coulomb correction as well.

The organization of this paper is as follows. We first give the matrix element definition for the dipole type photon TMDs in which the initial and final state multiple photon scattering in the BH process is encoded in a close loop gauge link. We compute the expectation value of the photon TMD matrix element in a boosted Coulomb potential and obtained a close form for the case of point-like charged

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particle. For an extended charge source, the photon TMDs have to be calculated numerically. The resulting photon TMDs clearly deviates from the widely used Weizsäcker-Williams (WW) distribution when Z is large. In Sec. 3, we compute the differential cross section of the BH process at EIC and EicC energies using the derived photon TMDs as the input. The Coulomb correction is signaled by the ratio of the cross sections in eA to that in ep collisions. Furthermore, we show that the $\cos 2\phi$ azimuthal modulation induced by the linearly polarized photons is slightly enhanced due to the Coulomb correction. The paper is summarized in Sec. 4.

2. The Coulomb correction to the photon TMDs

In the Bethe-Heitler process, the incoming electron multiple rescattering off the boosted Coulomb potential can occur either before emitting a photon or after a photon being radiated. At low virtuality, the exchanged photons coherently couple with charged heavy ion as a whole. The multiple coherent Coulomb scattering is much more pronounced in eA collisions than that in ep collisions, because the flux of coherent photons is enhanced by the factor Z^2 . If the calculation is carried out in TMD factorization, the cross section can be expressed as the convolution of the hard part and photon TMD distributions. The imaginary phase accumulated from the final and initial state interactions is summarized into a close loop gauge link in the photon TMD matrix element. In analogy to the gluon TMD distributions [31], the formal operator definition of photon TMDs is given by,

$$\int \frac{dy^- d^2 y_\perp}{P^+ (2\pi)^3} e^{ik_\perp \cdot y} \langle P | F_{+\perp}^\mu(0_\perp) U^\dagger(0_\perp) U(y_\perp) F_{+\perp}^\nu(y) | P \rangle \Big|_{y^+=0} = \frac{\delta_{\perp}^{\mu\nu}}{2} x f_1^\nu(x, k_\perp^2) + \left(\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \frac{\delta_{\perp}^{\mu\nu}}{2} \right) x h_1^{\perp\gamma}(x, k_\perp^2), \quad (1)$$

where the transverse tensor is commonly defined: $\delta_{\perp}^{\mu\nu} = -g^{\mu\nu} + p^\mu n^\nu + p^\nu n^\mu$ and $k_\perp^2 = \delta_{\perp}^{\mu\nu} k_{\perp\mu} k_{\perp\nu}$. Two photon TMDs, f_1^ν and $h_1^{\perp\gamma}$, are the unpolarized and linearly polarized photon distribution, respectively. $U^\dagger(0_\perp) U(y_\perp)$ and the transverse gauge link which is not explicitly shown here form a close loop gauge link. $U(y_\perp)$ is defined as,

$$U(y_\perp) = \mathcal{P} e^{ie \int_{-\infty}^{+\infty} dz^- A^+(z^-, y_\perp)}. \quad (2)$$

One should notice that the gauge link here plays the no role in ensuring gauge invariance as photon doesn't carry charge. Note that in analogy to the small x gluon TMDs [27,28], one can define a Weizsäcker-Williams type photon TMD. However, unlike the QCD case, the gauge link is absent in the WW type photon TMD, simply because photon doesn't carry charge. This also can be intuitively understood as follows: The WW photon TMD can be probed via lepton pair production in the correlation limit where lepton pair stay very close to each other in the position space. The produced lepton pair system can be effectively viewed as the charge neutral object when using coherent photons with long wave length as probe. Consequently, the final state interaction is power suppressed as explicitly shown in Ref. [21], leading to the absence of the staple like gauge link in the WW type photon TMD.

