String axions as the inflaton

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Abstract

We consider the inflation models where the string axions play the role of the inflaton. The simple two-field case is not sufficient to solve many cosmological problems, but with a large enough number of fields we can remove this difficulty. The mass distribution of the fields in this case is of crucial importance for observations and dynamics after inflation.

1 Introduction

Inflation is currently the most promising candidate to solve many cosmological problems. It is, however, still unclear yet how to implement a consistent and successful scenario in the context of string theory. Fortunately, there have been considerable advances to this end for recent years. An inflaton candidate of particular interest is the string axion, which is known to exist abundantly in string theory, and they have flat potentials even after all the moduli are stabilised.

Here, we briefly investigate the inflationary model where the string axion fields play the role of the inflaton. In Section 2, first we study the simple two-field case. In Section 3, we discuss the model where N fields drive inflation. We conclude in Section 4.

2 Two-field model

First, let us study a simple case where two decoupled axion fields derive inflation [1]. The reason why we are interested in the two-field model is twofold. First, there are plenty of axions, so we'd better build an inflation model of multiple fields of which a two-field model is the simplest. Also, when inflation is driven by a number of fields, fewer and fewer fields will be responsible for inflation because heavy fields will drop out soon.

The potential is written as

$$V(\phi,\psi) = \Lambda_\phi^4\left[1+\cos\left(\frac{2\pi\phi}{f_\phi}\right)\right] + \Lambda_\psi^4\left[1+\cos\left(\frac{2\pi\psi}{f_\psi}\right)\right], \tag{1}$$

where Λ_ϕ and Λ_ψ are the dynamically generated axion potential scales, and f_ϕ and f_ψ are the axion decay constants. We expect both $\Lambda_{\phi(\psi)}$ and $f_{\phi(\psi)}$ to be smaller than $m_{\rm Pl}$, especially the potential scale could be significantly so. There are many maxima, minima and saddle points, and they lead to a very rich structure of the potential depending on the values of the parameters. To compute the evolution of the scalar field on the potential given by Eq. (1), it is convenient to modify the equations of motion in terms of the number of e-folds N as

$$\phi_i'' + 3\phi_i' - \frac{\sum_j {\phi_j'}^2}{2m_{\rm Pl}^2}\phi_i' + \frac{V_{,i}}{2V}\left(6m_{\rm Pl}^2 - \sum_j {\phi_j'}^2\right) = 0, \tag{2}$$

where a prime denotes a derivative with respect to N, $V_{,i} = \partial V/\partial\phi_i$ and the indices i and j denote the canonical fields ϕ and ψ.

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Some results of the numerical evolution of the field according to Eq. (2) are shown in Fig. 1. Note that locked inflationary phase [2] hardly occurs. This could be seen from the fact that there is no interaction term in the potential, e.g., such as $\lambda\phi^2\psi^2$. Thus the oscillation of, say, ϕ is not transmitted to the effective mass of ψ to hold the field at a saddle point. Even if we include an interaction term, it is generally suppressed by $m_{\rm Pl}^4$ and does not lead to any significant change. Also we note that the dependence of the number of e-folds on the initial misalignment $\theta = \tan^{-1}\psi/\phi$ is very weak: in Table 2 several results depending on θ are shown.

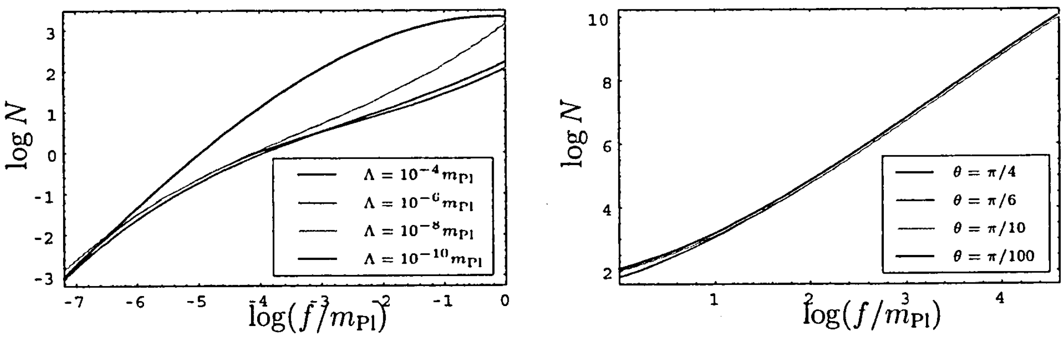

Figure 1: (Left) number of e-folds depending on Λ and $f < m_{\rm Pl}$, with $\theta = \pi/100$. We obtain larger N with smaller Λ and with bigger f, but generally N is not sufficient to solve cosmological problems. (Right) plot of N versus $f > m_{\rm Pl}$ and θ. The energy scale Λ is set to be $10^{-4}m_{\rm Pl}$. We can see that as f becomes bigger, we obtain larger N as expected.

θ	$\pi/100$	$\pi/40$	$\pi/20$	$\pi/10$	$\pi/8$	$\pi/5$
N	30.4465	30.2241	30.2100	30.1144	30.3783	30.0687

Table 1: Several results depending on different initial misalignment θ. Parameters are given by $\Lambda = 10^{-10}m_{\rm Pl}$ and $f = m_{\rm Pl}$. Note that the number of e-folds varies less than 1.

3 Many field model

In the previous section, we have seen that the number of e-folds obtained during inflation is not enough to solve many cosmological problems. The naive inflation model of the two string axions is thus not sufficient and we need some amendments to improve the situation.

