

High frequency gravitational waves: generation, detection

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Abstract. We consider the different approaches to the generation and detection of the high-frequency gravitational waves. The estimations of the amplitude of gravitational waves generated by these methods were made. The characteristics of the high-frequency gravitational waves induced by perturbations of matter and the electromagnetic field are compared. The possibility of the direct detection of gravitational waves on the basis of the different methods was considered as well.

1. Introduction

At present, various astrophysical processes [1] and cosmological perturbations in the early universe [1,2] are considered as natural sources of gravitational-wave radiation. The low-frequencies gravitational waves arising from black holes and neutron stars mergers with amplitude $h_0 \sim 10^{-20} - 10^{-21}$ were detected in the LIGO and Virgo experiments [3, 4] based on the interference method proposed in [5]. The relic gravitational waves of cosmological genesis waves have a wide frequency range for different models of the early universe, which are restricted by BBN bound and energy scale of inflation as well [6]. Various types of inflationary models of the early universe predict a significant increase in the amplitude of the high-frequency relic gravitational waves $f_g \sim 10^9$ Hz to the values of the order of $h_0 \sim 10^{-30} - 10^{-27}$ [7-9]. However, at the moment, relic gravitational waves have not been detected.

Also, various laboratory sources of gravitational waves are considered, based both on the interaction of electromagnetic radiation and matter [10, 11], and for the electromagnetic field itself as a source of gravitational waves [12, 13, 14].

Thus, gravitational-wave fluctuations of the space-time metric can be induced both by perturbations of the medium and the electromagnetic field, and the predominance of the first or second factor is associated with the specifics of the process of generating gravitational waves.

According to estimates given in [10,11], the interaction of a short pulse of high-power laser radiation and matter makes it possible to generate gravitational waves with an amplitude $h_0 \sim 10^{-40}$ which is much lower than the sensitivity limit of modern detectors.

Nevertheless, in [12,13,14] it was shown that taking into account the processes of conversion of intense laser radiation into a gravitational wave in nonlinear dielectric media makes it possible to consider these methods of generating and recording high-frequency gravitational waves as relevant ones.



In this paper we give brief review of the methods for generating the high-frequency gravitational waves by perturbations of the medium and the electromagnetic field $T_{\mu\nu} = T_{\mu\nu}^M + T_{\mu\nu}^{EM}$ in two regimes, namely for $T_{\mu\nu}^M \gg T_{\mu\nu}^{EM}$ and $T_{\mu\nu}^M \ll T_{\mu\nu}^{EM}$. The possibility of detecting high-frequency gravitational waves generated on the basis of these methods is also discussed.

2. The equations of a gravitational field

Artificial sources of gravitational waves imply gravitational radiation of small amplitude, and thus, weak gravitational waves are considered as small perturbations $|h_{\mu\nu}| \ll 1$ of Minkowski space-time $\eta_{\mu\nu}$, and the metric of perturbed space-time is defined as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

In this case, the equations of gravitational field can be written as follow [1]

$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \quad (1)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter, G is the gravitational constant, c is the speed of light in vacuum, \square is the d'Alembert operator, and tensor $\bar{h}_{\mu\nu}$ satisfies the following gauge conditions

$$\partial^\mu \bar{h}_{\mu\nu} = 0, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h. \quad (2)$$

The solution of equation (1) can be written as follows [1]

$$h_{ij}(t, \hat{x}) = \frac{4G}{rc^4} A_{ij,kl}(\hat{n}) \int d^4 x' T_{kl} \left(t - \frac{r}{c} + \frac{\hat{x}' \cdot \hat{n}}{c}, \hat{x}' \right), \quad (3)$$

where $r = |\hat{x} - \hat{x}'|$ and tensor $A_{ij,kl}$ is defined as

$$A_{ij,kl}(\hat{n}) = P_{ik} P_{jl} - \frac{1}{2} P_{ij} P_{kl}, \quad P_{ij}(\hat{n}) = \delta_{ij} - n_i n_j, \quad (4)$$

where vector n_i determines the direction of propagation of the gravitational wave.

For the case when the gravitational waves are induced by the perturbations of the medium with the small velocities $v \ll c$ of the particles oscillations, the quadrupole formalism implying approximate solutions of the gravitational field equations is usually used.

The leading non-zero (quadrupole) term in multipole expansion of expression (3) can be written as follow [1]

$$h_{ij}(t, \hat{x}) = \frac{2G}{rc^4} A_{ij,kl}(\hat{n}) \left[\frac{d^2}{dt^2} Q^{kl} \left(t - \frac{r}{c} \right) \right], \quad (5)$$

where quadrupole moment is defined as

$$Q^{ij} = \frac{1}{c^2} \int d^3 x T^{00} \left(x^i x^j - \frac{1}{3} r^2 \delta^{ij} \right). \quad (6)$$

Thus, to determine the characteristics of gravitational waves induced by perturbations of matter, it is sufficient to know the quadrupole moment of the system. The different types of the sources of the high-frequency gravitational waves were considered earlier on the basis of the quadrupole formalism (see, for example, in [1, 10, 11] and in many other works as well).

