

Cosmic String Evolution in Brane Inflation

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Abstract. There has been notable effort in recent years to obtain inflationary models from string theory. In the most well-studied scenario, brane inflation, the inflationary phase typically ends with a phase transition leading to the production of a network of cosmic (super)strings. These have different properties than ordinary cosmic strings and thus provide a potential probe to the physics of the early universe. In this letter we review recent work on the cosmological evolution of such string networks. Focusing on analytic velocity-dependent evolution models, we discuss the phenomenology of cosmic (super)string networks and obtain constraints for brane-inflation models.

1. Introduction

Recent years have seen major progress in string theory and phenomenology. The advent of branes [36] in particular has been an active area of fruitful research, which provided a link between theory and phenomenological model building. With the use of brane configurations one can now construct field theories resembling sectors of the standard model, or cosmological models of the early and late universe [29, 39]. A major success was the construction of inflationary models in string theory, with most well-studied example that of brane inflation [21, 20, 13, 22, 26, 27], where the role of the inflaton field is played by the distance between two branes or a brane and an anti-brane. The branes are initially placed at a distance apart and move towards each other due to an attractive potential arising from the exchange of bulk NS-NS and R-R modes (see for example Ref. [38]). As they come closer together, open string modes stretching between them start contributing more strongly to the inflaton potential and at a critical distance these modes become tachyonic, triggering an instability. Inflation then ends in a hybrid-type [31] fashion.

The instability at the last stages of brane inflation corresponds to a symmetry breaking phase transition related to the annihilation of the colliding branes. This leads to the production of topological defects via the Kibble mechanism, which can be identified [42, 19, 37] as lower dimensional D-branes extended in one dimension (D-strings) and fundamental F-strings, stretched over cosmological scales. F- and D- strings can bind together to form (p, q) composites, made of p F-strings and q D-strings [16]. These objects, collectively referred to

as ‘cosmic superstrings’, give rise to a network which has different properties than ordinary string networks [44]. Hence, cosmic superstrings provide a potential observational window to superstring physics.

An important distinction between cosmic superstrings and ordinary cosmic strings is that the former evolve in a higher dimensional spacetime so the strings can miss each other in the extra compact dimensions. This can dramatically affect the evolution of these objects because string collisions result in loop production, which is the main energy loss mechanism for string networks [44]. Thus, fewer string collisions would lead to a higher string number density relative to that of usual field theory strings. The first attempt to model this effect was made in Ref. [25], where an intercommuting probability-to account for the fact that strings can miss in the extra dimensions-was introduced in the simple one-scale string evolution model of [28]. In this model the string network was described by a single length-scale, the correlation length L , which is defined in terms of the string energy density ρ and the tension μ by $L = \sqrt{\mu/\rho}$. The result was an enhancement to the string density by a factor of order P^{-2} , where $P < 1$ is the string intercommuting probability. This can be as small as 10^{-3} [24], leading to an enhancement in string density of several orders of magnitude. Similar conclusions were drawn in Ref. [19] but with an enhancement of P^{-1} instead of P^{-2} , supported by flat space simulations [41, 40]. Recent simulations of string evolution in a matter- and radiation-dominated universe [3] suggest a significantly weaker dependence, namely $\rho \propto P^{-0.6^{+0.15}_{-0.12}}$, as will be discussed in section 3.

In the next section we describe a velocity-dependent one scale model, which can be used for quantitative string evolution in the presence of extra dimensions [5]. In section 3 we discuss the effect of small-scale-structure (string ‘wiggleness’) on string intercommutings and present numerical simulations of string evolution with small intercommuting probability. In section 4 we discuss cosmological constraints imposed on brane inflation models by considering monopole-like stringy objects.

2. The EDVOS Model

To have a quantitative picture of string evolution it is necessary to go beyond the one-scale model approximation. In particular, it is important to include the velocity dependence in the evolution equations because string velocities enter the loop production term, which is a crucial factor for string evolution. The velocity-dependent one-scale model of Refs. [33, 32, 34] (VOS model) has been particularly successful in three dimensions, being able to accurately fit numerical simulations throughout cosmic evolution. Here, we review the extra-dimensional velocity-dependent one-scale (EDVOS) model of Ref. [5], which is a quantitative model for higher-dimensional string evolution.

