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DETERMINATION OF $|V_{cb}|$

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Abstract

The present status of our knowledge of the magnitude of the quark mixing parameter V_{cb} is reviewed, with particular emphasis on the factors affecting experimental and theoretical errors and on prospects for a more precise determination.

1 Introduction

In the framework of the Standard Model, the quark sector is characterised by a rich pattern of flavour-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This report focuses on the quark mixing parameter $|V_{cb}|$.

Two different methods are used to extract this parameter from data: the **exclusive** measurement, where $|V_{cb}|$ is extracted by studying exclusive $B \to$

$D^\star \ell\nu$ and $B \to D\ell\nu$ decay processes; and the **inclusive** measurement, which uses the semileptonic width of b-hadron decays. Theoretical estimates play a crucial role in extracting $|V_{cb}|$, and an understanding of their uncertainties is very important.

2 Exclusive $|V_{cb}|$ determination

The exclusive $|V_{cb}|$ determination is obtained studying the $B \to D^\star \ell\nu$ and $B \to D\ell\nu$ decays, using Heavy Quark Effective Theory (HQET). HQET predicts that the differential partial decay width for $B \to D^\star \ell\nu$ process, $d\Gamma/dw$, is related to $|V_{cb}|$ through:

$$\frac{d\Gamma}{dw}(B \to D^\star \ell\nu) = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \mathcal{K}(w)\mathcal{F}(w)^2,$$

where w is the inner product of the B and $D^\star$ meson 4-velocities, $\mathcal{K}(w)$ is a known phase-space factor, and the form factor $\mathcal{F}(w)$ is generally expressed as the product of a normalisation constant, $\mathcal{F}(1)$, and a function, $g(w)$, constrained by dispersion relations [1].

The analytical expression of $\mathcal{F}(w)$ is not known a-priori. All recent published results use a non-linear shape for $\mathcal{F}(w)$, approximated with an expansion near $w = 1$ [2]. $\mathcal{F}(w)$ is parameterised in terms of the variable ρ^2, which is the slope of the form factor at zero recoil given in [2].

The decay $B \to D^\star \ell\nu$ has been studied in experiments performed at center-of-mass energies equal to the $\Upsilon(4S)$ mass and the Z^0 mass. At the $\Upsilon(4S)$, experiments have the advantage that the w resolution is quite good. However, they have more limited statistics near $w = 1$ in the decay $\overline{B}^o \to D^{\star +}\ell\nu$, because of the lower reconstruction efficiency of the slow pion, from the $D^{\star +} \to \pi^+ D^0$ decay. The decay $B^- \to D^{\star 0}\ell\overline{\nu}$ is not affected by this problem. In addition, kinematic constraints enable $\Upsilon(4S)$ experiments to identify the final state, including the $D^\star$, without a large contamination from the poorly known semileptonic decays including a hadronic system heavier than $D^\star$, commonly identified as $D^{\star\star}$. At LEP, B's are produced with a large momentum (about 30 GeV on average). This give a relatively poor w resolution and limited physics background rejection capabilities. By contrast, LEP experiments benefit from an efficiency that is only mildly dependent upon w.

Experiments determine the product $(\mathcal{F}(1) \cdot |V_{cb}|)^2$ by fitting the measured

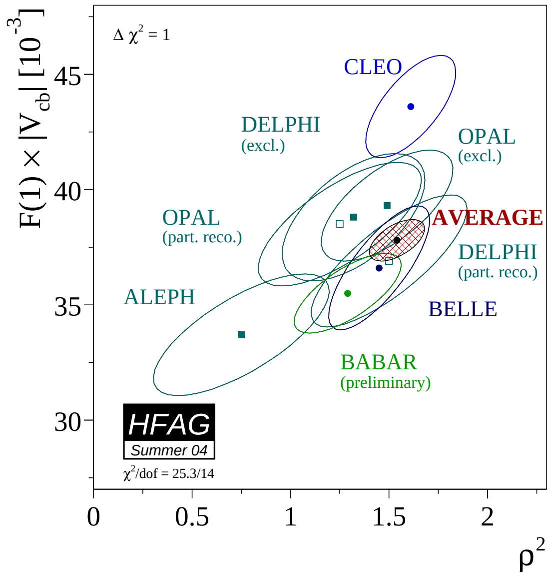

Figure 1: *The error ellipses for the corrected measurements and world average for $\mathcal{F}(1)|V_{cb}|$ vs ρ^2. The ellipses are the product between the 1 σ error of $\mathcal{F}(1)|V_{cb}|$, ρ^2, and the correlation between the two.*

