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Wiener Filter Deconvolution for Analog Signals

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Abstract

This project explores the use of Wiener deconvolution to recover an original signal that has been distorted by a known transfer function and by noise. A simulated Gaussian pulse was used as the test signal, and a transfer function was applied in the frequency domain to model the system distortion. Controlled noise was then introduced to approximate real-world signal degradation. A Wiener filter was implemented to reverse the effects of the transfer function while minimizing the influence of noise. The recovered signal was compared with the original pulse to evaluate the effectiveness of the filter. The results demonstrate that Wiener deconvolution offers a stable and effective approach to signal recovery, balancing complete transfer function inversion and noise suppression.

1 Introduction

In many analog systems, the collected signals are often distorted by the system's inherent transfer function and further degraded by noise. As a result, the recorded data do not directly reflect the true underlying signal. Accurate signal recovery is critical for diagnostics, control, and analysis in these systems. This project investigates a method for reconstructing the original signal by simulating a known input, applying a measured transfer function, introducing controlled noise, and then recovering the signal using a Wiener deconvolution filter. The goal is to assess the filter's ability to reverse system distortion while remaining robust in the presence of noise.

2 Motivation

Many instruments that collect data, such as the Wall Current Monitor, record signals that have been altered by the inherent transfer function of the system. Consequently, the resulting data often contain distortions from both the system response and external noise. The objective of this project is to reconstruct or estimate the original undistorted signal. To do this, a filter is developed that utilizes the system transfer function to recover the underlying input signal with minimal distortion.

3 Overview of the Process

This project aims to simulate and evaluate a signal recovery process using a known input signal and an instrument's measured transfer function. A Gaussian-like pulse is first generated to represent the original undistorted signal. A transfer function, representing the system response, is derived and applied to this pulse to simulate the effect of the system on the signal. Controlled noise is then introduced to approximate real-world measurement conditions. To recover the original signal, a Wiener filter is designed and applied, taking advantage of the known transfer function and noise characteristics. The performance of the filter is assessed by comparing the recovered signal with the original simulated pulse.

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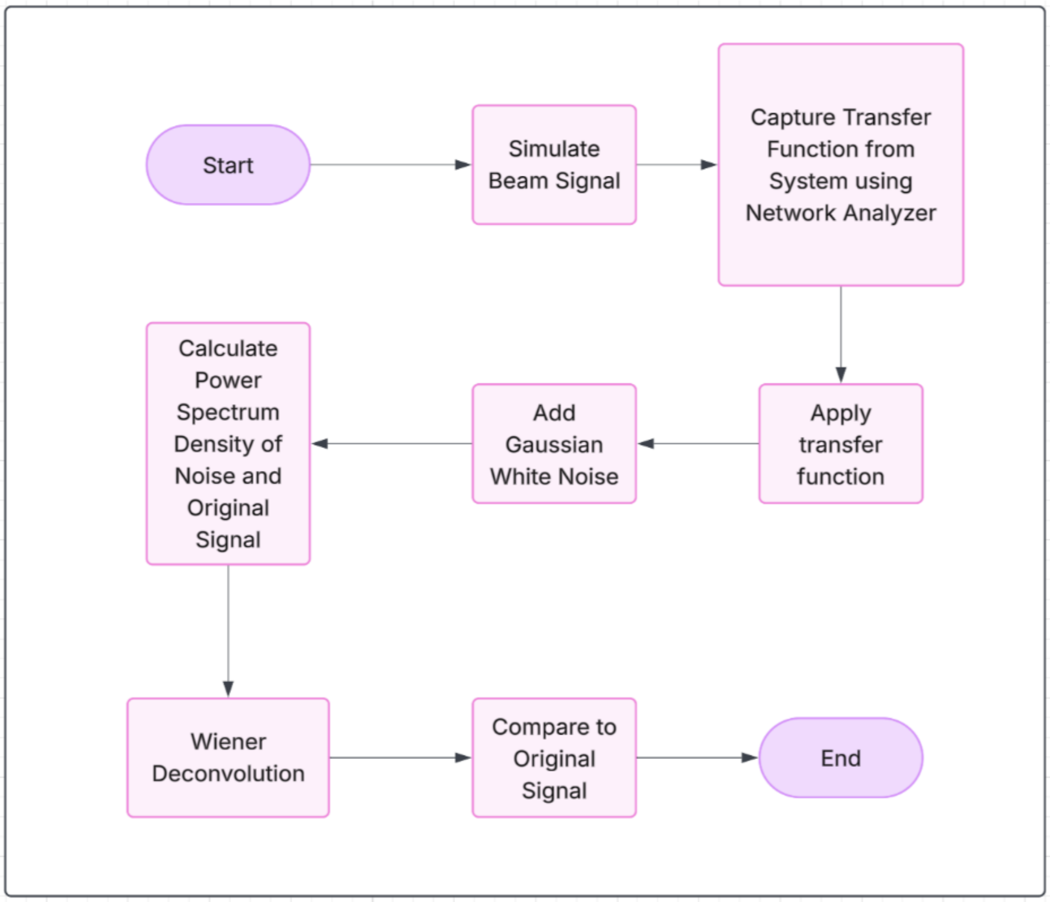

Figure 1: Overview of Project Timeline.