As argued above, at low transverse momentum, photons coherently generated by the charge source inside relativistic nuclei dominate the distribution. Both the unpolarized and polarized distributions of coherent photons can be computed with the equivalent photon method. If one neglects the gauge link contribution, the photon distributions associated with a boosted Coulomb potential are given by [29,30,32,33],

$$x f_{1,0}^\nu(x, k_\perp^2) = x h_{1,0}^{\perp\gamma}(x, k_\perp^2) = \frac{Z^2 \alpha}{\pi^2} k_\perp^2 \left[\frac{F(k_\perp^2 + x^2 M_p^2)}{(k_\perp^2 + x^2 M_p^2)} \right]^2 \quad (3)$$

where F is the nuclear charge form factor, and M_p is proton mass. The subscript "0" denotes the WW photon distributions. In the small x limit, two photon distributions $x f_{1,0}^\nu$ and $x h_{1,0}^{\perp\gamma}$ become identical [29,30].

The main purpose of this work is to investigate how the photon distributions are affected by the gauge link. To this end, we first express the gauge potential as,

$$\mathcal{V}(y_\perp) \equiv e \int_{-\infty}^{+\infty} dz^- A^+(z^-, y_\perp) = \frac{\alpha Z}{\pi} \int d^2 q_\perp e^{-iy_\perp \cdot q_\perp} \frac{F(q_\perp^2)}{q_\perp^2 + \delta^2} \quad (4)$$

where a photon mass δ is introduced for regulating the infrared divergence. The strength of the field appears in the photon TMD matrix element takes the similar form,

$$\mathcal{F}^\mu(x, y_\perp) \equiv \int_{-\infty}^{+\infty} dy^- e^{ix P^+ y^-} F_{+\perp}^\mu(y^-, y_\perp) = \frac{Ze}{4\pi^2} \int d^2 q_\perp e^{-iy_\perp \cdot q_\perp} (iq_\perp^\mu) \frac{F(q_\perp^2 + x^2 M_p^2)}{q_\perp^2 + x^2 M_p^2} \quad (5)$$

where x is the longitudinal momentum fraction carried by photon. The full expression of the photon TMD distributions incorporating the Coulomb correction (the gauge link contribution) is then given by,

$$\int \frac{d^2 y_\perp d^2 y'_\perp}{4\pi^3} e^{-ik_\perp \cdot (y_\perp - y'_\perp)} \mathcal{F}^\nu(x, y_\perp) \mathcal{F}^{*\mu}(x, y'_\perp) e^{i[\mathcal{V}(y_\perp) - \mathcal{V}(y'_\perp)]} = \frac{\delta_{\perp}^{\mu\nu}}{2} x f_1^\nu(x, k_\perp^2) + \left(\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \frac{\delta_{\perp}^{\mu\nu}}{2} \right) x h_1^{\perp\gamma}(x, k_\perp^2) \quad (6)$$

For a point-like charged particle, the close form solution of the above integration exists. By setting $F(q_\perp^2) = 1$ and $F(q_\perp^2 + x^2 M_p^2) = 1$, one readily obtains,

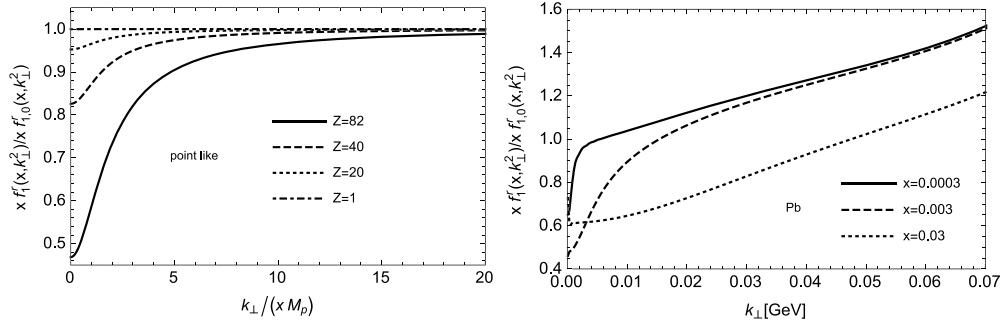


Fig. 1. The ratio $R = f_1^Y / f_{1,0}^Y$ is plotted as the function of $\frac{k_\perp}{xM_p}$ for a point like charged particle (left panel). The same ratio is plotted as the function of k_\perp for a Pb target at different x (right panel).

