Initial conditions in LQC/mLQCs

Bao-Fei Li*, Parampreet Singh* and Anzhong Wang†,‡

*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

†Institute for Theoretical Physics & Cosmology, Zhejiang University of Technology, Hangzhou, 310032, China

‡GCAP-CASPER, Department of Physics, Baylor University, Waco, TX 76798, USA

We compare the behavior of the effective masses in the Mukhanov-Sasaki equations of the linear scalar perturbations in the dressed metric and hybrid approaches in the standard loop quantum cosmology (LQC) and the modified LQC models (mLQCs). The effective mass of the Mukhanov-Sasaki equations depend on both the specific model and the approach we study. Analyzing the behavior of the effective masses plays an important role in the choice of the initial states for the scalar perturbations. Based on the properties of the effective masses in the contracting phase, we provide the initial states of the linear perturbations in LQC and mLQCs for the dressed metric and the hybrid approaches.

Keywords: Mukhanov-Sasaki equations; loop quantum cosmology.

1. Introduction

In loop quantum cosmology(LQC), the big bang singularity in the standard cosmology is resolved and replaced with a quantum bounce, and correspondingly the inflationary paradigm, which resolves several big bang puzzles and provides initial seeds for the formation of the large scale structure in the universe, can be extended to the Planck regime by using the effective dynamics.[1,2] Several approaches have been developed in LQC to compute the primordial power spectral produced by the quantum fluctuations which propagate on quantum geometries. However, such effects become negligible soon after the quantum bounce. In particular, at the beginning of inflation, where the energy density has decreased about 12 orders in comparing that at the bounce, $\rho_i/\rho_c \simeq 10^{-12}$, the quantum spacetime can be well approximated by a classical one, whereby the standard relativistic cosmology follows. Two of the most studied approaches are the dressed metric approach[3–5] and the hybrid approach.[6–9] Although these two approaches are formulated in different ways, the resulting equations of motion for the scalar modes in both approaches can be cast into the form

$$\nu_k^{''} + \left(k^2 + m_{\text{eff}}\right) \nu_k = 0, \tag{1}$$

here m_{eff} denotes the effective mass which accounts for the differences between two approaches. Similar to the situation in the classical cosmology, the initial condition problem persists in LQC in the form that up to now there is no consensus on whether the initial states should be imposed in the Planck regime or in the contracting phase where the background dynamics becomes classical again. In this paper, we take the viewpoint to impose the initial states in the deep contracting phase. As a result, we focus on the behavior of the effective mass in the deep contracting phase. Besides, in addition to the standard LQC, we also investigate the behavior of the effective mass in two modified LQC models (mLQCs) in order to study the potential effects of the quantization ambiguities on the observables.

The said mLQCs originate from a separate treatment of the Lorentz term in the classical Hamiltonian constraint in the spatially flat FLRW universe.[10] Two of the mLQCs which attract attention recently are the so-called mLQC-I and mLQC-II. Both the background dynamics and the primordial power spectra of these two mLQCs have been extensively studied in the literature.[11–17] From these studies, we learn that the background dynamics in mLQC-II is qualitatively similar to that in LQC while the contracting phase of mLQC-I has distinctive feature which is characterized by an emergent cosmological constant and a rescaled Newton's constant. When applying the dressed metric and the hybrid approaches to the mLQCs, Planck scale magnitude of the power spectrum is observed in the infrared(IR) regime of mLQC-I with the dressed metric approach while in the hybrid approach, the magnitudes of the power spectra in mLQCs and LQC become comparable. As a result, the modified LQC models also provide good examples to compare the two approaches. In the following, we compare the effective masses in LQC, mLQC-I and mLQC-II, respectively, in the dressed metric and the hybrid approaches, and from the properties of the effective masses in the contracting phase, we seek the appropriate initial states to be imposed during this phase. For more details, we refer readers to Ref. 17.