We begin by considering the Nambu-Goto action describing motion of a ‘thin’ string, tension μ , in a $(D + 1)$ -dimensional spacetime with metric $g_{\mu\nu}$. This is given by a non-linear σ -model from the two-dimensional worldsheet swept by the string to the spacetime:

$$S = -\mu \int \sqrt{-\gamma} d^2\zeta, \quad (1)$$

where $\zeta = (\zeta^0, \zeta^1)$ are the worldsheet coordinates and γ is the determinant of $\gamma_{\alpha\beta} = g_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu$, the pullback metric on the worldsheet.

We consider motion in a FRW spacetime, augmented by $D - 3$ toroidally compactified extra dimensions, with metric

$$ds^2 = N(t)^2 dt^2 - a(t)^2 d\mathbf{x}^2 - b(t)^2 d\mathbf{l}^2. \quad (2)$$

Here $t \equiv x^0$, $\mathbf{x} \equiv x^i$ with $i = 1, 2, 3$ and $\mathbf{l} \equiv x^\ell$ with $\ell = 4, 5, \dots, D$. The functions $a(t)$ and $b(t)$ are scalefactors of the three large (FRW) dimensions and the compact manifold respectively. The lapse function $N(t)$ allows us to switch from cosmic ($N(t)=1$) to conformal time τ by simply setting $N(\tau)=a(\tau)$. For stabilised extra dimensions, $b(t)$ is set to a constant.

Working in the gauge $\zeta^0 = t$, $\dot{x} \cdot x' = 0$ one can vary the action with respect to the bosonic fields x^μ to derive the equations of motion:

$$\dot{\epsilon} = -N^{-2}\epsilon \left\{ N\dot{N} + a\dot{a} \left[\dot{\mathbf{x}}^2 - \left(\frac{\mathbf{x}'}{\epsilon} \right)^2 \right] + b\dot{b} \left[\dot{\mathbf{l}}^2 - \left(\frac{\mathbf{l}'}{\epsilon} \right)^2 \right] \right\} \quad (3)$$

$$\ddot{\mathbf{x}} + \left\{ \frac{2\dot{a}}{a} - N^{-2} \left\{ N\dot{N} + a\dot{a} \left[\dot{\mathbf{x}}^2 - \left(\frac{\mathbf{x}'}{\epsilon} \right)^2 \right] + b\dot{b} \left[\dot{\mathbf{l}}^2 - \left(\frac{\mathbf{l}'}{\epsilon} \right)^2 \right] \right\} \right\} \dot{\mathbf{x}} = \left(\frac{\mathbf{x}'}{\epsilon} \right)' \epsilon^{-1} \quad (4)$$

$$\ddot{\mathbf{l}} + \left\{ \frac{2\dot{b}}{b} - N^{-2} \left\{ N\dot{N} + a\dot{a} \left[\dot{\mathbf{x}}^2 - \left(\frac{\mathbf{x}'}{\epsilon} \right)^2 \right] + b\dot{b} \left[\dot{\mathbf{l}}^2 - \left(\frac{\mathbf{l}'}{\epsilon} \right)^2 \right] \right\} \right\} \dot{\mathbf{l}} = \left(\frac{\mathbf{l}'}{\epsilon} \right)' \epsilon^{-1} \quad (5)$$

where ϵ is a scalar, the energy per unit coordinate length, defined by $\epsilon = -x'^2/\sqrt{-\gamma}$.

Varying with respect to the spacetime metric $g_{\mu\nu}$ yields the energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{Na^3b^{D-3}} \mu \int d\zeta (\epsilon \dot{x}^\mu \dot{x}^\nu - \epsilon^{-1} x'^\mu x'^\nu) \delta^{(D)}(\mathbf{x} - \mathbf{x}(\zeta, t), \mathbf{l} - \mathbf{l}(\zeta, t)), \quad (6)$$

from which we can define the energy of the cosmic string by integrating over a spacelike hypersurface $t = \text{const}$, with induced metric h and normal covector n_μ :

$$E = \int_{t=\text{const}} \sqrt{h} n_\mu n_\nu T^{\mu\nu} d^3\mathbf{x} d^{D-3}\mathbf{l} = N(t) \mu \int \epsilon d\zeta. \quad (7)$$

We can then define the string density ρ as the string energy per unit volume, in a statistical sense.