$d\Gamma/dw$ distribution. Averaging [3] all published results [4] we get:

$$\mathcal{F}(1)|V_{cb}| = (37.8 \pm 0.9) \times 10^{-3}$$

and

$$\rho^2 = 1.54 \pm 0.14$$

with a χ^2 per degree of freedom of 23.5/14. The error ellipses for the corrected measurements and for the world average are shown in Fig.1.

There are several different corrections to the infinite mass value $\mathcal{F}(1) = 1$ [5]. Estimates of these corrections have been performed with OPE sum rules [6], and with an HQET based lattice gauge calculation [7]. The central values obtained with both methods are similar. Consequently, we use $\mathcal{F}(1) = 0.91 \pm 0.04$ [8], from which we get $|V_{cb}| = (41.5 \pm 1.0_{\rm exp} \pm 1.8_{\rm theo}) \times 10^{-3}$, where the dominant error is theoretical.

The study of the decay $B \to D\ell\nu$ is challenging both from the theoretical and experimental point of view. The differential decay rate for $B \to D\ell\nu$ can be expressed as:

$$\frac{d\Gamma_D}{dw}(B \to D\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\mathcal{K}_\mathcal{D}(w)\mathcal{G}(w)^2,$$

where w is the inner product of the B and D meson 4-velocities, $\mathcal{K}_\mathcal{D}(w)$ is the phase space, and the form factor $\mathcal{G}(w)$ is generally expressed as the product of a normalisation factor, $\mathcal{G}(1)$, and a function, $g_D(w)$, constrained by dispersion relations [1].

The strategy to extract $\mathcal{G}(1)|V_{cb}|$ is identical to that used for the $B \to D^\star\ell\nu$ decay. However, $\mathcal{G}(1)$ is calculated with less accuracy than $\mathcal{F}(1)$ [9] [10], and $d\Gamma_D/dw$ is more heavily suppressed near $w = 1$ than $d\Gamma_{D^*}/dw$, due to the helicity mismatch between initial and final states. This channel is also hard to isolate from the dominant background $B \to D^\star\ell\nu$, as well as from fake D–ℓ combinations. Thus, the extraction of $|V_{cb}|$ from this channel is less precise than the one from the $B \to D^\star\ell\nu$ decay. Nevertheless, the $B \to D\ell\nu$ channel provides a consistency check.

Belle [11] and ALEPH [4] studied the $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ channel. CLEO [12] studied both $B^+ \to D^0\ell^+\overline{\nu}$ and $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ decays. Averaging all data [3], we get $\mathcal{G}(1)|V_{cb}| = (42.0 \pm 3.7) \times 10^{-3}$ and $\rho_D^2 = 1.15 \pm 0.16$, where ρ_D^2 is the slope of the form factor at zero recoil given in [2]. Using $\mathcal{G}(1) = 1.04 \pm 0.06$, we get $|V_{cb}| = (40.4 \pm 3.6_{\rm exp} \pm 2.3_{\rm theo}) \times 10^{-3}$, consistent with the value extracted from $B \to D^\star\ell\nu$ decay, but with a larger uncertainty.

3 $|V_{cb}|$ determination from inclusive B semileptonic decays

Alternatively, $|V_{cb}|$ can be extracted from the inclusive branching fraction for semileptonic b hadron decays $\mathrm{B}(B \to X_c\ell\nu)$ [13] [14]. Several studies have shown that the spectator model decay rate is the leading term in a well-defined expansion controlled by the parameter $\Lambda_{\rm QCD}/m_b$. Non-perturbative corrections to this leading approximation arise only to order $1/m_b^2$.

