OPEN ACCESS
IOP Publishing
Classical and Quantum Gravity
Class. Quantum Grav. 41 (2024) 245004 (31pp)
https://doi.org/10.1088/1361-6382/ad8b93

The accelerating universe in a noncommutative analytically continued foliated quantum gravity

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Received 14 August 2024; revised 25 September 2024
Accepted for publication 25 October 2024
Published 12 November 2024

Abstract
Based on an analytically continued Riemannian foliated quantum gravity super-Hamiltonian, known as branch cut quantum gravity (BCQG) we propose a

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novel approach to investigating the effects of noncommutative geometry on a minisuperspace of variables, influencing the acceleration behavior of the Universe's wave function and the cosmic scale factor. Noncommutativity is introduced through a deformation of the conventional Poisson algebra, enhanced with a symplectic metric. The resulting symplectic manifold provides a natural setting that enables an isomorphism between canonically conjugate dual vector spaces, spanning the BCQG cosmic scale factor and its complementary quantum counterpart. Using this formulation, we describe the dynamic evolution of the Universe's wave function, the cosmic scale factor, and its complementary quantum image. Our results strongly suggest that the noncommutative algebra induces late-time accelerated growth of the wave function, the Universe's scale factor, and its complementary quantum counterpart, offering a new perspective on explaining the accelerating cosmic expansion rate and the inflationary period. In contrast to the inflationary model, where inflation requires a remarkably fine-tuned set of initial conditions in a patch of the Universe, analytically continued non-commutative foliated quantum gravity captures short and long scales, driving the evolutionary dynamics of the Universe through a reconfiguration of the primordial cosmic content of matter and energy. This reconfiguration is encapsulated into a quantum field potential, which leads to the generation of relic gravitational waves, a topic for future investigation. Graphical representations and contour plots indicate a characteristic torsion (or twist) deformation of spacetime geometry. This result introduces new speculative elements regarding the reconfiguration of matter and energy as a driver of spacetime torsion deformation, generating relic gravitational waves and serving as an alternative topological mechanism for the Universe's acceleration. However, these assumptions require further investigation.

Keywords: foliated quantum gravity, symplectic metric, noncommutative geometry, accelerating Universe

1. Introduction

The chronology of the Universe indicates a primordial era dominated by inflation, followed by dominant phases of radiation, matter, and currently, a dark energy-dominated era, which is mostly assumed as the cause of the accelerated cosmic expansion.

This paper proposesss a new noncommutative approach to the accelerating expansion of the Universe, extending recent advances in quantum gravity theories. We build on the branch cut quantum gravity (BCQG) [1, 2] framework, based on the Wheeler DeWitt [3] and the Hořava-Lifshitz [4] formulations, incorporating elements of symplectic geometry and noncommutative algebra to investigate the interplay between quantum effects and large-scale cosmic behavior. Our model provides an alternative to inflationary theories by explaining cosmic acceleration through a fundamental restructuring of spacetime geometry rather than relying on specific initial conditions.

Moreover, this approach offers a fresh perspective on the generation of relic gravitational waves and a possible solution to unresolved cosmological puzzles, such as the horizon and flatness problems. By introducing noncommutative deformations into the standard Hamiltonian formalism, we extend the reach of quantum gravity models to address large-scale cosmological observations.

In the present formulation, the noncommutativity is imposed through a deformation of the conventional Poisson algebra structure incorporating a symplectic metric. The resulting symplectic manifold provides a natural setting that allows an isomorphism between canonically conjugate dual vector spaces, encompassing the BCQG cosmic scale factor and a complementary quantum counterpart, outlined in the perfect fluid domain of Hermann Weyl. We describe not only the temporal evolution of the Universe's wave function but also complementary equations that describe the temporal evolution of the cosmic scale factor and its dual quantum image in first and second order, yielding unprecedented outcomes.

The new algebraic formulation, compared to the (few) noncommutative formulations found in the literature, offers the consistent additional advantage of treating all degrees of freedom symmetrically, on the same footing.

Within the symplectic noncommutative algebraic framework, we investigate the Universe's wave function and scale factor, denoted as $\Psi(u,v)$ and $\eta(t)$, respectively. Using a reverse symplectic transformation, $\eta(t)$ and $\xi(t)$ represent commutative variables that span noncommutative complementary quantum spaces. The dynamic evolution of these quantities is governed by a modified Hamiltonian that includes terms for curvature, radiation, dark matter, and other components influencing cosmic acceleration. Furthermore, exploring the descriptive advantage of the new symplectic algebraic formulation we establish a point of contact with the inflation model by expanding this algebraic structure to incorporate a complex inflaton-type field into the formalism.

Based on this extended formulation, calculations of the temporal evolution of the wave function and scale factor of the Universe and its dual counterparts are performed, bringing to light new insights about theoretical attempts to account for cosmic acceleration. As we will see, the presence of a noncommutative algebra structure induces the capture of short and long spatiotemporal scales, driving not only the evolutionary dynamics of the Universe's wave function and the cosmic scale factor but also a reconfiguration of matter on small and intermediate scales, inducing the generation of relic gravitational waves, a topic for future investigation. Furthermore, the results indicate a twist-warping of the spacetime geometry, which may imply a spacetime torsion generator not only for relic gravitational waves but also as an alternative topological mechanism for the acceleration of the Universe, an assumption that requires further investigation. For a review of the branch-cut commutative formulation and the noncommutative approach based on the conventional Poisson algebra, with applications restricted to the time evolution of the wave function of the Universe, see [1, 2, 5–11].

2. BCQG

The BCQG extends the ontological domain of general relativity to the complex plane [1, 2, 5–13], offering a theoretical alternative to inflation models [14, 15]. This formulation is based on the mathematical augmentation technique and notions of closure and existential completeness [16], which have proved highly useful in both quantum mechanics [17–19] and pseudo-complex general relativity (pc-GR) [20, 21], with direct physical and cosmological manifestations. These findings have broadened our understanding, showing that expanding the descriptive domain of a theory by incorporating complex or pseudo-complex variables can provide insights into both infinitesimally small and large scales.

The line element in the BCQG quantum gravity, resulting from the complexification of the Friedmann Lemaître Robertson Walker (FLRW) metric [22–25], may be expressed as [5–7]

$$\mathrm{d}s^2_{[\mathrm{ac}]} = -N^2(t)\,c^2\mathrm{d}t^2 + \left(\ln^{-1}[\beta(t)]\right)^2\left[\frac{\mathrm{d}r^2}{(1-kr^2(t))} + r^2(t)\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]. \tag{1}$$

In this expression, [ac] denotes analytical continuation to the complex plane, where r and t represent real and complex space-time parameters, respectively, and k denotes the spatial curvature of the multiverse, corresponding to negative curvature ($k=-1$), flat ($k=0$), or positively curved spatial hypersurfaces ($k=1$). $\ln^{-1}[\beta(t)]$ represents the foliated scale factor, and $N(t)$ denotes the lapse function. The gauge invariance of the action in general relativity yields a Hamiltonian constraint that requires a gauge-fixing condition on the lapse function (see [26]). The BCQG formulation arises from the complexification of the FLRW metric, resulting in the Riemann superposition of field equations associated with continuously distributed single-poles arranged along a line in the complex plane (for details, see [1, 2, 5–11]), consistent with the concept of multiverse proposed by Hawking–Hertog [27]. Through a Riemann integration process, this complexification gives rise to the new scale factor, denoted as $\ln^{-1}[\beta(t)]$, and a topological foliated spacetime structure.

We introduce a mapping between the standard cosmological scale factor, $a(t)$, and its non-commutative counterpart, $\ln^{-1}[\beta(t)]$, which reveals new topological structures. This mapping reflects the complex interplay between the observed expansion of the Universe and the underlying quantum geometry.

To understand the role of foliation in the present formulation, we may assume a D-dimensional Euclidean manifold $\mathcal{M}$ associated to a metric $g_{\mu\nu}$ analytically continued to the complex plane, carrying coordinates x_α. Following the conventional steps of general relativity, we may set up a preferred time-direction by defining a time function $\tau(x)$ that assigns a specific time τ to each spacetime coordinate x. With this definition we may decompose the manifold $\mathcal{M}$ into a stack of spatial slices $\Sigma_{\tau_i} \equiv \{x : \tau(x) = \tau_i\}$ encompassing all points x with the same 'time-coordinate'. Then the gradient of the time function can be used to define a vector normal to the spatial slices, with the lapse function ensuring normalization with the metric, and the lapse defined in terms of the time coordinates τ and τ_Σ related to Σ_{τ_i}. In the projectable Hořava–Lifshitz gravity, N is restricted to be a function of Euclidean time only (see [28, 29]).

The BCQG provides a gateway to the evolutionary phase preceding the primordial singularity, often referred to as a *mirror world*. In both its classical and quantum versions, this concept replaces the singularity with a topological transition between the contraction and expansion phases of the evolutionary Universe. During this transition, spacetime acquires a helix-like foliated topological shape around a branch-point, preserving the fundamental conservation laws of thermodynamics.

In the classical scenario of the BCQG, the Universe evolves continuously from the negative complex cosmological time sector, prior to a primordial singularity, to the positive one, circumventing continuously the branch-cut, and no primordial singularity occurs in the imaginary sector, only branch-points. In this formulation, the foliated Universe involves a continuous sum of an infinite number of infinitesimally separated poles, surrounding a primordial branch-point, arranged along a line in the complex plane with infinitesimal residues. Similar to the primordial branch-point singularity, the argument of the resulting analytic function, can be mapped from a single point in the domain to multiple points in the range. In addition to branch-cuts, there are still 'singularities', - the branch-points -, but at the same time, there are multiple points that configure continuous paths in the Riemann sheets. This enables continuous solutions of the primordial singularity, which, in general relativity, is inescapable. For this to happen, the presumption at the level of a local continuity prevails, i.e. that there is some neighborhood of the branch-point, let us call it z_0, close enough although not equal to z_0, where one can find a small region around (local patches) where $\ln^{-1}[\beta(t)]$ is single valued and continuous. The cuts in the branch-cut are shaped by the $\beta(t)$ function which defines the range of $\ln^{-1}[\beta(t)]$.

In the primordial phase, the scale factor $\ln^{-1}[\beta(t)]$ shrinks to a finite critical size, shaped by the $\beta(t)$ function which, besides the range, characterizes also its foliation regularization and domain extension. It is important to emphasize that the presence of a regularization function $\beta(t)$ in this formulation does not imply a change in the integration limits of Friedmann's equations to avoid singularities since essential or real singularities at $t=0$ cannot be removed simply by any coordinate transformation. The technical procedure adopted here results in solutions conformed by branch-cuts that allow circumventing the singularities, which in turn become branch-points. Its range, in particular, extends beyond the Planck length as per the Bekenstein criterion [9]. In view of this criterion, the impossibility of confining energy and entropy within a finite size makes the transition between the contraction and expansion phases exceptionally peculiar, in which spacetime shapes itself topologically around a branch-point. In the contraction phase, as the patch size reduces linearly with $\ln[\beta(t)]$, light travels along geodesics within each leaf of the Riemann foliation, continually circumventing the analytically continued foliated domain. Although the horizon size increases with $\ln^{\epsilon}[\beta(t)]/\ln[\beta(t)]$, where ϵ denotes the dimensionless thermodynamic connection, the length of the path that light must traverse compensates for the difference in scale between the patch and the horizon sizes. Under these conditions, causality between the size of the horizon and the size of the patch can be achieved through the accumulation of branches in the transition region between the current state of the Universe and those of past events [9]. In addition to causality, cosmological dilemmas such as the flatness problem and the horizon problem come into focus. Technically, the flatness problem concerns the value of the ratio between the total density of the Universe and the critical density, resulting in a very small Planck value for the dimensionless and time-dependent cosmic spatial factor Ω_c [30–32]. This factor scales as $\ln^{2\epsilon}[\beta(t)]/\ln^{2}[\beta(t)]$. The horizon problem, on the other hand, arises precisely because the patch corresponding to the observable Universe has never been causally connected in the past [30–32]. The restoration of causality in BCQG provides an additional reliable perspective on the resolution of these cosmological puzzles [10].

