ELECTRIC STRUCTURE OF NUCLEONS

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In an age of giant accelerators, of complex experiments and of mystifying theories it is a pleasure to report on some simple experiments, made with simple equipment and having a simple interpretation - simple, that is, if one doesn't look too closely. The electron energies now available are about 1 Gev and the de Broglie wavelength of such electrons is about 0.2 fermis, hence we can expect to make out some of the details of the proton and neutron.

Last year at the Rochester Conference, as a result of scattering experiments made at Stanford University [1] with their Linac and at Cornell University [2] with our electron synchrotron, we were both able to report the beginning of a detailed structure in that for large momentum transfers the electric and magnetic form factors were no longer nearly equal - as had previously been believed to be true for energies less than about 500 Mev.

During the past year, the measurements at Stanford [3], [4] and at Cornell [5], [6] have been considerably refined and extended. As a result a remarkably clear and simple picture of the electric structure of the proton and neutron is developing. The most dominant feature of the nucleon according to these experiments is a meson cloud that is the same for the neutron and the proton except that it has a charge of + e/2 for the proton and - e/2 for the neutron. This isovector cloud seems to correlate well with the two-pion resonant state, $T = 1$, $J = 1$ that is also evident in meson experiments. Less clear is another mesonic cloud of larger radius but of smaller charge which is positive for both neutron and proton. This isoscalar cloud may be related to the three-pion resonant state, the one with $T = 0$, $J = 1$ although its size does not seem to correlate too well with the mass recently discovered for a similar state that is revealed in nucleon-antinucleon annihilation. Finally we may just be able to distinguish the charge, the size, and the magnetic moment of the central core of the nucleon strictly speaking, however, these properties of the core are just emerging from the shadows of the experimental and theoretic background. I will now discuss the scattering measurements, how they yield the form factors, and then make some remarks about the interpretation of these form factors in terms of the structure of the nucleon.

Figure 1 shows the experimental arrangement for the experiments at Cornell. My collaborators in the early measurements were Cassels, Berkelman, and Olson and in the present measurements they are Schopper, Littauer and Rouse. The electron source is the 1.3 Gev electron synchrotron. A thin target made of $C H_2$ or $C D_2$ is put directly in the beam in a straight section, where many traversals of the target can be made. The electrons being scattered at a particular angle are momentum analyzed by the single quadrupole magnet which forms a horizontal line image of the nearly point target. Along this line image is placed a long narrow scintillation counter that detects the electron : the narrow dimension of the counter together with the dimensions of the obstacle at the center of the magnet determine the momentum resolution of the magnet. A total absorption glass Cerenkov counter behind the thin scintillation counter separates electrons from protons and mesons : the electron makes a large pulse proportional to its energy but neither the proton or meson can give a very large pulse. In fact there are two such independent magnet and counter systems so that data can be taken simultaneously at two different angles. The solid angle of one of the quadrupoles, a so-called current-sheath quadrupole is 17 milli-steradians and its momentum resolution is 6 %. The solid angle of the magnets have been computed from their dimensions and in addition the magnets have been cross-calibrated by placing each of them at 90° but on opposite sides of the electron beam. The electron beam is monitored by measuring the absolute number of bremsstrahlung produced in the target. This gives just the proper product of the number of electrons times the effective target thickness that is obviously needed for calculating a cross section, however, the method is hazardous in that electrons striking anything except the target can contribute to a spurious reading.