In the EDVOS model one performs a statistical averaging procedure of the equations of motion (3-5) and the time derivative of (7), by integrating over the worldsheet. In the case of stabilised extra dimensions ($\dot{b}(t) = 0$) this yields the following macroscopic equations for the evolution of the correlation length $L = \sqrt{\rho/\mu}$ and the string rms velocities in the FRW and compact dimensions, v_x and v_ℓ respectively:

$$2\frac{dL}{dt} = [(2 + w_\ell^2) + (2 - w_\ell^2) v_x^2 + (1 - w_\ell^2) v_\ell^2] HL + \tilde{c}P_{\text{eff}}v_x \quad (8)$$

$$v_x \frac{dv_x}{dt} = \frac{k_x v_x}{R} (1 - v^2) - (2 - w_\ell^2) H v_x^2 (1 - v^2) - H v_x^2 v_\ell^2 \quad (9)$$

$$v_\ell \frac{dv_\ell}{dt} = \frac{k_\ell v_\ell}{R} (1 - v^2) - (1 - w_\ell^2) H v_\ell^2 (1 - v^2) + H v_\ell^2 v_x^2. \quad (10)$$

In the above equations, $H = \dot{a}/a$ is the Hubble constant and \tilde{c} is the 'loop production efficiency' [33, 32] quantifying string intersections which lead to the production of loops. w_ℓ is a 'string orientation parameter' describing the degree to which the strings explore the extra dimensions \mathbf{l} . It is defined by $w_\ell = \sqrt{\langle \mathbf{l}'^2 / (a^2 \mathbf{x}'^2 + \mathbf{l}'^2) \rangle}$, where angled brackets denote spatial averages obtained by integrating over the worldsheet. The 'curvature parameters' k_x and k_ℓ can be expressed in terms of the velocities v_x and v_ℓ (for detailed discussion on these see Ref.[5]).

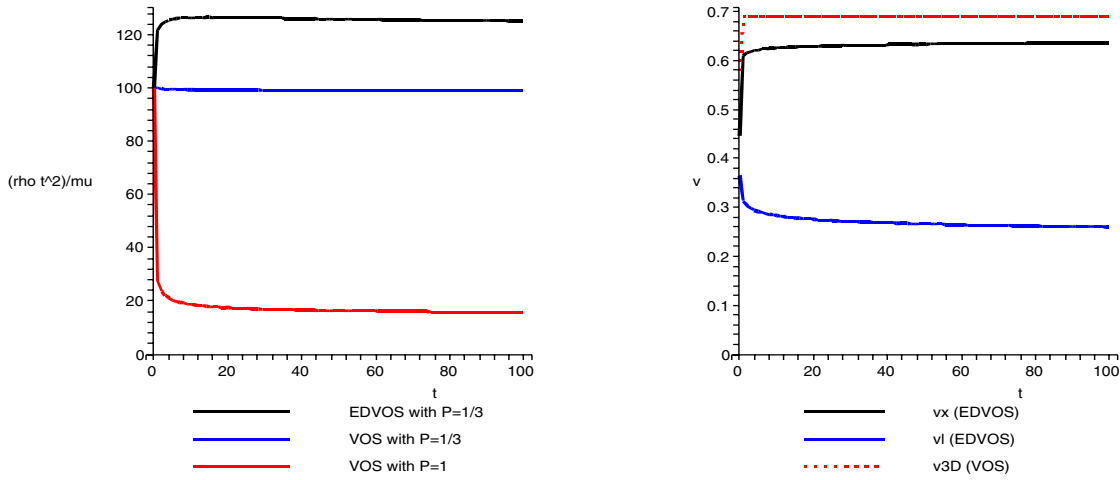


Figure 1. *Left:* Evolution of the dimensionless string density $\rho t^2/\mu$ for the EDVOS model with $P_{\text{eff}} = 1/3$ (black), the VOS model with the same P_{eff} (blue) and the VOS model for $P = 1$, i.e. purely 3D case (red). *Right:* String velocities for EDVOS and VOS models with $P_{\text{eff}} = 1/3$. The EDVOS model predicts a lower v_x , as part of the kinetic energy is taken by v_ℓ .