Regarding the presence of a mirror Universe, the model presents a similarity with the concepts of cyclical and bouncing models that experience infinitely alternating periods of rapid expansion and contraction, overcoming the primordial singularity, with no ending and no beginning. However, from the point of view of theoretical construction, the similarities between the two lines of investigation are quite remote. The main reason is related to the ontological and epistemological aspects of both lines of investigation. Cyclic bouncing models are proposed on basis of an analytical investigation of the Universe evolution and implemented mostly in an ad hoc way, through parameterizations, cosmic wedge diagrams and other aesthetic grounds. Analytic continuation foliated quantum gravity in turn, from the ontological and epistemological aspects, except for the complexification of the standard metric, combined with the concepts of multiverse, it contemplates the same theoretical foundations and theoretical investigation procedures as general relativity. BCQG shares with general relativity the same fundamental questions, the same objects of inquiry, the same claims about the nature of being and existence, that is, the same principles of its conceptual philosophy. In this sense, the realization of a transition that overcomes the primordial singularity is not the result of an ad hoc proposition, but is the natural outcome of an evolutionary process of the fundamental equations of general relativity, generated through theoretical ontological, methodological, and epistemological procedures based on field theory. Likewise, the realization of the transition region that overcomes the primordial singularity does not require the imposition of a mechanism, being the natural result of a topological restructuring of spacetime.

3. Noncommutative BCQG

The starting point of this study is the commutative BCQG action which depends on the branching scalar curvature of the Universe, $\mathcal{R}$, and on its covariant derivatives, $\nabla_i\Gamma$, in different orders [1, 2]:

$$\mathcal{S}_{\mathrm{HL}} = \int \mathrm{d}^3x\mathrm{d}t\mathcal{L} = \frac{M_{\mathrm{P}}^2}{2}\int \mathrm{d}^3x\,\mathrm{d}t N\sqrt{g}\times\left(K_{ij}K^{ij} - \lambda K^2 - g_0 M_P^2 - g_1\mathcal{R}\right.$$
$$-g_2 M_{\mathrm{P}}^{-2}\mathcal{R}^2 - g_3 M_{\mathrm{P}}^{-2}\mathcal{R}_{ij}\mathcal{R}^{ij} - g_4 M_{\mathrm{P}}^{-4}\mathcal{R}^3 - g_5 M_{\mathrm{P}}^{-4}\mathcal{R}\left(\mathcal{R}^i_j\mathcal{R}^j_i\right)$$
$$\left.-g_6 M_{\mathrm{P}}^{-4}\mathcal{R}^i_j\mathcal{R}^j_k\mathcal{R}^k_i - g_7 M_{\mathrm{P}}^{-4}\mathcal{R}\nabla^2\mathcal{R} - g_8 M_{\mathrm{P}}^{-4}\nabla_i\mathcal{R}_{jk}\nabla^i R^{jk}\right). \tag{2}$$

In this expression, g_i denote running coupling constants, M_{P} is the Planck mass, and the branching Ricci components of the three dimensional metrics may be determined by imposing a maximum symmetric surface foliation [11] which gives:

$$\mathcal{R}_{ij} = \frac{2}{\sigma^2 u^2(t)}g_{ij}, \quad \text{and} \quad \mathcal{R} = \frac{6}{\sigma^2 u^2(t)}, \tag{3}$$

where the variable change $u(t) \equiv \ln^{-1}[\beta(t)]$, with $\mathrm{d}u \equiv \mathrm{d}\ln^{-1}[\beta(t)]$, was introduced. $K = K^{ij}g_{ij}$ represents in expression (2) the trace of the extrinsic curvature tensor K_{ij} [1, 2, 11]:

$$K = K^{ij}g_{ij} = -\frac{3}{2\sigma N u(t)}\frac{\mathrm{d}u(t)}{\mathrm{d}t}. \tag{4}$$

In the context of the ADM formalism, for the metric introduced in equation (1), $N_i = 0$ and the three-metric $h_{ij} \to g_{ij}$ corresponds to a projectable analytically continued quantum gravity approach:

$$h_{ij} = \ln^{-1}[\beta(t)]^2\,\mathrm{diag}\left(\frac{1}{1-kr^2(t)}, r^2(t), r^2(t)\sin^2\theta\right). \tag{5}$$

Applying standard canonical quantization procedures to the action, the canonical momentum reads

$$\pi_u = \frac{\partial S}{\partial \dot{u}} = -\frac{2u(t)\dot{u}(t)}{N(t)}. \tag{6}$$

From these equations, the resulting Hamiltonian density

$$\mathcal{H} = \pi_u\dot{u} - \mathcal{L}, \tag{7}$$

becomes

$$= \frac{1}{2}\frac{N}{u}\left[-p_u^2 + g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2}\right], \tag{8}$$

where g_k, g_Λ, g_r, and g_s represent respectively the curvature, cosmological constant, radiation, and stiff matter running coupling constants [33, 34] defined as

$$g_k \equiv \frac{2}{3\lambda - 1}; \, g_\Lambda \equiv \frac{\Lambda M_{Pl}^{-2}}{18\pi^2 (3\lambda - 1)^2}; \, g_r \equiv 24\pi^2 (3g_2 + g_3);$$
$$g_s \equiv 288\pi^4 (3\lambda - 1)(9g_4 + 3g_5 + g_6). \tag{9}$$

The g_r, and g_s running coupling constants can be positive or negative, without affecting the stability of the solutions. Stiff matter contribution in turn is determined by the $p = \omega\rho$ condition in the corresponding equation of state (EoS). We supplemented the Hamiltonian with two additional terms, $g_m u$, that describes the contribution of baryon matter combined with dark matter, and $g_q u^3$, a quintessence contribution, a time-varying, spatially-inhomogeneous and negative pressure component of the cosmic fluid, which allows approaching the 'coincidence problem' [35, 36]:

In order to introduce a noncommutative formalism, based on the perfect fluid conception of Weyl [37], using the formalism of Schutz [38], an additional term is inserted in the Hamiltonian density associated to a field $v(t)$, which obeys the perfect fluid Weyl equation, characterized by a dimensionless quantity ω, whose canonically conjugated momentum to $v(t)$ is denoted as p_v:

$$\mathcal{H}_v \equiv \frac{1}{2} N \frac{p_v}{u(t)^{3\omega}}, \quad \text{with} \quad p_v = -\frac{2v(t)\dot{v}(t)}{N(t)}. \tag{10}$$

The variables $u(t)$ and $v(t)$ span complementary quantum dual variable spaces. After simplifying the notation and combining (8) and (10), the following super-Hamiltonian density results:

$$\mathcal{H} = \frac{1}{2}\frac{N}{u}\left[-p_u^2 + g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2} + \frac{1}{u^{3\omega-1}} p_v\right]. \tag{11}$$

The fields u and v obey the following commutative Poisson algebra

$$\{u, v\} = \{p_u, p_v\} = \{u, p_v\} = \{v, p_u\} = 0;$$
$$\{u.p_u\} = \{v, p_v\} = 1. \tag{12}$$

4. Faddeev–Jackiw symplectic approach for a noncommutative algebra: two fields approach

The geometrically motivated formalism of Faddeev and Jackiw's depends on a phase-space symplectic structure and represents an alternative to Dirac's quantization method for constrained and unconstrained systems. The focus of the Faddeev and Jackiw's approach is to overcome the generally assumed mandatory quantization requirement based on bracketing and constraints categorization [39]. As an additional outcome Faddeev and Jackiw's formalism furthermore represents a robust procedure for constructing a noncommutative algebraic structure of symplectic variables.

The steps followed by the authors for quantization of physical systems follow as a starting point the particular case of a Lagrangian density with first order time derivatives dependence, describing a constrained system. Upon introducing phase-space canonical variables and the corresponding conjugated momenta, the authors introduce an equivalent Lagrangian density expressed in terms of a symplectic matrix, which represents the key element of the formulation for generating a noncommutative formulation. When considering a more general case, the approach gives rise to Euler–Lagrange equations, and the resulting non-singular inverse of the symplectic matrix directly provides the Hamiltonian of the system which is expressed in terms of the phase-space variables, without the imposition of any constraint, as usually imposed by the Dirac quantization method. In short, the Faddeev and Jackiw's method introduces a noncommutative algebra involving the phase-space variables, which may be expressed in terms of the symplectic matrix [39], although, as indicated by [40], this alternative is not mandatory. The realization of the underlying geometric structure synthesized by the method can in short be materialized from the elements of the inverse of the mentioned symplectic matrix, whose components are in line with the corresponding Dirac brackets, thus providing a consistent connection with the commutators of the quantized theory.

In what follows, we will elaborate on a two fields transformations from non-commuting coordinates to commuting variables on basis on the Faddeev–Jackiw symplectic approach, limited to the dual quantum fields $u(t)$ and $v(t)$. The procedure adopted below follows a reverse logic of the conventional Faddeev–Jackiw formalism, presupposing a character change for the variables u and v, assuming that they obey the noncommutative algebraic structure described by a non-zero Poisson bracket formalism.

Let us assume the commutative, labeled by tilde, and non-commuting coordinates are represented in general as

$$(\tilde{x}_i) = (\tilde{u}, \tilde{p}_u, \tilde{v}, \tilde{p}_v) \quad \text{and} \quad (x_i) = (u, p_u, v, p_v) \ . \tag{13}$$

The commuting and non-commuting coordinates satisfy both the respective Poisson-brackets

$$\{\tilde{x}_i\,, \tilde{x}_j\} = \tilde{g}_{ij} \quad \text{and} \quad \{x_i\,, x_j\} = g_{ij}\,, \tag{14}$$

where on the right side of these equations we have the symplectic metrics, satisfying respectively

$$\tilde{g}_{ji} = -\tilde{g}_{ij} \quad \text{and} \quad g_{ji} = -g_{ij} \ . \tag{15}$$

This symmetry property corresponds, for natural reasons, to a symplectic space. The matrix structure corresponding to these metrics are

$$(\tilde{g}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} ;$$
$$(g) = \begin{pmatrix} 0 & 1 & 0 & -\gamma \\ -1 & 0 & \chi & \alpha \\ 0 & -\chi & 0 & 1 \\ \gamma & -\alpha & -1 & 0 \end{pmatrix} . \tag{16}$$

In this expression, σ, α, γ and χ represent the parameters of the noncommutative non-zero Poisson algebra. Using (14), this leads to the non-zero commutative algebra Poisson brackets

$$\begin{aligned} &\{\tilde{u}, \tilde{v}\} = 0; \quad \{\tilde{u}, \tilde{p}_u\} = 1\,; \quad \{\tilde{v}, \tilde{p}_v\} = 1\,; \\ &\{\tilde{u}, \tilde{p}_v\} = 0; \quad \{\tilde{v}, \tilde{p}_u\} = 0; \quad \text{and} \quad \{\tilde{p}_u, \tilde{p}_v\} = 0\,. \end{aligned} \tag{17}$$

For the noncommutative case the following relations apply:

$$\begin{aligned} &\{u, v\} = \sigma; \quad \{u, p_u\} = 1\,; \quad \{v, p_v\} = 1; \\ &\{u, p_v\} = -\gamma\,; \{v, p_u\} = -\chi\,; \quad \text{and} \quad \{p_u, p_v\} = \alpha. \end{aligned} \tag{18}$$

In what follows, we seek for a linear transformation involving both commutative and noncommutative elements that complies with the above algebraic requirements, defined as:

$$x_i = \sum_j M_{ij}\tilde{x}_j \,. \tag{19}$$

Applying the requirements of the Poisson algebra brackets above, we obtain the structural conditions the matrix elements of M_{ij} should fulfill. As a result, equation (19) can be cast into the form

$$\begin{aligned} u &= M_{11}\tilde{u} + M_{12}\tilde{p}_u + M_{13}\tilde{v} + M_{14}\tilde{p}_v\,; \\ p_u &= M_{21}\tilde{u} + M_{22}\tilde{p}_u + M_{23}\tilde{v} + M_{24}\tilde{p}_v\,; \\ v &= M_{31}\tilde{u} + M_{32}\tilde{p}_u + M_{33}\tilde{v} + M_{34}\tilde{p}_v\,; \\ p_v &= M_{41}\tilde{u} + M_{42}\tilde{p}_u + M_{43}\tilde{v} + M_{44}\tilde{p}_v\,. \end{aligned} \tag{20}$$

Using the condition $\sigma = 0$ which implies $\tilde{u} = u$ and $\tilde{v} = v$, we can reduce the matrix M to the expression

$$(M) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & M_{24} \\ 0 & 0 & 1 & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} . \tag{21}$$

We can also take a second path, namely transforming the commuting variables to the non-commuting ones. However, the present path chosen is more effective and it gives us directly the non-commuting variables in terms of the commuting ones, as will be needed in the next section.