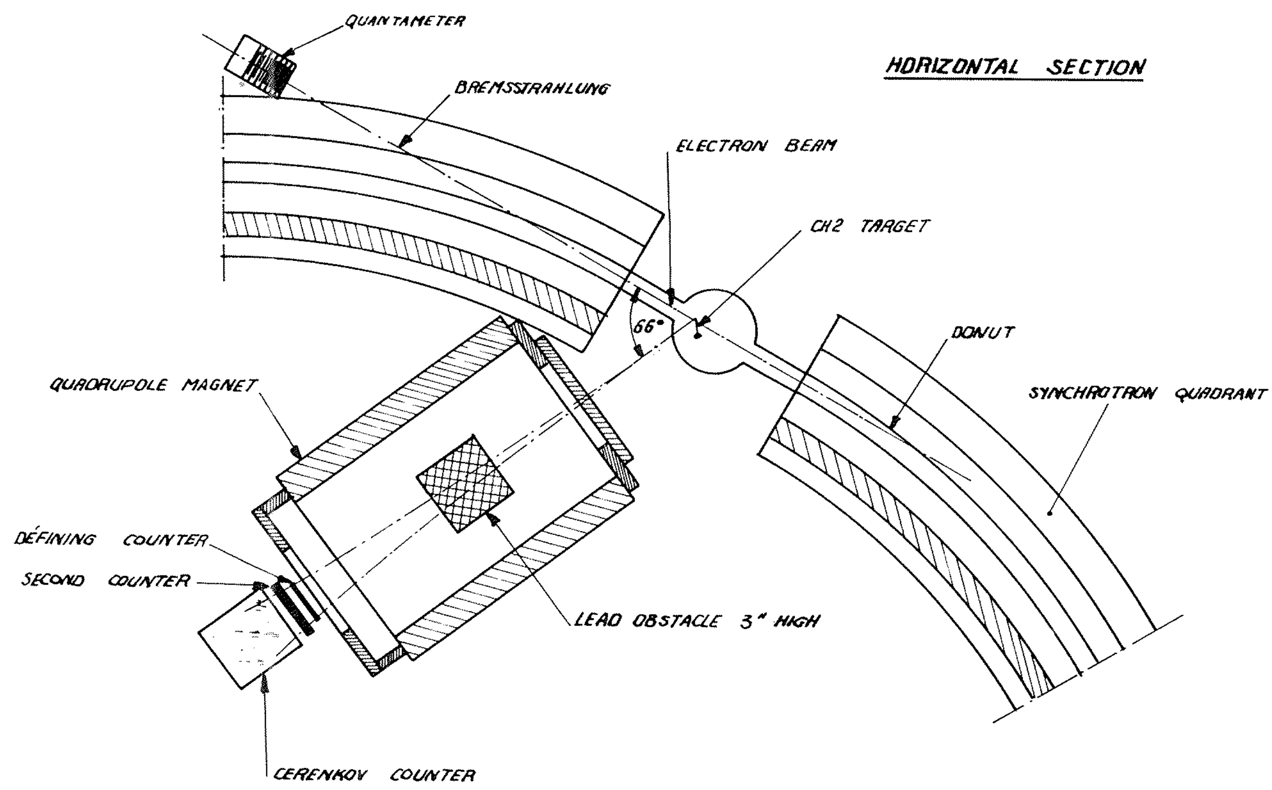

Figure 1 - Plan view of the scattering experiment.

The experiments carried out at Stanford [1], [3] at energies up to about one Gev have more of a quality of elegance. There they luxuriate in the direct beam of a linac and there they use larger magnets having better momentum resolution.

At a particular angle the procedure is to trace out a curve of counting rate v.s. magnet current or momentum of the electrons. For a target of $C H_2$ one typically recognizes a peak corresponding to a momentum of an elastically scattered electron. A carbon target does not show the narrow peak but does enable one to subtract the background in $C H_2$ due to C. Reversing the magnet allows us to examine the discrimination against mesons, something which becomes increasingly difficult at large angles and at high energies.

Figure 2a shows the Cornell measurements of R.M. Littauer, H.F. Schopper and R.R. Wilson [5] for the scattering by hydrogen of electrons at 45, 90, 112, and 135° where the cross-section is plotted against the angle of scattering. Our data do not differ appreciably from the Stanford data [3], [4] even though we do not use quite the same radiation correction. We have applied the Schwinger correction [7]- not too different from the correction of Tsai.

The determination of the cross section of scattering from deuterium is complicated by the internal motion of the nucleons in the deuteron. This causes the peak that is observed in the counting rate v.s. magnet current to be spread out, of course. Although one might think it best to measure a complete counting rate curve and then to integrate it in order to get a cross section, it turns out instead to be best to measure the value of the counting rate at the energy where the electrons scattered from hydrogen give an elastic peak. Then the total deuteron cross section is calculated from this peak counting rate using the impulse approximation as given by Goldberg [8]. Recently, Durand [9] has derived Goldberg's formula more rigorously and has shown it to be accurate to within a few percent. I have heard that at very low energy, the final state interactions may become important (See the Hofstadter paper of this meeting). The deuteron cross sections obtained using the simple impulse approximation are plotted in figure 2b - again our data are in good agreement

