Study of photon orbit around magnetic black holes coupled to nonlinear electrodynamics

A S. Habibina and H. S. Ramadhan

Department of Physics, FMIPA, University of Indonesia, Depok, Indonesia

E-mail: a.sayyidina@sci.ui.ac.id

Abstract. In this work we study the orbit of light particles around a asymptotically at, magnetically charged black hole which is coupled to a model of nonlinear form of Maxwell electrodynamics. The matter source is characterized by two parameters, β and γ, which determine the behaviour of photon around the corresponding spacetime. We serve the effective potential and analyse its change by varying the charge and the model parameter. Finally, we solve its orbit equation and show how photon moves according to the black hole potential.

1. Introduction

The way light behave around black hole (BH) is one of the most fascinating phenomena in the realm of astrophysics, with one example being null bound orbit which is a particular condition where light rays orbit between two radius. It is known that the classic static vacuum and charged BH (Schwarzchild and Reissner-Nordström solution) can not perform a condition where stable photon orbit outside their horizons is physically possible. It is also important to note that the extremal case of RN can possess stable photon orbit exactly on its horizon [1]. One study around this problem claims that test particles can escape the horizons and appear into another universe because of the influence of potential barrier [2]. One of the many ways to crack the mentioned issue is by employing alternative models of linear Maxwell electrodynamics, which is widely known as Nonlinear Electrodynamics (NLED).

Since the beginning of the twentieth century, there has been many extensive studies in the realm of NLED. Born and Infeld (BI) suggested a nonlinear model of Maxwell's electrodynamics to resolve the problem of singularity in electron's self-energy [3]. This proposal is followed by Euler and Heisenberg who predicted the existence of vacuum magnetic birefringence in quantum electrodynamics (QED) [4]. The NLED theory later attracted many interests to various topics, in which one of them is gravitational physics. The extensions of BI model and other NLEDs to black holes or compact stars have been widely examined [5, 6, 7]. The expansion of NLED theory as charged black hole is also followed with its phenomenological study. It is stated that, in the presence of NLED, photon propagates along the null geodesic of its effective geometry [8]. The consequence of corresponding theory has been inspected in various applications, such as particles trajectories [9], deflection angle [10, 11], and gravitational lensing [12]. Other studies about particle motion in NLED-sourced black hole can be found in [13, 14].

Driven by our curiosity of the NLED theory, we decide to study their effects as magnetically charged black hole and verify the possibility of null bound orbits around them. For this article, we choose the generalized BI model which is proposed by Kruglov [15]. It has been proven that for its one horizon (extremal) case, this model is predicted to have a stable photon orbit outside its horizon [16], which become the main concern of our study. The summary of this work is given by the following. In Section 2 we give a brief description of the models and the resulting black hole solution. In section 3 we reveal the spacetime properties for two different cases. In Section 4, we show the derivation of the geodesic equation as well as the effective line element. Then, we observe the effective potential and bound orbit for both cases in 5. Finally, we summarize our result in Section 6.

2. Magnetically Charged Black Hole Solution

The generalized BI model was first proposed by Kruglov [15] in the form of

$$\mathcal{L} = \frac{1}{\beta}\left[1 - \left(1 + \frac{\beta F}{q}\right)^q\right], \tag{1}$$

where q is an arbitrary dimensionless parameter and β is a parameter with dimension of $[L]^4$. Under the condition q=1/2, the model reduces to BI electrodynamics while q=1 gives the linear Maxwell lagrangian. The magnetically charged black hole solution is gained using the following action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \mathcal{L}\right], \tag{2}$$

where $\kappa^2 \equiv 8\pi G$. Here we employ the magnetic monopole ansatz [6],

$$A_t = A_r = A_\theta = 0, \quad A_\phi = Q(1 - \cos\theta), \tag{3}$$

and spherical symmetry, in the form of line element

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \tag{4}$$

which in that corresponding spacetime, gives the field equation as

$$\partial_\mu(\Gamma^{q-1}F^{\mu\nu}) = 0, \quad \Gamma \equiv 1 + \frac{\beta\mathcal{F}}{q}. \tag{5}$$