From equations (18), (19) and (21), we obtain

$$\begin{aligned} &\{u,p_u\} = 1 = M_{22}\,; \quad \{v,p_v\} = 1 = M_{44}\,; \\ &\{u,p_v\} = -\gamma = M_{42}\,; \quad \{v,p_u\} = -\chi = M_{24}\,, \end{aligned} \tag{22}$$

where the Poisson-bracket $\{p_u,p_v\} = 0$ will be posteriorly determined.

In the following we seek to resolve the set of equations given in (22), which leads to the following conditions:

$$M_{22} = M_{44} \ = \ 1\,; \quad M_{42} = -\gamma \; M_{24} \ = -\chi \,. \tag{23}$$

Additionally, from the commutator condition $\{p_u,p_v\} = \alpha$, we obtain :

$$\{p_u,p_v\} = \alpha = M_{21}M_{42} - M_{41}M_{22} + M_{23}M_{44} - M_{24}M_{43}, \tag{24}$$

which can be simplified, using the former results $M_{22} = M_{44} = 1$ and $M_{42} = -\gamma$, $M_{24} = -\chi$. leading to

$$\alpha = -\gamma M_{21} - M_{41} + M_{23} - \chi M_{43}\,. \tag{25}$$

The transformation implemented shows still some freedom in choosing the remaining matric elements M_{21}, M_{23}, M_{41} and M_{43}. We can then implement a particular gauge fixing condition in finding the remaining metric elements (see discussion below). We then make the choice:

$$M_{21} = M_{23} = 0\,; \quad M_{41} = -M_{43} = \alpha\,. \tag{26}$$

The application of the Faddeev and Jackiw method implies an additional symmetry associated with the super-Hamiltonian which implies a gauge fixing given as $\Sigma = N - 1$ (for details we recommend reading [39]). This implies that the ongoing method is subject to gauge fixing conditions which suggests, more broadly, ambiguities in the determination of the remaining matrix elements. In this sense, we visualize two main paths: the first is to select a particular choice for $\tilde{p}_u$ and $\tilde{p}_v$ in terms of the non-commuting variables; the second one is to express the non-commuting momenta p_u and p_v in terms of the commuting ones. We used the second path, as explained above. The choice then gives conditions to the remaining matrix elements, which can be resolved. The present deduction also implies that there are several options in choosing p_u and p_v, depending on the particular choice of M_{21}, M_{23} and M_{43}. Still, for each alternative one has to verify the corresponding relations of the Poisson brackets.

In conclusion, we have presented a novel framework for understanding the accelerating expansion of the Universe based on noncommutative quantum gravity. This model provides a unified explanation for both early inflation and late-time acceleration without the need for fine-tuning. By introducing noncommutative deformations to the conventional Poisson algebra, we have shown that the Universe's wave function and scale factor can evolve in ways that align with observed acceleration. Future work will focus on further refining this model, particularly its implications for relic gravitational waves and its compatibility with observational data.

5. The wave function of the Universe in the noncommutative two-fields formalism

With the choice of gauge taken in the previous section, the transformation of the commuting variables to the noncommuting ones is given by

$$p_u = (\tilde{p}_u - \chi\tilde{p}_v)\,; \quad p_v = (-\gamma\tilde{p}_u + \alpha\tilde{u} - \alpha\tilde{v} + \tilde{p}_v)\,, \tag{27}$$

we arrive at the following expression for the super-Hamiltonian (11) expressed in terms of commutative variables and the parameters of the noncommutative algebra:

$$\mathcal{H} = \frac{1}{2}\frac{N}{\tilde{u}}\Big[-(\tilde{p}_u - \chi\tilde{p}_v)^2 - \frac{1}{u^{3\alpha-1}}(\gamma\tilde{p}_u - \alpha\tilde{u} + \alpha\tilde{v} - \tilde{p}_v)$$
$$+ \left(g_r - g_m\tilde{u} - g_k\tilde{u}^2 - g_q\tilde{u}^3 + g_\Lambda\tilde{u}^4 + \frac{g_s}{\tilde{u}^2}\right)\Big]\,. \tag{28}$$

By adopting the reverse mapping path proposal, the above equation materializes the effects of reconfiguration of the original super-Hamiltonian through the imposition of a noncommutative algebra. The resulting equation, although dependent on the original commutative variables, highlights this reconfiguration through the imposition of a structural composition that inserts new dynamic components into the original formalism, modulated by the parameters σ, χ, γ, α. In the following, for notation simplicity, we eliminate the tilde identification of the commutative variables.

Canonical quantization procedures applied to the Hamiltonian (28), allow the variables $u(t)$ and $v(t)$ along with their corresponding conjugate momenta p_u and p_v, to be treated as operators. Making the replacements $p_u \to -i\,\frac{\partial}{\partial u}$ and $p_v \to -i\,\frac{\partial}{\partial v}$, we obtain the following differential equation to describe the evolution of the wave function of the Universe, giving the condition $\mathcal{H}\Psi(u,v) = 0$, which results in the wave equation:

$$\left[\left(\frac{\partial^2}{\partial u^2} - 2\chi\frac{\partial}{\partial u}\frac{\partial}{\partial v} + \chi^2\frac{\partial^2}{\partial v^2}\right) + \frac{1}{u^{3\alpha-1}}\left(i\gamma\frac{\partial}{\partial u} - i\frac{\partial}{\partial v} + \alpha u - \alpha v\right)\right.$$
$$\left. + \left(g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2}\right)\right]\Psi(u,v) = 0. \tag{29}$$

The parameters χ and γ are treated as complex numbers. Moreover, to make contact with conventional formulations, in special regarding the insertion of a set of factor-ordering parametrization to overcome ambiguities in the time-ordering of quantum operators [1, 2, 41–43]. To maintain the complex nature of the variables u and v, only the real component of the parameter α was taken into account. In addition, the following notation was adopted: $\gamma = i|\gamma| \mapsto i\gamma$, with $i\gamma = i^2|\gamma| = i\gamma = -|\gamma|$, so the previous equation then becomes:

$$\left[\left(\frac{\partial^2}{\partial u^2} - 2\chi\frac{\partial}{\partial u}\frac{\partial}{\partial v} + \chi^2\frac{\partial^2}{\partial v^2}\right) - \frac{1}{u^{3\alpha-1}}\left(|\gamma|\frac{\partial}{\partial u} + i\frac{\partial}{\partial v} - \alpha u + \alpha v\right)\right.$$
$$\left. + \left(g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2}\right)\right]\Psi(u,v) = 0\,. \tag{30}$$

Following the steps shown in appendix, we reduce equation (30) to a canonical form

$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{\gamma}{\eta^{3\alpha-1}}\frac{\partial}{\partial \eta} + g_r - g_m\eta - g_k\eta^2 - g_q\eta^3 + g_\Lambda\eta^4 + \frac{g_s}{\eta^2} + \frac{\alpha}{\eta^{3\alpha-2}}\right.$$
$$\left. - \frac{\alpha\xi}{\eta^{3\alpha-1}} - \frac{i}{\eta^{3\alpha-1}}\frac{\partial}{\partial \xi}\right]\Psi(\eta,\xi) = 0. \tag{31}$$

In the Faddeev-Jakiw original formalism, the variables u and u are noncommutative, and after the variable transformation η and ξ are commutative.However, due to a reverse symplectic transformation and the subjacent noncommutative algebraic structure, $\eta(t)$ and $\xi(t))$ represent commutative variables that span noncommutative complementary quantum spaces. Thus, the following relation between these variables holds:

$$\xi = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(\eta)\, e^{i\xi\eta} \mathrm{d}\eta\,. \tag{32}$$

Expression (31) may be cast as

$$\left[-p^2_{\eta,\gamma,\alpha} + g_r - g_m\eta - g_k\eta^2 - g_q\eta^3 + g_\Lambda\eta^4 + \frac{g_s}{\eta^2} + \frac{\alpha}{\eta^{3\alpha-2}}\right.$$
$$\left. - \frac{\alpha\xi}{\eta^{3\alpha-1}} + \frac{1}{\eta^{3\alpha-1}}p_\xi\right]\Psi(\eta,\xi) = 0\,. \tag{33}$$

In this expression, $-p^2_{\eta,\gamma,\alpha}$ is defined as

$$-p^2_{\eta,\gamma,\alpha} \equiv \frac{\partial^2}{\partial\eta^2} + \frac{\gamma}{\eta^{3\alpha-1}}\frac{\partial}{\partial\eta}$$
$$= -\left(-i\frac{\partial}{\partial\eta}\right)\left(-i\frac{\partial}{\partial\eta}\right) + \frac{i\,|\gamma|}{\eta^{3\alpha-1}}\frac{\partial}{\partial\eta} \equiv -p^2_\eta - p_{\eta,\gamma,\alpha}\,, \tag{34}$$

with

$$p_{\eta,\gamma,\alpha} \equiv -\frac{i\,|\gamma|}{\eta^{3\alpha-1}}\frac{\partial}{\partial\eta} \mapsto \frac{\gamma}{\eta^{3\alpha-1}}\left(-i\frac{\partial}{\partial\eta}\right) = \frac{\gamma}{\eta^{3\alpha-1}}p_\eta\,. \tag{35}$$

With the particular choice $\alpha = 1/3$, that allows a separation of variables, with $\Psi(\xi,\eta) = \Psi(\xi)\Psi(\eta)$, equation (33), reduces to the following equations

$$\left(\frac{\partial^2}{\partial\eta^2} + \gamma\frac{\partial}{\partial\eta} + V(\eta)\right)\Psi(\eta) = 0\,, \tag{36}$$

where

$$V(\eta) = \tilde{g}_r - \tilde{g}_m\eta - g_k\eta^2 - g_q\eta^3 + g_\Lambda\eta^4 + \frac{g_s}{\eta^2}\,, \tag{37}$$

with $\tilde{g}_r \equiv g_r - \mathcal{C}$ and $\tilde{g}_m \equiv g_m - 1/3$, and

$$\left(i\frac{\partial}{\partial\xi} - \frac{1}{3}\xi + \mathcal{C}\right)\Psi(\xi) = 0; \tag{38}$$

in these equation, $\mathcal{C}$ is a separation constant.