Finally, we have introduced an intercommuting probability $P_{\text{eff}} < 1$ to account for the fact that strings can miss each other in the presence of extra dimensions, but we have been careful to call it an *effective* probability because small-scale-structure effects can affect string interactions changing the probability dependence from P to $P_{\text{eff}} = f(P)$. Indeed, wiggly strings may have more than one opportunities to interact during each crossing time so one expects a weaker than linear dependence of P_{eff} on P . Such effects can only be quantitatively modelled by numerical simulations, and in section 3 we will summarise numerical results of Ref. [3], which show evidence for $P_{\text{eff}} \propto P^{0.3 \pm 0.1}$.

The EDVOS equations presented above are of the same form as in the 3D VOS model [34] but include corrections due to the presence of extra dimensions. In particular, the coefficients appearing in these equations depend on the string orientation parameter w_ℓ , giving rise to quantitative corrections to the 3D VOS model (Fig. 1(left)). Furthermore, by taking into account string velocities in the extra dimensions the equations demonstrate that these take a share of the total available kinetic energy, resulting in a reduction of the 3D string velocities as shown in Fig. 1(right). This suggests that cosmic superstrings may have smaller 3D velocities due to excitations in the extra dimensions. Hubble damping is very weak in these non-expanding dimensions and so it is inefficient in damping v_ℓ . Therefore, this effect is likely to be relevant for many orders of magnitude in time, until moduli stabilisation, expected to kick in after supersymmetry breaking, freezes the motion of strings in the compact directions¹.

In the above figures we have taken P_{eff} as a constant parameter to illustrate the results of applying the EDVOS, rather than the VOS, model. We have mentioned however that P_{eff} must be some function of the theoretical intercommuting probability P , which in turn can be calculated from first principles [24]. In the following section we discuss how this function can be determined from numerical simulations.

¹ For strings propagating in a warped spacetime, the warping introduces a non-trivial potential which can also stabilise the strings at one of its minima [24].

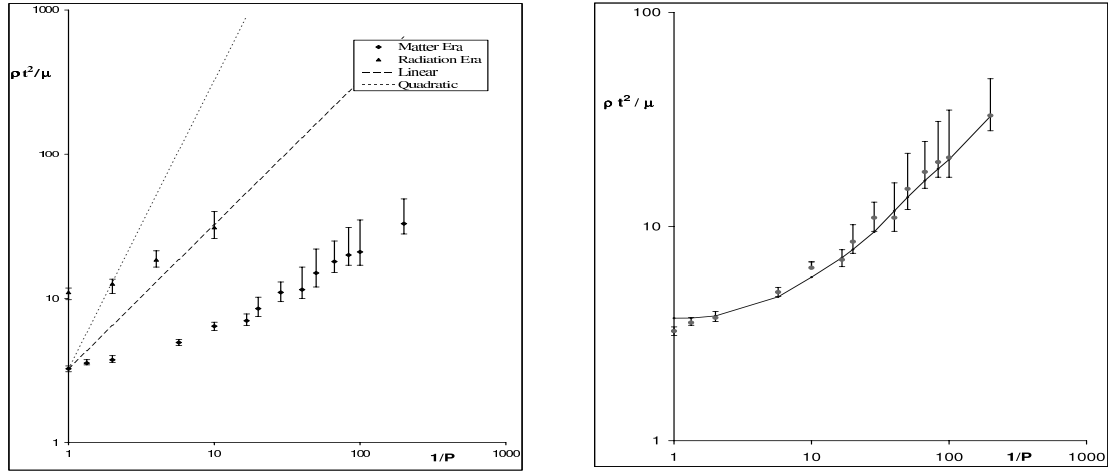


Figure 2. *Left:* Dimensionless string scaling density plotted against the inverse intercommuting probability $1/P$, for matter and radiation era runs. The constant slope part of the matter era data can be fitted by a power law with exponent $0.6^{+0.15}_{-0.12}$. The overall dependence of ρ on P is much weaker than the previously suggested $\rho \propto 1/P^2$ and $\rho \propto 1/P$ forms. *Right:* Scaling string density obtained from simulations (data points with error bars) and from the analytic two-scale model (11) (solid line).