4 Tools

4.1 File Types

This project primarily utilized .s2p and .csv file formats. Touchstone .s2p files, a standard in radio frequency (RF) engineering, were used to store S-parameter data representing the frequency-domain behavior of two-port networks. These files were parsed using the scikit-rf (skrf) library to extract the S21 transmission coefficient, which served as the system transfer function. Additionally, .csv files were employed to store and manage time-domain and frequency-domain signal data for further analysis and visualization.

4.2 Physical Components

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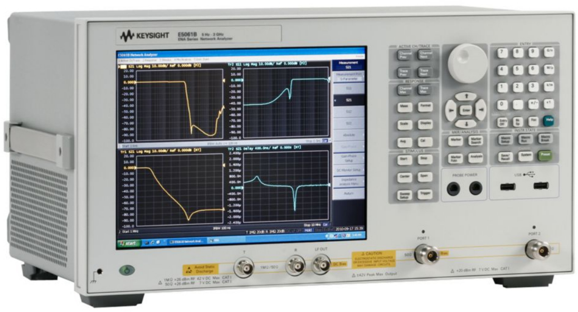

Figure 2: Image of Vector Network Analyzer.

Figure 3: Image of Signal Analyzer.

4.3 Libraries

The following Python libraries were used to support signal simulation, processing, visualization, and RF data analysis throughout the project:

- NumPy was used for efficient numerical operations, including array manipulation, vectorized computations, and synthetic signals such as Gaussian pulses.
- Pandas provided structured data handling and analysis tools that make it possible to conveniently store, inspect, and manipulate intermediate and final datasets, including time-domain and frequency-domain representations.
- Matplotlib was utilized to generate visualizations of signals in various processing stages. Time-domain waveforms, frequency spectra, and recovery comparisons were plotted to support qualitative analysis and presentation of results.
- SciPy FFT (Fast Fourier Transform Module) was used to convert signals between the time and frequency domains. This transformation was essential for applying frequency-domain filtering techniques, such as convolution with a transfer function or Wiener filter implementation.
- scikit-rf (skrf) was used to import and process S-parameter data from touchstone (.s2p) files. Specifically, it allowed extraction of the S21 transmission coefficient, which was used to construct the transfer function representing the system's frequency response.

5 Simulating a Signal

To simulate beam passing through the instrument, a Frequency Modulated Train of Gaussian Pulses was created. The following equation was used:

$$x(t) = A \cdot \exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right) \cdot \left[1 + \cos\left(2\pi f_c (t - t_0)\right)\right]$$

The following settings were used:

- amplitude = 1.0
- standardDeviation = 30e-9
- sampleRate = 200e6
- totalTime = 300e-9
- numPoints = 6000

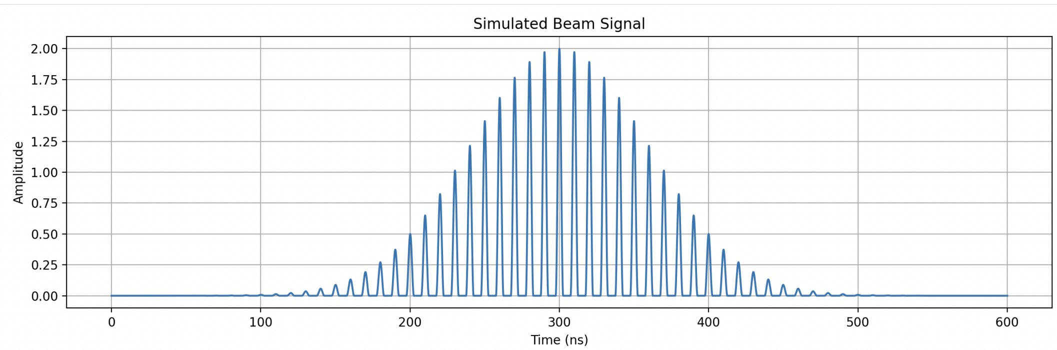

Figure 4: Image of Frequency Modulated Gaussian Pulse.