On the other hand, equations (1) can be represented as

$$\square h_{\mu\nu} = 2\kappa \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T_\rho{}^\rho \right), \quad (7)$$

where $\kappa = (8\pi G/c^4)$ and $T_\rho{}^\rho = T_{\rho\sigma} \eta^{\rho\sigma} = T$ is the trace of the energy-momentum tensor.

Also, we note, that gauge conditions (2) are correct when the energy-momentum conservation

$$\partial^\mu T_{\mu\nu} = 0, \quad (8)$$

is satisfied.

This representation of the gravitational field equations is useful for the case when the electromagnetic field is considered as a source of gravitational waves, since for electromagnetic field one has $T = 0$, and equations (7) are reduced to the following form

$$\square h_{\mu\nu} = 2\kappa T_{\mu\nu}, \quad (9)$$

implying the analysis of the characteristic of gravitational waves based on the exact solutions of these equations.

The wave solutions of the gravitational field equations outside the source in empty space $T_{\mu\nu} = 0$ can be noted as the plane gravitational waves propagating with the speed of light [1]

$$h_{\mu\nu} = A_{\mu\nu} \cos(k_\rho x^\rho), \quad (10)$$

where $A_{\mu\nu}$ is the amplitude of gravitational waves which is orthogonal to the wave vector $A_{\mu\nu} k^\rho = 0$, and $k^0 = \omega/c$, where ω is the frequency of gravitational waves.

The characteristic of the gravitational waves in empty space are defined by the type of the energy-momentum tensor of material source.

3. Perturbations of a medium as the source of gravitational waves

Irradiation of matter with the high-power laser radiation is considered as a possible mechanism for generating high-frequency gravitational waves, which implies an increase in the characteristics of gravitational wave radiation with an increase in the power of modern lasers [10]. In this case, one can neglect the gravitational field of electromagnetic waves and consider the reaction of the medium to external influence as the only source of gravitational waves.

Shock waves arising from the irradiation of matter with short pulses of high-power laser radiation were considered as sources of high-frequency gravitational waves in [10].

The amplitude of flat gravitational waves induced by the shock waves was estimated on the basis of equations (5)-(6) as follows

$$h_0 = \frac{7}{2} \frac{GP_L \tau}{rc^4} \sqrt{\frac{\rho_c}{\rho_0}}, \quad (11)$$

where P_L is the laser power and τ is the laser pulse duration, ρ_0 is the density of the medium and

$$\rho_c = \frac{\epsilon_0 m_e m_i (2\pi c)^2}{Ze^2 \lambda_L^2}, \quad (12)$$

where ϵ_0 is permittivity of vacuum, m_e is the rest mass of the electron, m_i is the mass of the ion, e is the charge of electron, λ_L is the wavelength of the laser, Z is the atomic number.

The maximal value of the amplitude of gravitational waves on the distance $r = 10$ m from such a source for realistic parameters of experiment was estimated as $h_0 \sim 10^{-40}$ [10].

The other approach to generation of the high-frequency gravitational waves by the perturbations of the medium based on a model of an ionic crystal interacting with an electromagnetic wave was considered in [11].

The amplitude of flat gravitational waves induced by this method was estimated on the basis of equations (5)-(6) as

$$h_0 = \frac{G}{rc^4} \gamma_\omega l^2 \omega_T^2 m_i N, \quad (13)$$

where $\gamma_\omega \equiv \frac{\varepsilon_\infty - \varepsilon_0}{2\pi}$ is the dispersion factor, l is the electromagnetic field-induced ion movement, m_i is the mass of ion, N is the number of ions affected by electromagnetic radiation, ω_T is the natural frequency of the elastic transverse oscillations of the crystal.

For realistic values of the parameters in expression (13) the maximal amplitude of gravitational waves on the distance $r = 10\text{m}$ from source was estimated as $h_0 \sim 10^{-40}$ [11], which is coincide with result in previous approach.

4. Electromagnetic field as the source of gravitational waves

The other approach to analyze the generating high frequency gravitational waves is considering the electromagnetic field itself as the source of gravitational waves in the electromagnetic resonators. In the context of this approach one can neglect the gravitational waves induced by the action of electromagnetic field on the walls of the resonator.

In the papers [15, 16] the electromagnetic waves in the Fabry-Perot resonator are considered as the source of coupled gravitational waves inside resonator and free gravitational waves in empty space outside resonator.

After obtaining the inner exact solutions of equation (9) corresponding to gravitational waves coupled with electromagnetic ones, one can reconstruct the characteristics of the gravitational waves in empty space on the basis of the continuity of the energy density flux

$$ct^{01} = -\frac{c}{4\kappa} \left(\partial_0 h_{\alpha\beta} \partial_1 h^{\alpha\beta} \right), \quad (14)$$

after reflection of electromagnetic waves by the walls of resonator.