Solving Eq. \ref{field} and \ref{action} gives the solution as [7]

$$F_{\theta\phi} = Q\sin\theta, \tag{6}$$

$$f(r) = 1 - \frac{2M}{r} - \frac{\kappa^2 r^2}{3\beta}\left[{}_2F_1\left(-\frac{3}{4}, -q; \frac{1}{4}; -\frac{Q^2\beta}{2qr^4}\right) - 1\right], \tag{7}$$

where Q is the magnetic charge and ${}_2F_1(a,b;c;d)$ is the hypergeometric function. In the limit of $\beta \to 0$ the solution reduces to the magnetic RN, while the limit of $q \to 1/2$ reduces it to the magnetic BI black holes [17].

3. Spacetime Properties

In this section, we choose to examine two scenarios of spacetime with q=1/2 (BI) and q=1/4. We show the plot of the metric function in Fig. 1 and Fig. 3, where the red dashed line represents the radius of the extremal horizon (r_{EH}). To further investigate how our spacetime affect lights, we examine the null radial geodesic of the spacetime inside the horizon in the Eddington-Finkelstein (EF) coordinate. We introduce a pair of null coordinate

Journal of Physics: Conference Series **2243** (2022) 012117 doi:10.1088/1742-6596/2243/1/012117

$$u \equiv t + r' \quad , \quad v \equiv t - r' \quad \text{where} \quad r' = \int f(r)^{-1}\, dr, \tag{8}$$

which give the new temporal coordinate t' as

$$t' = \begin{cases} u - r, & for\ ingoing\ photon, \\ v + r, & for\ outgoing\ photon. \end{cases} \tag{9}$$

The behaviour of the ingoing null coordinates is then visualized as a series of worldline in EF diagram (Fig. 2 and Fig. 4). We divide the spacetime into two regions: inside horizon ($r < r_{EH}$) and outside horizon ($r > r_{EH}$) where r_{EH} is indicated by the dashed red line. From a quick glance, it is clear that the region inside horizon is different with RN spacetime. The worldlines suggest that light particles are allowed to travel back outside the horizon, which mean it is physical for an orbit to exist over the mentioned two regions.

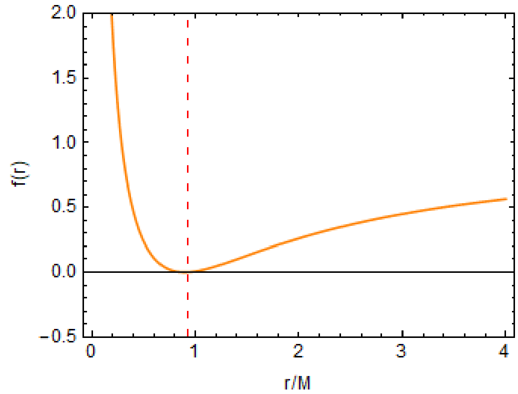

Figure 1. Plot of metric function with q=1/2, Q=1.4500 and β=0.5.

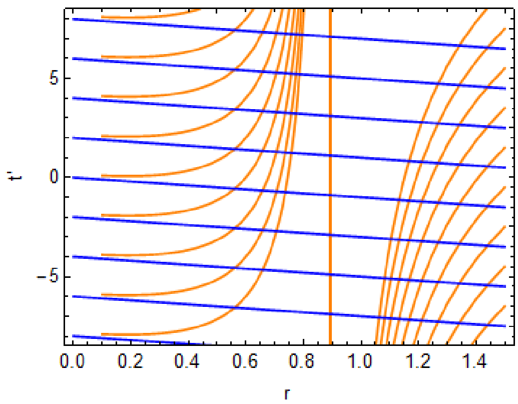

Figure 2. Plots of EF diagram with similar parameter values.

