Squashed Kerr-Gödel Black Holes

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Abstract

Applying *squashing transformation* to Kerr-Gödel black hole solutions, we present a new type of a rotating Kaluza-Klein black hole solution to the five-dimensional Einstein-Maxwell theory with a Chern-Simonss term. The new solutions generated via the squashing transformation have no closed timelike curve everywhere outside the black hole horizons. The spacetime is asymptotically locally flat. One of the remarkable features is that the solution has two independent rotation parameters along an extra dimension associated with the black hole's rotation and the Gödel's rotation. The space-time also admits the existence of two disconnected ergoregions, an inner ergoregion and an outer ergoregion. These two ergoregions can rotate in the opposite direction as well as in the same direction.

1 Summary

In recent years, some non-BPS black hole solutions have also been found in addition to supersymmetric black hole solutions. Although no one has found higher-dimensional Kerr-Newman solutions in Einstein-Maxwell theory yet, Cvetič *et al* [1] found a non-extremal, charged and rotating black hole solution with asymptotic flatness in the five-dimensional Einstein-Maxwell theory with a Chern-Simons term. In the neutral case, the solution reduces to the same angular momenta case of the Myers-Perry black hole solution [2]. Exact solutions of non-BPS Kaluza-Klein black hole solutions are found in neutral case [3, 4] and charged case [5]. These solutions have a non-trivial asymptotic structure, i.e., the spacetime is asymptotically locally flat and approaches a twisted S^1 metric over a four-dimensional Minkowski spacetime, topologically not a direct product. The horizons are deformed due to this non-trivial asymptotic structure and have a shape of a squashed S^3, where S^3 is regarded as a S^1 bundle over a S^2 base space. The ratio of the radius S^2 to that of S^1 is always larger than one.

As was proposed by Wang, a kind of Kaluza-Klein black hole solutions can be generated by the *squashing transformation* from black holes with asymptotic flatness [6]. In fact, he regenerated the five-dimensional Kaluza-Klein black hole solution found by Dobiasch and Maison [3, 4] from the five-dimensional Myers-Perry black hole solution with two equal angular momentum (The solution generated by Wang coincides with the solution in Ref.[3, 4].). In the previous work [7], applying the squashing transformation to the Cvetič *et al*'s charged rotating black hole solution [1] in vanishing cosmological constant case, we obtain the new Kaluza-Klein black hole solution in the five-dimensional Einstein-Maxwell theory with a Chern-Simons term. This is the generalization of the Kaluza-Klein black hole solutions in Ref. [3, 4, 5]. This solution has four parameters, the mass, the angular momentum in the direction of an extra dimension, the electric charge and the size of the extra dimension. The solution describes the physical situation such that in general a non-BPS black hole is boosted in the direction of the extra dimension. As the interesting feature of the solution, unlike the static solution [5], the horizon admits a prolate shape in addition to a round S^3. The solution has the limits to the supersymmetric black hole solution and a new extreme non-BPS black hole solutions and a new rotating black hole solution with a constant S^1 fiber.

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Applying this squashing transformation to Kerr-Gödel black hole solutions [8], we have constructed [9] a new type of rotating Kaluza-Klein black hole solutions to the five-dimensional Einstein-Maxwell theory with a Chern-Simons term. We also have investigated the features of the solutions. Though the Gödel black hole solutions have closed timelike curves in the region away from the black hole, the new Kaluza-Klein black hole solutions generated by the squashing transformation have no closed timelike curve everywhere outside the black hole horizons. At the infinity, the space-time is asymptotically a Kaluza-Klein spacetime. The solution has four independent parameters, the mass parameter, the size of an extra dimension and two kinds of rotations parameters in the same direction of the extra dimension. These two independent parameters are associated with the rotations of the black hole and the universe. In the case of the absence of a black hole, the solution describes the Gross-Perry-Sorkin (GPS) monopole which is boosted in the direction of an extra dimension and has an ergoregion by the effect of the rotation of the universe.

2 Squashed Kerr-Gödel black hole solution

The action is given by

$$S = \frac{1}{16\pi} \int \left(R * 1 - 2F \wedge *F - \frac{8}{3\sqrt{3}} F \wedge F \wedge A \right). \tag{1}$$

The metric and the gauge potential for the squashed Kerr-Gödel black hole solution are given by

$$ds^2 = -f(r)dt^2 - 2g(r)\sigma_3 dt + h(r)\sigma_3^2 + \frac{k(r)^2 dr^2}{V(r)} + \frac{r^2}{4}[k(r)(\sigma_1^2 + \sigma_2^2) + \sigma_3^2], \tag{2}$$

and

$$\mathcal{A} = \frac{\sqrt{3}}{2} jr^2 \sigma_3, \tag{3}$$

respectively, where the functions in the metric are

$$f(r) = 1 - \frac{2m}{r^2}, \tag{4}$$

$$g(r) = jr^2 + \frac{ma}{r^2}, \tag{5}$$

$$h(r) = -j^2 r^2(r^2 + 2m) + \frac{ma^2}{2r^2}, \tag{6}$$

$$V(r) = 1 - \frac{2m}{r^2} + \frac{8jm(a + 2jm)}{r^2} + \frac{2ma^2}{r^4}, \tag{7}$$