$$\mathcal{V}(y_\perp) = 2Z\alpha \lim_{\delta \rightarrow 0} K_0(|y_\perp| \delta) \approx Z\alpha \left(-2\gamma_E + \ln \frac{4}{y_\perp^2 \delta^2} \right) \quad (7)$$

$$\mathcal{F}^\mu(x, y_\perp) = \frac{Ze}{2\pi} \frac{y_\perp^\mu}{|y_\perp|} x M_p K_1(|y_\perp| x M_p) \quad (8)$$

Inserting these results into Eq. (6),

$$\begin{aligned} & Z^2 \alpha \int \frac{d^2 y_\perp d^2 y'_\perp}{4\pi^4} e^{-i(y_\perp - y'_\perp) \cdot k_\perp} \frac{y_\perp^\nu y'^\mu}{|y_\perp| |y'_\perp|} x^2 M_p^2 K_1(|y_\perp| x M_p) K_1(|y'_\perp| x M_p) \left(\frac{y'^2_\perp}{y_\perp^2} \right)^{iZ\alpha} \\ &= \frac{\delta_\perp^{\mu\nu}}{2} x f_1^Y(x, k_\perp^2) + \left(\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \frac{\delta_\perp^{\mu\nu}}{2} \right) x h_1^{\perp Y}(x, k_\perp^2) \end{aligned} \quad (9)$$

One notices that the infrared cutoff scale δ dependence now drops out. Carrying out the integration over y_\perp and y'_\perp , we arrive at,

$$\begin{aligned} & x f_1^Y(x, k_\perp^2) = x h_1^{\perp Y}(x, k_\perp^2) \\ &= \frac{Z^4 \alpha^3 (1 + Z^2 \alpha^2) k_\perp^2}{M_p^2 x^4} {}_2F_1 \left[1 - iZ\alpha, 2 - iZ\alpha, 2, \frac{-k_\perp^2}{M_p^2 x^2} \right] {}_2F_1 \left[1 + iZ\alpha, 2 + iZ\alpha, 2, \frac{-k_\perp^2}{M_p^2 x^2} \right] \left(\frac{2}{e^{Z\alpha\pi} - e^{-Z\alpha\pi}} \right)^2 \end{aligned} \quad (10)$$

where ${}_2F_1$ is the hypergeometric function. The unpolarized and the linearly polarized photon TMDs remain the same after taking into account gauge link contribution. The similar relations between the dipole type gluon TMDs have been established in earlier work [34–36] for both cases of unpolarized target and transversely polarized target. In the limit $Z \rightarrow 1$, the above result is reduced to,

$$x f_1^Y(x, k_\perp^2) = x h_1^{\perp Y}(x, k_\perp^2) \approx \frac{Z^2 \alpha}{\pi^2} \frac{k_\perp^2}{(k_\perp^2 + M_p^2 x^2)^2} \quad (11)$$

which recovers Eq. (3) as it should. Furthermore, when $k_\perp^2 \gg x^2 M_p^2$, the photon TMDs are simplified as,

$$x f_1^Y(x, k_\perp^2) = x h_1^{\perp Y}(x, k_\perp^2) \approx \frac{Z^2 \alpha}{\pi^2} \frac{1}{k_\perp^2} \quad (12)$$

This indicates that the gauge link contribution, i.e. the Coulomb correction, to the photon TMDs is vanishing with increasing transverse momentum for the case of point like particle. The photon distributions are altered by the multiple re-scattering effect only in the low transverse momentum region. However, this is no longer true for an extended charge source as shown below.