2. The initial states in LQC/mLQCs in the dressed metric approach

In the dressed metric approach, the quantum fluctuations are described as propagating on a quantum background spacetime whose geometry can be well approximated by a manifold with a dressed metric for the sharply peaked states. Using the effective dynamics, it can be shown that the effective mass in (1) takes the same form as in the classical perturbation theory while the difference originates from two sources.[15] Firstly, the background quantities in $m_{\rm eff}$ should come from the effective dynamics governed by the effective Hamiltonian constraint in LQC. Secondly, the regularization of the conjugate momentum of the scale factor in $m_{\rm eff}$ should be also consistent with the effective Hamiltonian constraint for the background dynamics. With the above two points in mind, we impose the kinetic-energy-dominated initial conditions for the background evolution at the bounce and study the behavior of the effective mass for the chaotic and the Starobinsky potentials. In practice, one can define the comoving Hubble radius as,[17]

$$\lambda_H^2 = -\frac{1}{m_{\rm eff}}. \tag{2}$$

When the wavelength of the relevant modes is larger than λ_H^2, the mode function is oscillating. On the other hand, when the wavelength of the relevant modes are less than λ_H^2, the mode function is exponentially decaying or growing.

From our numerical results, we find the behavior of the comoving Hubble radius in mLQC-II is very similar to that in LQC. Its evolution is almost symmetric with respect to the bounce point. During the deflationary stage in the contracting phase, the comoving Hubble radius keeps increasing until it reaches positive infinity at some time $t = t_i < 0$ where a'' vanishes. After $t = t_i$, it is the intermediate regime where the comoving Hubble radius becomes negative and correspondingly all the modes are oscillating in this regime. Near the bounce, the comoving Hubble radius becomes positive and reaches its minimal right at the bounce, which corresponds to the characteristic wavelength in LQC/mLQC-II. As a result, we find the Bunch-Davies vacuum can be chosen in the intermediate regime of the contracting phase where $\lambda_H^2 < 0$ in LQC and mLQC-II. On the other hand, we find a different behavior of λ_H^2 in the contracting phase of mLQC-I. In particular, λ_H^2 is always positive in the contracting phase and it can be well approximated by $\eta^2/2$ with η denoting the conformal time in the quasi de Sitter regime of the contracting phase. As a result, we can choose the de Sitter state as the initial state in the contracting phase. The above analysis on the behavior of the comoving Hubble radius is robust with respect to the choice of the inflaton potentials and the regularization of the conjugate momentum of the scale factor in the effective mass as long as the bounce is dominated by the kinetic energy of the inflaton field.

3. The initial states in LQC/mLQCs in the hybrid approach

The application of the hybrid approach to the mLQCs is studied in detail in[16] where the equation of motion for the scalar perturbations are given explicitly. Using the effective mass given in the mode equation, we study its properties in the contracting phase when the initial conditions of the background dynamics are chosen the same as in the dressed metric approach. From our numerical results, we find the evolution of the comoving Hubble radius is generally not symmetric with respect to the bounce in any model and it also depends on the specific inflationary potential. Surprisingly, with the chaotic potential, the comoving Hubble radius behaves similarly in all three models. It increases monotonically from some negative value in the deep contracting phase and tends to positive infinity at time $t = \tilde{t}_i < 0$ where $a'' = 0$. Afterwards, it remains negative until the onset of inflation in the expanding phase. This implies the effective mass is always negative near the bounce. As a result, with the chaotic potential, one can choose the BD vacuum in the deep contracting regime or near the bounce in all three models. However, since the comoving Hubble radius becomes infinity at $t = \tilde{t}_i$, choosing BD vacuum at different times, either in the deep contracting regime or near the bounce is expected to give different power spectra in the IR regime due to particle creation effect.

On the other hand, with the Starobinsky potential, we find the comoving Hubble radius has similar behavior in LQC and mLQC-II. It remains negative throughout the contracting phase until the onset of inflation in the expanding phase. As a result, the choice of the BD vacuum can be made either in the deep contracting phase or near the bounce and the resulting power spectra are expected not to be significantly different from one to another. In contrast, in mLQC-I, although the comoving Hubble radius is still negative near the bounce, there exists a finite time interval in the contracting phase where it become positive. In the deep contracting phase, it becomes negative again. So similar to the case of the chaotic potential, imposing the BD vacuum at different times, either in the deep contracting phase or near the bounce is expected to yields different power spectra in the IR regime.

If we compare different approaches for the same model, the most striking difference lies in mLQC-I where the choice of the de Sitter state in the contracting phase in the dressed metric approach yields a Planck scale magnitude of the power spectra in the IR regime, while the choice of the BD vacuum in the deep contracting phase in the hybrid approach highly suppresses the magnitude of the resulting power spectrum in the IR regime. This serves as a concrete example where the different approaches can result in different qualitative behaviors in the power spectra.

