Quark gluon plasma in the early universe expansion with quasi-particle approach

Y Kumar[1], R Sharma[1], G Kuksal[1], P Jain[2,*], V Kumar[3] and P Bangotra[4]

[1] Department of Physics, Deshbandhu College, University of Delhi, Kalkaji, New Delhi-110019, India
[2] Department of Physics, Sri Aurobindo College, University of Delhi, Malviya Nagar, New Delhi-110017, India
[3] Department of Physics, University of Lucknow-226007, India
[4]Atmospheric Research Laboratory, School of Basic Sciences and Research, Sharda University, Greater Noida, India
[*]Corresponding author: poonam.jn1@gmail.com

Abstract. To understand the behaviour of Quark Gluon Plasma (QGP) in the early stages of universe, a precise temporal evolution of different thermodynamic parameters is studied. Out of many indirect signatures used for the detection of QGP, we compute the Equation of State (EoS) by solving the Friedmann equations. A phenomenological model is used with the value of thermal dependent finite quark mass. The variation of temperature, as well as the energy density with respect to time, are provided which predicts a suitable transition temperature for the phase transition. These results can also be used to calculate other thermodynamic observables. The evolution of early universe and its related properties are thus important in the detection of QGP.

1. Introduction

The unique structure of quark gluon plasma (QGP) during the primordial universe has become the most important problem among researchers. Due to the complex behaviour of quantum chromo-dynamics (QCD) phase diagram, it is still a challenging part of investigation to resolve various unusual and interesting properties of QGP. It was assumed that the exotic state of QGP exists when the temperature is above and around 150MeV. This makes the study of deconfined QGP and confined hadronic medium important to understand the evolution of the universe [1,2]. Since long, the exotic matter of QCD was investigated via the QGP EoS within the Lattice QCD computations. For producing the equation of temporal evolution of thermodynamic quantities within the QGP medium, most of the work has been done at zero or small chemical potential. At critical temperature, there is a transformation of hadronic matter into QGP phase. The work of Aoki and other authors suggested that the Lattice QCD results from cosmic quark-hadron transition showed a nature of crossover [1,2,3,4,5], while several other groups claimed a transition from hadrons to QGP to be a first order type [6,7,8,9,10]. Due to the several factors involved in the early universe as well as computer simulation errors, it is difficult to determine the exact order of phase transition. However, researchers are still struggling to ascertain the order of phase shift whether it is a first, second or crossover. Although there are some indications of a crossover transition, some significant studies reported a first order phase transition because of extensive research on bubble nucleation [11,12]. Also, some work has been done on quark nuggets and their consequences in the early universe. Earlier, it was studied that in the early universe, the quark nuggets are generated only if the phase shift is of the order one [13,14,15,16,17,18].

Over the past decade, authors [19,20] pointed out the possibility of QCD first order phase shift in the beginning of universe. This outstanding work was followed by many useful studies which inspected the outcome of an order one type in the early universe [21,22]. The work of Boeckel and his co-workers was revisited to search the possibility of an order one type phase transition by authors [23,24]. The above studies provided us the opportunity for the further investigation of QGP. The EoS is a relationship among various thermodynamic state functions, such as pressure, temperature, and etc. Therefore, the EoS directly describes the behaviour of the particular state of matter, with which it is associated. RHIC and LHC have increased our understanding on this unique QGP signature [25], which has been used extensively in cosmology, especially to understand the thermodynamic evolution of the early universe. Thus, the plasma of quarks and gluons can be described by thermodynamic observables, which are energy density, pressure and temperature varying with respect to time. By using cosmology models derived from the general relativity, a study of the time variation of equation of state can be done easily. The equations given by Einstein are the fundamental field equations of the general theory of relativity. To compute the variation of thermodynamic quantities mentioned above, we need to solve the time evolution equation from the Friedmann equation [26] (eqn. (1)):

$$-\frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon+p)}=\sqrt{\frac{8\pi G}{3}}dt \tag{1}$$