6 Adding Noise

Gaussian White Noise was added to the original signal with a Noise Standard Deviation of 0.1. Gaussian Noise was chosen because its Power Spectral Density is flat and all the noise is normally distributed.

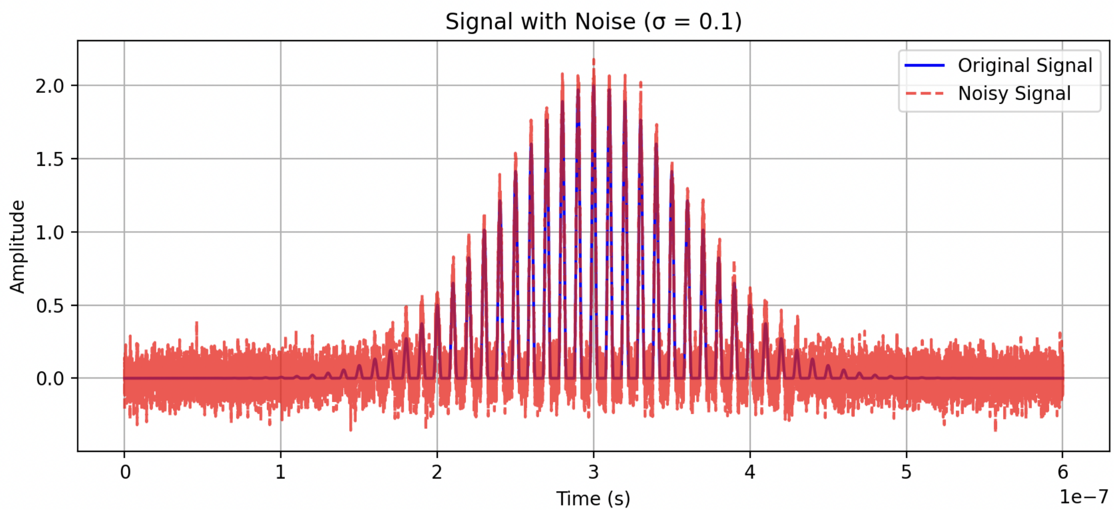

Figure 5: Before and After Added Noise.

7 Transfer Function

7.1 Collection

The system used as a case study during this investigation was the Wall Current Monitor (WCM). The Transfer Function from the WCM was obtained using a signal analyzer and a vector network analyzer. To deal with transfer functions, Signals and Systems [1] was used. The Transfer Function looks as follows:

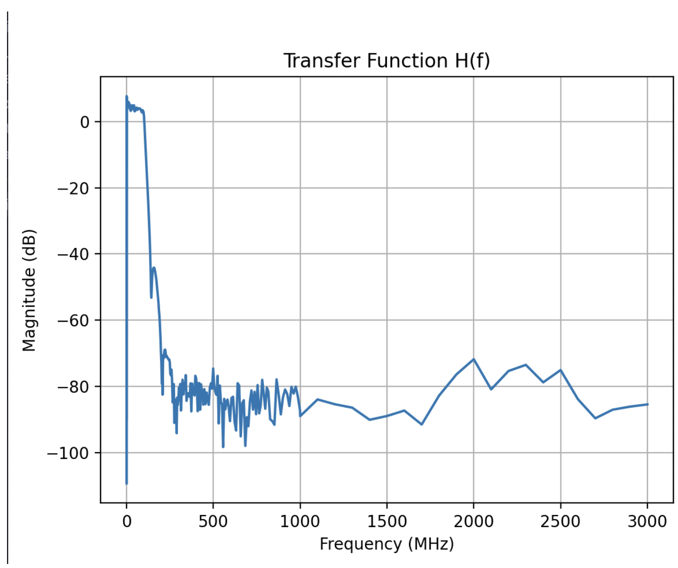

Figure 6: Image of Transfer Function.

7.2 Implementation

In the signal processing pipeline, the system's transfer function was applied in the frequency domain. First, the Fast Fourier Transform (FFT) of the simulated Gaussian pulse was computed to convert the signal from the time domain to the frequency domain. The resulting spectrum was then multiplied by the transfer function, which had been derived from the S-parameter data. This operation simulates how the system modifies the input signal in the frequency domain. Finally, the inverse FFT (IFFT) was applied to convert the modified signal back to the time domain, yielding the system-distorted version of the original pulse.