By using the successive approximation method, we found the following algebraic solution for equation (39):

$$\Psi(\eta) = a_1 e^{-\frac{\gamma}{2}(\eta-\log(\eta)/\gamma)} J_{\frac{i\sqrt{3}}{2}}\left(-\frac{i\gamma x}{2}\right) + a_2 e^{-\frac{\gamma}{2}(\eta-\log(\eta)/\gamma)} Y_{\frac{i\sqrt{3}}{2}}\left(-\frac{i\gamma x}{2}\right)$$
$$+ b_1 + b_2\gamma\sum_{m=1}^{5}\frac{\eta^m}{m!} + \frac{1}{\gamma}\sum_{n=6}^{11} b_n\frac{\eta^n}{n!} + \mathcal{O}\left(\eta^{12}\right). \tag{39}$$

The solution to the equation (38) in turn, up to an additional constant is

$$\Psi(\xi) = c\exp\left(\mathcal{C}\xi - \frac{1}{6}\xi^2\right). \tag{40}$$

5.1. Naturalness

In the study of the evolution of the Universe, we come across an epistemic limitation of realism: the under-determination of theory by evidence. Data under-determination poses a substantial problem for the high-energy frontier of fundamental physics, most notably within the fields of particle physics and quantum gravity. Therefore, an organizing and guiding principle is required to assess the viability of models in a non-empirical manner, enabling consistent and accurate calculations. As first proposed by Weinberg [44] the principle of naturalness serves as a conventional method for classifying and organizing the terms within highly intricate approaches. It also provides guidance for understanding the various interaction couplings associated with the dynamic composition of matter, energy, and the primordial sources of gravitational waves. This principle, which suggests that the underlying parameters are all of the same size in appropriate units or, more precisely, that a given quantum field theory can only describe nature at energies below a certain scale or cutoff [44], will be adopted. In this context, we adhere to the principles of naturalness, normalizing them to unity.

5.2. Solutions

In what follows, we use the successive approximation method to find algebraic solutions for equation (36), in order to describe the dynamical evolution of the wave function of the Universe, $\Psi(\eta,\xi)$. The boundary conditions of the solutions are based on the Bekenstein criterion, which provides an upper limit for the Universe's entropy, following the proposition presented in [1, 2]. The total entropy of a black hole, according to the Bekenstein limit, is proportional to the number of Planck areas needed to cover the event horizon, where each area corresponds to one unit of entropy. In noncommutative branched gravitation, we assume that the primordial singularity is equally covered by a certain number of Planck areas, whose numerical value in turn corresponds to the total primordial entropy of the Universe. We assume that the dimensions of this boundary region correspond to the farthest points observable while still respecting causality. For this, we consider an appropriate distance, denoted as $d(t)$, between a pair of objects at any given time instant t, and the corresponding distance, denoted as $d(t_0)$, at a reference time t_0. We establish this relationship as $d(t) = |\eta(t)|d(t_0)$. This means that the relationship between the two distances is modulated by the scale factor of the BCQG Universe. This implies that for $t = t_0$ we have $|\eta(t_0)| = 1$. From a quantum probabilistic point of view, this condition implies a maximum probability of observation, $|\Psi(1)|^2 = 1$, assuming a normalized wave function. Thus, the boundary conditions considered in this contribution are, in the contraction sector $\Psi(-1) = -1$, while in the expansion region, $\Psi(1) = 1, \Psi'(\pm 1) = 0$. The other possibility in turn corresponds to $\Psi(1) = 0, \Psi'(\pm 1) = \pm 1$.

Figure 1 depict family sample solutions of equation (36), for different γ values. Figure 2 shows the plot of the solutions of equation (38), which corresponds to $\Psi(\xi)$, the dual quantum counterpart of $\Psi(\eta)$. The behavior of the solutions highlights the process of acceleration of the Universe as a result of the noncommutative algebraic structure. These results align with the previous predictions of the BCQG, suggesting that the present Universe did not originate from nothing as stressed by [14, 15, 45]) or event from a quantum loop as indicated by [46]. Instead, it appears to have emerged from a prior phase before the current expansion phase.

Various explanations have been proposed for the accelerating expansion of the Universe. Traditional models, such as Einstein's cosmological constant or the concept of dark energy, attribute this acceleration to an intrinsic property of spacetime or a new form of energy. However, our model suggests a more fundamental origin: the noncommutative geometry of spacetime itself. This quantum modification leads to a late-time accelerated growth of the cosmic scale factor, offering a compelling alternative to dark energy and other external driving forces.

When we examine the plots of the configuration of matter and energy in the early Universe (see (49) and (51)), we may identify noncommutative imprints of the spacetime structure implying non-symmetrical redistribution of matter and energy which captures, in our conception, the short- and long-range spacetime scales. Moreover, the transition region between the two universes could serve as a source of matter/particles and energy, which drives the acceleration of the Universe. Briefly, our results involving the contraction phase as well as the

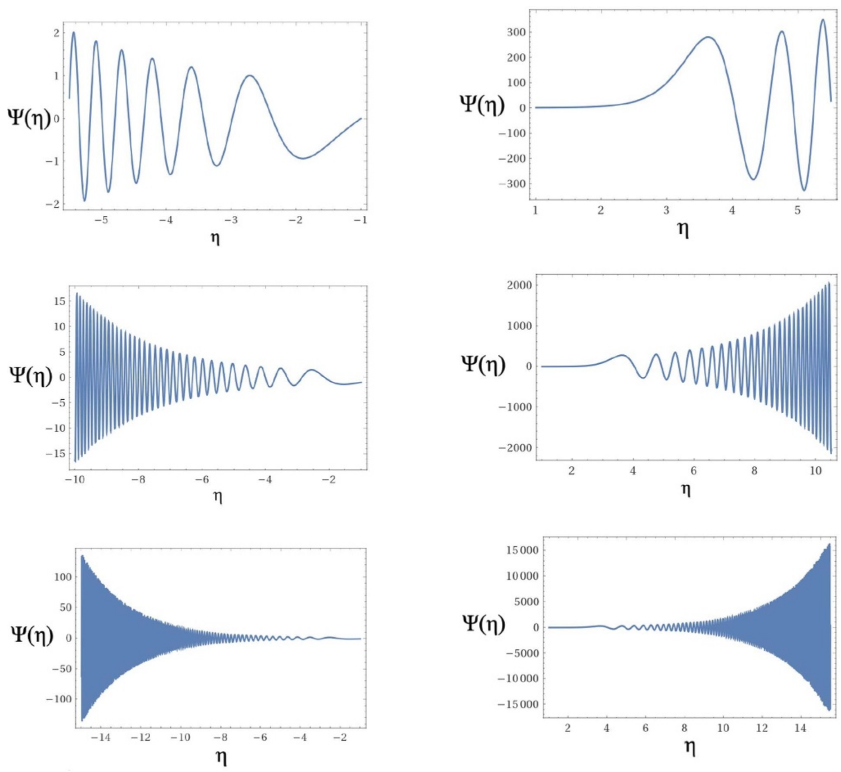

Figure 1. Sample solutions of the wave function $\Psi(\eta)$ for the Universe, derived using the noncommutative algebraic formulation in equation (39). The left figures represent solutions for positive values of the running coupling constants, while the right figures show solutions for negative values. These plots illustrate the role of noncommutative geometry in driving the Universe's acceleration, with key differences between positive and negative parameter sets influencing the wave function's behavior. On the left figures, the values of the running coupling constants and parameters are as follows: $\tilde{g}_r = 0.4$; $\tilde{g}_m = 0.6185$; $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_s = -0.03$; $\alpha = 1/3$; $\gamma = 1$; $\mathcal{C} = 1$. The boundary condition set as $\Psi(-1) = 0, \Psi'(-1) = -1$. On the right figures, the values of the running coupling constants and parameters are: $\tilde{g}_r = 0.4$; $\tilde{g}_m = 0.6185$; $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_s = -0.03$; $\alpha = 1/3$; $\gamma = -1$; $\mathcal{C} = 1$. The boundary conditions are set as $\Psi(1) = 0, \Psi'(1) = 1$.

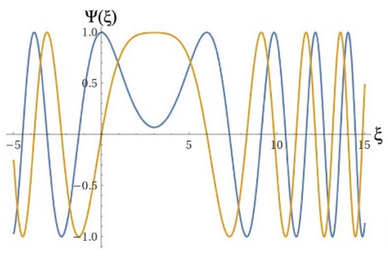

Figure 2. Solution of equation (38) corresponding to $\Psi(\xi)$, the dual quantum counterpart of $\Psi(\eta)$. The blue lines correspond to real solutions while the orange lines correspond to imaginary solutions.

expansion phase of the analytically continued foliated quantum Universe in the noncommutative domain, indicate an acceleration of the wave function $\Psi(\eta)$ in the expansion phase, in tune with the predictions of the inflation model, as well as a deceleration in the contraction phase, both predictions in tune with the BCQG prognosis.

This heterogeneous distribution of matter and energy and its implications in terms of the expansion dynamics of the Universe can be visualized, as we will see later, particularly in the configuration of the color palette of the effective potentials of the noncommutative formulation of the BCQG. The intensities of these potentials, which are reflected in the accelerated evolution of the wave function and the scale factor of the Universe, are intrinsically linked to the spectrum of these color palettes. In the 3D colored graphical representations of these potentials, lighter colors identify regions of greater intensity, while darker colors represent regions of lower intensity. The palettes also allow us to visualize the asymmetry of these color combinations, which reflect the heterogeneity of the distribution of matter and energy in the primordial Universe.

5.3. Probability interpretation of the wave function of the Universe

For Rovelli [47–50] the absence of time is a feature of the classical Hamilton–Jacobi formulation of general relativity, and the wave function is only a function of the '3-geometry', namely the equivalence class of metrics under a diffeomorphism, and not of the specific coordinate dependent form of the metric tensor. According to the second law of thermodynamics, forward in time represents the direction in which entropy increases and in which we obtain information, so the flow of time would represent a subjective feature of the Universe, not an objective part of physical reality [47–50]. In this realm, in which the observable Universe does not exhibit time-reversal symmetry, events, rather than particles or fields, are the basic constituents of the Universe, implying that the evolution of physical quantities is related to the description of the relationship between events [47–50]. For instance, given the wave function of the Universe as a functional constrained to a super-space that contains a three-surface Σ and matter fields configuration, represented by ϕ, where the metric is given by h_{ij}, the corresponding WdW wave function $\Psi(h_{ij},\phi)$ may be interpreted according to [47–50], as stressed before, as describing the $\Psi(\phi)$ evolution not in a temporal sense but in terms of the physical variable ϕ.

For Hawking [42] for which $|\Psi[h_{ij},\phi,\Sigma]|^2$ is proportional to the probability $P(\mathcal{A})$ of finding a three-surface Σ with metric h_{ij} and matter field configuration ϕ:

$$P(\mathcal{A}) \propto \int_{\mathcal{A}} |\Psi[h_{ij},\phi,\Sigma]|^2 \, \mathrm{d}\mathcal{V} \tag{41}$$

where $\mathcal{V}$ corresponds to a volume element [42]. $\Gamma(N)$ in turn represents the superspace metric which does not depend linearly on N.