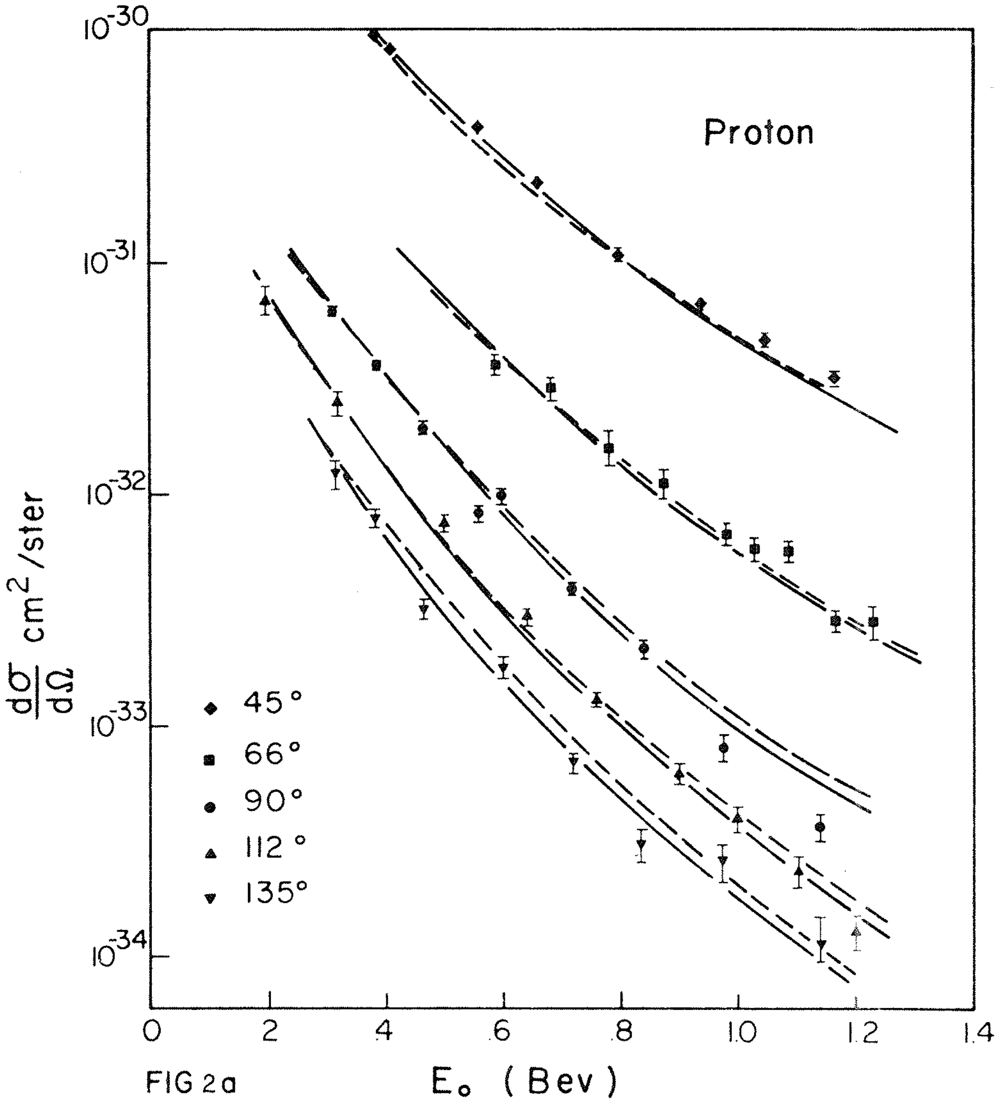

FIG 2a

Figure 2 - Differential elastic scattering cross section for the proton (a) and the deuteron (b), as a function of incident electron energy and laboratory scattering angle. Full curves computed with core model, dashed curves according to BSFV (cf. the following Letter).

with Stanford where they overlap - at low energy our data are not as accurate as the Stanford data and have not been corrected for final state interactions.

The procedure used to reduce the cross sections to form factors is straightforward. The Rosenbluth formula for the differential scattering cross section is almost as simple as Rutherford's formula ; it can be written :

$$\sigma = \sigma_M \left\{ G_1^2 + \frac{q^2}{4M^2}\left[2(G_1 + G_2)^2 \tan^2\frac{\vartheta}{2} + G_2^2 \right] \right\} \qquad (1)$$

where σ_M is Mott scattering per unit solid angle from a point charge, q is the momentum-energy transfer (approximately equal to E $\sin\frac{\vartheta}{2}$), G_1 is the Dirac electric form factor normalized to a unit charge at $q^2 = 0$ and G_2 is the Pauli magnetic form factor which is normalized at $q^2 = 0$ to

