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EIC LUMINOSITY MODELS FOR VARIOUS HADRON COOLING SCENARIOS*

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Abstract

We have developed a simulation to model the evolution of proton and heavy-ion bunches stored in the Electron-Ion Collider's Hadron Storage Ring (HSR) over the course of several hours, taking into account intrabeam scattering, the beam-beam effect, and particle loss. This has enabled us to predict how various cooling schemes, including microbunched electron cooling and microwave stochastic cooling, would impact the collider's luminosity. We discuss the details of this code and show the luminosity evolution for various scenarios.

INTRODUCTION

The Electron-Ion Collider (EIC) is a new collider which will be built at Brookhaven National Laboratory to probe nuclear structure through the collision of electrons with various ion species. In order to improve luminosity, various cooling scenarios are considered, including a low-energy cooler to create initial flat hadron beams [1] as well as stochastic and microbunched electron cooling at store to maintain the beam emittance [2, 3]. Since not all of these systems may be available at startup, it is desirable to have a simulation code which will enable us to quantify the performance of the various cooling systems and operational modes.

To achieve this goal, we have developed a long-term luminosity model. This evolves a bunch of hadrons[1] over the course of the several-hour store and uses this information to compute changes in the luminosity. We discuss the details of this model and the various physical effects included, then show luminosity evolution for various physics cases.

MODEL DETAILS

Our model is an extension of the long-term tracking developed for studies of microbunched electron cooling [3]. We initialize a bunch of hadron macroparticles and then use a small number of transfer matrices to transport them between a handful of key locations in the EIC's hadron storage ring (HSR), allowing the macroparticles to sample their full phase space. We include both 591 and 197 MHz RF cavities to provide sinusoidal energy kicks to the hadrons each turn. We manually add in the interesting dynamic effects, including intrabeam and Touschek scattering, the beambeam effect, and cooling, as will be described in the subsequent subsections. In order to simulate many real turns in a small number of simulated turns, we let each simulated turn represent $N >> 1$ real turns by increasing coherent cooling kicks by a factor N and incoherent kicks by $\sqrt{N}$.

Intrabeam Scattering (IBS)

We calculate intrabeam scattering rates using the formulas in [5]. These are equivalent to the usual Bjorken-Mtingwa formalism [6], but written in terms of elliptic integrals which can be quickly evaluated using the methods of [7]. Due to our explicit implementation of large-amplitude Touschek kicks, as will be discussed later, we restrict the maximum momentum kick in the Coulomb logarithm to be the minimum momentum kick we use for our calculations of Touschek scattering.

As inputs to the IBS calculation, we require the optics around the ring as well as information about the bunch distribution. The optics are simply sampled at 1000 equidistant points of the HSR, with coarser sampling not giving any advantage. The beam transverse emittances are calculated by fitting the distributions of the particles' transverse actions to exponentials, and the bunch length and energy spread are found by fitting the distributions of longitudinal position and energy offset to Gaussians, with the longitudinal bunch positions further constrained to lie within the central 591 MHz RF bucket. This allows us to focus our studies on the core of the beam as opposed to the potentially large tails.

Once the IBS rates are calculated, we apply Gaussian random kicks to each macroparticle in each of the 3 planes in a zero-dispersion location. These kicks have mean 0 and standard deviation chosen to produce the correct heating rates. Additionally, the sizes of the kicks to a given macroparticle are scaled by the square root of the hadron's local density relative to the average density in the 591 MHz RF bucket, since we expect that the heating rate for an individual particle will increase as it moves toward the core of the bunch.

Beambeam Effect

The beambeam effect results in an emittance growth in the hadron beam due to the strong nonlinear force exerted by the electrons. Since this is a complex phenomenon, we simply assume a constant beambeam growth time of 20 hours horizontally and 5 hours vertically for protons and no beambeam growth for heavy ions due to their smaller beambeam parameter [4]. The implementation of the kicks is the same as described for IBS in the previous subsection, except that we do not perform the density scaling, since this is not expected to be the case for beambeam.