One obvious modification is to add more contribution to H so that the field receives greater friction and can roll down the potential for many e-folds [3]. If each axion field is displaced not too far from the minimum, the potential is approximately [4]

$$V = \sum_{i=1}^{N}\frac{1}{2}m_i^2\phi_i^2\,, \tag{3}$$

where we assume that there are N axion fields.

To investigate inflation with N string axions, we must specify the masses and the initial amplitudes of those fields. The mass of string axion depends on the details of the compactification which involves many factors not yet (or formidable to be) computed. Nevertheless, irrespective of the microscopic detail, the mass distribution is known to follow the Marčenko-Pastur law [5]: the probability density function for m^2 whose average value is $\bar{m}$ is given by

$$p(m^2) = \frac{\sqrt{\left[\bar{m}^2\left(1+\sqrt{\beta}\right)^2 - m^2\right]\left[m^2 - \bar{m}^2\left(1-\sqrt{\beta}\right)^2\right]}}{2\pi\beta\bar{m}^2 m^2}, \tag{4}$$

where β is the model dependent parameter which is expected to be 1/2. For simplicity, however, we will adopt the mass distribution uniform on logarithmic scale [4, 6], i.e.

$$m_i^2 = m_{\rm Pl}^2 e^{-(i-1)/\sigma}, \tag{5}$$

where i is the indices of fields, and σ is the density of fields per logarithmic interval. For the initial amplitudes, we can choose the condition which gives the largest possible number of e-folds, because it will be preferred posteriori. Thus we take $\phi_1 = \cdots = \phi_N = m_{\rm Pl}$. In this case, generally the power spectrum of density perturbations will be rather red compared with single field case, because more and more fields drop out of inflationary regime and no more contribute to the energy density of the universe. The detail of the tilt of the spectrum depends on the mass distribution: if fields with heavy masses are more abundant, more fields will quit early and consequently H is decreasing faster. This is illustrated in Fig. 2.

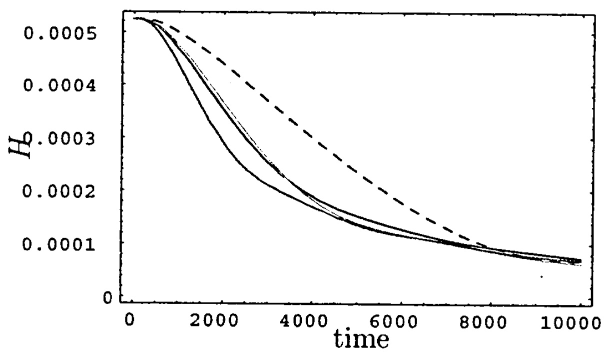

Figure 2: H's depending on different mass distributions. The masses are equally distributed (red), densely distributed at heavy (blue) and light (green) regions. For comparison, we plot the single field case (black dashed). $H(t=0)$'s are set to be the same.

One important thing is that, at the end of inflation, *not* all the fields are relaxed at the minima. For single field inflation with quadratic potential, inflation ends when $\phi \sim m_{\rm Pl}$. Likewise, inflation proceeds no more although there are still some fields whose expectation values are non-zero. After inflation, these fields may lead to interesting or dangerous cosmological consequences, e.g. they may produce dark matter or additional perturbations. At the end of inflation, the Hubble parameter $H_{\rm end}$ and the amplitude of the lightest field $\phi_N^{\rm end}$ are [7]

$$H_{\rm end} \simeq \frac{\sqrt{2}}{3} m_N \,,$$
$$\phi_N^{\rm end} \simeq \frac{4}{3\sqrt{n}} m_{\rm Pl} \,, \tag{6}$$

where n is the number of fields that have not yet been relaxed at the minima. This number depends on the underlying mass distribution, but roughly those fields lighter than $2m_N$ retain non-zero values. For example, about 1/10 of the fields are so if masses are distributed according to Eq. (5). Fig. 3 shows the evolution of several fields.

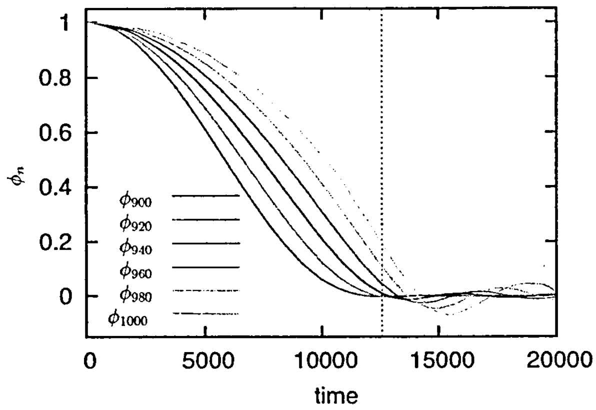

Figure 3: The evolution of 6 fields by a numerical calculation of total 1000 fields. The mass distribution is given by Eq. (5), with the lightest mass being $10^{-3}m_{\rm Pl}$. Vertical dotted line indicates the end of inflation.

4 Conclusions

We have investigated a simple inflation model which consists of two or more non-interacting axions. For two-field model, the number of e-folds N achieved while the field rolls off from a maximum is dependent on both the energy scale Λ and the decay constant f but is hardly affected by the initial misalignment θ: we obtain larger N with smaller Λ and with bigger f, but generally N is not sufficient to solve cosmological problems. To alleviate this situation, we can increase H by adding more axion fields. The power spectrum depends on the mass distribution. And the number of fields with non-zero values at the end of inflation may lead to many interesting consequences after inflation.

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