According to Eq. (10), $k_\perp^2 x f_1^Y(x, k_\perp^2)$ is a function of the single variable $\frac{|k_\perp|}{xM_p}$ rather than of two variables $|k_\perp|$ and x . In Fig. 1, we plot the ratio $R = f_1^Y / f_{1,0}^Y$ as the function of $\frac{|k_\perp|}{xM_p}$ for a point-like particle with the various choices of Z . One sees that the photon TMD is significantly reduced by the Coulomb correction at the low value of $\frac{|k_\perp|}{xM_p}$. In the case of an extended particle, the photon distribution is no longer the function of the single variable $\frac{|k_\perp|}{xM_p}$. The ratio as the function of k_\perp at different x for a Pb target is displayed in Fig. 1(right). In our numerical estimation, the nuclear charge form factor is taken from the STARlight MC generator [37],

$$F(|\vec{k}|) = \frac{4\pi\rho^0}{|\vec{k}|^3 A} \left[\sin(|\vec{k}| R_A) - |\vec{k}| R_A \cos(|\vec{k}| R_A) \right] \frac{1}{a^2 \vec{k}^2 + 1} \quad (13)$$

where $R_A = 1.1 A^{1/3}$ fm, and $a = 0.7$ fm. This parametrization is very close to the Woods-Saxon distribution. Our numerical results demonstrate that the photon distribution of the charged heavy ion is also suppressed at low k_\perp due to multiple Coulomb re-scattering. However, in a sharp contrast with the point like particle case, one notices that the ratio exceeds 1 at relatively large k_\perp .

It is also interesting to investigate how the integrated photon distribution is modified by the Coulomb phase. The integration over k_\perp has to be carried out with extreme caution [11]. For a point like particle, the difference between the integrated dipole type photon distribution (with the Coulomb correction) and the integrated WW photon distribution (without the Coulomb correction) is given by,

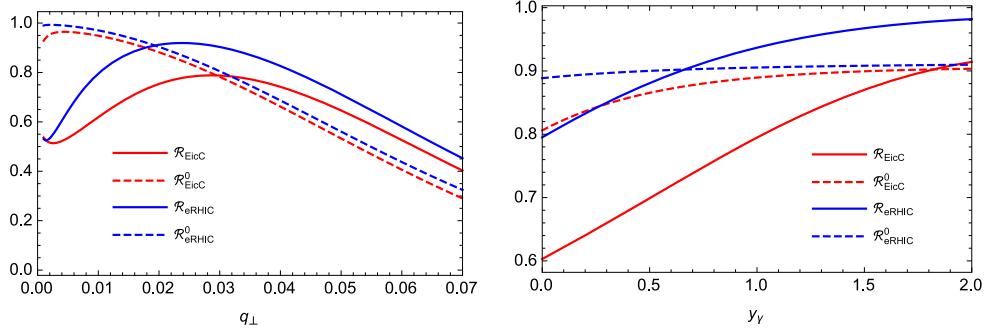


Fig. 2. The ratio \mathcal{R} and \mathcal{R}^0 as the function of q_\perp (left panel) and y_γ (right panel) for a Pb target at EicC and EIC. P_\perp is integrated over the regions [300 MeV, 400 MeV] for EicC and [1.5 GeV, 2 GeV] for EIC. In the left plot, the emitted photon rapidity y_γ is integrated over [0.5, 1]. In the right plot, the total transverse momentum q_\perp is fixed to be 20 MeV.

$$\int d^2k_\perp \left[xf_1^\gamma(x, k_\perp^2) - xf_{1,0}^\gamma(x, k_\perp^2) \right] = -\frac{2Z^2\alpha}{\pi} f(Z\alpha) \quad (14)$$

where $f(Z\alpha) \equiv R\psi(1 + iZ\alpha) + \gamma_E$ with $\psi(x) = d \ln \Gamma(x)/dx$ is just the well known universal function derived in the Bethe-Maximon theory [1]. We also numerically test this relation and confirm its validation.