As highlighted by Hartle [51], in standard quantum mechanics, the probabilities associated with wave functions are represented by squares of amplitudes, and additionally, a criterion is needed to specify which sets of histories can have probabilities consistently assigned to them. And yet, as highlighted by Hartle, in standard quantum mechanics that criterion is measurement, so probabilities can be consistently assigned to histories of measured alternatives and usually not otherwise. Hartle further emphasizes that the application of quantum mechanics to cosmology also requires another kind of generalization of the usual formulation. Usual quantum mechanics predicts the outcomes of 'measurements' carried out on a system by another system outside it. But in cosmology there is no system outside. So, cosmology requires a quantum mechanics interpretation of closed systems as a generalization of the usual theory. The most general predictions of this formulation of quantum mechanics are the

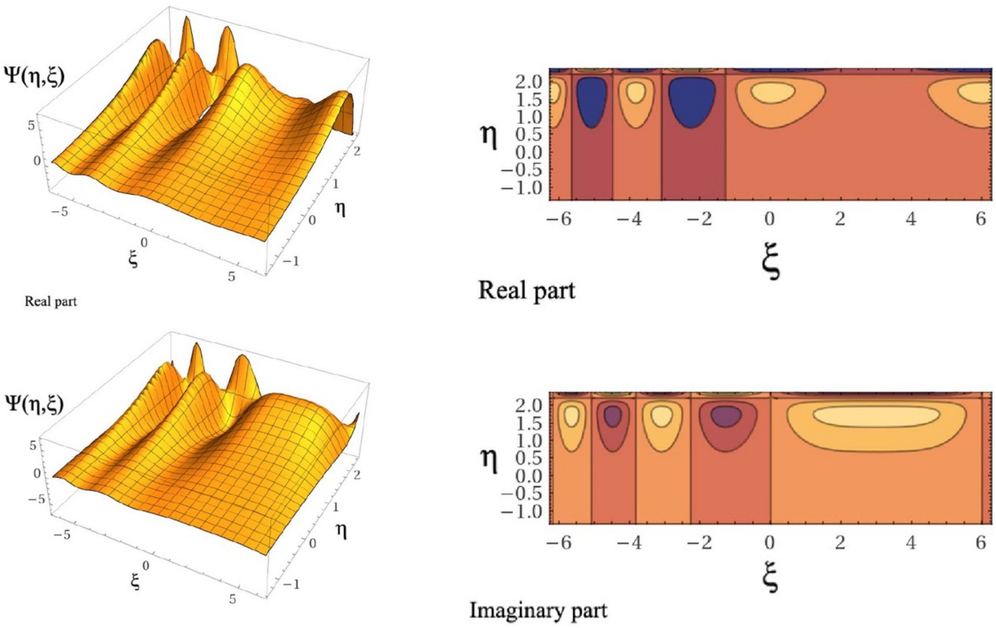

Figure 3. The upper-left and lower-left figures depict 3D plots illustrating the real and imaginary solutions of equation (33) for the wave function of the Universe, $\Psi(\eta,\xi) = \Psi(\eta)\Psi(\xi)$. The solutions $\Psi(\eta)$ and $\Psi(\xi)$ are obtained in the naturalness regimen, $\mathcal{C} = 1$ and $\gamma = -1$. Correspondingly, the upper-right and lower-right figures display the associated contour plots.

probabilities of the individual members of sets of alternative histories of the closed system. Consistency of probability sum rules is the criterion determining the sets of histories to which may be assigned probabilities rather than any notion of measurement. The absence of quantum mechanical interference between histories, or decoherence, is the sufficient condition for this consistency. The initial condition of the closed system and Hamiltonian determine which sets of histories decohere rather than the action of any external observer [51].

This form of interpretation, in our view, is more in harmony with Hawking's prescription. In what follows, we consider both interpretations, Rovelli's and Hawking's prescription for $|\Psi(\eta)|^2$ and $|\Psi(\xi)|^2$, and to establish a connection with standard quantum mechanics we use the denomination 'probability density'.

The upper-left and below-left images in figure 3 show the 3D plots of the real and imaginary solutions of equation (33) for the wave function of the Universe $\Psi(\eta,\xi)$, while the upper-right and below-right images show the corresponding contour plots. Figure 3 in turn show the corresponding density distributions of the wave function of the Universe $\Psi(\eta,\xi)$. The results describe the expansive acceleration of the Universe similarly to previous cases. The most unexpected results correspond to the minimum values of the Universe's wave function, separated symmetrically, whose realization occurs in the mirror Universe, separated by maximum values that are more intense at the initial point of the transition to the current Universe, presenting a behavior similar to that of the roll on inflation potential on the inflaton field. In particular, the format of this potential presents points of similarity with the argument of the equation, more specifically, $V(\xi) = (\mathcal{C}(\xi-\xi_0) - (1/6)(\xi-\xi_0)^2)^2$. This result indicates a strong correlation involving the scale factor η and quantum fluctuations of its dual partner ξ, that behave as a kind of noncommutative symplectic phase space background.

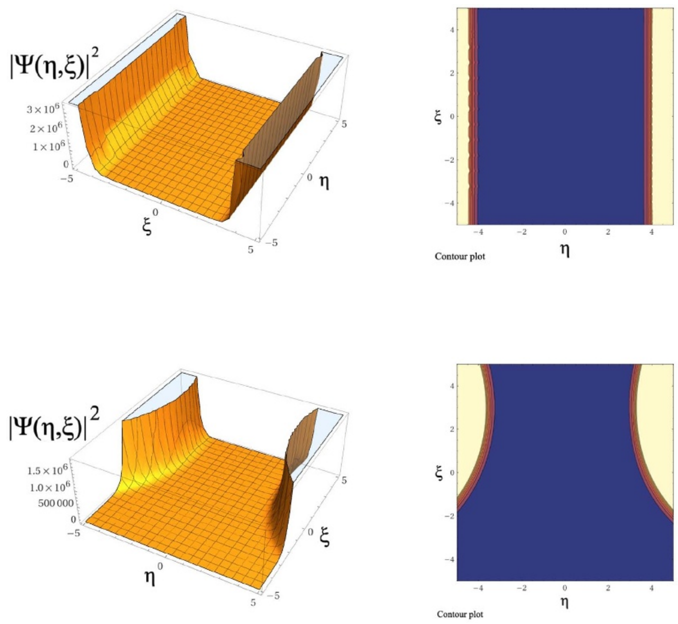

Figure 4. The upper-left and lower-left figures show the 3D plots of the probability density distributions of the real and imaginary solutions of equation (33) for the complete wave function of the Universe, denoted as $\Psi(\eta,\xi)$. The upper-right and lower-right figures show the corresponding contour plots. The parametrization of the solutions are similar to those of the previous figure.

Figure 4 shows 3D plots of the probability density for the real and imaginary solution of equation (33) for the complete wave function of the universe, denoted by $\Psi\left(\eta,\xi\right)$. The parametrization of the solution is similar t that in the previous figure.

As a summary, complex equations similar to Friedmann's equations underlie the branched gravitation scenarios, in which the primordial singularity is replaced by a foliated transition region, described by helix-shaped cosmological factor $\eta(t)$, analytically continued to the complex plane, interposing two distinct evolutionary stages of the Universe, a contraction phase and an expansion phase. The consequences of these scenarios on the behavior of the Universe's wave function are notable in that they imply the evolutionary description of $\Psi(\eta,\xi)$ in both regions.

6. Dynamical equations: two-fields noncommutative approach

In what follows, we analyze the role of the noncommutative algebraic structure in the expansion of the Universe. In the analysis that we will carry out, we seek to examine the role of the noncommutative structure in the early time accelerated expansion of the Universe, more precisely in the inflation period.

6.1. Second-order dynamical equations: two-fields noncommutative approach

In the following, we consider second-order dynamical equations for the scale factor $\eta(\tau)$ and its counterpart $\xi(\eta)$.

From expression (28), assuming the gauge fixing condition $N = 1$, the following dynamical equations for $\eta(\tau)$ and $\xi(\tau)$ result:

$$\dot{\eta} = \frac{\partial \mathcal{H}^{NC}}{\partial p_\eta} = -\frac{p_\eta}{\eta} - \frac{\gamma}{2\eta^{3\alpha}}\,; \tag{42}$$

$$\dot{\xi} = \frac{\partial \mathcal{H}^{NC}}{\partial p_\xi} = \frac{1}{2\eta^{3\alpha}}\,; \tag{43}$$

$$\dot{p}_\xi = -\frac{\partial \mathcal{H}^{NC}}{\partial \xi} = -\frac{\alpha}{2\eta^{3\alpha}}\,; \tag{44}$$

$$\dot{p}_\eta = -\frac{\partial \mathcal{H}^{NC}}{\partial \eta} = -\frac{1}{2\eta}\left[\frac{1}{\eta}p_\eta^2 + \frac{3\alpha\gamma}{\eta^{3\alpha}}p_\eta - \frac{g_r}{\eta} - g_k\eta \right.$$
$$\left. -2g_q\eta^2 - 3g_\Lambda\eta^3 - 3\frac{g_s}{\eta^3} - \frac{\alpha(3\alpha-1)}{\eta^{3\alpha-1}} + \frac{3\alpha^2 t}{\eta^{6\alpha-1}}\right], \tag{45}$$

where we have assumed an explicit conformal time-dependence on the variables η and ξ; thus, all known solutions are of the separation of variables type, where time and space dependence are treated separately. Thus, from the previous equations, we obtain the following expressions for η and ξ:

$$\xi = \int \mathrm{d}t\dot{\xi} = \frac{t}{2\eta^{3\alpha}}\,, \quad p_\xi = \int \mathrm{d}t\dot{p}_\xi = -\frac{\alpha t}{2\eta^{3\alpha}}\,. \tag{46}$$

The time derivative of the conjugate canonical momentum $p_\eta = -\eta\dot{\eta}/N$ in the $N = 1$ gauge may be written as

$$\dot{p}_\eta = \frac{\partial p_\eta}{\partial t} = -\frac{\partial(\eta\dot{\eta})}{\partial t} = -\dot{\eta}^2 - \eta\ddot{\eta}\,. \tag{47}$$

Equation (45) combined with (47) may be recast in the form

$$2\eta(t)\ddot{\eta}(t) + \dot{\eta}^2(t) + \frac{3\alpha\gamma\dot{\eta}}{\eta^{3\alpha}} + V(\eta,t) = 0\,, \tag{48}$$

where the potential $V(\eta,t)$ has an explicit time dependence:

$$V(\eta,t) = g_k + 2g_q\eta - 3g_\Lambda\eta^2 + \frac{g_r}{\eta^2} + 3\frac{g_s}{\eta^4} + \frac{\alpha(3\alpha-1)}{\eta^{3\alpha}} - \frac{3\alpha^2 t}{\eta^{6\alpha-1}}\,. \tag{49}$$

Figure 5 show 3D graphical illustrations of the potential $V(\eta,t)$ of equation (49) and the corresponding contour plots characterizing the dependence of the scale factor $\eta(t)$ on time. Figure 6 presents graphical representations of the potential $v(\eta,t)$ from equation (51), indicating the presence of a torsion deformation. More specifically, it indicates a deformation that resembles a kind of torsional conformation of an object due to the application of an 'external' torque. In the noncommutative formulation, this symmetry is broken, indicating a mixture of intensities or amplitudes of the potential $V(\eta,t)$. The potential $V(\eta,t)$ simulates the presence of different compositions of matter in the primordial Universe that imply structural modifications of the spacetime structure, shaping this way its curvature that depends locally on the amount and distribution of matter or, equivalently, energy. This symmetry breaking reveals the potentiality of a noncommutative formulation in terms of its implications in affecting not only the curvature of space-time, but furthermore, the capture of short and long scales, boosting the evolution dynamics of the wave function of the Universe and the cosmic scale factor. Insofar, as the presence of the potential is associated with a force, of a gravitational character, which may constitute the propelling element of the acceleration of the primordial Universe. These