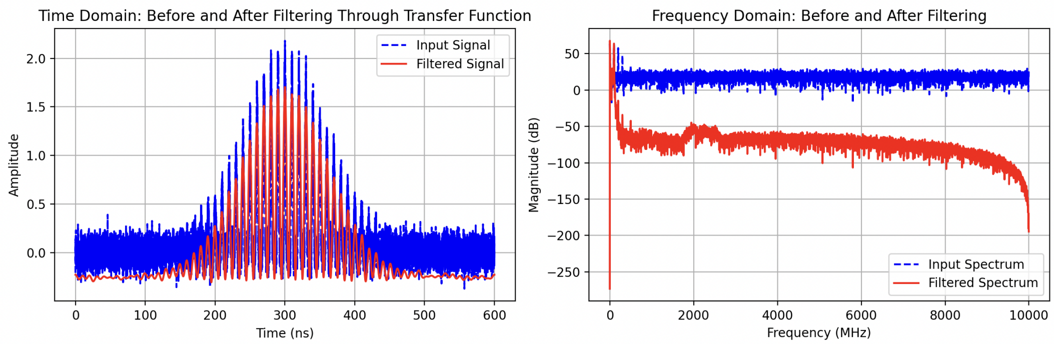

Figure 7: Image of Signal Before and After Transfer Function.

8 Wiener Filter

8.1 Choosing the Filter

The goal was to design a filter capable of reversing the effects of the transfer function while remaining stable and effective in the presence of noise. A key requirement was that the filter should not amplify noise or become unstable when applied to real-world signals. Given these criteria, the Wiener filter emerged as a strong candidate due to its ability to suppress noise while preserving signal fidelity.

In exploring this approach, a Wiener deconvolution filter was implemented. In the absence of noise, this filter behaves similarly to an equalizer, effectively inverting the transfer function and recovering the original signal. However, when noise is present, the Wiener filter operates more conservatively: It seeks an optimal balance between reversing the system response and minimizing the amplification of noise, making it well-suited for practical signal recovery applications.

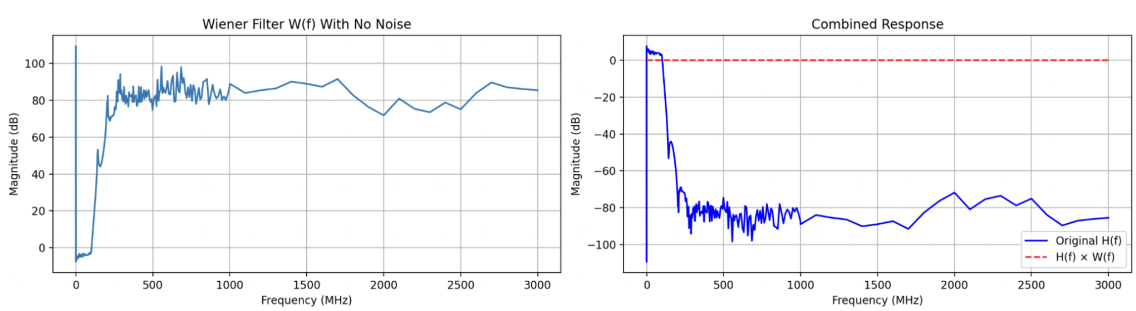

Figure 8: Image of Wiener Filter without accounting for Noise.

As shown by Figures 8 and 9, when not accounting for noise, the Wiener filter acts similarly to a zero-equalizer filter.

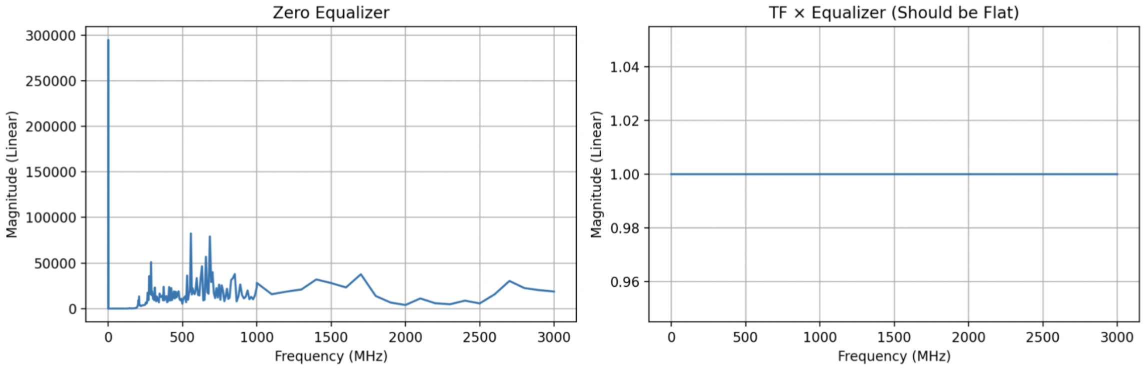

Figure 9: Image of Zero Equalizer.