Touschek Scattering

While IBS models the emittance growth due to many small-angle scattering events, there is also a non-zero probability that a given hadron will receive a single large energy

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[1] We do not need to do anything special for the electron beam, since its emittance is held constant by radiation damping and the bunches are frequently replaced for polarization reasons [4].

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kick that will instantaneously cause a significant change in its phase-space coordinates. The probability of receiving an energy kick of a given size or larger is given by Eq. 41 of [8]. We see empirically that the probability of receiving a kick of at least δ_m scales approximately as δ_m^{-3}. Letting the initial momentum spread of the bunch be δ_0, we calculate the probability P_0 of receiving a momentum kick equal to or greater than δ_0 within a given timestep. Then, at each timestep, we draw a random number in the range $r \in [0, 1)$ for each macroparticle and give it a momentum kick of size $\delta = \pm[-P_0\delta_0^3/\ln(1-r)]^{1/3}$ if $|\delta| > \delta_0$, with both signs having equal probability.

Stochastic Cooling

Stochastic cooling is a well-developed method for cooling low-density hadron beams [9] and has been successfully used at a number of facilities, including the Relativistic Heavy Ion Collider (RHIC) [10]. It uses pickups to measure the positions of the particles in the bunch in 3 dimensions within some bandwidth then uses a kicker to apply momentum kicks to the bunch in order to correct the particle offsets.

We simulate the process directly in our code. At the location of the transverse pickups, we create histograms of the macroparticles' longitudinal positions weighted by either their horizontal or vertical positions. We filter this through the 16 available cavity frequencies, equally spaced between 5 and 8 GHz [10], and apply an appropriate gain factor to obtain the kicks which we then apply at the kicker location. In order to have the transverse offsets at the pickup correspond to transverse angles at the kicker, we ensure that there is a $\pi/2$ phase advance between these locations. For the longitudinal case, we have two pickups separated by $1/6$ of a turn. We take the histogram of the longitudinal macroparticle distribution at each location. Since the shift in particle positions between the two pickups is due to their energy offsets, taking the difference in these two distributions gives us information about the momentum distribution of the particles. We then filter this through the 16 available cavity frequencies and apply the appropriate gain and phase shift to get the kick we need to apply at the longitudinal kicker 1/6 of a turn after the second pickup.

Since the noise scales with the bunch population, we need to be careful about how we perform the time and particle-number rescaling. Letting Δp_c be the coherent cooling kick, Δp_h be the diffusive heating kick, g be the amplifier gain, N be the number of macroparticles, and T be the timestep length, we have that $\Delta p_c \propto gT$ and $\Delta p_h \propto g\sqrt{NT}$. Therefore, if we increase the gain by some constant R and decrease the number of macroparticles by this same factor R, we see that it is equivalent to increasing the length of the timestep by R, allowing an effective time rescaling [11].

Microbunched Electron Cooling

For the dense proton bunches planned to be stored in the EIC, stochastic cooling will not work due to its limited bandwidth. Coherent electron cooling is a method of cooling such dense proton bunches which operates by using a co-propagating beam of electrons as both the pickup and kicker. The protons provide energy kicks to nearby electrons, producing energy fluctuations which are then amplified and turned into density fluctuations. These provide energy kicks to the protons which tend to fix their initial phase space actions [12].

Microbunched electron cooling (MBEC) is a specific implementation of coherent electron cooling which amplifies the electron density fluctuations using the microwave instability [13]. Details of the design for the EIC can be found in [3] and the references provided therein.

We once again simulate this process directly. We compute the wake function which tells us the energy kick that a proton receives in the kicker as a function of its phase space coordinates in modulator and kicker. We also can compute a diffusion term which tells us the size of the random kick it would receive due to the wakes of the other protons and due to noise in the electron beam. We then simply track the proton macroparticles directly between the modulator and kicker using the relevant transfer matrix and apply the appropriate cooling kick and a Gaussian random heating kick, with RMS value given by the diffusion term,. Further details are given in [3].