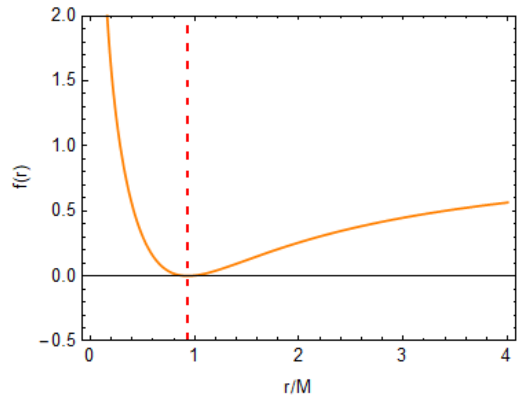

Figure 3. Plot of metric function with q=1/4, Q=1.4365 and β=0..

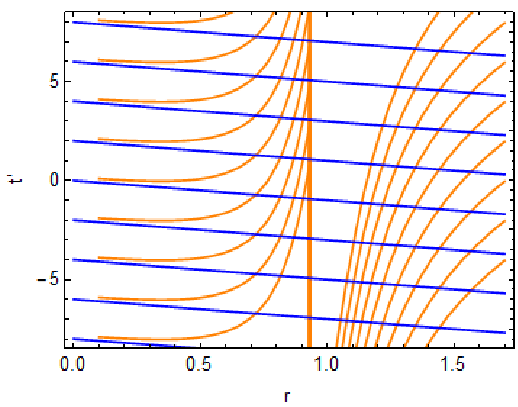

Figure 4. Plots of EF diagram with similar parameter values.

4. Geodesic Equation

Novello et al shows in their paper that in the presence of NLED, photon follows the null geodesic of its effective geometry given by [8]

Journal of Physics: Conference Series **2243** (2022) 012117 doi:10.1088/1742-6596/2243/1/012117

$$g^{\mu\nu}_{eff} = \mathcal{L}_{\mathcal{F}} g^{\mu\nu} - 4\mathcal{L}_{\mathcal{F}\mathcal{F}} F^{\mu}_{\alpha} F^{\alpha\nu}. \tag{10}$$

For the generalized BI case, it is given by

$$g^{\mu\nu}_{eff} = \left(1 + \frac{\beta F}{q}\right) g^{\mu\nu} - \frac{4\beta(q-1)}{q} F^{\mu\alpha} F_{\alpha}{}^{\nu}. \tag{11}$$

We define a conformal factor $h(r) \equiv \frac{2qr^4+\beta Q^2}{\beta(8q-7)Q^2+2qr^4}$, so then our rescaled effective line element can be written as

$$ds^2_{eff} = -f(r)dt^2 + f(r)^{-1}dr^2 + h(r)r^2 d\Omega^2.. \tag{12}$$

It has been established that the behavior of test particle with mass μ and (electric/magnetic) charge ε around compact object can be described by the geodesics equation

$$\frac{d^2x^\nu}{d\tau^2} + \Gamma^{\nu}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = -\frac{\epsilon}{\mu}F_{\sigma}{}^{\nu}\frac{dx^\sigma}{d\tau} \tag{13}$$

According to the mentioned equation, the null rays in our new line element (12) then follow the trajectories given by

$$0 = f\dot{t}^2 - f^{-1}\dot{r}^2 - hr^2\dot{\phi}^2. \tag{14}$$

Furthermore, due to the spherical symmetry and staticity, Eq. \ref{geodesic} gives two integrals of motion:

$$\dot{t} = \frac{E}{f} \quad , \quad \dot{\phi} = \frac{L}{r^2}. \tag{15}$$

where E and L are the energy and angular momentum per unit mass of the charged test particles, respectively. Inserting (15) into (14) gives the radial geodesic equation

$$\dot{r}^2 = E^2 - V_{eff} \quad , \quad V_{eff} = \frac{fL^2}{hr^2}, \tag{16}$$

where $V_{eff}(r)$ is the effective potential. We perform rescaling in which r→r/M, L→M^2/ L^2, Q→ Q/M, and β → βM, so the effective potential and the orbital equation can be cast as

$$\left(\frac{dr}{d\phi}\right)^2 = E^2 L r^4 - \frac{f}{hLr^2} = R(r). \tag{17}$$

The orbital equation R(r) can not be solved analytically due to the hypergeometric term, so is then solved numerically. We obtain the required values by observing the effective potential and use the corresponding values to plot the predicted orbit, which are discussed in next section.

