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Musing on extreme quantity values in physics and the problem of removing infinity

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Abstract. Many physical quantities display range values apparently extending to infinity (unbounded on one or on both sides). In this respect, unit systems and measurement conventions do not place any constraint to their validity for a maximum (or minimum) value. In general, this happens because such extreme values are far from being reached on the earth or yet are reached in experimental settings. Nevertheless, the issue of extreme values (not in the usual mathematical analysis meaning here) is not irrelevant, since the same units are used also in countless fields of physics, chemistry or technology where extreme values do occur—namely, in the description of the universe in one frame, and in pico/nano-scale or particle physics in another. The issue, of direct interest also of measurement science and specifically of metrology, is discussed here illustrating, as an example, our currently accepted concept of temperature, i.e., the kelvin temperature scale based on Lord Kelvin's second definition, which encompasses the full range between bounds (0, +∞). In general, the occurrence of infinite values in physical equations, such as singularities in the description of black holes, is a painstaking problem that causes many theories to break down and/or being incapable of describing extreme events. Different methods, such as re-normalization (scaling) or logistic/geometrical, have been used in the assessment of physical observables in order to avoid the undesirable infinity.

1. Introduction

Physicists have already raised the point of infinity long since. For example, almost one century ago the physicist Bridgman discussed the issue for time in the frame of measurement science and, specifically, of metrology ("What is the meaning in saying that an electron when colliding with a certain atom is brought to rest in 10^{-18} s?" in 1927) [1] and length ("What is the possible meaning of the statement that the diameter of an electron is 10^{-13} cm?" in 1955), [2] then opting for operational definitions.

The same issue also attracted the philosophers of science. For example, in a recent book, Chang [3] asked "Is a [scale] definition valid for a quantity's full range?" and introduced the concept of "metrological extension", then proposing a "compatibility requirement" for measurement standards in different ranges, most often satisfied by "patching up disconnected standards". However, in principle, only a theory-based definition—i.e. a model-based definition with operational method(s) ("realisations") available—might satisfy the necessary conditions.

In the following, the issue is discussed illustrating, as an example, our currently accepted concept of temperature, i.e., the kelvin temperature scale based on Lord Kelvin's second definition, which encompasses the full range between bounds (0, +∞). In the following, some modern viewpoints in the frames where the present definition is not applicable are introduced.

In general, in physical equations the occurrence of infinite values, such as, e.g., singularity in the description of black holes, is a painstaking problem that causes many theories to break down and/or being incapable of describing extreme events. Different methods, such as re-normalization (scaling), have been used in the assessment of physical observables in order to avoid the undesirable infinity.

2. An example: the concept of temperature at extreme values

Let us take temperature as a good example of the above cases, for both its asymptotes for extreme low and extreme high values of the scale (present experimental limits: $\approx 10^{-9}$ K and $\approx 3 \cdot 10^{9}$ K). The question applies to the present kelvin scale, based on Lord Kelvin's second (II) definition (indicated in the following with T^{II}), based, as the first one, to (mechanical) work, historically linked to the use of a gaseous media and heat for producing work, i.e., "power motion". [4] The paper is specifically focusing on the lower extreme only, $-\infty$ (as in his first [I] definition, T^{I} —see Table 1), $\rightarrow 0$ in the kelvin scale T^{II}.

Table 1. Temperature values in Kelvin's first definition T^{I} and second definition T^{II} [a].

T^{II}/K	T^{I}/L	T^{II}/K	T^{I}/L
$\rightarrow +\infty$ [b]	$\rightarrow +\infty$ [b]	100	–322
1 000 000	2 630	10	–1 060
100 000	1 892	1	–1 798
10 000	1 154	0.1	–2 536
1 000	416	0.01	–3 274
373.15	100	0.001	–4 012
273.15	0	$\rightarrow 0$ [b]	$\rightarrow -\infty$ [b]

[a] T^{I} in unit L, T^{II} in unit K.
[b] But see text about approaching the asymptotes.

For the issue of the upper extreme, $+\infty$, it is enough to recall here that, quite far from experimental science, contemporary models of physical cosmology postulate an "absolute hot", i.e. that the highest possible temperature is the Planck temperature, 1.416 808(33) 10^{32} K (energy of the Planck mass for Boltzmann constant $k_{\mathrm{B}} = 1$). [5] Above about 10^{32} K, particle energies become so large that gravitational forces between them would become as strong as other fundamental forces according to current theories. A quantum theory of gravity would be required [6] ("The point at which our physical theories run into most serious difficulties is that where matter reaches a temperature of approximately 10^{32} degrees, also known as Planck's temperature. The extreme density of radiation emitted at this temperature creates a disproportionately intense field of gravity. To go even farther back, a quantum theory of gravity would be necessary, but such a theory has yet to be written." [7] At those high energies, consequences may arise for some electromagnetic units of the revised SI, [8, 9] due to an increasing variability with energy of the hyperfine constant α value [10]).