The amplitude and frequency of the gravitational waves for this case can be noted as follow [16]

$$h_0 = \frac{GE_0^2}{4\omega^2 c^2} = \frac{G}{c^4} \left(\frac{E_0 L}{2\pi q} \right)^2, \quad (15)$$

$$\omega_g = 2\omega = \frac{2\pi qc}{L}, \quad (16)$$

where E_0 is the electric field strength (or magnetic field $H_0 = E_0$) of the electromagnetic wave, L is the length of the Fabry-Perot resonator and $\omega = q(\pi c/L)$ is the frequency of electromagnetic wave, where $q = 1, 2, 3, \dots$ correspond to the number of the longitudinal optical mode in the resonator.

For the optical range $\omega \sim 10^{14} - 10^{15} \text{sec}^{-1}$ of electromagnetic waves with realistic electric field strength E_0 and realistic Fabry-Perot resonator length L , the amplitude of gravitational waves induced by such a way was estimated as $h_0 \sim 10^{-44} - 10^{-46}$ [16].

After placing the Fabry-Perot resonator in a constant magnetic field H_{00} transverse to the direction of propagation of electromagnetic waves, it is possible to significantly increase the amplitude of the induced gravitational waves due to the process of conversion of electromagnetic waves into gravitational ones [17-19].

The amplitude of the gravitational waves induced by this approach is [20, 21]

$$h_0 = \frac{2G}{c^3 \omega} E_0 H_{00} L, \quad (17)$$

and its maximal realistic value is estimated as $h_0 \sim 10^{-32} - 10^{-30}$ [20].

Thus, the amplitude of gravitational waves induced by an electromagnetic field is ten orders of magnitude higher than in the case of the amplitude of gravitational waves induced by the matter perturbations, which was considered earlier in [22].

5. The detection of the high-frequency gravitational waves

There are different methods for detecting the high-frequency gravitational waves (see, for example, in [23-27]). As the promising method for detection of the high-frequency gravitational waves in optical range of frequencies one can use the electromagnetic response of gravitational waves passing through the magnetic field or inverse Gertsenshtein effect [18, 19].

This approach is based on the detection of photon flux arising as the result of graviton to photon conversion when gravitational waves passing through the magnetic field. The minimal detectable amplitude of the high-frequency gravitational waves by this method can be defined as follows [27]

$$h_0^{min} \simeq \left(\frac{4N_{exp}}{AH_{00}^2 L^2 \varepsilon_\gamma(\omega) \Delta\omega} \right)^{1/2}, \quad (18)$$

where $\omega = 2\pi f = \omega_g$ is the main frequency of the photons, N_{exp} is the total detected number of photons per second in the bandwidth $\Delta\omega$, A is, H_{00} is the magnetic field amplitude, L is the distance extension of the magnetic field and $\varepsilon_\gamma(\omega)$ is the quantum efficiency of the detector.

The minimal detectable amplitude of the high-frequency gravitational waves with frequencies $f_g = (2.7 - 14) \times 10^{14}$ Hz is estimated as $h_0^{min} \simeq 6 \times 10^{-26}$ [27].

Also, for detection of the gravitational waves with frequencies $f_g < f_{opt}$ one can use the other methods, for example, the gravitational-optical resonance in a multi-beam interferometers [28-30], where condition

$$f_g = \frac{n}{2} \left(\frac{c}{L} \right), \quad n = 1, 2, 3, \dots, \quad (19)$$

is fulfilled.

For the case $n=1$ the relative variation of the power of the laser radiation in the Fabri-Perrot interferometer in the field of a gravitational wave is [30]

$$\frac{\delta W(t)}{W} = \frac{QL}{\lambda_e} h(t), \quad (20)$$

where Q is the quality factor of interferometer, λ_e is the laser wavelength and W is the laser power at the input to the Fabry-Perot interferometer.

For the following parameters $Q = 10^6$, $\lambda_e = 1.064 \times 10^{-6}$ m, $L \sim 1$ m, $f_g \sim 10^8$ Hz and sensitivity $(\delta W/W)_{min} \sim 10^{-12}$ one has $h_0^{min} \sim 10^{-24}$. We also note that averaging the spectral density of variations in the power of laser radiation for the case of long-term measurements can improve the sensitivity by two orders of magnitude [30].

Thus, at the moment, the discrepancy between the maximum amplitude of artificial high-frequency gravitational waves and the sensitivity of the detectors is at least several orders of magnitude.

6. Conclusion

In this paper, we considered the methods for generating the high-frequencies gravitational waves by perturbations of medium and the electromagnetic field. The maximum amplitude of gravitational waves obtained by the first method was estimated as $h_0 \sim 10^{-40}$, while the generation of high-frequency gravitational waves by means of electromagnetic field gives a much higher value $h_0 \sim 10^{-32} - 10^{-30}$.

It is necessary to note, that modern detection methods do not allow direct observation of artificial high-frequency gravitational waves in contrast to low-frequency gravitational waves from astrophysical sources [3, 4]. Thus, the development of methods for generating and detecting artificial high-frequency gravitational waves requires fundamentally new approaches.

Nevertheless, the further development of the existing methods for detecting high-frequency gravitational waves implies the possibility of direct observation of relic gravitational waves of cosmological genesis [7-9], which can be considered as a promising direction for further research in the field of gravitational wave astronomy.

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