TOP PAIR PRODUCTION AT THRESHOLD

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I give an overview of the third-order calculations for the heavy-quarkonium parameters in the nonrelativistic effective theory framework and its application to the phenomenology of top quark threshold production. The focus is on the ultrasoft contribution [1].

1 Introduction

The theoretical study of nonrelativistic heavy quark-antiquark systems is among the earliest applications of perturbative quantum chromodynamics (QCD) [2] and entirely relies on the first principles. The nonperturbative effects are well under control and the reliable theoretical predictions can be obtained within the perturbation theory. This makes the heavy quark-antiquark systems an ideal laboratory to determine fundamental parameters of QCD, such as the strong coupling constant α_s and the heavy-quark mass m_q.

The binding energy of the heavy-quarkonium state and the value of its wave function at the origin are among the characteristics of the heavy quarkonium system that are of primary phenomenological interest. The former determines the mass of the bound state resonance, while the latter controls its production and annihilation rates. The heavy-quarkonium ground state energy has been computed through $\mathcal{O}(\alpha_s^5 m_q)$ including the third order correction to the Coulomb approximation in Refs.[3,4,5] This result has been extended to the excited states in Refs.[19,20] For the wave function at the origin however a complete result is only available through $\mathcal{O}(\alpha_s^2)$.[6,7,8,9,10,11,12] The second order correction is huge and for a reliable perturbative prediction one has to to gain full control over the next order. In this order the complete result is available only for logarithmically enhanced terms which include the double logarithmic $\mathcal{O}(\alpha_s^3 \ln^2 \alpha_s)$ contribution [13] and the single logarithmic $\mathcal{O}(\alpha_s^3 \ln \alpha_s)$ contribution [14,15] (see also Refs.[16,17,18]). The calculation of the most difficult non-logarithmic term has been started in Refs.[19,20], where the contribution to the wave function at the origin from the loop corrections to the Coulomb potential have been evaluated. In Ref.[21] the contributions from the non-Coulomb potentials have been obtained. The last breakthrough is the calculation of the contribution due to the emission and absorption of an ultrasoft gluon by the quarkonium bound state,[1] which completes the analysis of the non-relativistic quarkonium bound-state dynamics in the third order. The full third-order correction to the wave function at the origin is now expressed in terms of a few yet unknown matching coefficients, which can be obtained by standard fixed-order loop calculations. In this paper I outline the effective theory approach to the theory of heavy quarkonium and present the rsult for the ultrasoft contribution to the top quark-antiquark resonance cross section. The ultrasoft correction is of special interest, because it constitutes a qualitatively new effect, which shows up for the first time in the third order. No other such effects are expected in higher orders of the perturbative expansion.

2 Nonrelativistic effective theory

Near the threshold, the heavy quarks are nonrelativistic, so that one may consider the quark velocity v (or inverse quark mass) as a small parameter. An expansion in v may be performed directly in the QCD Lagrangian by using the framework of effective field theory, [22,23,24] or diagrammatically with the threshold expansion.[25] The relevant momentum regions are the hard region (energy k^0 and momentum $\boldsymbol{k}$ of order m), the soft region ($k^0, \boldsymbol{k} \sim mv$), the potential region ($k^0 \sim mv^2$, $\boldsymbol{k} \sim mv$), and the ultrasoft region ($k^0, \boldsymbol{k} \sim mv^2$). By integrating out the hard modes of QCD, one arrives at the effective theory of nonrelativistic QCD (NRQCD).[23] If one also integrates out the soft modes and the potential gluons, one obtains the effective theory of potential NRQCD (pNRQCD), which contains potential heavy quarks and ultrasoft gluons as dynamical particles.[24] The propagation of the quark-antiquark pair in pNRQCD is described by the Green function of the Schrödinger equation

$$(\mathcal{H} - E)\, G(\boldsymbol{r}, \boldsymbol{r}', E) = \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}')\,, \tag{1}$$

where $\mathcal{H}$ is the nonrelativistic effective Hamiltonian of the following form

$$\mathcal{H} \;=\; -\frac{\partial_{\boldsymbol{r}}^2}{m_q} + V_C(r) + \ldots\,, \tag{2}$$

with $r = |\boldsymbol{r}|$. The ellipses stand for the higher order terms in α_s and v. For the color singlet state the leading order Coulomb potential is attractive, $V_C(r) = -C_F\alpha_s/r$, where $C_F = (N_c^2 - 1)/(2N_c)$, $N_c = 3$. As a consequence the color singlet Green function gets a contribution from an infinite number of approximately Colombic bound states of the following form:

$$G(\boldsymbol{r}, \boldsymbol{r}', E) \;=\; \sum_{n=1}^{\infty} \frac{\psi_n^*(\boldsymbol{r})\psi_n(\boldsymbol{r}')}{E_n - E - i\varepsilon} + \ldots\,, \tag{3}$$