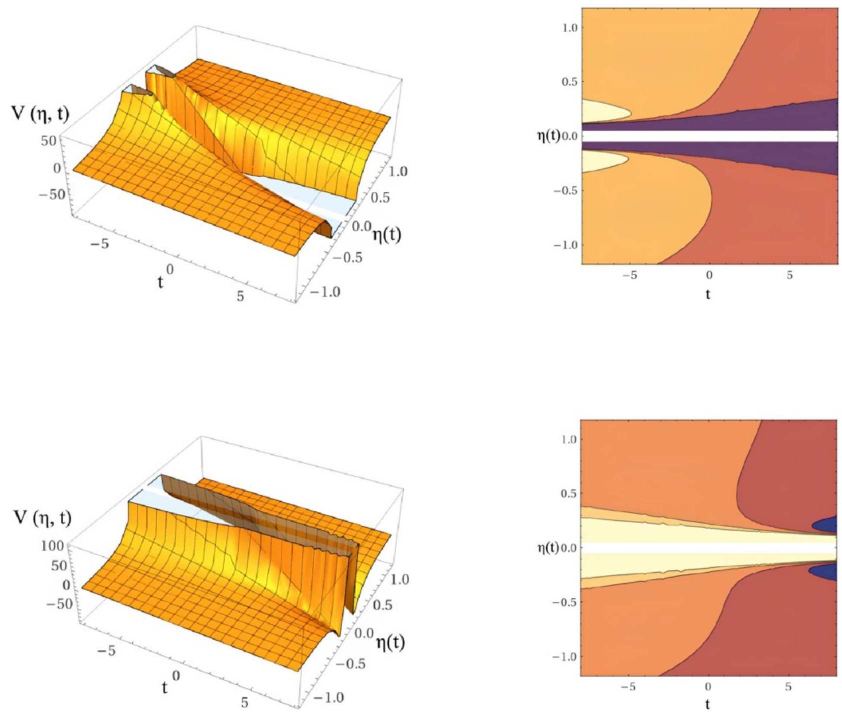

Figure 5. Graphical illustrations of the potential $V(\eta, t)$ from equation (49) and corresponding contour plots are presented. In the upper pair of figures, the values of the running coupling constants and the parameters of the noncommutative algebra are as follows: $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_r = 0.4$; $g_s = -0.03$; $\alpha = 1/2$; $\mathcal{C} = 1$. In the below pair of figures, the values of the running coupling constants and parameters are: $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_r = 0.4$; $g_s = 0.03$; $\alpha = 1/2$; $\mathcal{C} = 1$.

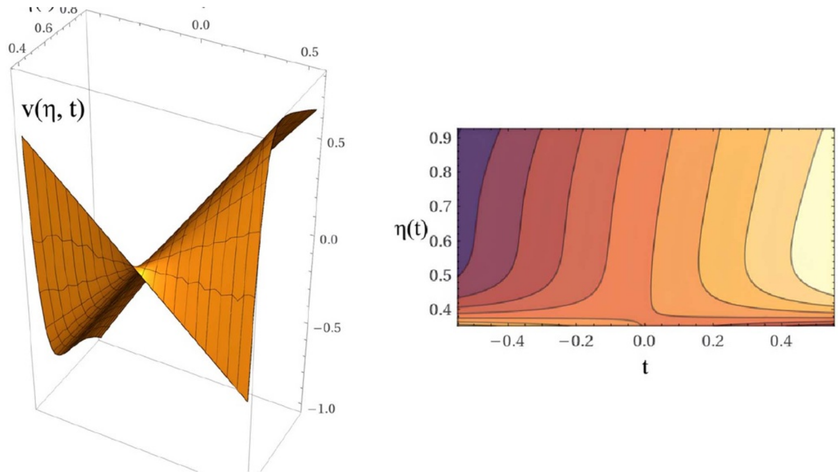

Figure 6. Graphical illustrations of the potential $v(\eta, t)$ of equation (51) and contour plots, indicating the presence of a torsion deformation The values of the running coupling constants and the parameters of the noncommutative algebra for the figure are: $g_k = 1$; $g_q = 0.7$; $g_\Lambda = 0.333$; $g_r = 0.4$; $g_s = -0.03$; $\alpha = -5/6$; $\gamma = 3/5$; $\mathcal{C} = 1$.

results provide conceptual elements that indicate a reconfiguration of matter on small scales of dimensions. This reconfiguration of matter, as previously observed, represents a typical characteristic of a noncommutative formulation, which drives, through a non-symmetric gravitational force, the acceleration of the Universe. Furthermore, due to the dual role of gravity, spacetime is both warped by matter and matter in turn experiences a warped spacetime due to the presence of other distributions of matter, which intensifies the effects of higher curvature terms. The color palette used in the contour plots is indicative of these dynamic processes. Lighter colors suggests the materialization of greater amplitudes or intensities of the effects associated with the composition of matter and energy, while darker colors signify the opposite effects. It is precisely within the interplay of these effects, where symmetry breaks along the axis $\eta(t) = t = 0$, that the accelerated evolutionary dynamics of the analytically continued foliated quantum Universe finds the necessary components for its manifestation.

6.2. First-order dynamical equations: two-fields noncommutative formulation

As mentioned earlier, the dynamic equations governing the evolution of the scale factor $\eta(t)$, both in terms of first-order and second-order time derivatives, are crucial for describing primordial gravitational waves. Thus, in the following, we will elaborate on the corresponding first-order dynamic equations.

Although it may seem unnecessary or redundant to seek for the behavior of the solutions by determining first- and second-order equations, we adopt this procedure fundamentally for reasons of formal consistency. This is because, as previously noted, we adopt an explicit conformal time-dependence on the variables η and ξ; thus, all known solutions are of the separation of variables type, where time and space dependence are treated separately. In this sense, the deductions of the first- and second-order wave equations present distinct characteristics, especially with regard to time dependence. However, it is expected, and this has been proven as will be observed later, that the predictions regarding the temporal evolution of the Universe are consistent.

Upon integrating equation (47), while assuming that η possesses explicit time dependence allowing for separation from time, and further considering the $\dot{\eta}$ and η function as independent variables, we obtain the following expression relating the time-derivative dependence of the scale factor $\eta(t)$ and a time-dependent potential $v(\eta,t)$:

$$\dot{\eta} + v(\eta,t) = 0\,. \tag{50}$$

In the expression the potential $v(\eta,t)$ is defined as

$$v(\eta,t) = \frac{1}{3\eta}\left\{-\frac{3\alpha\gamma}{(3\alpha-1)\eta^{3\alpha-1}} + \left(g_k + 2g_q\eta - 3g_\Lambda\eta^2 + \frac{g_r}{\eta^2} + 3\frac{g_s}{\eta^4} + \frac{\alpha(3\alpha-1)}{\eta^{3\alpha}}\right)t - \frac{3}{2}\frac{\alpha^2t^2}{\eta^{6\alpha-1}}\right\}. \tag{51}$$

6.3. Results: noncommutative dynamical equations

Figure 7 presents typical sample family solutions of the scale factor of the analytically continued foliated quantum Universe, $\eta(t)$, obtained through the combination of second-order in time $\ddot{\eta}(t)$ and first-order in $\dot{\eta}(t)$ noncommutative approach given by equation (48). These solutions are for variations in the initial conditions $\eta(0)$ and $\eta'(0)$. Figure 7 shows also typical sample family solutions of $\eta(t)$ using the first-order in time $\dot{\eta}$ noncommutative approach given

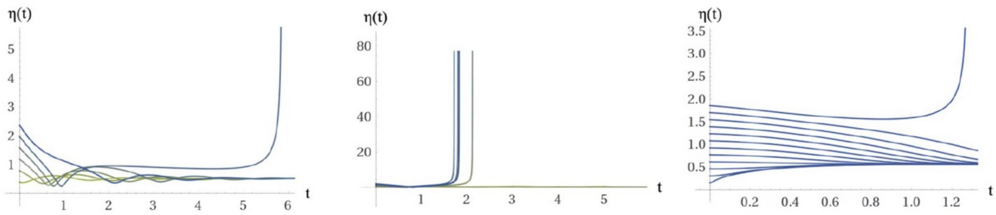

Figure 7. The left and central figures depict typical sample family solutions for the scale factor of the analytically continued foliated quantum Universe, denoted as $\eta(t)$, utilizing the second-order time derivative noncommutative approach equation (48). These solutions are for varying initial conditions $\eta(0)$ and $\eta'(0)$. In the right figure, the values of the running coupling constants and parameters are as follows: $\tilde{g}_r = -0.4$; $g_k = 1$; $g_q = 0.7$; $g_\Lambda = -0.333$; $g_s = -0.03$; $\alpha = -1/2$; $\gamma = 1$; $\mathcal{C} = 1$. In figure (b), the values of the running coupling constants and parameters remain the same, except for $\alpha = -3/4$; $\gamma = -1$. Figure (c) displays a typical sample family corresponding to the firs-order time derivative noncommutative approach, following equation (50), with variations in the initial condition $\eta(0)$. The values of coupling constants and parameters are: $\tilde{g}_r = -0.4$; $g_k = 1$; $g_q = 0.7$; $g_\Lambda = -0.333$; $g_s = -0.03$; $\alpha = -3/4$; $\gamma = -1$; $\mathcal{C} = 1$.

by equation (50), with variations in the value of $\eta(0)$. These results demonstrate the significant impact of the noncommutative formulation on the dynamics of the early Universe's evolution. The findings reveal that the behavior of the scale factor $\eta(t)$ is notably different from what is predicted in a commutative formulation, particularly concerning the acceleration components of the early Universe.

While the Universe may experience a deceleration phase in its initial moments of expansion for specific parameter sets, it abruptly undergoes a drastic acceleration. This acceleration is characterized by an evolutionary curve that approaches a ninety-degree angle with the x-axis, nearly parallel to the y-axis. These results align with one of the key propositions of this work, which aims to understand the mechanisms driving the acceleration of the Universe's expansion.

The formal structure of the super-Hamiltonian obtained enables us to associate the new variable $\eta(t)$ with the scale factor of the BCQG, $\ln^{-1}[\beta(t)]$. However, it is evident that, as a result of the imposed variables transformations, this incorporates elements characterizing a noncommutative algebra at a fundamental level. And given its noncommutative nature, this algebraic structure allows us to identify the $\xi(t)$ variable as the dual quantum counterpart of $\eta(t)$, with both variables scanning reciprocal quantum complex spaces. It is important to note that, even though the new variables η and ξ are treated as linearly independent dual variables, they carry new effective identities when compared to the original bare variables. This is due to the nature of the coordinate transformations, which imbue these variables with underlying mutual and complementary properties and identities. In classical formulations, the statistical distribution of matter is typically assumed to be homogeneous and is determined by the dynamics of the Hamiltonian. However, systems with higher complexity can exhibit topological constraints that may be independent of the Hamiltonian and can affect the shape of statistical matter distribution functions [52]. In general relativity, the curvature of spacetime arises as a source of heterogeneity in the statistical distribution of matter [52]. In our formulation we can identify an additional source of statistical distribution heterogeneity, specifically, the noncommutative algebraic representation of the BCQG combined with the Hořava–Lifshitz formulation. This formulation leads to the realization of a potential that incorporates different matter composition contributions. These contributions are represented by algebraic terms dependent on $\eta(t)$ and appear in different orders. Their formal structure, apart from the derivative terms, is similar to the dependence of the Hořava–Lifshitz formulation on the scalar curvature of the Universe. The implications of this formulation suggest a significant impact not only on the curvature of spacetime and the statistical distribution of matter but also on the evolutionary dynamics of the early Universe, which drives its acceleration.

In revisiting the cosmological implications of the results presented in Figure 6, which indicate the presence of a torsion or twist deformation of spacetime in the context of the analytically continued foliated noncommutative algebra, we encounter an intriguing outcome. The color palette associated with the contour plots unveils a remarkable pattern. There appears to be a progressive transition in color intensity, ranging from darker colors (representing lower potential intensity values) to lighter colors (indicating higher potential intensity values) when examining the contour plots of the potential $v(\eta,\tau)$. When we relate these colors to the regions corresponding to the 3D graph of the potential, a significant revelation emerges. The point where $t = 0$, the boundary region between the present Universe and its mirrored counterpart, aligns with the point of maximum torsion in the potential $v(\eta,\tau)$. This suggests that the accelerated expansion of our Universe may be the result of a folded memory shared by both universes. This concept implies that spacetime possesses a fold-memory (or twist-memory or torsion-memory), which, when subjected to a twist in the mirrored counterpart and subsequently regulated and shaped by this fold-memory, spontaneously unfolds in response to 'external' stimuli. This unfolding process propels the acceleration of our Universe's expansion.