$$k(r) = \frac{V(r_\infty)r_\infty^4}{(r^2 - r_\infty^2)^2} \tag{8}$$

and the 1-forms on S^3 are given by

$$\sigma_1 = \cos\psi d\theta + \sin\psi \sin\theta d\phi, \tag{9}$$

$$\sigma_2 = -\sin\psi d\theta + \cos\psi \sin\theta d\phi, \tag{10}$$

$$\sigma_3 = d\psi + \cos\theta d\phi. \tag{11}$$

The coordinates r, θ, ϕ and ψ run the ranges of $0 < r < r_\infty$, $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$, $0 \leq \psi < 4\pi$, respectively. m, a, j and r_∞ are constants. The space-time has the timelike Killing vector fields ∂_t and two spatial Killing vector fields with closed orbits, ∂_ϕ and ∂_ψ. Note that this metric can be obtained from the Kerr-Gödel black hole solution [8] by the transformation $\sigma_1 \to \sqrt{k(r)}\sigma_1$, $\sigma_2 \to \sqrt{k(r)}\sigma_2$, $\sigma_3 \to \sqrt{k(r)}\sigma_3$ and $dr \to k(r)dr$, which is called squashing transformation. In the limit of $k(r) \to 1$, i.e., $r_\infty \to \infty$ with the other parameters kept finite, the metric coincides with that of the original Kerr-Gödel

black hole solution [8] with CTCs. Here we assume that the parameters j, m, a and r_∞ appearing in the solutions satisfy the following inequalities

$$m > 0, \tag{12}$$

$$\frac{r_\infty^2}{m} > 1 - 4j(a + 2jm) > \sqrt{\frac{2}{m}}|a|, \tag{13}$$

$$r_\infty^4 - 2m(1 - 4j(a + 2jm))r_\infty^2 + 2ma^2 > 0, \tag{14}$$

$$-4j^2 r_\infty^6 + (1 - 8j^2 m)r_\infty^4 + 2ma^2 > 0. \tag{15}$$

These are the necessary and sufficient conditions that there are two horizons and no CTCs outside the horizons. Eqs. (12)-(14) are conditions for the presence of two horizons, and Eq.(15) is the condition for the absence of CTCs outside the horizons. It is noted that in the limit of $r_\infty \to \infty$ with the other parameters finite, Eq.(15) can not be satisfied. Let us normalize the parameters a, j and r_∞ as $A = a/\sqrt{m}, J = \sqrt{m}j$ and $R_\infty = r_\infty/\sqrt{m}$, respectively and furthermore, we fix the value of R_∞. Then, in the cases of $R_\infty^2 < 2$, $R_\infty^2 = 2$ and $R_\infty^2 > 2$, the quadratic curve $R_\infty^4 - 2(1 - 4J(A + 2J))R_\infty^2 + 2A^2 = 0$ in the condition (14) becomes an ellipse, a line and a hyperbola, respectively. The curve $R_\infty^2 = 1 - 4J(A + 2J)$ in the condition (13) has different shapes in the cases of $R_\infty^2 < 1$, $R_\infty^2 = 1$ and $R_\infty^2 > 1$. Hence we consider the cases of (i) $0 < R_\infty^2 < 1$, (ii) $R_\infty^2 = 1$ (iii) $1 < R_\infty^2 < 2$ (iv) $R_\infty^2 = 2$ and (v) $R_\infty^2 > 2$. The shaded regions in Figure1-5 show the parameter region (12)-(15) for a given R_∞ in each case of (i)-(v), respectively. Thus, applying the squashing transformation to the Kerr-Gödel black hole solution, we can obtain such a Kaluza-Klein black hole solution without CTCs everywhere outside the black hole.
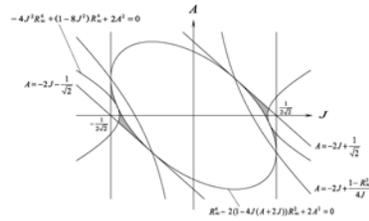

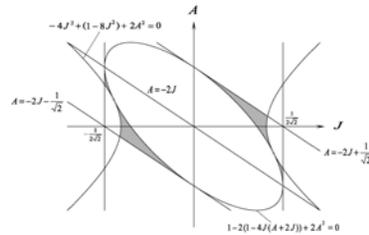Figure 1: The parameter region in the (J, A)-plane in the case of $0 < R_\infty^2 < 1$.

Figure 2: The parameter region in the (J, A)-plane in the case of $R_\infty^2 = 1$.

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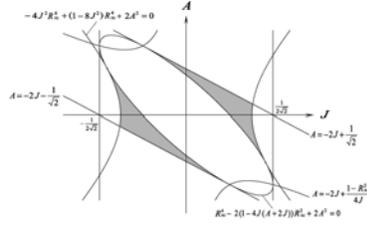M. Cvetič, H. Lü and C.N. Pope, Phys. Lett B **598**, 273 (2004).

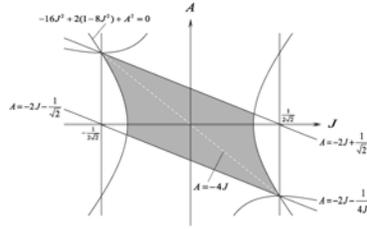Figure 3: The parameter region in the (J, A)-plane in the case of $1 < R_\infty^2 < 2$.

Figure 4: The parameter region in the (J, A)-plane in the case of $R_\infty^2 = 2$.
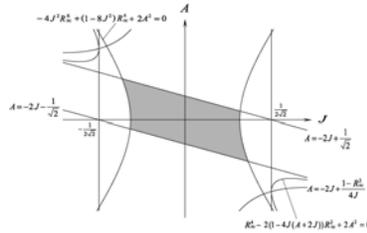

Figure 5: The parameter region in the (J, A)-plane in the case of $2 < R_\infty^2$.

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