Here, the symbols used in this equation are energy density *(ε)* of the universe, pressure *(p)* of the system, Gravitational constant *(G)* and time evolution *(t)* of the universe. Since the system is homogenous, the expansion of universe is well defined by Friedmann and Einstein, provided the field equations [27], from which the Friedmann equations are derived (eqn.(2) and (3)):

$$\frac{\dot{a}^2+k_c c^2}{a^2}=\frac{8\pi G\varepsilon+\lambda c^2}{3} \tag{2}$$

$$\frac{\ddot{a}}{a}=\frac{-4\pi G}{3}\left(\varepsilon+\frac{3p}{c^2}\right)+\frac{\lambda c^2}{3} \tag{3}$$

In the above equations, we have used natural units ($c=1$). The symbol a is the scale factor that allows the knowledge about distance ratios at different times and a dot on a is its time derivative. Equation (3) has a double time derivative of symbol a which can be considered as equation of motion. The symbol k_c represents a curvature constant which is used to fix the character of spatial curvature such as k_c=0 for flat space, k_c=+1 for closed space and k_c=-1 for open space. It is important to note that whatever be the expansion of universe, the space-time geometrical metric should be of the form of Friedmann-Lemaitre-Robertson-Walker (FLRW) due to isotropy and homogeneity of the system. Another parameter λ is used as cosmological constant. The time evolution equation (eqn. (1)) assumes that the universe's expansion is isentropic and it aids us in finding the time variation of energy density once we ascertain the pressure quantity which depends on energy density, $p(\varepsilon)$ [28]. In this study, a quasi-particle approach is adopted, in which, a finite (thermal dependent) quark mass is considered instead of the earlier assumed dynamic quark mass. Earlier studies in the same area have been conducted, a review is provided in Ref. [29]. Various variants

of the initially developed MIT bag model [30] were previously incorporated to solve the Friedmann equation, which allows us to see the trends between the time and the thermodynamic observables taken as energy density term, pressure term, and sound speed, etc. In Ref. [31], a realistic equation of state was used for the initial stages of the universe and relativistic massive ion collisions. The change of thermodynamic quantities with respect to time varies for different temporal evolution equations, and the equations we are using depend on the choice of our model. Any progress along this line of query is important since an EoS is necessary to conduct theoretical studies of QGP state, the source of it could either be through relativistic massive ion collisions or the primordial universe. Other developments like finite temperature QCD numerical simulations on a lattice also played a crucial role in the study of QGP.
The gravitational wave (GW) emission [32,33] also varies with the nature of the EoS [34,35], since the nature of evolution of the early universe changes by considering EoS using different models. In this work, we attempt to predict the temperature of transition from QGP to the hadronic phase from our model results, which is currently to be determined accurately although it is still a matter of further discussion. The validity of the model can only be checked if the theoretical model results match well with the experimental results. But unfortunately, data is still not very much clear at RHIC and LHC. Therefore, theoretical models are the reliable tools to give the best outcome to explain the phase structure of QCD. Finally, our model works well to explain the evolution of early universe of QGP.

2. Model description

Several models have been used to explain the various properties of early universe expansion of QGP. Since long, extensive research in this field indicated the range of transition temperatures, i.e., $T \sim$ (110-200) MeV, although the temperature varies according to the applied model. So, active research is still going on to claim the phase shift order and the exact measure of transition temperature at which phase transition occurs. In this work, we try to obtain the EoS and predict the value of transition temperature using a quasi-particle approach with various initial conditions.
For that, we use the Friedmann equations which are derived by substituting the FLRW metric in Einstein's Field Equations [36] (eqn. (4)):

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} = \frac{8\pi G}{c^4} T_{\mu\upsilon} \quad (4)$$

Here, $R_{\mu\nu}$ is Ricci tensor which depends on the metric tensor and its derivatives, $T_{\mu\nu}$ is the stress-energy tensor and $g_{\mu\nu}$ is the space-time metric. These equations (eqn. (2) and (3)) provide us with the energy density-time evolution equation (eqn. (1)), which has been mentioned in the previous section. Various works to determine the relationship between thermodynamic quantities like pressure, temperature, sound speed started from solving the basic MIT Bag model. The simple expressions of pressure and the energy density in the MIT Bag model are used by [37,38].
A phenomenological model as a quasi-particle model (with thermal quark mass) is used to explain the evolution of early universe. Earlier, we have used an expression for free energy of quarks and gluons, F_i [39,40]. The expression of F_i is written as:

$$F_i = \mp T g_i \int dk \rho_i(k) \, ln\left(1 \pm e^{-\sqrt{[m_i^2 + k^2]}/T}\right) \quad (5)$$