8.2 Filter Design

The Wiener deconvolution filter is designed to optimally recover a signal that has been distorted by a system (represented by a transfer function) and corrupted by noise. The filter operates in the frequency domain and is defined as follows:

$$G(f) = \frac{H^*(f)S(f)}{|H(f)|^2 S(f) + N(f)}$$

The equation was gathered from standard techniques in digital image processing [2].

Each component of this expression has a specific meaning:

- G(f): The Wiener filter itself. This is the frequency-domain function applied to the distorted signal in an attempt to recover the original, undistorted signal.
- S(f): The power spectral density (PSD) of the original signal. This represents how the signal's power is distributed across different frequencies. It helps the filter give more weight to frequencies where the true signal is strong.
- N(f): The power spectral density of the noise. This quantifies how noise is spread across the frequency spectrum. The filter uses this to avoid boosting frequencies dominated by noise.
- H(f): The transfer function of the system, describing how the system modifies signals at each frequency. It includes both amplitude and phase information.
- H*(f): The complex conjugate of the transfer function. This term is used to "undo" the phase shift introduced by the system and is essential in inverting the system response.

8.3 Power Spectral Densities

The following equation was used to calculate power spectral densities of the Gaussian White Noise and the Original Simulated Pulse:

$$S_{xx}(f_k) = \frac{1}{N f_s} |X[k]|^2$$

where:

- $X[k]$ is the discrete Fourier transform (DFT) of the signal at frequency bin k,
- N is the total number of samples,
- $f_s = \frac{1}{\Delta t}$ is the sampling frequency,
- $S_{xx}(f_k)$ is the power spectral density (PSD) estimate at frequency f_k.

[3]

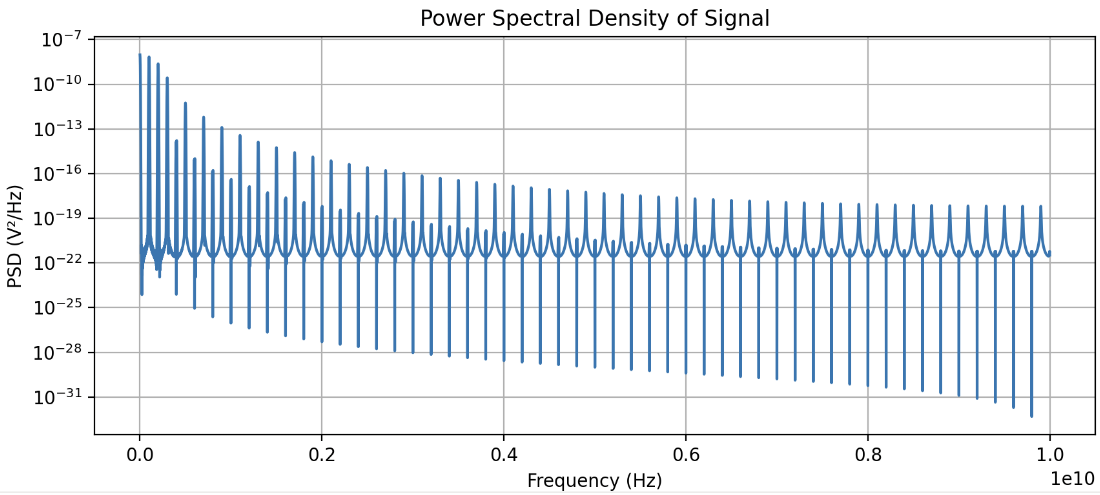

Figure 10: Image of Original Signal Power Spectral Density.

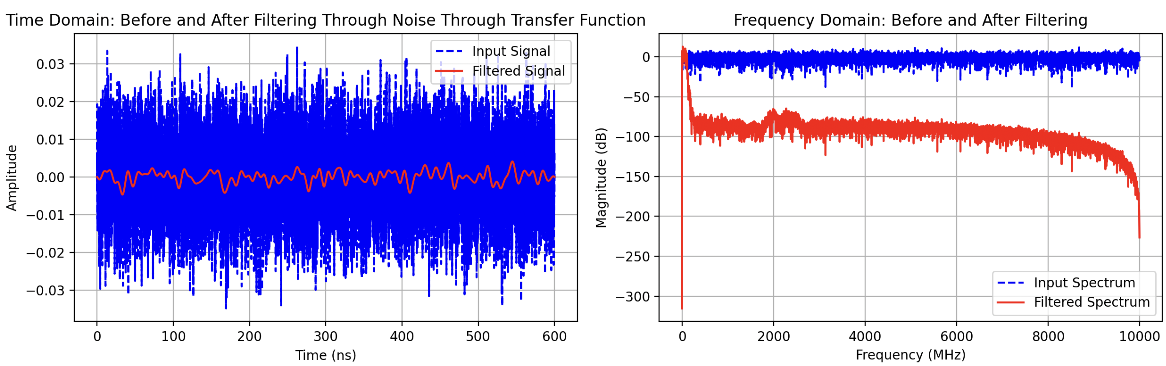

Figure 11: Image of Gaussian White Noise After Going Through Transfer Function.