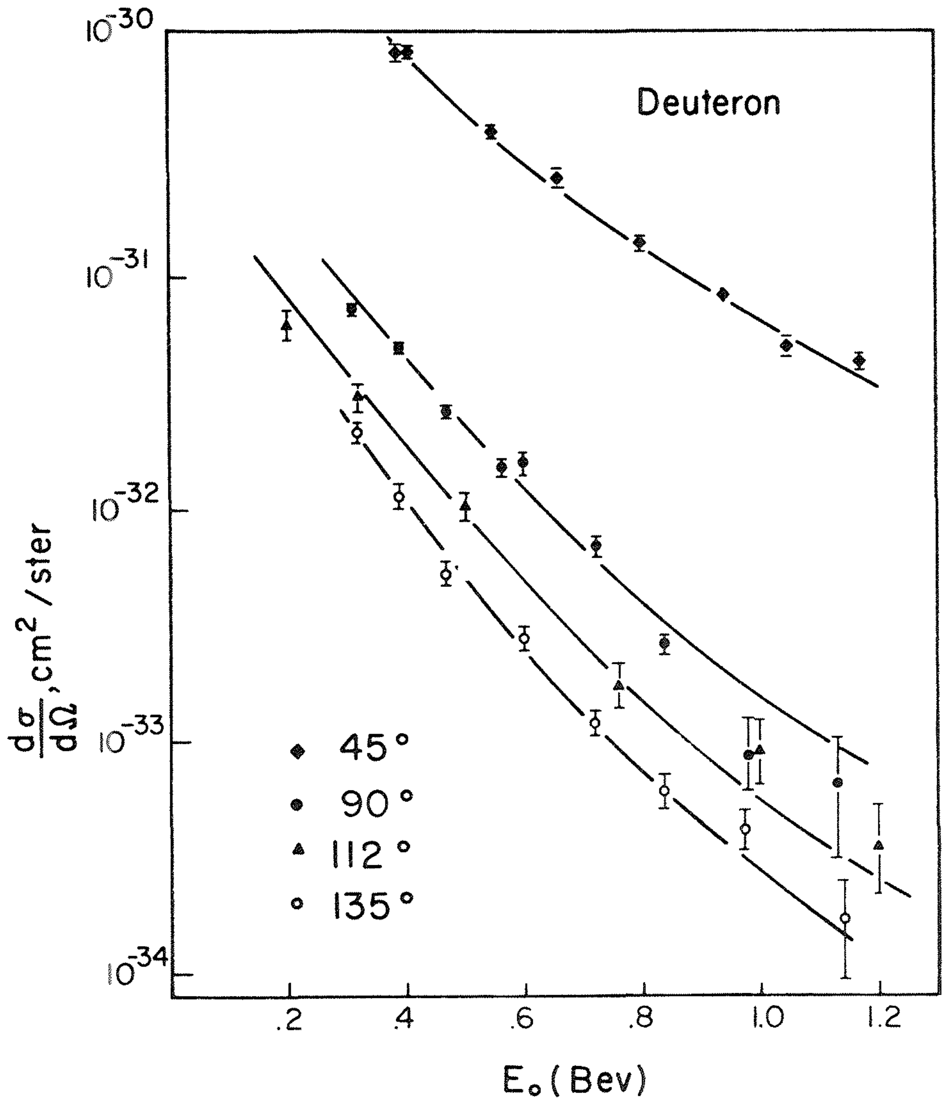

Figure 2 b

the anomalous magnetic moment in nuclear magnetons. In the static model it is just the Fourier transform of the form factors that gives the radial distribution of charge or magnetic moment in the nucleon. Theory instructs us furthermore that G_1 and G_2 should be functions only of q^2, if they are to be relativistically invariant a tremendous simplification if true.

The first objective of the experiment should be to ascertain the validity of the Rosenbluth formula. This is especially important for electron energies above one Gev : the formula in addition to assuming the validity of quantum electrodynamics also assumes the absence of fourth-order processes in which, for example, two photons are exchanged between the electron and nucleon ; this latter assumption becomes particularly suspect at high energy. Does it not seem strange that photon-proton scattering is so completely dominated by nucleonic processes such as the (3-3) resonant nucleon state and yet that these effects should be absent from electron-proton scattering ? Drell [10] assures us that such is the case, to within a few percent at least. Nevertheless, we should be anxious to test the validity of his calculation.

Let us rewrite (1) in the form :

$$\sigma/\sigma_M = \left(G_1^2 + \frac{q^2}{4M^2} G_2^2\right) + \frac{q^2}{2M} (G_1 + G_2)^2 \tan^2 \frac{\vartheta}{2} \tag{2}$$

then we see that if we measure cross sections at different angles while keeping q^2 constant by varying the electron energy, and if we plot the resulting values of σ/σ_M as a function of $\tan^2 \vartheta/2$ as has been done for some typical values in figure 3, then the points should fall on a straight line. If indeed the plot is linear, then we have at least some evidence for the validity of the formula so that we can then go on and determine values of G_1 and G_2. This kind of plot is most useful for such a determination inasmuch as the intercept at $\tan^2 \vartheta = 0$ which is equal to $\left(G_1^2 + \frac{q^2}{4M^2} G_2^2\right)$ turns out to be approximately equal to G_1^2 because the second term in the parenthesis is always quite small. The measured slope of the line then gives the combination $(G_1 + G_2)^2$ and hence G_2.