3. Simulations in an Expanding Universe

In Ref. [3] we have performed high-resolution (16 points per correlation length) simulations of strings, with reduced intercommuting probabilities, evolving in a matter- and radiation-dominated FRW spacetime, using a modified version of the Allen-Shellard code [1]. Each of the evolved networks was given a different intercommuting probability in the range $5 \times 10^{-3} \leq P \leq 1$ and its evolution was followed for a dynamical range ($\tau_{\text{final}}/\tau_{\text{initial}}$) of order 3, taking several days of CPU time. By plotting the time evolution of the string density for different initial conditions, one can bracket the scaling solution, getting successively more accurate convergence with subsequent runs. We have thus obtained, within errors, the scaling density ρ of each of these networks, characterised by a given value of P .

Plotted in Fig. 2(left) is the dimensionless parameter $\rho t^2/\mu$ versus the inverse intercommuting probability $1/P$ for runs in the matter and radiation eras. In the former case P ranges from 5×10^{-3} to 1, but in the latter our limited dynamical range has at present only allowed us to bracket the scaling densities for probabilities in the range $0.1 \leq P \leq 1$. We see that for $P \gtrsim 0.1$, the function $\rho(1/P)$ is approximately flat, but for smaller probabilities it develops a constant slope on a log-log scale. A weighted fit gives a slope of $0.6^{+0.15}_{-0.12}$ for the matter runs, and the radiation era data are consistent with this picture, although more data points are needed to confirm the value of the slope. Comparing to the $\rho \propto P^{-2}$ and $\rho \propto P^{-1}$ forms of Refs. [25] and [41] respectively (also plotted), one sees that the enhancement of string densities due to a reduced intercommuting probability is less prominent than initially anticipated. A probability of 5×10^{-3} for example, leads to an enhancement in string density by only a factor of 10, to be contrasted with the predictions of 200 (resp. 10^4) obtained from $\rho \propto P^{-1}$ (resp. $\rho \propto P^{-2}$).

The simulations also show evidence that for $P \ll 1$ a second scale becomes important in the

problem so that the one-scale approximation becomes less and less accurate as we reduce P . Two- and three-scale generalisations of the one-scale model have already been considered in [15] and [2] respectively. Our results can be fit by a velocity dependent two-scale model, which describes the string network in terms of two length scales, the interstring distance L (related to string energy density, as before) and a persistence length ξ (corresponding to the average distance on the string beyond which string directions become uncorrelated). These obey the macroscopic evolution equations [3]

$$2\frac{dL}{dt} = 2\frac{\dot{a}}{a}L(1 + v^2) + \tilde{c}v \left(\frac{\xi}{L} - \frac{L}{\xi} \right) \quad , \quad \frac{dv}{dt} = (1 - v^2) \left(\frac{k}{\xi} - 2\frac{\dot{a}}{a}v \right) \quad (11)$$

in the notation of section 2. The persistence length ξ is taken to scale with the horizon, based on numerical evidence. The striking agreement of this model with the numerical results over the whole range of probabilities probed is illustrated in Fig. 2(right).

By introducing the function $P_{\text{eff}} = f(P)$ in the EDVOS model, one can obtain reliable results for string evolution, using the theoretical probability P as input. Note that since $f(P)$ is weaker than linear and since $P > 10^{-3}$ [24], one can ignore the dependence on the second scale (which is only important for $P \ll 1$) in most of the parameter space.

4. Cosmological Constraints from Stringy Objects

Long cosmic strings are not the only means of probing brane inflation. There are a number of other objects present in the theory which can be used to constrain brane inflation models. Ref. [7] for example obtains strong constraints on cosmic superstring networks from dilaton emission. Further, Refs. [8, 35] identify stringy objects, in particular stable cosmic loops and kinks, which can be viewed as massive monopoles from the 3D point view. In these references however, the cosmological evolution or the constraints that these objects impose are not fully investigated.