7. Discussion of results

In simple words, a symplectic manifold is equipped with a symplectic form, i.e. a closed nondegenerate two-form manifold [53]. Symplectic forms allow the definition of symplectic bases, which are the analogues of orthonormal bases in Euclidean geometry. Let E be a real vector space; its generic vector will be denoted by z. A symplectic form (or: skew-product) on E is a mapping ω: $E \times E \to \mathbb{R}$ [54]. The symplectic geometry in turn is the geometry of symplectic manifolds. Symplectic manifolds are necessarily even-dimensional and orientable, due to the nondegeneracy condition which states that the top exterior power of a symplectic form is a volume form. The closedness condition is a natural differential equation, which forces all symplectic manifolds to being locally indistinguishable. In short, symplectic geometry is the geometry of manifolds equipped with a symplectic form, that is, with a closed and nondegenerate form of degree 2. Hamiltonian geometry, in turn, is the geometry of (symplectic) manifolds equipped with a moment map, that is, with a collection of quantities conserved by symmetries [53].

Noncommutative symplectic geometry, parallel to the usual calculus of differential forms, Poisson brackets and operator algebras, corresponds to a generalization of symplectic geometry in the domain of noncommutative algebra [55]. The noncommutative symplectic geometry was motivated by the cohomology (or compactification) study of the properties of topological spaces, a complementary way to homology theory, with applications to moduli spaces of smooth complex algebraic curves, as well as to cohomology of foliations and by perturbation expansions of Chern–Simons theory. According to Kontsevich [55], when applied to a generalized version of Lie theory, the resulting approach is reflected in differential graded algebra with finite dimensional cohomology and a rational Poincaré duality [55]. It is important to highlight that the Poincaré Duality Theorem is valid for the basic cohomology of taut Riemannian foliations, more precisely, for harmonic foliations with minimal leaves on a Riemannian manifold [56]. In short, the Poisson brackets have the form $\{q_i, p_j\} = g_{ij}$. Because of the antisymmetry of the Poisson brackets, the g_{ij} automatically correspond to a symplectic metric, no further assumptions required. As seen in equation (16), the metric has the block structure of a symplectic metric.

Regarding this topic, in our formulation we have two foliation levels. The first level was addressed in section 2. The second level is characterized by an analytically continued Riemannian foliation which corresponds to the reciprocal of a complex multi-valued function, the natural complex logarithm function $\ln[\beta(t)]$, a helix-like superposition of cut-planes, which correlates Riemann sheets, with an upper edge cut in the nth plane joined with a lower edge of cut in the $(n+1)$th plane. The BCQG Universe's scale factor $\ln[\beta(t)]$ maps an infinite number of Riemann sheets onto horizontal strips, which represent in the branch-cut cosmology the evolution of the time-parameter dependence of horizon sizes. The patch sizes in turn maps progressively the various branches of the $\ln[\beta(t)]$ function which are *glued* along the copies of each upper-half plane with their copies on the corresponding lower-half planes. In the branch-cut cosmology, the cosmic singularity is replaced by a family of Riemann sheets in which the scale factor shrinks to a finite critical size, — the range of $\ln^{-1}[\beta(t)]$, associated to the cuts in the branch cut, shaped by the $\beta(t)$ function —, well above the Planck length. In the contraction phase, as the patch size decreases with a linear dependence on $\ln[\beta(t)]$, light travels through geodesics on each Riemann sheet, circumventing continuously the branch-cut, and although the horizon size scale with $\ln^{\epsilon}[\beta(t)]$, the length of the path to be traveled by light compensates for the scaling difference between the patch and horizon sizes. Here, $\epsilon(t)$ represents the dimensionless thermodynamical connection between the energy density E and the pressure P of a perfect fluid thus enabling the fully description of the EoS of the system. Under these conditions, causality between the horizon size and the patch size may be achieved through the accumulation of branches in the transition region between the present state of the Universe and the past events.

Conventionally, the theory deals exclusively with finite-dimensional real symplectic spaces. The BCQG in turn extends the ontological domain of general relativity to the complex plane.

In the BCQG approach, as previously emphasized, noncommutativity was introduced by means of a deformation of the conventional Poisson algebra, enhanced with a symplectic metric, based in the Faddeev–Jackiw symplectic two fields formulation. As an alternative exemplification, the deformation to the minisuperspace, in order to incorporate noncommutativity, may be introduced by means of Moyal brackets, which are based on the Moyal product [57]. A key insight from this formulation is that the introduction of noncommutative geometry creates a natural asymmetry in the early Universe, potentially explaining both the inflationary phase and late-time acceleration without requiring separate mechanisms. The evolution of the dual spaces, η and ξ, reveals a topological twist in the spacetime fabric, which drives this expansion. Additionally, the model suggests the emergence of relic gravitational waves from the early Universe, which may offer observable signatures in upcoming cosmic microwave background (CMB) experiments.

The limitations presented by standard cosmology arise not only from the fact that, when extrapolating back in time, the ratio of the horizon size to the patch size shrinks in such way that the horizon size approaches zero significantly faster, but essentially from the existence of singularities that break down this ratio and thus make it impossible to restore causality, making conventional cosmology geodesically incomplete. In the standard big bang model, this vertex corresponds to the cosmic singularity, a point in spacetime where, formally, the density and temperature diverge and the geometry shrinks to zero, a sign that the equations used to describe the evolution of the Universe cannot be trusted near this point. The horizon represents the maximal region that is causally connected by means of interactions with light or any other particles; as a corollary, a patch corresponding to the observable Universe today was increasingly disconnected in the past. Finding a suitable fix corresponds to overcoming the cosmic singularity and horizon problems (see [30–32]).

In the branch-cut cosmology, these problems are overcome by the presence of a branch-cut and a branch point, defining this way the domain of the analytically continued to the complex sector Universe's scale factor $\ln^{-1}[\beta(t)]$. In BCQG, going back in time, the scale factor shrinks as stressed before to a finite critical size, keeping causality restored. Furthermore, our proposal assumes the description of the background evolution of branch-cut cosmological scenarios in leading order by classical equations of motion.

The new symplectic algebraic quantum formulation presented in this contribution offers, in comparison with traditional methods, another descriptive advantage by overcoming the imposition for an explicit ad hoc insertion of an inflaton-type field to describe the cosmic inflationary period of the Universe. And additionally, BCQG allows us to overcome still controversial aspects of the inflationary model, such as the ambiguities in defining probabilities in eternally inflating spacetimes, with emphasis on the youngness paradox that results from a synchronous gauge regularization technique. Moreover, taking into account that although inflation is generically eternal into the future, it is not eternal into the past. This implies that the inflating region must be incomplete in past directions, which has implications in settling past boundaries of the inflating region. Additionally, according to the authors of [58], as a descriptive additional advantage, the noncommutative symplectic algebraic formulation gives a mechanism to end inflation and also permits that a non accelerating Universe after a period of time can start a reacceleration period. This is because the noncommutative symplectic algebraic formulation induces the capture of short and long spatiotemporal scales, driving not only the evolutionary dynamics of the Universe's wave function and the cosmic scale factor but also a reconfiguration of matter on small and intermediate scales, inducing additionally the generation of mass-quadrupole moment configurations and relic gravitational waves (an investigation still in progress).

There are numerous research works in the literature that address the topic of quantum gravity in a noncommutative environment. However, with regard to the description of the wave function of the Universe based on the formalism of Hořava–Lifshitz quantum gravity and the Wheeler-DeWitt equation, or based on other alternative approaches, the number of articles dealing with this topic are not significant. Most of the published works deal with the temporal evolution of the scale factor of the Universe, a topic of studies commonly designated as 'dynamical equations'. In some articles the authors address both topics. A few references on these subjects are [59–70]. We emphasize once again that concerning the wave function of the Universe in a noncommutative environment, most authors, due to the inherent computational difficulties imposed by the formalism, use approximations that significantly limit their conclusions and direct their studies towards formal aspects without a numerical approach or algebraic results.

Regarding dynamic equations, some studies involving non-commutative formulations present different degrees of improvement from a theoretical point of view, but the corresponding results are limited by the introduction of computational approximations that imply the shortcoming of the corresponding conclusions. It is important to mention, although this is not a central topic in the present work, studies involving strings and noncommutative gauge theories which have contributed significantly to a better understanding of the influence of noncommutative algebra on the deformation of geometric structures and the impacts of these studies in understanding the accelerated evolution of our Universe in string theory. Edward Witten, precursor of this line of investigation [71], was followed by many scientists who have continued this line of research (see, for instance [72, 73], and references therein).

In our study, we concentrate on numerically solving the equations describing the evolution of the Universe's wave function, as well as the dynamic equations involving the Universe's scale factor and relic gravitational waves. We aim to achieve this without resorting to any computational approximations. To this end, we employ a numerical approach based on the Runge–Kutta–Fehlberg method. This method is known for its effectiveness in solving differential equations accurately without the need for numerical approximations. Despite the significant formal complexity of the theory and resulting equations, this computational approach enables us to conduct a comprehensive numerical investigation of the evolutionary process examined in our work. We explore the theory's parameter space, allowing for a broad analysis of the evolution of functions such as $\Psi(\eta)$, $\Psi(\xi)$, $\eta(t)$, and $\xi(t)$, along with a wide range of solutions. Due to space constraints, we present only a select few figures representing the most significant results. The most expressive results involving both the contraction phase and the expansion phase of the branched Universe indicate an accelerating cosmic expansion.

8. Conclusions

The paradigm supporting the theory of renormalization groups—organizing physical phenomena based on energy or distance scales—holds firm in commutative quantum field theories. In their noncommutative counterparts, however, one encounters uncertain territory. One of the features of noncommutative field theories is the mixing of short and long scales. A striking illustration of this phenomenon can be found in UV/IR mixing [74]. In cases where noncommutativity exists at a small scale, the UV/IR mixing effect is anticipated to manifest at an earlier epoch in the Universe's history, thereby raising new questions about the hierarchy problem. The authors of [57, 75] address the features of noncommutative field theories related to UV/IR mixing, as a result of the capture of short and long scales, emphasizing on the same line of investigation that these effects might be present at an older time of the Universe. Another striking example pertains to the inhomogeneities within the distribution of large-scale structures and anisotropies observed in the CMB radiation. These anomalies bear traces of the noncommutative nature of the early Universe. Specifically, the power spectrum of these structures becomes direction-dependent in noncommutative spacetime [76].

In 1947, Hartland Sweet Snyder introduced the concept of quantizing spacetime in a seminal paper [77]. While this paper has received relatively few citations, it sparked one of the most remarkable inquiries in the realm of physics, namely the possibility of discretizing spacetime. In alignment with Snyder's proposal, the uncertainty principle of Werner Heisenberg suggests that spacetime possesses a noncommutative structure, which can be represented as

$$[x^{\mu}, x^{\nu}] = i\theta^{\mu\nu} \,. \tag{52}$$

This noncommutative property implies a minimum scale of approximately $\sim \sqrt{\theta}$. From a cosmological perspective, to assess the implications of this concept on the dynamics of the Universe, it is most appealing to investigate how the noncommutativity of coordinates affects the deformation of spacetime algebra. We assume that this equation holds within a comoving frame, a coordinate system in which galaxies are freely falling.