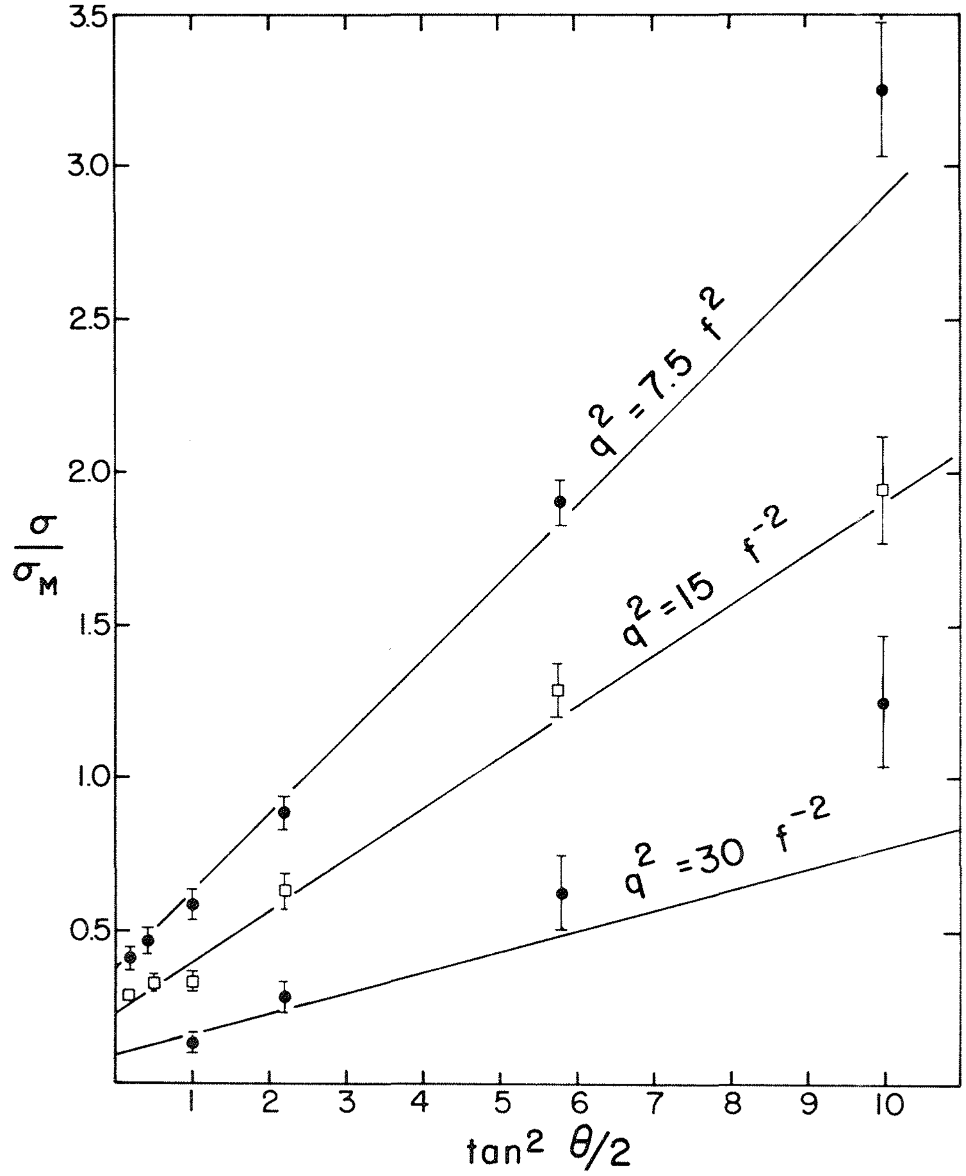

Figure 3 - The ratio σ/σ_M plotted as a function of $\tan^2 \frac{\theta}{2}$

Professor Hofstadter has introduced an equivalent method [11] in which he plots ellipses in G_1 and G_2 space that correspond to cross sections given by the Rosenbluth formula. Each experimental cross section gives one ellipse and the intersection of two ellipses give specific values of G_1 and G_2 ; a test of the formula is to notice whether more than two ellipses intersect at a point. We have also used this method in reducing our data. Generally speaking, the ellipses do intersect in a point for $q^2 < 20\ f^{-2}$; furthermore, as figure 3 shows, the experimental values of σ/σ_M do fall along straight lines. However, for $q^2 > 25$ the ellipses do not all intersect and there are inconsistencies in the straight lines. Thus for $q^2 = 30$ of figure 3, the intercept $\left(G_1^2 + \frac{q^2}{4 M^2} G_2^2\right)$ is nearly zero, only possible if both G_1 and G_2 are both zero. Nevertheless, the slope of the line, proportional to $(G_1 + G_2)$, is still quite large - obviously not consistent.

This possible deviation from the Rosenbluth formula was first pointed out by the Stanford workers on the basis of their work combined with ours at somewhat higher energies. They have also found an interesting trend in their data taken at 145° which indicate a dramatic flattening-out of the cross sections above an energy of about 850 Mev. This summer, we decided to test this trend by extending their measurements, which stop a little below one Gev, to our top energy of 1.3 Gev. Our measurements are still in progress but are in accord with the Stanford work where they overlap. At our highest preliminary point at 1 120 Mev, the cross section is nearly a factor two above what one would expect from the form factors as determined at smaller angles. This is an indication, very tentative, that the Rosenbluth formula is no longer entirely valid. If this trend develops, life can be especially exciting for the physicists building high energy electron machines.

Finally, the values of G_1 and G_2 corresponding to the straight lines drawn in figure 3 are given in figure 4 as a function of q^2. The Stanford values for G_1 and G_2 are also indicated on the curve for the most recent Stanford results for the neutron see their contribution of this meeting.

Basic to our interpretation of the form factors has been the idea that the neutron and proton have structural components that are essentially the same except for a change of the sign of the charge of some particular component. We express this by analyzing the form factors into an isoscalar component which is the same for neutron and proton and an isovector component for which the sign changes. Thus we write :

$$G_{1P} = G_{1S} + G_{1V} \quad ; \quad G_{1N} = G_{1S} - G_{1V}$$
$$G_{2P} = G_{2S} + G_{2V} \quad ; \quad G_{2N} = G_{2S} - G_{2S} \qquad (3)$$

clearly these components can be obtained directly from the experimental values of G_1 and G_2 for example,

$$2\,G_{1S} = G_{1P} + G_{1N} \qquad 2\,G_{1V} = G_{1P} - G_{1N} \qquad (4)$$

and similarly for G_{2S} and G_{2V}. Having done this we then try to interpret these components with structural details of the nucleons.

As illustrative of this approach let us assume a very simple model of the nucleon and compare it to the data. Consider one, for example, in which both the neutron and the proton have a point core of positive charge $+\frac{e}{2}$. This will give rise to an isoscalar from factor G_{1S} of value 1/2 which remains a constant as q^2 is changed. Let us also postulate that surrounding the point core is an extended meson cloud of charge + 1/2 for the proton and - 1/2 for the neutron. This gives rise to a G_{1V} term whose value at $q^2 = 0$ is 1/2 and we expect G_{1V} to decrease as q^2 increases in a manner characteristic of the particular charge distribution that we have assumed. Because the form factor can be expanded for small q^2 in the static approximation :

$$G = 1 - \frac{q^2 a^2}{6} + \dots \qquad (5)$$

where a is the rms radius of the distribution, the derivative $\partial G/\partial(q^2)$ at $q^2 = 0$ should be equal to $-a^2/6$.