Let us start now a bit of history, then this paper will tackle some of the current and modern views. In order to keep acceptable the length of this paper, the reader is directed for specific contents to the relevant references.

One can ask, for example, if mechanical work does really fit all needs of thermometry and the corresponding metrology. Actually, since the Caloric concept was defeated, the modern way-out is to use rather Energy, [11, 12] inclusive of Mechanical Work and Heat. [13, 14, 15] Energy is a "subtle concept", [16] possibly too subtle and pervasive, and so not so easy to define. For replacing "living force", Young proposed the term "energy" since 1807, basically meaning "potential work". Then Lord Kelvin introduced it formally only in 1851 (later, in 1865, Clausius introduced the term 'entropy'). Feynman popular definition is: "Energy is that-which-is-conserved". For Coopersmith, [16] "the energy of a system is the capacity of the system to do work"—a definition that requires the concept of 'force', not anymore popular in some branches of physics ("the concept of force is conspicuously absent from our [physical] most advanced formulations of the basic laws", e.g., see [17]); also, "energy has extensive (entropy) and intensive (temperature) attributes" [16] — but temperature is not an attribute of energy in the same way that in a reservoir level is not an attribute of an amount of substance. The recent revision of the definition of the International System of Units (SI) [18] endorses an energy-based temperature, where T^{II} is linear in energy, as indicated before: $\Delta T = 1$ K for $\Delta Q = k_{\mathrm{B}}$, with $[k_{\mathrm{B}}] = [\mathrm{J\ K^{-1}}]$, the unit of a quantity called heat capacity.

2.1. Modern views on the classical domain
A well-known picture of statistical mechanics, here reported in figure 1, indicates the basic three statistical models on which the T^{II} kelvin scale is presently based at the lowest temperatures, where the statistics splits into different possibilities; note that there classical Boltzmann statistics are just limiting values of either the Fermi and Bose branches at low occupancy of available quantum states. This fact was commented since many years. For example, Simon, [19] while appreciating the Lord Kelvin First Definition (in more recent literature [20–24] one can find other examples of illustrations, and possible redeeming, of the Lord Kelvin first definition), noted specific basic problems related to the Second Definition in approaching the "absolute zero": (i) the modern justification of the choice of T^{II} is the kinetic theory and statistical mechanics. However, for $T^{\mathrm{II}} \rightarrow 0$ one reaches a point where the statistical hypotheses are not anymore respected (and before that fluctuations occur); (ii) the above theories are normally dealing with material's lattice, while for $T^{\mathrm{II}} \rightarrow 0$ one needs to distinguish between specific sub-systems. Actually, going toward $T^{\mathrm{II}} \rightarrow 0$, energy is going toward zero logarithmically—and the level $E =: 0$ is unreachable except possibly for the whole universe energy balance, according to some recent theories (e.g., [25]).

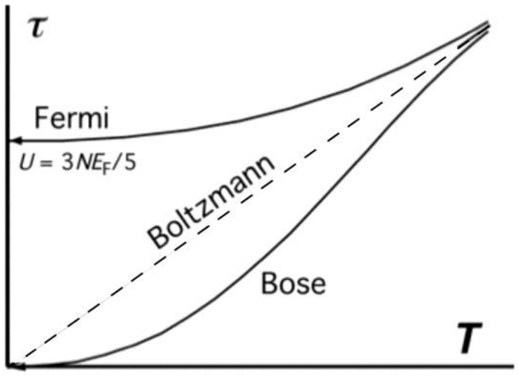

Figure 1. Statistical models for gas toward 0 K: for Bose gas $U \propto T^{5/2}$. The arrows indicate that the $T^{\mathrm{II}} = 0$ cannot be reached. (modified from [26])

2.2. Two contemporary views on possible future temperature concepts
In nano-thermodynamics and the quantum frame, the recent extension of experimental work and technologies to very small dimensions (nano-technologies) and to very low temperatures (nano-temperatures), prompted new problems and the need of rethinking the very concept of temperature in the lowest range. When the concept of temperature does not apply to the whole system, the concept of "local temperature" is introduced, and one should study the "minimal length scales for the existence of local temperature" (e.g., see [27–29]). In these studies it is claimed, for example, that "This length scale is found to be constant for temperatures above the Debye temperature and proportional to T^{-3} below" so that "high temperatures can exist quite locally, while low temperatures exist on larger scales only" and, e.g., "in quasi one-dimensional systems, like carbon-nanotubes, room temperatures (300 K) exist on length scales of 1 mm, while very low temperatures (10 K) can only exist on scales larger than 1 mm". [27]