Particle Losses and Initialization

In order to simulate particle losses, we define an ellipse in the 6-dimensional phase space and eliminate particles which fall outside the ellipse. Specifically, a macroparticle with actions in the three planes equal to J_x, J_y, and J_z is removed if $J_x/J_{x,max} + J_y/J_{y,max} + J_z/J_{z,max} > 1$, where the maximum transverse actions $J_{x,max}$ and $J_{y,max}$ are those which would bring a particle to transverse coordinates 6 times the beam's initial transverse size and $J_{z,max}$ is defined by the RF bucket. We use the full 197 MHz bucket for the heavy ions, but limit the protons to stay within the 591 MHz bucket due to their larger sensitivity to the beambeam effect [14].

In order to get a realistic distribution of particles filling the RF bucket, we seed the particles longitudinally using the rejection algorithm. We randomly place a trial macroparticle within some area of z-pz phase space including the entire RF bucket. It is then kept with probability $(1 - J_z/J_{z,max})^2$, so that macroparticles at the bucket center are always kept and those near the edges are rarely retained. This is repeated until the desired number of macroparticles has been seeded.

Beta-Function Rescaling

During the store, the emittances of the beam will change, and so we adjust the beta functions at the interaction point (IP) in order to maintain a number of constraints. First, we must rescale the beta functions of the electrons so that their beam size matches that of the hadrons. We also will have forward detectors near the IP to measure particles which are scattered at very small angles. In order to keep backgrounds here low, we need to tightly constrain the divergence of the beam at the IP. Therefore, if the hadron beam emittance

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would increase above its initial value, we increase the corresponding beta function at the IP to maintain the divergence equal to the initial divergence. Conversely, if the hadron emittance is reduced (primarily due to cooling), we also increase the corresponding beta function at the IP to avoid needing to shrink the beta functions of the electron beam below their initial values.

Since the hadron beam may have a non-Gaussian profile, particularly due to the effects of cooling, we need to define the emittance appropriately when evaluating the above constraints. Since we care about the tails for the divergence constraint, we compute the emittance as the average of the particle actions for that case, while we compute the emittance using an exponential fit to the actions when evaluating the other two constraints related to matching beam sizes, since we care there mainly about the core of the bunch.

Crabbing

The EIC will collide electrons and hadrons with a 25 mrad crossing angle [4]. Without any correction, the particles in the two bunches will have poor overlap during the collision, significantly reducing the luminosity. In order to help fix this issue, we will install crab cavities which provide a transverse kick to particles in the two beams as a function of their longitudinal coordinate, effectively rotating the bunches so that they collide head-on. However, during early operations, we may not yet have any cavities for the electron beam and only the 197 MHz cavities for the hadrons, which will have a significant nonlinear x-z distribution.

Luminosity Evaluation

In order to evaluate the luminosity, we make use of Eq. 1 from [15], evaluated in the limit when the electron and hadron beams have negligible transverse momenta,

$$\mathcal{L} = 2cf_{rep}\int d^3\vec{x}dt\rho_+(\vec{x},t)\rho_-(\vec{x},t) \quad (1)$$

where c is the speed of light, f_{rep} is the frequency of bunch crossings, and $\rho_\pm(\vec{x},t)$ are the densities of the hadrons and electrons.

In order to evaluate this integral numerically, we use a transfer matrix to bring the hadron bunch to the IP, backtrack them some distance upstream, provide a z-dependent transverse kick to model the effect of the crab cavities, and then track them as they move forward across the IP. We assume that the electron beam is perfectly Gaussian, potentially with a tilt due to the crab cavities. While tracking the hadron macroparticles across the IP, we can analytically determine the local electron density that each of them sees. We then need only numerically integrate this over the time that the hadron macroparticle is near the IP, average over the macroparticles, and multiply by the relevant prefactors to recover the instantaneous luminosity.