Comparing this model to the experimental values of G_{1S} and G_{1V} given in figure 4, we see that it fails on a number of counts. G_{1S} is not a constant of value 1/2, rather it decreases to about 0.25 by the time q^2 has reached 20 or 30 f^{-2}. The variation of G_{1V} does not seem to be inconsistent with our model and its form corresponds to an exponential charge distribution of r.m.s. radius

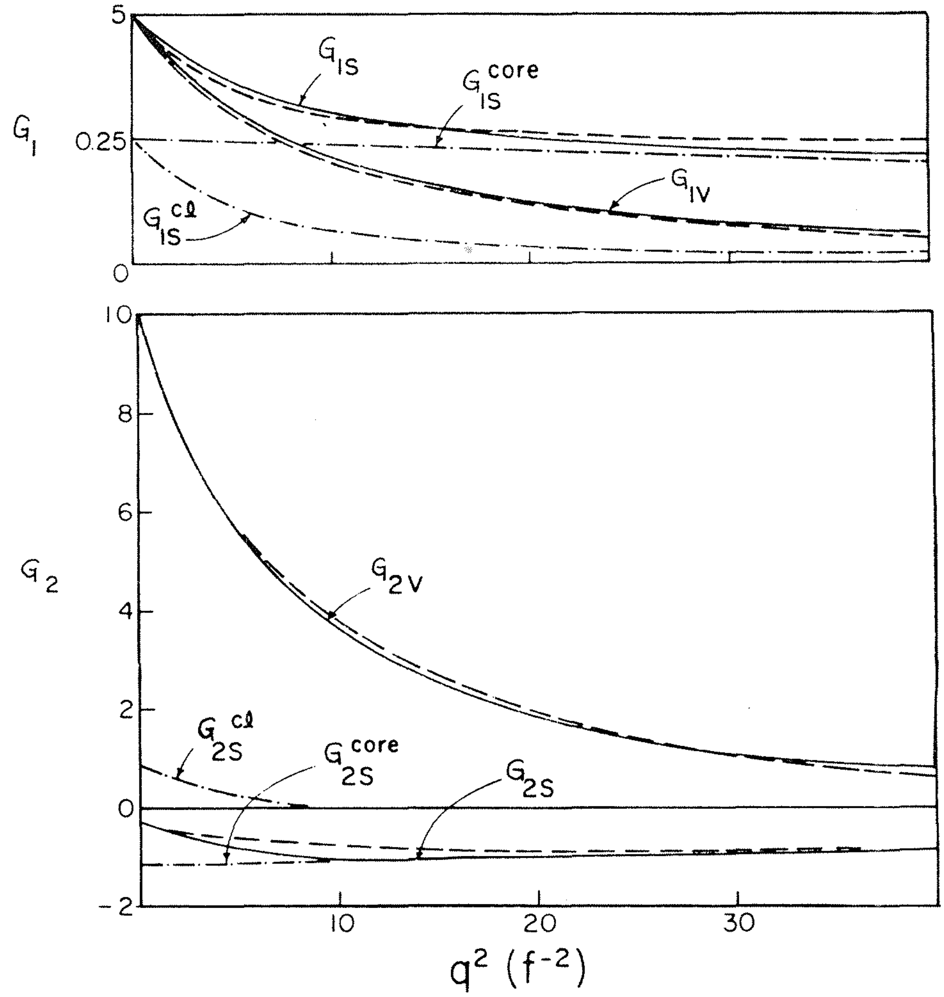

Figure 4 - Partial form factors for G_1 and G_2/μ, each form factor is resolved into isoscalar and isovector parts, whose sum and difference give, respectively, the neutron and proton form factors. The solid curves indicate the fit according to the core model, where the scalar partial form factors have been further split into terms corresponding to a core and an extended cloud, each of exponential distribution. The dashed curves indicate the best fit obtained by the Clementel-Villi form.

equal to 0.80 f. The simple model also conflicts with the result of experiments on the scattering of low energy neutrons by electrons. Foldy [11] has shown that these experiments can be interpreted to mean that the mean squared radius of the neutron is zero or, more accurately, $0 \pm 0.006\ f^2$, whereas our model would give $0.84\ f^2$ for the neutron. Then, you might ask, what changes can be made in this model to bring it into agreement with these considerations. In the first place, we can assign the core a radius ; this causes the form factor to decrease at large values of q^2 just as observed. In addition to this, we can say that the isoscalar part of the form factor also has some kind of meson cloud associated with it.

With this more complicated core model we have six parameters, i.e. the partial charges and the radii of the core and of the two meson-like clouds that we have postulated. However, we have four conditions : two that are set by the charge of the neutron and proton, and two by the radii of the particles, thus the radius of the neutron is known to be zero from the neutron-electron scattering while the r.m.s. radius of the proton is given by the slope of the G_1^p curve at $q^2 = 0$. This leaves two parameters to be determined from our data. Our procedure has been to assume that the behavior of G_{1S}^p at very large q^2 is dominated by the properties of the core and so we fit this part of the curve by assigning a partial charge and a radius to the core. Then all the other parameters are determined by means of simple algebraic relations [5]. Table I gives the values of the various parameters that best fit our data, and the resulting partial form factors are plotted in figure 5. I must hasten to add, however, that we have also assumed rather arbitrarily that all the radial charge distributions are simple exponentials. Of the forms without a singularity at the

