Optimized fitting procedure to estimate nuclear matter properties within the relativistic mean-field formalism

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Introduction

Moving across the nuclear landscape away from the β-stability line, it is interesting to study the variation of nuclear matter properties. The advancement in the radioactive ion-beam (RIB) facilities has increased our rather crude understanding of nuclear properties. Owing to the present operating cost and technological limitations of the functioning of the RIB facility, it is still highly challenging to explore the nuclei near and beyond the drip line. Modern theoretical studies involving relativistic mean-field (RMF) formalism have shown great strides in helping expand the present limitation of available experimental data. Recently there has been increasing interest in studying the nuclear symmetry energy (NSE) over the finite nuclei within the coherent density fluctuation model (CDFM) [1, 2, 3]. The NSE is a quantity of infinite nuclear matter defined in momentum space and poses great difficulty in translating it to the coordinate space needed for studying finite nuclei. The CDFM divides the nucleus into tiny spherical pieces of fluctons and makes it accessible to convert the nuclear matter quantities from momentum space to coordinate space. Many previous studies used the non-relativistic Brückner energy density functional energy (Brückner-EDF) using the CDFM to study the symmetry energy over isotopic and isotopic chains [1, 3]. Some of these studies were performed within the RMF formalism by using the RMF densities as the input to the CDFM. However, the use of Brückner's prescription fails to replicate the exact empirical binding energy per nucleon, referred to as the Coester-band problem. To resolve this Coester-band problem, recently, an effective mean-field theory motivated relativistic mean-field (E-RMF) fitting procedure of binding energy per nucleon for different neutron-proton asymmetry was proposed [2]. The fitting procedure suffered from a severe case of overfitting consisting of 24 terms. It is widely known that a complex overfitted model is too closely aligned to a limited training data sample, and its performance degrades for other data sets. The inclusion of additional terms brings with it unpredictable errors in the output of the modelling algorithm. Thus, it is quintessential to resolve the overfitting problem. In this work, we have reduced the number of terms from 24 to 8 for the case of the DD-ME2 parameter set while providing the best fit without over-parametrization. The new fitting procedure has all the benefits of E-RMF and removes the chances of undetected error encountered with the addition of unnecessary inter-dependent terms.

Theoretical Formalism

The effective nuclear symmetry energy (S^A) is calculated within the CDFM as [1]

$$S^A = \int_0^\infty dx |\mathcal{F}(x)|^2 S^{NM}(x). \qquad (1)$$

The term $S^{NM}(x)$ refers to the symmetry energy at local density, and $|\mathcal{F}(x)|^2$ refers to the weight function given as:

$$|\mathcal{F}(x)|^2 = -\left(\frac{1}{\rho_0(x)}\frac{d\rho(r)}{dr}\right)_{r=x}. \qquad (2)$$

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Following the Danielwics method over the liquid drop model, we estimate the surface (S_S) and volume symmetry energy(S_V) [1] as:

$$S_S = \frac{S}{\kappa}\left(1 + \frac{1}{\kappa A^{1/3}}\right), S_V = S\left(1 + \frac{1}{\kappa A^{1/3}}\right). \quad (3)$$

The term κ implies the ratio S_V and S_S. More details related to the CDFM formalism are provided in Ref. [1, 3].

Results and discussion

Firstly, we obtain the spherical RMF densities for chosen nuclei with the DD-ME2 parameter set. Secondly, following the CDFM formalism, we calculate the weight function. Next, using the newly optimized fitting procedure [4] we find the expression of symmetry energy at local density $S^{NM}(x)$ and calculate the effective nuclear symmetry energy S along with its surface and volume components within CDFM. The graph in Fig. 1 shows a spherical equivalent density ρ and the corresponding weight function $|\mathcal{F}(x)|^2$ of ^{16}O as a function of nuclear distance r. It is observed that the weight function transverse a bell-shaped form having the maximum value lying close to the central region where corresponding density has a significantly low value indicating the importance of surface effects in the region.

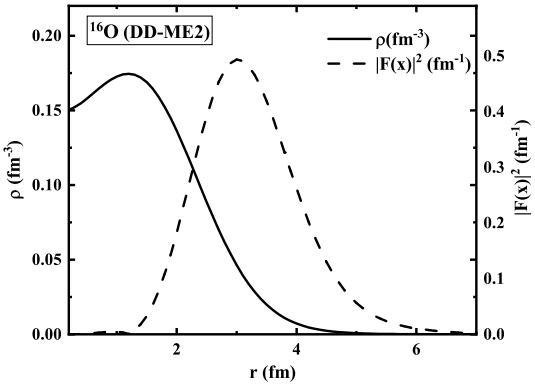

FIG. 1: The variation of RMF density ρ and corresponding weight function $|\mathcal{F}(x)|^2$ are shown as a function of the nuclear distance r for ^{16}O with DD-ME2 parameter set.

TABLE I: The nuclear symmetry energy S^A along with surface S_S and volume symmetry energy S_V of ^{16}O and ^{40}Ca nuclei using the optimized fitting procedure for DD-ME2 parameter set.

DD-ME2	^{16}O	^{40}Ca
S^A	27.56	29.82
S_S	23.38	24.69
S_V	34.89	35.82

Table I shows the calculated effective nuclear symmetry energy along with surface and volume terms for ^{16}O and ^{40}Ca with the DD-ME2 parameter set. The results show that with a large increase in the atomic number Z and mass number A, the symmetry energy and its surface and volume terms also increase. It is important to note here that along the isotopic or isotonic chain, there may be sharp discontinuity or kink in the value of symmetry energy and its components, indicating the possible existence of shell and/or sub-shell closure. The preliminary results based on the newly optimized fitting procedure is being extended to a range of nuclei encompassing light, heavy and super-heavy region and a systematic study is under process.

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