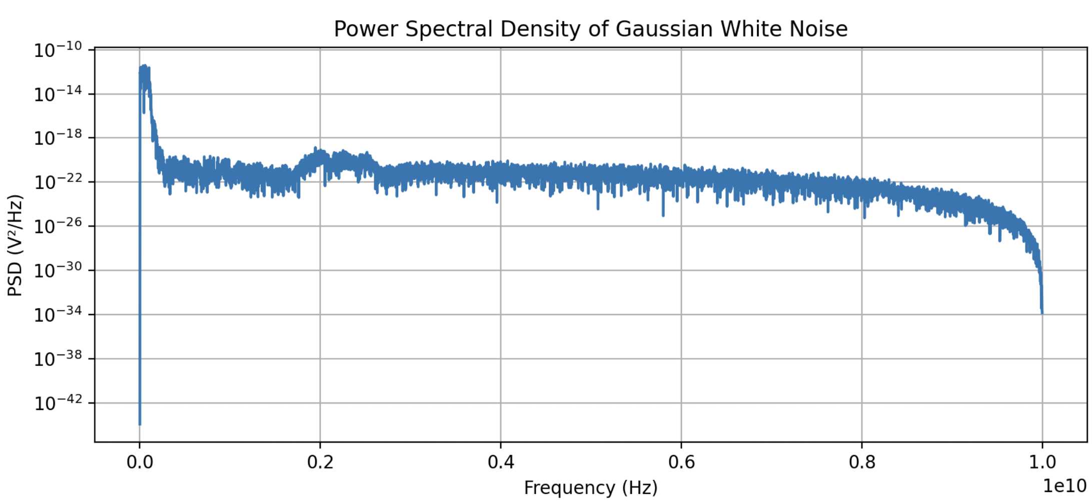

Figure 12: Image of Noise Power Spectral Density used as Input.

8.4 Implementation

The Wiener filter was implemented in the frequency domain to recover the original signal from its distorted and noisy version. First, the Fast Fourier Transform (FFT) of the distorted signal was computed to convert it from the time domain to the frequency domain. The Wiener filter, calculated using the transfer function, signal power spectral density, and noise power spectral density, was then elementally multiplied by the FFT of the distorted signal. This operation applies the frequency-dependent weighting of the filter to attenuate noise and invert the system response. Finally, the result was transformed back to the time domain using the inverse FFT (IFFT), yielding the recovered signal.

8.5 Results

In this section, we can see the Wiener Filter resulting from the inputs gathered during earlier processes in the timeline. In Figure 13, the final Wiener filter is visible on the left. The Wiener Filter multiplied by the Transfer Function is visible on the right. The Wiener Filter times the Transfer Function is not flat in this case due to the noise the Wiener Filter is accounting for. The Wiener Filter is more conservative than a zero equalizer now that it is accounting for noise, so that it does not blow up areas of noise.

In Figure 14, the distorted curve is recovered using the Wiener Filter. The curve can be seen before filtering and after in both the time and frequency domains.

In the final Figure, the Recovered signal is compared to the original signal in both the time and frequency domains.

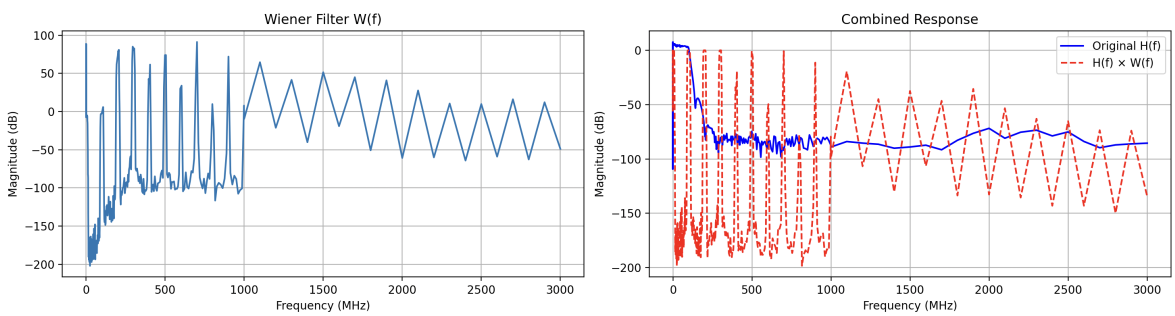

Figure 13: Image of Wiener Filter (left) and Wiener Filter Multiplied by Transfer Function (right).