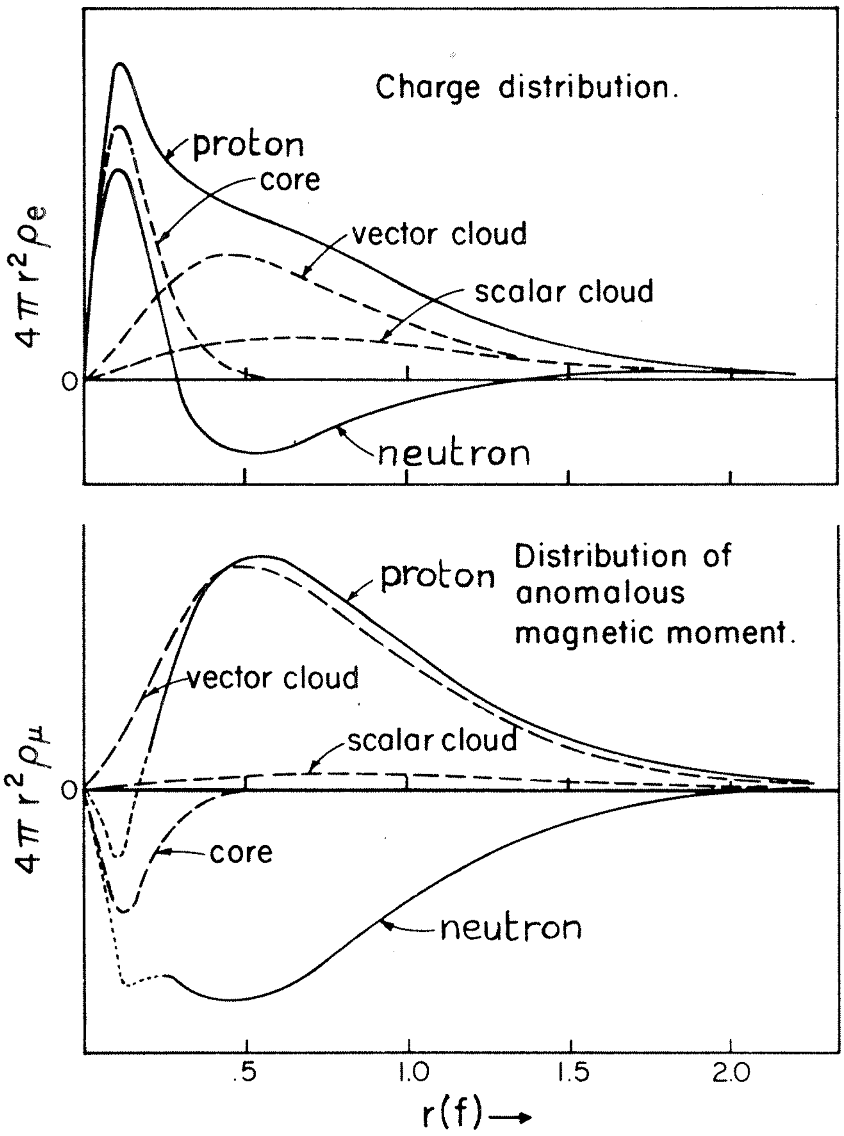

Figure 5 - Spatial distribution of charge and anomalous magnetic moment for proton and neutron, according to the core model, are shown for the sentimentalists only.

origin, we have tried gaussian distributions, but then the overall fit of the data is worse. Of course nothing at all can be said about the shape of the core, the main question here being whethe it is spread out with an r.m.s. radius of about 0.2 f or not.

Table I

Best-Fit Parameters for Core Model with Exponential Density Distributions. See Text and Reference [6] for Definition of Symbols.

e_s^{core}	= 0.25 e	$a_{s,core}$	= 0.2 f
e_s^{cl}	= 0.25 e	$a_{s,cl}$	= 1.13 f
e_v	= 0.5 e	a_v	= 0.80 f
μ_s^{core}	= -0.22 n.m.	$b_{s,core}$	undetermined
μ_s^{cl}	= 0.16 n.m.	$b_{s,cl}$	= 1.30 f
μ_v	= 1.853 n.m.	b_v	= 0.89 f
a_p	= 0.80 f	b_p	= 0.98 f
a_n	= 0	b_n	= 0.79 f

In exactly the same manner one can apply independently the same model to fit the Pauli magnetic form factors. Again there are six parameters consisting of the three partial magnetic moments and their three ranges that must be assigned to the core and the two meson clouds. But again algebraic conditions relating the anomolous magnetic moments of the neutron and proton, the values of the radii (determined by $\partial G/\partial(q^2)$ at $q^2 = 0$) leave only two parameters to fit to the experimental curve at high q^2. Again we have chosen to fit the partial magnetic moment of the core and its radius to the G_{2S} curve at large values of q^2. All the values pertaining to G_2 are also shown in Table I and figure 5.

The qualitative result of applying the core model to the magnetic form factor is that it is necessary to attribute a small magnetic moment of negative sign to the core. A point moment would give a satisfactory fit to the data as would an extended core with a radius of 0.2 f. The negative sign of this magnetic moment does not seen unreasonable : for if the angular momentum of the meson cloud is unity ; then necessarily the core will be left with a spin of - 1/2, which then might rather naturally give rise to a negative magnetic moment, for example, by dissolution of the core into a $K^+\Lambda$ system in direct analogy to the pionic origin of the Pauli moment. It is interesting that the isovector magnetic cloud has a comparable range 0.89 f to that of the isovector charge cloud, and that it gives rise to most of the magnetic moment of the nucleon, as one would expect from the near equality of the Pauli moments of the neutron and proton. The isoscalar magnetic and charge clouds, although far less well determined, also show a similar range.