SELECTED RESULTS

We show here plots of luminosity and integrated luminosity for 3 potential operational modes, discussed in detail below. Since the experimentalists care about the integrated luminosity over real time, ie, including the downtime between stores, we include plots of average luminosity as a function of store length. This is computed by dividing the integrated luminosity up to that time by the length of the store plus some amount of turnaround time to account for the time needed to ramp down the machine, refill the hadrons, do low-energy cooling, and ramp to collision energy. We additionally set the luminosity to zero during the first 15-30 minutes of the store, since this is the time when we would be filling electrons and turning on the detector. However, the hadron beam still deteriorates during this time. We assume that the turnaround time is 1 hour for heavy ions without low-energy cooling, 1.5 hours for heavy ions with low-energy cooling, and 2 hours for protons, to account for the time needed for low-energy cooling and the necessity of slowing down the ramp due to the presence of the Siberian snakes [16]. For the heavy ions, where we only have 290 electron bunches to fill, we assume that filling the electrons and turning on the detector takes 15 minutes, while we assume 30 minutes for the protons, when we have 1160 bunches. We do not directly simulate low-energy cooling, but simply adjust the initial emittances of the beam.

For the first year of EIC operation, we are exploring the use of heavy ions such as niobium which have charge-to-mass ratios which will let us store them in the center of the beampipe with the dipole magnets at their maximum strength. We assume that only the 197 MHz crab cavities are installed for the ions, and none for the electrons. Plots of instantaneous and average luminosity are shown in Fig. 1 with and without low-energy cooling and/or stochastic cooling. We see that the inclusion of each of these roughly doubles the average luminosity.

We show in Fig. 2 luminosities for the highest energy heavy ion collisions foreseen for the EIC, 110 GeV/u gold colliding with 18 GeV electrons with or without stochastic cooling. In this case, we assume that low-energy cooling and all crab cavities are installed. Once again, we see that stochastic cooling doubles the average luminosity.

Finally, we consider the highest luminosity case for the EIC: 275 GeV protons colliding with 10 GeV electrons. We show in Fig. 3 the luminosity for this mode with or without microbunched electron cooling at store. Again, low-energy cooling and all crab cavities are assumed to be operational. We once again see that cooling at store doubles the average luminosity we can expect.

CONCLUSION

We have developed a simulation program which allows us to model the evolution of hadron bunches stored in the EIC with a wide variety of physical effects included, and in particular see the effects of stochastic cooling and microbunched electron cooling at store and the impact of the initial emittances which could be achieved with a low-energy cooler. For the parameters shown here, including cooling either at injection or at store roughly doubles the average

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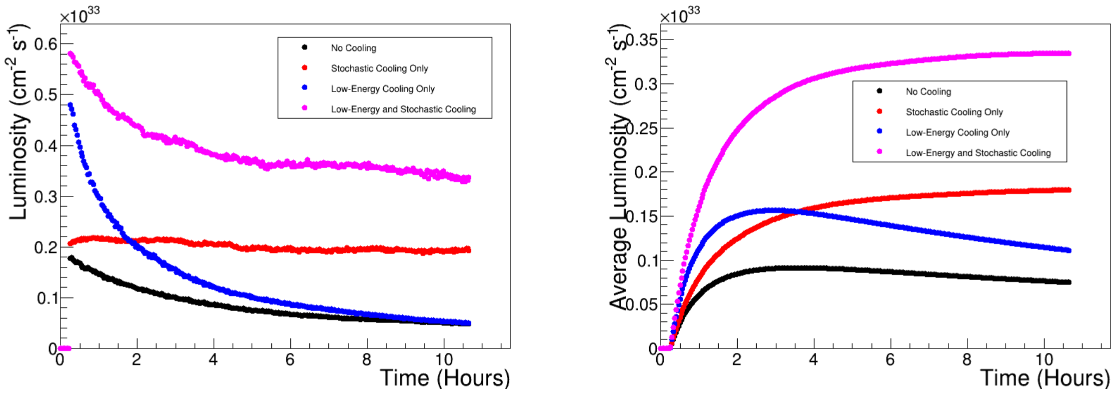

Figure 1: Instantaneous (left) and average (right) luminosity for 121 GeV/u niobium on 10 GeV electrons. We achieve an average luminosity of at most $0.9 \times 10^{32}\text{cm}^{-2}s^{-1}$ without any cooling, but this increases to $1.8 \times 10^{32}\text{cm}^{-2}s^{-1}$ with stochastic cooling alone, $1.6 \times 10^{32}\text{cm}^{-2}s^{-1}$ with low-energy cooling alone, and $3.3 \times 10^{32}\text{cm}^{-2}s^{-1}$ with both methods.