More generally concerning nano-thermodynamics, [30] and "small systems", [31] the issue was initially prompted by studies on thermodynamic "fluctuations", experimentally observed only since

1992, before the nanotechnology field started to develop. Several theories have been developed, [29, 32] like the non-extensive statistical mechanics, Hill's theory, [30] and tensorial approach. These studies also involve an effort to reconcile the quantum with the classical thermodynamics, [33] with controversial positions, similarity between quantum mechanics and thermodynamics. (e.g., see [34, 35]) It is found that if the Clausius equality is imposed on the Shannon entropy and the analogue of the quantity of heat, then the value of the Shannon entropy comes to formally coincide with that of the von Neumann entropy of the canonical density matrix, and pure-state quantum mechanics apparently transmutes into quantum thermodynamics. The corresponding quantum Carnot cycle of a simple two-state model of a particle confined in a one-dimensional infinite potential well was studied. However, some authors (e.g. [35]) contended that the statement is incorrect. In particular, they claim to have proved that the state at the beginning of the cycle is mixed due to the process of measuring energy. The imposition of the Clausius equality allows the connection between quantum mechanics and thermodynamics, thus resulting in quantum thermodynamics. An asserted experimental evidence of connection of the classical to quantum world is also available. [36]

Another approach (especially [37-39, 40]), also bringing to the analysis of negative temperatures, involves the concept of temperature under another perspective, with the introduction of a Gibbs thermodynamic temperature T_G, alternative to the Boltzmann one, T or T_G, an issue still controversial too (e.g., see [41–43]). For classical systems with many degrees of freedom, the difference in the value of the temperature based on entropies S_B and S_G is considered negligible. Yet, concerning entropy, some findings indicate that Gibbs entropy satisfies the three fundamental laws, while Boltzmann does not. [39] However, other authors [44] argue that Gibbs' entropy fails to satisfy a basic requirement of thermodynamics, that when two bodies are in thermal equilibrium they should be at the same temperature, while Boltzmann's one does.

The above discussion involves, as a consequence the acceptance, or not (in T^{II}), of negative temperatures—e.g., see [45, 46]. A recent paper [47], introducing generalised entropy, intended to be inclusive of Gibbs, Boltzmann and Shannon definitions, supports the latter position.

For sure one can already state that no "ultimate" solution exists. Any new Kuhn's "revolution" can provide, in one year or in 10^X years, new knowledge that, while extending the range where the concept of temperature can be managed, could also innovate, at least partially, in the ranges where today we think to be confident that no innovation is needed. This evidence already exists. After all, the previous concepts of time (subjective—of the observer), space (e.g., force (e.g., [18]), vacuum (e.g., [48]) — and related ones— and even 'ether' (e.g., [49]), 'universe' (e.g., [25] and space: "the space of astronomy is not a physical space of meter sticks, but is a space of light waves" [1, 50]),) have already been revisited.

3. Infinity in general physics

In their theories and models dealing with formulas that describe finite, measurable quantities, physicists overlook the occurrence of unwished infinite values. Indeed, with the exception of various forms of conformal infinity [51, 52], mathematical infinity (indeterminate infinite results in which, e.g., solutions of the gravitational field equations cannot be continued, [53] prevents scientific issues to provide practical formulas that correspond to, or at least approximate, the real observables). For example, in the case of bodies with infinite gravitational mass and/or energy, equations become intractable and useless, since their results would be always the same, regardless of objects' position, mass and movement. In some cases, infinite results mean that a theory is approaching the point where it fails. Therefore, although infinity can be used in physics, scientists require for practical purposes the final result to be physically meaningful: e.g., in quantum field theory, infinities are treated through procedures such as renormalization [54].

4. Infinity as a straight line in a geometrical approach

As stated above, unqualified infinity cannot be any of the physical observables that one either can assess or measure: when one sets out to investigate the infinity, one must leap beyond simple physical concepts and use mathematics. A way to use mathematical and geometrical features to undertake physical infinity is illustrated in [55]. If one wants to assess finite physical measurements leading to infinity, one needs at first to consider finite mathematical figures and topological manifolds, together with their features and relations. Next, one must apply these relations in a projective way. Thirdly, one must thereafter, in a still more highly transformed way, apply the relations of these infinite figures to the general concept of mathematical infinity, which is altogether independent even of all figures and manifolds.