5. Bound Orbit

We give the plot of V_{eff} in Fig. 5 and Fig. 7. The stability of an orbit is defined by the potential derivatives, where the local maximum ($V_{eff}''(r) < 0$) correspond to the radius of unstable circular orbit

r_{UCO}, while the local minimum of ($V_{eff}''(r) > 0$) represents the radius of Stable Circular Orbit r_{SCO}. In our case, we notice there exist two r_{UCO} and an r_{SCO}, in which the first r_{UCO} coincides with (and so is denoted as) r_{EH} (dashed red line) and the line V_{eff}=0. There is also a radius inside the region where the potential goes from positive to negative, which we refer as transition point r_t (dashed magenta line). We predict the bound orbit to lay in between the two local maxima and set the energy value (blue line in Fig. 5) such that the intersection point is rested inside the region, which we denote as r_i (dashed purple line). By denoting the outer r_{UCO} as r_u and the r_{SCO} as r_s, we can say that particles approach the mentioned region with energy level of $E^2 > V_{eff}$ and lose its energy as it travels past r_u. It is important to note that the particular energy level ($E^2 = V_{eff}(r_i)$) is chosen above $V_{eff}(r_s)$ and $V_{eff}(r_t)$ (not negative) to maintain its physicality.

Inserting all required values to R(r), we obtain the predicted orbit in Fig. 8 and Fig. 9. The first thing to notice is the orbit does cross the horizon (red dashed circle), which have stated to be physical by our EF diagram. It also can be observed that the photon travels back-to-back in three apsis: r_i, r_t, and r_{EH}, which is caused by the turn in r_{SCO} point. What strange in this situation is the fact that photon is shown to be able to reach the singularity and bounce back, continuing the orbit. While our spacetime suggest that this behaviour might be permitted, it is still a troublesome condition when it comes to avoiding the point of singularity. The possible explanation might be that lights come out in different universe. However, it is better to leave this phenomenon to be verified by future studies.

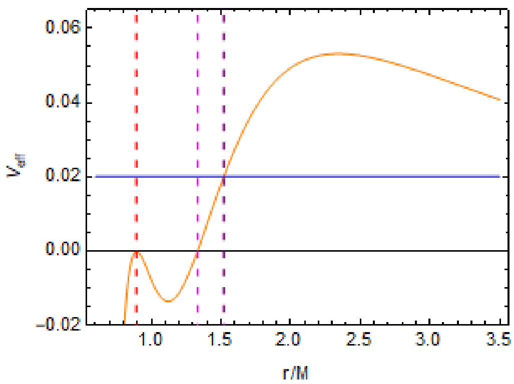

Figure 5. Plot of effective potential for q=1/2 case with L=1 and E^2=0.02.

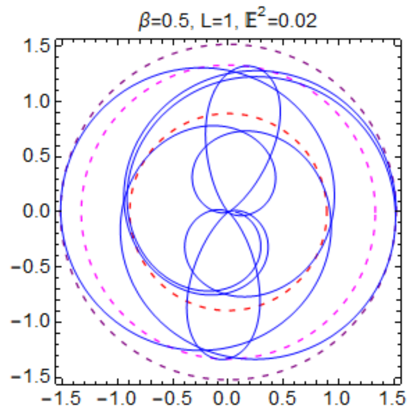

Figure 6. Plot of null bound orbit resulted from the corresponding V_{eff}.

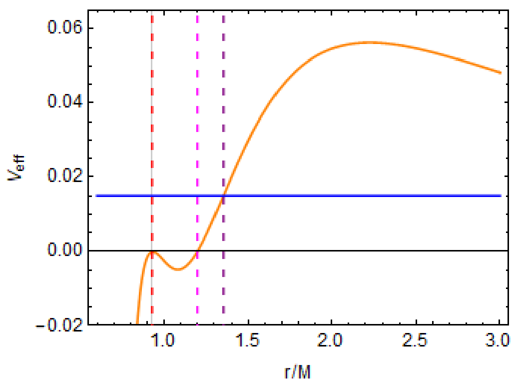

Figure 7. Plot of effective potential for q=1/4 case with L=1 and E^2=0.005.