This is a quite puzzling result in the sense that photon PDF seems to be process dependent. For example, the photon PDF enters into the cross section of dilepton production in ultra-peripheral heavy ion collisions receives no coulomb correction. This is because the produced lepton pair is a charge neutral object and the final state interactions with soft photons are absent. As a consequence, the gauge link in the WW type photon TMD is absent. Correspondingly, the photon PDF that is obtained by integrating over k_\perp for the WW photon TMD is free from the Coulomb correction and thus different from the one under consideration. We will thoroughly explore this issue in a future publication.

3. Observables

The photon TMDs with a close loop gauge link can be probed in the Bethe-Heitler process,

$$e(\bar{P}) + \gamma(xP + k_\perp) \rightarrow \gamma(p_1) + e(p_2), \quad (15)$$

where $xP + k_\perp$ is understood as the total momentum transfer via multiple photon exchange. We focus on a specific kinematical region, the so-called correlation limit where the total transverse momentum $k_\perp = p_{1\perp} + p_{2\perp}$ of the final state produced particles ($\gamma + e$) is much smaller than $P_\perp = \frac{p_{1\perp} - p_{2\perp}}{2} \approx p_{1\perp} \approx -p_{2\perp}$. The possible background to the BH process under consideration is the process where the real photon is emitted from large nuclei instead of electron. But this channel is either lack of Z^2 enhancement or suppressed due to nuclear charge form factor in the kinematical region that we are interested in. Therefore, this background can be safely neglected. In the correlation limit, the calculation of the cross section can be formulated either in the CGC framework or in the TMD formalism. The equivalence of the two approaches has been verified for the gluon initiated bremsstrahlung process [34,38]. Obviously, all the analysis can be extended to the corresponding QED process. Since there are two well separated scales in the correlation limit, large logarithm terms arise from unobserved soft photon radiations show up in higher order QED calculations. It is conventional to express the differential cross section in the impact parameter space to facilitate resumming these large logarithms,

$$\frac{d\sigma}{dP.S} = H_{\text{Born}} \int \frac{d^2r_\perp}{(2\pi)^2} e^{ir_\perp \cdot q_\perp} e^{-\frac{\alpha_e}{2\pi} \ln^2 \frac{P_\perp^2}{\mu_r^2}} \int d^2k_\perp e^{ir_\perp \cdot k_\perp} xf_1^\gamma(x, k_\perp^2) \quad (16)$$

with $\mu_r = 2e^{-\gamma_E}/|r_\perp|$. Here the Sudakov factor $e^{-\frac{\alpha_e}{2\pi} \ln^2 \frac{P_\perp^2}{\mu_r^2}}$ takes care of all order soft photon radiation effect up to the double leading logarithm accuracy. The phase space factor is defined as $dP.S = dy_\gamma d^2P_\perp d^2q_\perp$, where y_γ is the rapidity of the emitted photon. The hard coefficient is given by,

$$H_{\text{Born}} = 2\alpha_e^2 z^2 \frac{1 + (1 - z)^2}{P_\perp^4} \quad (17)$$

where z is the longitudinal momentum fraction of the incoming electron carried by the final state photon. In the above formula, the electron mass has been neglected. This is a very good approximation at EIC and EicC energies.