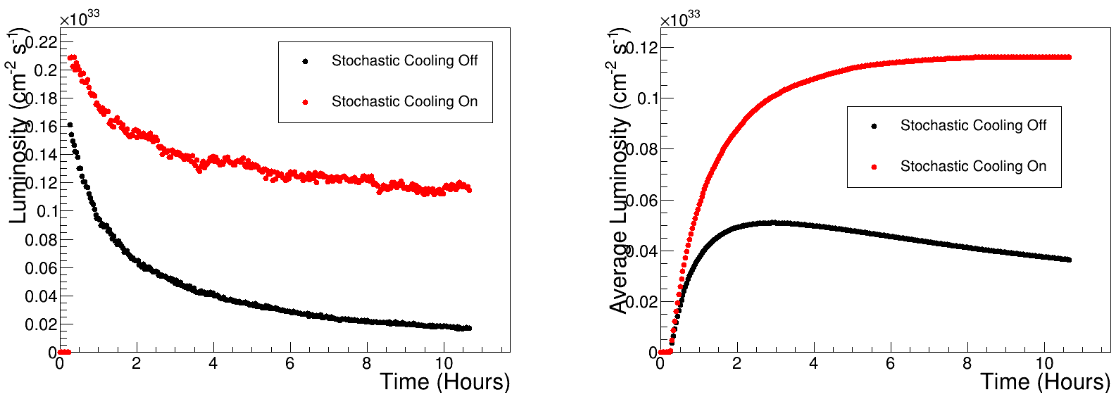

Figure 2: Instantaneous (left) and average (right) luminosity for 110 GeV/u gold on 10 GeV electrons. We see that adding stochastic cooling increases the average luminosity from 0.5 to $1.2 \times 10^{32}\text{cm}^{-2}s^{-1}$.

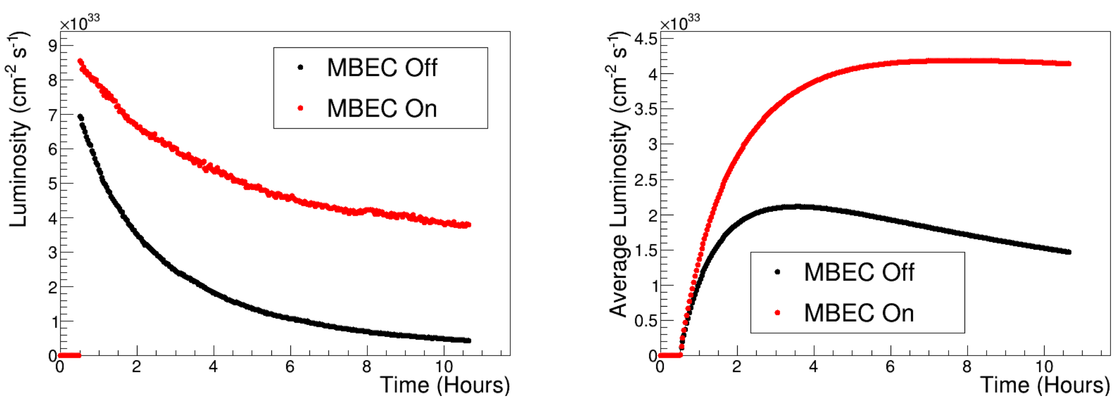

Figure 3: Instantaneous (left) and average (right) luminosity for 275 GeV protons on 10 GeV electrons. We see that adding microbunched electron cooling increases the average luminosity from 2.1 to $4.2 \times 10^{33}\text{cm}^{-2}s^{-1}$.

luminosity we can achieve, and including both quadruples the average luminosity. While we have only included detailed models of microbunched and stochastic cooling, we can in principle extend this code to handle other methods, such "regular" electron cooling.

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