where E_n and ψ_n are the energy and the wave function of a bound states, respectively, n is the principal quantum number, the spin and orbital quantum numbers are suppressed and the ellipsis stands for the contribution of the spectral continuum. The leading order approximation of the quarkonium bound state is given by the Coulomb solution of Eq. (1), *e.g.* the leading order binding energy is $E_n^C = -C_F^2\alpha_s^2 m_q/(2n)^2$. The corrections due to the high order terms in the nonrelativistic Hamiltonian can be systematically computed by means of the time ordered quantum mechanical perturbation theory. In addition, the Green function gets the correction due to the multipole interaction of the quark-antiquark pair to the ultrasoft gluons. The leading ultrasoft effect is due to the chromoelectric dipole interaction, which results in a NNNLO correction.[26,27] The PNRQCD diagram representing this correction is shown in Fig. 1.

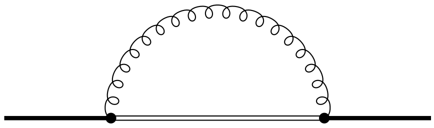

Figure 1: The ultrasoft correction as a PNRQCD Feynman diagram. The bold and the double lines stand for the singlet and octet Coulomb Green functions, respectively, the curly line represents the propagator of the ultrasoft gluon, and the black circles represent the chromoelectric dipole interaction $g_s\boldsymbol{rE}$.

3 Ultrasoft contribution to top-quark production near threshold

For top quarks the nonperturbative effects are negligible and its decay width $\Gamma_t \approx 1.4$ CeV smears out the Coulomb resonances below the threshold.[28] The NNLO analysis of the cross section [29] shows that only the ground-state pole gives rise to a prominent resonance. Although the calculation of the normalized cross section $R = \sigma(e^+e^- \to t\bar{t}X)/\sigma(e^+e^- \to \mu^+\mu^-)$ requires the full Green function the height of the resonance can be estimated from the wave function at the origin of the *would-be* toponium ground state. In the leading-order approximation $R_1^{\rm LO} \approx 6\pi N_c e_t^2 |\psi_1^C(0)|^2/\,(m_t^2\Gamma_t)$, where $|\psi_n^C(0)|^2 = (m_t\alpha_s C_F)^3/(8\pi n^3)$ is the value of the Coulomb wave function at the origin. The ultrasoft correction to the wave function results in the following variation of the resonance cross section [1]

$$\begin{aligned}\delta^{us}R_1 \quad &= \quad \alpha_s^3\Big\{ -18.71\ln^2\alpha_s + 52.03\ln\alpha_s + 112.38 \\ &\qquad + \Big[23.52\ln\alpha_s - 30.98\Big]\,\ln\frac{\mu}{m} - 6.55\ln^2\frac{\mu}{m}\Big\}R_1^{\rm LO}. \end{aligned} \tag{4}$$

The scale of the coupling α_s is most naturally of order of the inverse Bohr radius $m_t\alpha_s C_F$ in two of the three powers of the overall factor α_s^3, and of order of the ultrasoft scale $m_t\alpha_s^2$ in the third. However, any other scale choice is formally equivalent at this order. In the following we evaluate α_s at $\mu_B = m_t C_F \alpha_s(\mu_B)$, wherever it appears. The scale μ in the $\ln(\mu/m_t)$ terms is related to scale-dependent potentials and hard matching coefficients. We vary μ/m_t between $\alpha_s C_F$ (corresponding to the scale μ_B) and 1 (hard scale). Adopting $\alpha_s = 0.14$, which corresponds to $\mu_B \approx 32.5\,\text{GeV}$, we obtain $\delta^{us}R_1/R_1 \approx 0.31$ from the nonlogarithmic correction alone. Including an estimate of the logarithmic terms we find

$$\delta^{us}R_1 \approx \Big[(-0.17) - (+0.13)\Big]\,R_1^{\rm LO}. \tag{5}$$

It therefore appears that the large nonlogarithmic term leads to a large enhancement of the width. Whether or not perturbation theory is out of control (as may be suggested by the upper limit of the given range) can be decided only after combining all third-order terms.

4 Summary

The problem of evaluating the total $\mathcal{O}(\alpha_s^3)$ corrections to the top quark threshold production is reduced to the fixed-order loop calculation of of a few yet unknown matching coefficients in dimensional regularization. The nonlogarithmic ultrasoft contribution is large and significantly increases the production rate. It might limit the accuracy of the perturbative analysis of the quarkonium even for top quarks. We should however emphasize that a definite conclusion can only be drawn once the full NNNLO result is available. In this respect the sizable negative third-order correction from the perturbation potentials [19,20,21] should be mentioned.

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