The most exciting note that has been sounded on the subject of nucleonic structure has been the hypothesis of Frazer and Fulco [13] that the radial extent of the isovector cloud is due to the T = 1, J = 1 resonant state of the two pion system. Fubini will discuss his own and his collaborators more refined calculations [14] based on dispersion theory and a similar two-pion interaction ; however, for a comparison with the experimental data I will simply say that his theory yields expressions for the four partial form factors of the Clementel - Villi form :

$$G_1^{S\ or\ V} = \frac{e}{2}\left[(1 - \alpha) + \frac{\alpha}{1 + q^2/q^2_{S\ or\ V}}\right] \tag{6}$$

The constant α specifies the fraction of the partial charge that is in the cloud or the core and has the value α_S or α_V for G_1^S or G_1^V. The two constants q_S or q_V also refer to G_1^S or to G_1^V : q_V is the resonant energy of the two-pion state with T = 1, J = 1 ; and q_S is the resonant energy of a postulated three-pion state with T = 0, J = 1. The same form of equation applies for F_2 except that the charge e/2 is now replaced by the partial magnetic moment 1.85 n.m. for G_2^V and by -0.06 for G_2^S. The constant α in (6) becomes β_V for G_2^V and β_S for G_2^S. These six parameters are reduced to five because of the extra condition that the neutron charge radius is zero. Table II gives values of the five independent parameters that best fit our data plus the r.m.s. radii of the neutron and proton. The same parameters have also been determined at Stanford [6] and their values in our nomenclature are shown for comparison ; the general agreement is good. Slight discrepancies arise mainly because our experimental values for G_{2p} at high q^2 - values are close to zero whereas an extrapolation of the Stanford form factors gives negative values. Our experiments indicate also a somewhat smaller difference between G_{2p} and G_{2n}.

Table II

Parameters for Best Fit According to BSFV [14] Form

	Cornell	Stanford (old values)
α_v	1.10	1.20
β_v	1.14	1.20
α_s	0.58	0.56
β_s	-1.5	-3.0
a_v	0.85 f	0.77 f
a_s	1.16 f	1.13 f
a_p	0.88 f	0.85 f
b_p	0.95 f	0.94 f
b_n	0.87 f	0.76 f
q_v^2	$8.3\ f^{-2} = 16\ (m_\pi/c)^2$	$10\ f^{-2} = 19.6\ (m_\pi/c)^2$
q_s^2	$4.4\ f^{-2} = 8.5\ (m_\pi/c)^2$	$4.7\ f^{-2} = 9\ (m_\pi/c)^2$

The experimental data are fit equally well by the BSFV [14] form factors or by those following from our simple core model.

According to the interpretation of BSFV, our values of q_v and q_s would imply that the resonant energy of the two-pion state is 4 m_π and that of the three-pion state is about 3 m_π. The first value is not too different from that obtained from the meson experiments of Walker [15] et al. which give 5.5 m_π. Professor Hofstadter has already remarked in his paper that a choice of 5.3 for the isovector mesonic state and 4.5 for the isoscalar state fits the data almost as well. It is relevant to remark that as the isovector meson mass is raised, then the isoscalar mass must also be raised to fit the data. Our best values, though, are those shown in Table II.

It is reasonable at this point to become skeptical of how well the various parameters in the above analyses habe been determined or indeed if they are even unique. It is gratifying that the Stanford and Cornell measurements do give such similar results. Nevertheless, the measurements are not easy, and with regard to the Cornell measurements Johnson's famous comment about the walking dog applies : "The wonder is not that she does it well, but that she does it at all'". We must refine our work considerably before we can honestly stand behind any detailed conclusions that are to be drawn from it. Even worse, I fear, is that the basic concepts of the interpretation are subject to grave doubts and that the way abounds with dangerous pit falls. Particularly is the core model suspect. In discussing spatial distributions, we are flying exactly in the face of dire warnings from many of our theoretical friends who will consider nothing except the raw form factors. Recoil of the nucleon, they point out, has become perilously relativistic at these energies. Sachs [16] has warned of this and points out that the only quantities that have meaning if we are to make spatial models are what he calls the electric form factor G_e defined as $G_1^2 + \frac{q^2}{4 M^2} G_2^2$ and magnetic form factor defined as $(G_1 + G_2)$. In fact he asserts that a Fourier transform of G_e and G_M will give the spatial distributions of the charge and magnetic moments. In this regard perhaps it is relevant that the Rosenbluth formula can be rewritten in terms of Sach's form factors [17] :

$$\frac{\sigma_{exp}}{\sigma_M} = \frac{\left(G_e^2 + \frac{q^2}{4 M^2} G_M^2\right)}{\left(1 + \frac{q^2}{4 M^2}\right)} - \frac{q^2}{2 M^2} G_M^2 \tan^2 \frac{\vartheta}{2} \qquad (6)$$

This has the same pleasing simplicity as the form involving G_1 and G_2. But Yennie, Levy, and Ravenhall tell us that it is ambiguous as to whether to use G_1 and G_2, G_e and G_M, or, worse yet, even something else as characteristic of static distributions. The theorists would be kind if they were to explain this mystery in gentle terms.

The procedure of Fubini in using a dispersion relation to calculate G_1 and G_2 directly is surely on firmer ground. I hope that he will reassure us that a different and unique prescription would be used were he to calculate instead say, G_e and G_M.

Despite this caveat, it seems evident that shining out from the form factors, no matter how they are considered, are three gross characteristics : (1) that the neutron and proton are states of a fundamental nucleon (charge independence) ; (2) the existence of meson-like clouds ; (3) the mysterious core itself. If we have only "through a glass seen darkly" these characteristics of the nucleon, still our esthetic appetite has been whetted for the revelations that future and more precise measurements will bring.

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