In this work we adopt an alternative approach by considering a noncommutative quantum cosmological scenario based on the deformation of a mini-superspace of variables obeying Poisson algebra. We aim to examine whether such a perspective can help identify the mechanism that drives the acceleration of the Universe [78]. While the results presented here are preliminary, they are promising in suggesting that noncommutative BCQG offers an algebraic framework that impacts both the statistical distribution and gravitational dynamics of the matter constituting the primordial Universe.

As previously emphasized, the implications of noncommutative algebra are evident in the solutions presented in this study. These implications manifest in various aspects, including the wave function of the early Universe, the dynamical equations involving the cosmic scale factor and its dual counterpart, as well as relic gravitational waves. The results point to a dynamic acceleration driven by a force that, in our view, arises from the reconfiguration of matter in the early Universe due to the algebraic structure of noncommutative geometry. This structure captures the intrinsic properties of spacetime at short distances, with significant implications for the dynamical symmetries and conventional duality symmetries of quantum spacetime geometry.

As we have seen, we adopt in our proposal a reverse logical approach to the Faddeev-Jackiw symplectic formalism, implying a change of character for the original quantum fields $u(t)$ and $v(t)$, by imposing a noncommutative algebraic structure described by a nonzero Poisson bracket formalism. Then, we carry out a linear transformation of the noncommutative phase space configuration into a commutative representation; this transformation allows the incorporation of the noncommutative algebra, through the insertion of a new set of variables and the consequent modification of the quantum gravity phase space, conforming a new intrinsic structure of the cosmic quantum dynamics. After reducing the corresponding equations of motion to a canonical form, the resulting commutative variables, - the cosmic scale factor $\eta(t)$ and its quantum dual counterpart $\xi(t)$, encompassing a non-commutative underlying strutcure, and the corresponding separable solutions, $\Psi(\eta)$ and $\Psi(\xi)$, present a compelling tenue. By taking the product $\Psi(\eta)\Psi(\xi)$, the $\Psi(\eta)$ amplitude is modulated by $\Psi(\xi)$, whose probability density presents a similar behavior of the inflaton potential field of the inflation theory. In other words, although we do not explicitly incorporate the presence of an inflaton-type scalar-complex field, our approach describes the exponential growth expansion of space in the early Universe, without the need to insert, in an ad hoc manner, an inflaton-type field. In short, the most unexpected results correspond to the behavior of the Universe's wave function counterpart $\Psi(\xi)$, whose more intense realization occurs in the mirror Universe, in which the maximum values of the probability density are symmetrically separated and are more intense at the initial point of the transition from the mirror Universe to the current one, presenting a behavior similar to the roll-on inflation Guth's potential. In our conception, the inflaton roll-on potential represents a simple field parametrization-realization of the structural effects of a noncommutative algebra structure, one of the most fundamental features of spacetime.

Due to its structural character, the algebraic structure of the formalism allows identifying the complex variable $\xi(t)$ as the complex dual quantum counterpart of $\eta(t)$, both scanning reciprocal quantum complex spaces. In quantum mechanics, conjugate dual variables correspond to pairs of mathematical variables which comprise dual-Fourier transforms, or more generally, they are related through the Pontryagin duality, more precisely, a locally compact abelian topological group formed by the continuous homomorphisms group and the topology of uniform convergence on compact sets. Duality relations naturally lead to an uncertainty relation conformed by the Heisenberg uncertainty principle. In mathematical terms, conjugate variables are part of a symplectic basis, and the corresponding uncertainty relation are akin to a symplectic form. Furthermore, conjugate variables are related by Noether's theorem, which states that if the laws of physics are invariant with respect to a change in one of the conjugate variables, then the other will remain invariant, or more precisely, represents a conserved quantity. These concepts then apply to the variables $\eta(t)$ and $\xi(t)$, although the deepening of its content and cosmological meaning requires a more rigorous approach, an aspect still open in our analysis, allowing for broad and speculative interpretative possibilities in the future. Even more so when we come across Hawking's conception of the multiverse, whose theoretical potentialities have been permanently the object of the most diverse speculations, involving for instance, dual complementary quantum universes, among others. This interpretation would follow the many-worlds proposed by Hugh Everett [79], one of the mainstream interpretations of quantum mechanics, along with quantum decoherence, which implies that there are most likely an uncountable number of universes, for which time corresponded to a many-branched tree, wherein every possible quantum outcome is realized.

As stressed before, in our formulation we do not explicitly incorporate the presence of a complex scalar field of inflaton-type. Its explicit additional inclusion, we can foresee, would certainly imply an enhancement of the acceleration of the primordial Universe. As previously mentioned, the probability density associated to $\Psi(\xi)$ presents a behavior similar to the inflaton potential field of the inflation theory. It is important to note that the presence of the variables $\xi(t)$ as well as the wave function $\Psi(\xi)$ are the result of the incorporation, - taking as starting point the commutative version of the Horava–Lifshitz Hamiltonian formalism -, of the perfect fluid conception of Hermann Weyl combined with the formalism of Bernard F. Schutz. Later, using the Faddeev–Jackiw symplectic two fields approach, the canonical noncommutative version of a super Hamiltonian was developed and the corresponding wave equation of the Universe was solved. The particular choice $\alpha = 1/3$ allowed the separation of the variables in the form $\Psi(\eta)\Psi(\xi)$. The resulting equations were solved algebraically, using the successive approximation method to obtain the solution corresponding to$\Psi(\eta)$ and the pure integration method to obtain the solution corresponding to $\Psi(\xi)$. In other words, we do not impose, in any of the stages of treating the problem in focus, explicitly and in an ad hoc way, any type of solution corresponding to an inflaton-type field, whether of a scalar or scalar-complex nature. Even so, the results obtained reveal that $\Psi(\xi)$ presents a behavior similar to an inflaton-type potential in the transition region between the present Universe and its mirror homonymous. Evidently, this behavior is not a coincidence, being a result of the noncommutative algebra that induces, as we stated before, the capture of short and large spacetime scales, which in the case of the conventional model of cosmic inflation was modeled by a baseline slow-roll potential. In the case of the explicit inclusion of a complex scalar field of the same nature as the inflaton field, which in conventional formalism configures a state that drives inflation, in a local (not global) configuration, the superposition of the effects of the three fields, in phase, we can foresee, would ignate a superposed level of inflation. However, for such a process to be configured, it would be appropriate to construct an extension of the two-fields noncommutative version as proposed in the present contribution. Such an extension would cause the original fields to acquire a quantum dual nature, evidently sharing their original natures among themselves. This is a proposal that deserves a technically rigorous approach in the near future. We thank the referee once again for raising this possibility that deserves further study as well as an extension of the present study involving two fields, to a three-fields configuration.

In conclusion, the results suggesting a primordial dynamic acceleration of spacetime demonstrate that noncommutative BCQG provides a viable theoretical alternative to models such as inflation [14, 15] and bouncing [30–32]. This exploration aligns with a fundamental characteristic of noncommutative algebra, namely the interplay between small and large scales. As a result, if the effects of noncommutative algebra were indeed present in the primordial Universe, it is reasonable to anticipate their persistence in the present day.

A fundamental question then emerges: Can inhomogeneities in the distribution of large-scale structures and anisotropies in the stochastic gravitational wave background (SGWB), if they indeed exist, carry traces of the noncommutative nature of the early Universe? Our results indicate a scenario during the early stages of the Universe, characterized by an SGWB distribution that deviates significantly from the homogeneity expected to be observed today. As a consequence, inhomogeneities in the SGWB distribution, if influenced by traces of noncommutativity, could serve as crucial windows into the initial phases of the early Universe, preceding the recombination era. However, answering this question necessitates further observations, and this theme will remain the primary focus of our ongoing investigations.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

P O H acknowledges financial support from PAPIIT-DGAPA (IN116824). F W is supported by the U.S. National Science Foundation under Grant PHY-2012152. M N-M acknowledges support of the Margarete und Herbert Puschmann-Stiftung.

Author contributions

Conceptualization, C A Z V ; methodology, C A Z V and B A L B and P O H and J A deF P and D H and F W and M N-M ; software, C A Z V and B A L B and M R and M N-M ; validation, C A Z V and B A L B and D H and P O H and J A deF P and F W ; formal analysis, C A Z V and B A L B and P O H and J A deF P and D H and F W ; investigation, C A Z V and B A L B and P O H and J A deF P and M R and M N-M and F W ; resources, C A Z V ; data curation, C A Z V and B A L B ; writing-original draft preparation, C A Z V ; writing-review and editing, C A Z V and B.A L B and P O H and J A deF P and D H and M R and M N-M and F W ; visualization, C A Z V and B A L B ; supervision, C A Z V ; project administration, C A Z V ; funding acquisition (no funding acquisition). All authors have read and agreed to the published version of the manuscript.

Appendix. Canonical form of the two-fields noncommutative Hamiltonian

Equation (30) can be rewritten in the general form:

$$\left\{\left[a(u,v)\frac{\partial^2}{\partial u^2}-2b(u,v)\frac{\partial^2}{\partial u\partial v}+c(u,v)\frac{\partial^2}{\partial v^2}\right]\right.\\ \left.-\left[d(u,v)\frac{\partial}{\partial u}+e(u,v)\frac{\partial}{\partial v}+F(u,v)\right]\right\}\Psi(u,v)=0\,, \tag{A1}$$

with

$$a(u,v)=1\,;\quad b(u,v)=\chi\,;\quad c(u,v)=\chi^2\,;\\ d(u,v)=\frac{|\gamma|}{u^{3\alpha-1}}\,;\quad e(u,v)=\frac{i}{u^{3\alpha-1}}\,;\\ F(u,v)=-\left(g_r-g_m u-g_k u^2-g_q u^3+g_\Lambda u^4+\frac{g_s}{u^2}-\frac{\alpha}{u^{3\alpha-2}}+\frac{\alpha v}{u^{3\alpha-1}}\right)\,, \tag{A2}$$

where, for simplicity, we do not use the symbol *tilde* in the partial derivatives as identification of the new variables in the scope of the noncommutative algebra. In the above expression,

$a(u,v)$, $b(u,v)$, and $c(u,v)$ represent functions of the independent variables u and v, and have continuous derivatives up to second-order. Moreover, since these variables obbey the condition $b^2(u,v) - a(u,v)\,c(u,v) = 0$, expression (A1) belongs to the mathematical group of parabolic differential equations. In order to reduce this equation to a canonical form, one should first write out the characteristic equation [80]

$$a\,\mathrm{d}u^2 - 2b\,\mathrm{d}u\,\mathrm{d}v + c\,\mathrm{d}v^2 = 0\,, \tag{A3}$$

which splits into two equations:

$$a\,\mathrm{d}\eta - \left(b \pm \sqrt{b^2 - ac}\right)\mathrm{d}\xi = 0\,. \tag{A4}$$

The next step corresponds to find the general integrals of the differential equation above. In case of a parabolic equation, the two previous solutions coincide, resulting in a common general integral $\varphi(u,v) = \mathcal{I}_G$. This allows the variables u and v to be replaced by two new independent variables, which we denote as ξ and η, in accordance with

$$\xi = \varphi(u,v)\,, \quad \text{and} \quad \eta = \eta(u,v)\,, \tag{A5}$$

where $\eta(u,v)$ is a differentiable function that satisfies the non-degeneracy condition of the Jacobian $D(\xi,\eta)/D(u,v)$ in the given domain. As a result, equation (A1) is reduced to the canonical form

$$\frac{\partial^2\Psi(\xi,\eta)}{\partial\eta^2} = G\left(\xi,\eta,\Psi,\frac{\partial}{\partial\xi},\frac{\partial}{\partial\eta}\right)\,. \tag{A6}$$

For η one can take u or v. We take, for convenience, u.

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