The coefficients of the $1/m_b$ power terms are expectations values of operators that include non-perturbative physics. There are two ways [15] [16] [13] [14] to handle the energy scale μ used to separate long-distance from short-distance physics. HQET is most commonly renormalised in a mass-independent scheme, thus making the quark masses the pole masses of the underlying theory (QCD). The second group of authors prefer the definition of the non-perturbative operators using a mass scale $\mu \approx 1$ GeV.

The corresponding equations for the semileptonic width can be found in [13] [17] and [25].

4 HQE and moments in semileptonic decays

Experimental determinations of the HQE parameters are important in several respects. Non-calculable quantities are parametrised in terms of expectation values of hadronic matrix elements, which can be related to the shape (moments) of inclusive decay spectra. Furthermore, redundant determinations of these parameters may uncover inconsistencies.

CLEO [18] determines the parameter $\bar{\Lambda}$ from the first moment of the γ energy in the decay $b \to s\gamma$, which gives the average energy of the γ emitted in this transition, using the formalism of [17].

Babar, CLEO and DELPHI performed moments measurements the hadronic mass M_X^2 spectrum. Babar measures up to the fourth moment of this distribution, DELPHI up to the third moment.

Babar [21] and CLEO [26] explored the moments of the hadronic mass M_X^2 as a function of the lepton momentum cuts. CLEO performs a fit for the contributions of signal and backgrounds to the full three-dimensional differential decay rate distribution as a function of the reconstructed quantities q^2, M_X^2, $\cos\theta_{W\ell}$. BaBar uses a sample where the hadronic decay of one B is fully reconstructed and the charged lepton from the other B is identified. In this case the main sources of systematic errors are the uncertainties related to the detector modelling and reconstruction. Moments of the M_X distribution without an explicit lepton momentum cut have been extracted from DELPHI data [24] and give consistent results.

The shape of the lepton spectrum provides further constraints on OPE. Moments of the lepton momentum with a cut $p_\ell^{CM} \geq 1.5$ GeV/c have been measured by the CLEO collaboration [28]. Babar [22] extract up to the third moment of this distribution, using a low momentum cut of $p_\ell^{CM} \geq 0.6$ GeV/c. Moments of the lepton momentum without an explicit lepton momentum cut have been extracted from DELPHI data [24] and give consistent results.

The results are compared with theory and they are consistent.

Babar [23] determine the non-perturbative parameters and $|V_{cb}|$ simultaneously from a fit to the moments of the hadronic-mass and electron-energy distributions from B($B \to X_c \ell \nu$) using the calculation in Ref. [25]. This fit yields significantly improved measurements of the inclusive branching fraction B($B \to X_c \ell \nu$) and $|V_{cb}|$. Using Babar only data, we get [23]:

$$|V_{cb}|_{\rm incl} = (41.4 \pm 0.4_{\rm exp} \pm 0.4_{\rm HQE} \pm 0.6_{\rm theo}) \times 10^{-3}$$

where the first error is experimental, and the second is from the measured value of the moments assumed to be universal up to higher orders. The third error is from $1/m_b^4$ corrections and from the ambiguity in the α_s scale definition. The error on the average b-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching ratio.

5 Conclusions

The values of $|V_{cb}|$ obtained both from the inclusive and exclusive method agree within errors. The value of $|V_{cb}|$ obtained from the analysis of the $B \to D^\star \ell \nu$ decay is:

$$|V_{cb}|_{\rm exclusive} = (41.5 \pm 1.0_{\rm exp} \pm 1.8_{\rm theo}) \times 10^{-3} \ ,$$

where the first error is experimental and the second error is from the $1/m_Q^2$ corrections to $\mathcal{F}(1)$. The value of $|V_{cb}|$, obtained from inclusive semileptonic branching fractions is:

$$|V_{cb}|_{\rm incl} = (41.4 \pm 0.4_{\rm exp} \pm 0.4_{\rm HQE} \pm 0.6_{\rm theo}) \times 10^{-3},$$

where the first error is experimental, the second error is from the measured HQE values, and the last is from $1/m_b^4$ corrections and α_s.

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