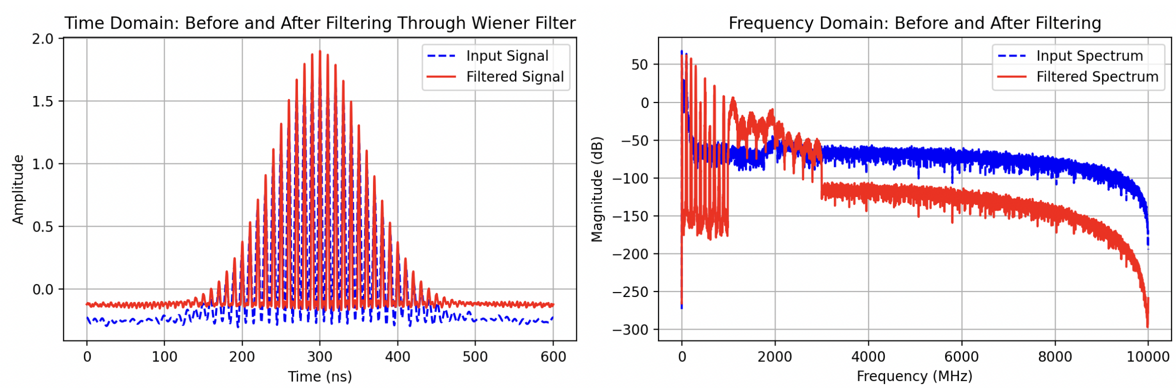

Figure 14: Before and After Filtering Through Wiener Filter in Time Domain (Left) and Frequency Domain (Right).

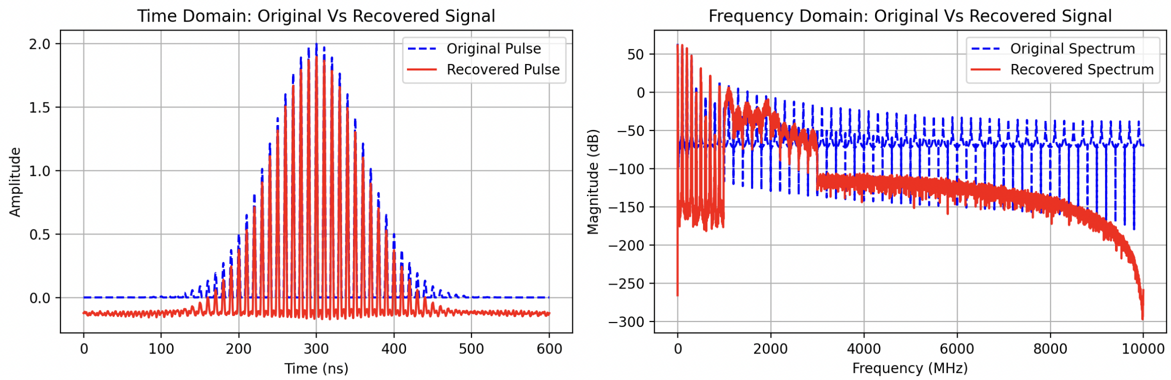

Figure 15: Image of Original Versus Recovered Signal in Time Domain (Left) and Frequency Domain (Right).