4. Summary

We study the behavior of the effective masses in the equation of motion for the scalar mode in LQC and mLQCs in the dressed metric and the hybrid approaches. With the same kinetic-energy-dominated initial conditions for the background dynamics, we find the properties of the comoving Hubble radius in the contracting phase. In the dressed metric approach, the comoving Hubble radius has similar behavior for the chaotic and the Starobinsky potentials. One can choose the BD vacuum in the deep contracting phase of LQC and mLQC-II and the de Sitter state in the contracting phase of mLQC-I. On the other hand, the properties of the comoving Hubble radius in the hybrid approach depends on the specific inflationary potentials. One common feature of the comoving Hubble radius for all the models is that it is negative near the bounce and we find it appropriate to choose the BD vacuum in the hybrid approach for all three models in the deep contracting phase.

We are grateful to Robert Brandenberger and David Wands for valuable comments and helpful discussions. We thank Javier Olmedo and Tao Zhu for various discussions. This work is supported by NSF grant PHY-1454832.

References

1. A. Ashtekar and P. Singh, *Loop Quantum Cosmology: A Status Report*, Class. Quant. Grav. **28**, 213001 (2011).
2. I. Agullo and P. Singh, *Loop Quantum Cosmology*, in Loop Quantum Gravity: The First 30 Years, edited by A. Ashtekar and J. Pullin (World Scientific, Singapore, 2017).

3. I. Agullo, A. Ashtekar and W. Nelson, *Quantum gravity extension of the inflationary scenario*, Phys. Rev. Lett. **109**, 251301 (2012).
4. I. Agullo, A. Ashtekar and W. Nelson, *Extension of the quantum theory of cosmological perturbations to the Planck era*, Phys. Rev. D**87**, 043507 (2013).
5. I. Agullo, A. Ashtekar and W. Nelson, *The pre-inflationary dynamics of loop quantum cosmology: Confronting quantum gravity with observations*, Class. Quantum Grav. **30**, 085014 (2013).
6. L. Castelló Gomar, M. Fernández-Méndez, G. A. Mena Marugan and J. Olmedo, *Cosmological perturbations in hybrid loop quantum cosmology: Mukhanov-Sasaki variables*, Phys. Rev. D**90**, 064015 (2014).
7. M. Fernández-Méndez, G. A. Mena Marugán and J. Olmedo, *Hybrid quantization of an inflationary universe*, Phys. Rev. D**86**, 024003 (2012).
8. M. Fernández-Méndez, G. A. Mena Marugán and J. Olmedo, *Hybrid quantization of an inflationary model: The flat case*, Phys. Rev. D**88**, 044013 (2013).
9. L. Castelló Gomar, M. Martín-Benito, G. A. Mena Marugán, *Gauge-invariant perturbations in hybrid quantum cosmology*, JCAP 1506 (2015) 045.
10. J. Yang, Y. Ding and Y. Ma, *Alternative quantization of the Hamiltonian in loop quantum cosmology II: Including the Lorentz term*, Phys. Lett. B**682** (2009) 1.
11. B.-F. Li, P. Singh, A. Wang, *Towards cosmological dynamics from loop quantum gravity*, Phys. Rev. D**97**, 084029 (2018).
12. B.-F. Li, P. Singh, A. Wang, *Qualitative dynamics and inflationary attractors in loop cosmology*, Phys. Rev. D**98**, 066016 (2018).
13. I. Agullo, *Primordial power spectrum from the Dapor-Liegener model of loop quantum cosmology*, Gen. Relativ. Grav. **50** (2018) 91.
14. B.-F. Li, P. Singh, A. Wang, *Genericness of pre-inflationary dynamics and probability of the desired slow-roll inflation in modified loop quantum cosmologies*, Phys. Rev. D**100**, 063513 (2019).
15. B.-F. Li, P. Singh, A. Wang, *Primordial power spectrum from the dressed metric approach in loop cosmologies*, Phys. Rev. D**100**, 086004 (2020).
16. B.-F. Li, J. Olmedo, P. Singh, and A. Wang, *Primordial scalar power spectrum from the hybrid approach in loop cosmologies*, Phys. Rev. D**102**, 126025 (2020).
17. B.-F. Li, P. Singh, and A. Wang, *Phenomenological implications of modified loop cosmologies: An overview*, Front. Astron. Space Sci. 8 (2021) 701417.