The Fig. 2 displays the two ratios $\mathcal{R}(q_\perp)$ and $\mathcal{R}^0(q_\perp)$ defined as follows,

$$\mathcal{R}(q_\perp) = \frac{d\sigma_{eA}}{Z^2 d\sigma_{ep}} \quad \mathcal{R}^0(q_\perp) = \frac{d\sigma_{eA}^0}{Z^2 d\sigma_{ep}^0} \quad (18)$$

where $d\sigma_{ep}$ and $d\sigma_{eA}$ are the exact cross sections of the BH process in ep and eA scatterings respectively. For a comparison, the cross sections $d\sigma_{ep}^0$ and $d\sigma_{eA}^0$ are computed using the conventional equivalent photon approximation (see Eq. (3)). The parametrization $F(\vec{k}) = 1/(1 + \frac{\vec{k}^2}{Q_0^2})^2$ with $Q_0^2 = 0.71 \text{ GeV}^2$ for the proton charge form factor is used for determining the photon distribution of proton. Note that

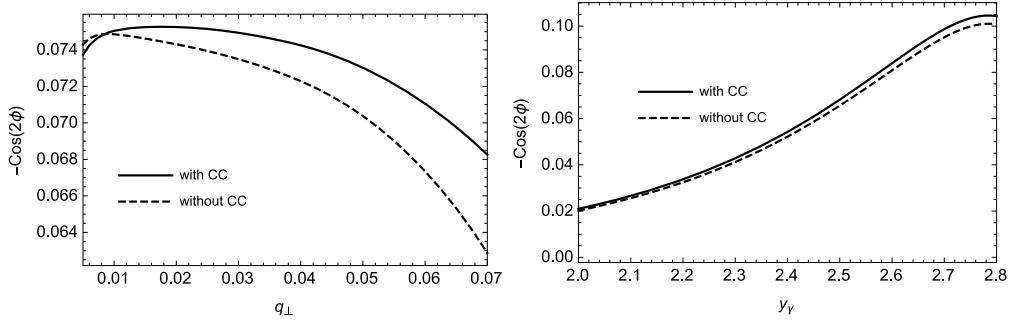


Fig. 3. The azimuthal asymmetry as the function of q_{\perp} (left panel) and y_{γ} (right panel) with and without taking into account the Coulomb corrections for a Pb target at EIC. Q^2 is fixed to be $Q^2 = 4 \text{ GeV}^2$. The asymmetry is averaged over the P_{\perp} region [1.5 GeV, 2 GeV]. In the left plot, the emitted photon rapidity y_{γ} is integrated over the region [2, 2.8]. In the right plot, the total transverse momentum q_{\perp} is fixed to be 50 MeV.

the Coulomb correction to photon distribution of proton is negligible. Therefore, we simply use the WW photon distribution to calculate the cross section in ep collisions. The proton magnetic moment contribution to the BH cross section at low q_{\perp} can be neglected. The rapidity y_{γ} in the Fig. 2(right) is defined in the lab frame where electron beam and heavy ion beam energies are 18 GeV and 100 GeV for EIC respectively, while they are 3.5 GeV and 8 GeV for EicC respectively. From Fig. 2, one sees that the ratios \mathcal{R} and \mathcal{R}_0 are rather different in the most kinematical regions at EIC and EicC energies. If the experimentally measured ratios deviate from these dashed lines (\mathcal{R}_0) presented in Fig. 2, it would be a clear evidence of the Coulomb correction.

We now turn to study the impact of the Coulomb correction on the polarization dependent observable in the BH process. A $\cos 2\phi$ azimuthal modulation in the BH cross section is induced by the linearly polarized photons if a virtual photon instead of real one is emitted in the final state. The similar phenomena in QCD have been studied in Ref. [34,38], from which one can readily recover the azimuthal dependent cross section in the QED case,

$$\frac{d\sigma}{dP.S.} = \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{ir_{\perp} \cdot q_{\perp}} e^{-\frac{\alpha_e}{2\pi} \ln^2 \frac{Q^2}{\mu_r^2}} \times \int d^2 k_{\perp} e^{ir_{\perp} \cdot k_{\perp}} \left\{ H'_{\text{Born}} x f_1^{\gamma}(x, k_{\perp}^2) + H_{\text{Born}}^{\cos(2\phi)} \left[2(\hat{k}_{\perp} \cdot \hat{P}_{\perp})^2 - 1 \right] x h_1^{\perp\gamma}(x, k_{\perp}^2) \right\} \quad (19)$$