Signal Recovery Performance Evaluation

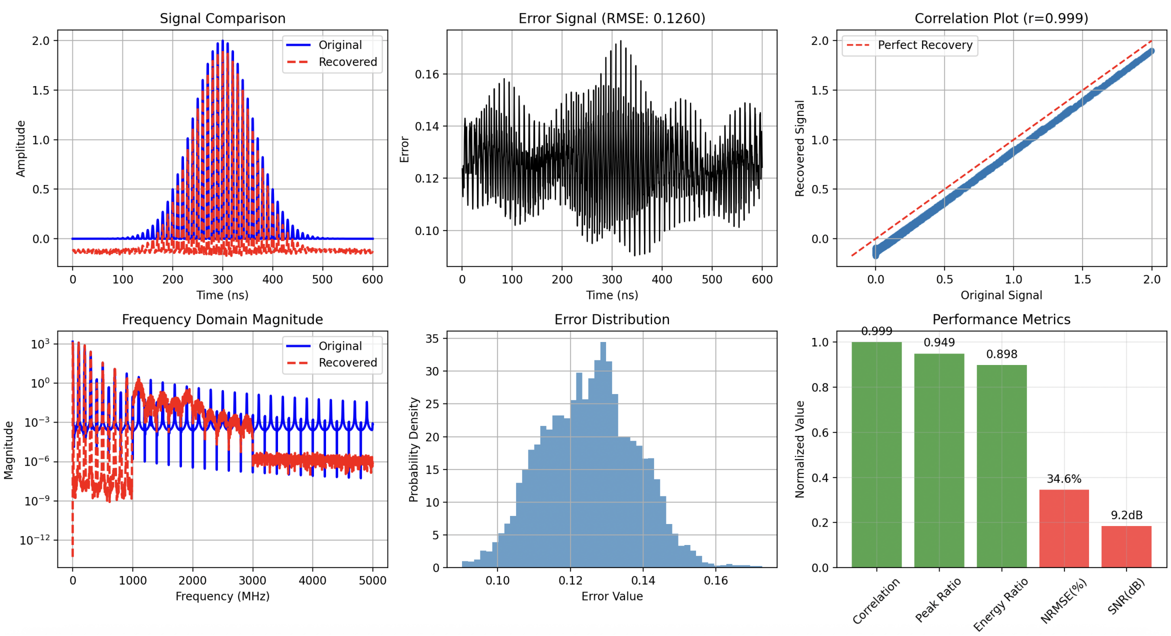

Figure 16: Image of Error Analysis.

To evaluate the effectiveness of the proposed signal recovery technique, we present a comprehensive set of time-domain, frequency-domain, and statistical analyses comparing the original, distorted, and recovered signals. Figure 16 consists of six subplots, each assessing a different aspect of recovery quality.

- **(a) Signal Comparison:** The original and recovered signals are plotted in the time domain. The original signal (blue) shows a modulated Gaussian pulse, while the recovered signal (red dashed) closely matches its shape and timing. The peak alignment and overall envelope preservation suggest high temporal fidelity, although slight amplitude discrepancies remain.

- **(b) Error Signal:** The pointwise error between the original and recovered signals is shown, with a Root Mean Square Error (RMSE) of 0.1259. The error remains relatively stable and bounded across the signal duration, indicating minimal fluctuation and consistent performance.

- **(c) Correlation Plot:** This scatter plot compares the amplitude of each sample in the original signal against the corresponding value in the recovered signal. A near-perfect linear relationship is evident, with a Pearson correlation coefficient of $r = 0.999$, suggesting exceptional structural similarity. The red dashed diagonal line represents ideal recovery, and the clustering of data points around this line confirms high reconstruction accuracy.

- **(d) Frequency Domain Magnitude:** The frequency spectra of both signals, computed via Fast Fourier Transform (FFT), are shown on a logarithmic magnitude scale. The original signal (blue) displays well-defined harmonic content, while the recovered signal (red dashed) replicates the major spectral components. Some deviations are observed at higher frequencies, indicating partial loss or distortion of fine-grained spectral information.
- **(e) Error Distribution:** A histogram of the pointwise error values provides insight into the statistical characteristics of the reconstruction error. The distribution is narrow and centered around 0.125, confirming that most of the error values are small and consistently distributed. This further supports the stability and predictability of the recovery process.
- **(f) Performance Metrics:** Quantitative metrics are summarized in a bar chart to assess overall performance:
  - **Correlation:** 0.999 - The original signal closely matches the curve of the original signal
  - **Peak Ratio:** 0.952 (recovered peak vs. original peak) - The amplitude of the recovered signal closely matches that of the original signal
  - **Energy Ratio:** 0.900 (recovered energy vs. original energy) - The total area under the recovered curve closely matches that of the original
  - **Normalized RMSE (NRMSE):** 34.5% - The error of the recovered signal in respects to the original signal is low-moderate
  - **Signal-to-Noise Ratio (SNR):** 9.2 dB - There's a moderate amount of noise compared to the amount of signal

The table below compares the Pre-Wiener and Post-Wiener metrics when weighed against the original signal.

Table 1: Performance metrics before and after Wiener filtering

Metric	**Pre-Wiener**	**Post-Wiener**	Δ
Correlation	0.857	**0.999**	+0.142
Peak Ratio	0.853	**0.952**	+0.099
Energy Ratio	1.498	**0.900**	−0.598
NRMSE (%)	72.7	**34.5**	−38.2 pp
SNR (dB)	2.8	**9.2**	+6.4

All metrics indicate strong alignment between the original and recovered signals, with particularly high correlation and peak preservation. The modest NRMSE and SNR values highlight areas where further improvements could enhance reconstruction precision, especially in preserving signal energy and mitigating residual noise.

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References

[1] B. Girod, R. Rabenstein, and A. Stenger, *Signals and Systems.* Wiley, 2nd ed., 2001.

[2] R. C. Gonzalez, R. E. Woods, and S. L. Eddins, *Digital Image Processing Using MATLAB.* Pearson Education, 1st ed., 2004.

[3] W. B. J. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise.* New York: Wiley-IEEE Press, 1987.