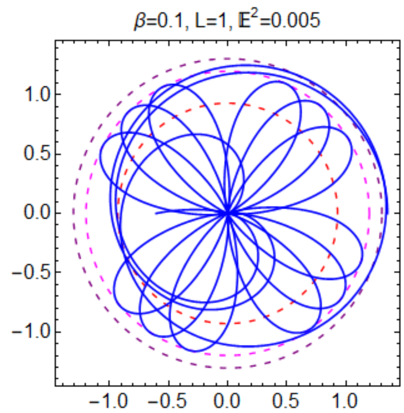

Figure 8. Plot of null bound orbit resulted from the corresponding V_{eff}.

6. Conclusion

In this article, we analyse a magnetically charged black hole with a model of nonlinear electrodynamics (NLED) which is proposed by Kruglov [15]. The model is a generalized form of Born-Infeld model, which is parametrized by two parameters: q and \beta. The condition q=1/2 reduce the model back to BI electrodynamics, while q=1 case gives the original Maxwell one. These parameters are then determine how the spacetime behave, which in our case is specified for the extremal (one horizon) scenario with two different values of q.

We examine the spacetime by introducing the EF coordinate, which leads to us obtaining the EF diagram. From our inspection, it is evident that the resulting spacetime is different with the usual RN spacetime. The region inside horizon possesses worldlines such that photon can travel back into the region outside horizon. This situation leads to assumption that for our spacetime, it is physical for the null bound orbit to rest in the two mentioned region. However, our observation in the resulted orbit comes to a bizarre territory. We manage to acquire null bound orbits, and the orbits do cross the horizon. The shape of the orbit is unusual which shows the photon going back-to-back in 3 radii instead of 2, which we suspect is caused by the bump in the r_{SCO}. What peculiar is the orbit shows light particles being able to hit the point of singularity and casually continue move in its orbit. While the orbit is physical, the action of bouncing back from singularity has not yet been confirmed to be physical, therefore we hold our judgement to be re-examined by more studies in the future.

Acknowledgments

We acknowledge grants from Universitas Indonesia through Hibah Riset PPI Q1 No.NKB-583/UN2.RST/HKP.05.00/2021.

References

[1] P. Pradhan and P. Majumdar 2011, *Phys. Lett. A* **375** 474
[2] S. Grunau and V. Kagramanova 2011, *Phys. Rev. D* **83**, 044009
[3] M. Born and L. Infeld 1934, *Proc. Roy. Soc. Lond. A* **144**, no. 852, 425
[4] W. Heisenberg and H. Euler 1936, *Z. Phys.* **98** no.11-12, 714
[5] M. Hassaine and C. Martinez 2008, *Class. Quant. Grav.* **25**, 195023
[6] K. A. Bronnikov 2001, *Phys. Rev. D* **63**, 044005
[7] S. I. Kruglov 2017, *Annals Phys.* **383**, 550
[8] M. Novello, V. A. De Lorenci, J. M. Salim and R. Klippert 2000, *Phys. Rev. D* **61**, 045001
[9] S. Fernando 2012, *ISRN Math. Phys.* **2012**, 869069
[10] E. F. Eiroa 2006, *Phys. Rev. D* **73**, 043002
[11] P. Amore, S. Arceo and F. M. Fernandez 2006, *Phys. Rev. D* **74**, 083004
[12] H. J. Mosquera Cuesta, J. A. de Freitas Pacheco and J. M. Salim 2006, *Int. J. Mod. Phys. A* **21**, 43-55
[13] R. Linares, M. Maceda and D. Martínez-Carbajal 2015, *Phys. Rev. D* **92**, no.2, 024052
[14] F. Atamurotov, S. G. Ghosh and B. Ahmedov 2016, *Eur. Phys. J. C* **76**, no.5, 273
[15] S. I. Kruglov 2017, *Mod. Phys. Lett. A* **32** no.36, 1750201
[16] A. S. Habibina and H. S. Ramadhan 2020, *Phys. Rev. D* **101**, no.12, 124036
[17] S. Fernando and D. Krug 2003, *Gen. Rel. Grav.* **35**, 129