where $\hat{k}_{\perp} = k_{\perp}/|k_{\perp}|$ and $\hat{P}_{\perp} = P_{\perp}/|P_{\perp}|$ are unit transverse vectors. The hard parts take the form,

$$H'_{\text{Born}} = 2\alpha_e^2 z^2 \left[\frac{1 + (1-z)^2}{(P_{\perp}^2 + (1-z)Q^2)^2} - \frac{2Q^2 P_{\perp}^2 z^2 (1-z)}{(P_{\perp}^2 + (1-z)Q^2)^4} \right]$$

$$H_{\text{Born}}^{\cos(2\phi)} = 2\alpha_e^2 z^2 \frac{-2Q^2 P_{\perp}^2 z^2 (1-z)}{(P_{\perp}^2 + (1-z)Q^2)^4} \quad (20)$$

To avoid having to deal with a three scale problem, we restrict to the kinematical region where Q^2 is of the order of P_{\perp}^2 . This happens to be the optimal region to observe $\cos 2\phi$ azimuthal asymmetry as suggested by our numerical estimation. Moreover, as long as Q^2 is sufficiently large, the Coulomb multiple rescattering effect can be neglected for the lepton pair production via the virtual photon decay as explained above.

We plot the azimuthal asymmetries computed for EIC energy in Fig. 3. Here the azimuthal asymmetries, i.e. the average value of $\cos 2\phi$ is defined as,

$$\langle \cos(2\phi) \rangle = \frac{\int \frac{d\sigma}{dP.S.} \cos 2\phi dP.S.}{\int \frac{d\sigma}{dP.S.} dP.S.} \quad (21)$$

As shown in Fig. 3(right), the asymmetry becomes larger with the increasing photon rapidity. The maximal value of the asymmetry reaches roughly 10%. According to Eq. (10), the linearly polarized photon TMD and the unpolarized photon TMD are modified by the multiple Coulomb rescattering effect in the same way, and thus remain identical. If the Sudakov effect were not considered, the azimuthal asymmetry would not be affected by the Coulomb correction. However, the azimuthal averaged cross section and $\cos 2\phi$ dependent part evolve with the scale P_{\perp} following a different pattern. The different initial conditions for the photon distributions would lead to the different $\cos 2\phi$ asymmetries at higher scale P_{\perp} . Fig. 3 displays the $\cos 2\phi$ asymmetries computed with the photon distributions given in Eq. (6) and the WW photon distributions. One sees that at EIC, the deviation caused by the Coulomb correction is visible though tiny. On the other hand, at EicC, the difference (not shown here) is completely negligible since the evolution effect is much weaker at low energy scale. Therefore, it appears to be not optimistic to observe the Coulomb correction effect via the polarization dependent observable at neither EIC nor EicC.

4. Summary

In this paper, we performed the detailed analysis of the dipole type photon TMDs associated with a boosted Coulomb potential. Our main focus is on the contribution of the close loop gauge link to photon transverse momentum distributions, which is conventionally

referred to as the Coulomb correction in the study of strong field QED. Due to the large Z enhancement, the Coulomb correction (or gauge link contribution) alters transverse momentum distributions of photons substantially for a charged heavy ion target, as compared to the Weizsäcker-Williams photon distribution. The photon TMDs under consideration can be accessed in the BH process. Our numerical results show that it is promising to observe the Coulomb correction at EIC and EicC. The investigation of the Coulomb correction in the BH process will offer us a clean way to test the TMD formulation of initial/final state multiple re-scattering effects, and would be beneficial for deepening our understanding of the gauge link contribution in QCD processes. Moreover, the accurate account of the Coulomb correction to the BH process is also important for the determination of luminosity at EIC and EicC.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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