Let us start, [55, 56] with the picture of mathematical infinite, which will be represented by a straight line. One can maintain that, if there were an infinite line, it would be a straight one, or, for example, an infinite triangle, circle or sphere. Since the latter three figures display infinite sides, as will be shown, they can also be described in terms of infinite lines. First of all, an infinite line would be a straight one. The circle's diameter is a straight line, and its circumference is a curved line greater than the diameter. If the curved line becomes less curved in proportion to the increased circle's circumference, then the maximum circle's circumference, which cannot be greater, is minimally curved and therefore maximally straight (Figure 2. *A:* for a positive-curvature manifold; *B:* for a positive-negative curvature manifold. [57]). Indeed, in the figure, the arcs of the larger circle are less curved than the smaller ones. Therefore, the straight line will be the arc of the maximum circle, which cannot be greater. An infinite line is necessarily the straightest; and to it no curvature is opposed. In the same way, every manifold with positive curvature, such as, for example, a triangle, or a circumference, or a sphere, can be described in terms of an infinite line standing for a maximum triangle, or a maximum circle, or a maximum sphere. In fact, an infinite line is whatever is present in the curvature of a finite line: a line finite in length can be longer and straighter; therefore the maximum line is the longest and straightest. If a finite line can describe figures, and if an infinite line is all-the-things-with-respect-to-which a finite line is in infinity, then it follows that an infinite line stands also for a triangle, a circle, and a sphere.

How is it possible that an infinite line is a side of a triangle? Since any two sides of any triangle cannot, if conjoined, be shorter than the third, this means that, in the case of a triangle whose one side is infinite, the other two sides are not shorter, i.e., they are both infinite. Further, since there cannot be more than one infinite thing, an infinite triangle cannot be composed of a plurality of lines, even though it is the greatest and simplest triangle. And because it is a triangle—something which it cannot be without three lines—it will be necessary that the one infinite line be three lines, and that the three lines be one most simple line. And similarly, regarding the angles: for there will be only one infinite angle; and this angle is three angles, and the three angles are one angle. Nor will this maximum triangle be composed of sides and angles; rather, both the infinite line and angle are one and the same thing, so that the line is the angle, because the triangle is the line. The larger the one angle is, the smaller are the other two. Now, any one angle can be increased almost but not completely up to the size of two right angles. Nevertheless, let us make the hypothesis that it is increased completely up to the size of two right angles, while the triangle remains nonetheless a triangle. In that case, it will be obvious that the triangle has one angle that comprises the three angles and that the three angles are one. In the same manner, one can state that a triangle is a line and an infinite line is a maximum triangle. For any two sides of a quantitative triangle are, if conjoined, as much longer than the third side as the angle which they form is smaller than two right angles. Hence, the larger the angle is, the less the lines and the smaller its surface. Therefore, if, by hypothesis an angle could be two right angles, the whole triangle would be resolved into a simple line. Hereby it is evident that an infinite line is a maximum triangle. Next, by applying the same reasoning and the proper rotations, it is feasible to show that an infinite triangle is also an infinite circle and an infinite sphere.

In sum, an infinite line has been shown to be all that which is in the possibility of every finite line and manifold: a triangle is educed from a line, and an infinite line from an infinite triangle. Hence, an important speculative consideration can be inferred: infinity is correlated with finite manifolds.

Because infinite curvature is infinite straightness, this means that an infinite manifold can be described in opposite terms: it is not a thing and is not any other thing; it is not here and is not there; it is unqualifiedly free from all things and is beyond all things; is above the negation of all things. By a physicist's standpoint, this explains why physical theories leading to infinite values are awfully problematic and difficult to cope with.

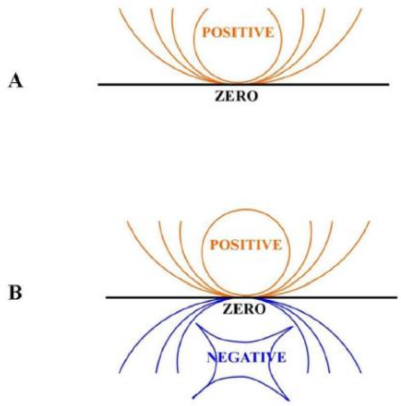

Figure 2. Manifolds toward infinity.
A: given a physical system described by progressively increasing curves on a positive-curvature manifold, the occurrence of infinity (straight line) can be removed by taking into account progressively decreasing curves on a negative-curvature manifold.
B: by placing physical observables or equations on a toroidal manifold, one achieves a correspondence between positive and negative curvatures, thus erasing the unwanted occurrence of infinity (from [57]).

It is assumed by physicists that, due to pragmatic issues, no measurable quantity or event has infinite values. Indeed, any physical theory needs to provide operational tools that correspond to, or at least approximate, reality. This also reflects on the corresponding models and methods of measurement science, in particular of metrology.

To solve the problem of the occurrence of infinity in physical equations and quantities novel conceptual frameworks are needed.

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