Here we discuss the cosmological constraints imposed by a similar monopole-like object, the cycloop [6]. Cycloops are closed cosmic strings (loops) wrapping 1-cycles in the compact dimensions. If the internal manifold admits non-trivial 1-cycles, these loops can become topologically trapped and behave like stable monopoles or, in closer analogy, vortons [18, 11, 14]. This gives rise to a potential monopole problem, which can be used to constrain those models of brane inflation which allow cycloops. The production and cosmological evolution of cycloops was studied in Ref. [6]. By requiring that the cycloop energy density does not exceed that of matter at the time of matter-radiation transition t_{eq} , the following constraint was obtained

$$\tilde{c}w_\ell^2 \left(\frac{t_{\text{eq}}}{L} \right)^3 \left(\frac{\mathcal{N}_s(T_s)}{\mathcal{N}_s(T_f)} \right)^{1/4} \frac{\mathcal{N}_s(T_{\text{eq}})}{\mathcal{N}(T_{\text{eq}})} \left(\frac{\mu T_s}{T_{\text{eq}} M_{pl}^2} \right) \lesssim 1, \quad (12)$$

where w_ℓ is the string orientation parameter of section 2 and M_{pl} is the Planck mass. T_{eq} , T_s and T_f are respectively the temperatures of the universe at t_{eq} , at the time where cosmic string production is peaked and at the time when cycloops, after radiating energy away, reach their minimum, topologically stable size and freeze. \mathcal{N} and \mathcal{N}_s are the effective number of degrees of freedom for energy density and entropy respectively, which arise by forming the ratio of energy to entropy density in order to ensure that we take into account changes in the number of degrees of freedom near mass thresholds [30]. For typical choices of parameters in brane inflation models this leads to the following constraint for the temperature of the universe at the time where string production is peaked:

$$T_s \leq (10^{-2} T_{\text{eq}} M_{pl}^2)^{1/3} \sim 10^9 \text{ GeV}. \quad (13)$$

This is several orders of magnitude smaller than the typical energy scale of brane inflation (GUT scale) and since string production is peaked at the time of brane collision, it rules out the majority of brane inflation models allowing cycloops. However, small regions in parameter space exist in which the cycloop density is comparable to that of matter. In these models the cycloop provides an interesting dark matter candidate.

Similar constraints have been presented recently [12] by considering D-string vortons, which are D-string loops stabilised by worldsheet currents. Refs. [9, 10] find similar monopole-like objects, namely string loops stabilised by angular momentum in the compact dimensions and by fermionic currents. It would be interesting to follow the cosmological evolution of these objects in detail and explore the constraints they impose.

5. Discussion

Cosmic (super)strings provide a potential probe to brane inflation and the physics of the early universe in general. Apart from having observable (in principle) properties, as for example tension, velocity and string number density, which could be used to obtain information about certain parameters (e.g. the string and compactification scales), they also give rise to monopole-like objects which can be used to impose strong constraints and rule out some models. We have reviewed recent work in the subject and presented analytic models that can be used for higher dimensional string evolution in this context. We have used these to obtain cosmological constraints on brane inflation by studying the evolution of monopole-like objects, namely string loops wrapping non-trivial one-cycles in a compact dimension.

There has been significant activity recently focused on the study of entangled networks where links can be created between colliding string segments to form Y-type junctions. The key issue is whether such a network reaches a scaling regime that is, one in which the string energy density stays constant with respect to that of the dominant fluid in the universe. It is crucial that such a behaviour occurs because otherwise the strings could dominate the energy density of the universe leading to an analogue of the monopole problem. Evidence is now accumulating, both from analytic [43] and numerical [17, 23] studies, pointing towards a scaling behaviour of such networks. Preliminary results [4] from analytic models which are extensions of those we presented here also support this view.

Acknowledgments

A.A. is funded by EPSRC and the Cambridge Newton Trust. The work reviewed here was done in collaboration with E.P.S. Shellard. The simulations of section 3 were performed on COSMOS supercomputer, which is supported by SGI, Intel, HEFCE and PPARC.

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