Here, g_i is the factor of degeneracy for quarks and gluons and $\rho_i(k)$ is the state density of quarks and gluons. m_i is thermal mass of a quark and a gluon generated due to the interactions among these particles, k is the momentum and T is the parameter for temperature. Negative sign used in the above integral refers for quarks (fermions) and positive sign indicates for gluons (bosons). These masses strongly depend on temperature and coupling parameters in QGP medium. It is defined as [40,41]:

$$m_i^2(T) = \gamma_{q,g}(g^2(k))T^2$$

In the above equation, $\gamma_{q,g}$ is the factor used to relate hydrodynamic features of hot QGP flow and $g(k)$ is order one QCD running coupling invariant [42,43,44]. Parameters used in thermal mass are taken as: $\gamma_q=1/6$ and $\gamma_g= 6\,\gamma_q$ or $8\,\gamma_q$ [41,42]. In addition to free energy term for the deconfined particles, another term is included as interface energy which plays a significant role in order to separate QGP medium from hadronic medium. This term can be obtained via scalar Weyl surface with proper modification to take care hydrodynamical aspect of QGP. It is expressed as [45,46,47,48]:

$$F_{interface}=\frac{1}{4}\gamma R^2 T^3. \quad (6)$$

Here, R is the radius of QGP droplet, γ is the effective rms quantity of flow parameter, defined in Ref. [41]. The total free energy can be calculated using free energy of interface term, and quarks and gluons term given in Ref. [25]. Equations (5) & (6) is useful to create a stable QGP droplet, and further, the same is required so as to calculate the expression for pressure and energy density using thermodynamic relations [49]:

$$p = -\frac{dF_i}{dv} \quad (7)$$

The resultant pressure (p) is the addition of the pressures due to individual entities obtained by total free energy and v is the QGP volume. Further, the energy density can be calculated using eqn. (7). It is expressed as:

$$\varepsilon = T\frac{dp}{dT} - p \tag{8}$$

Using the relations of free energy (eqn. (5) & (6)) and pressure (eqn. (7)), we can now express the time evolution of energy density *(ε)* (above as eqn. (1)) and time evolution of temperature *(T)*. The time (t) evolution of temperature can be obtained using eqn. (8) with the help of Ref. [50]:

$$\frac{dT}{dt} = \frac{1}{(\frac{d\varepsilon}{dT})}\cdot\frac{d\varepsilon}{dt} \tag{9}$$

With the help of eqn. (9), we solve time evolution of temperature $T(t)$. One should note that since the EoS are different, keeping a fixed starting point of energy density suggests that, the early universe evolution begins at different starting temperatures for separate models [28,50]. Finally, we obtained the results of time evolution of temperature and energy density which help us to understand the evolution of early universe.

3. Results

To understand how the early universe of QGP changes with time, a study of temporal variation of temperature and energy density are presented. Instead of using the previously used MIT bag model, we have used a quasi-particle approach to solve the Friedmann equation in order to obtain the thermodynamic relations between energy density with time and temperature with time. Considering the time from the starting phase (origin) of early universe, i.e., t=0 to the initial time of QGP creation, i.e., t_0. We have chosen initial conditions such as energy density ~10^4 GeV/fm^3 at $t_i=10^{-9}$ sec and the time evolution vary from $t_i=10^{-9}$ sec to $t_f=10^{-4}$ sec [50]. These conditions are well suited into the evolution of early universe. Due to complexity of system, some error occurred on these conditions. This is due to the fact that these initial conditions may vary model to model for the fine tuning of parametrization factors to show the evolution of early universe. Therefore, in order to investigate unusual properties of early universe, it is interesting to see how the system affects thermodynamic observables with respect to time in the expansion of early universe. These equations provide us more valuable information about the early expansion of the universe in the fraction of time just after Big Bang.

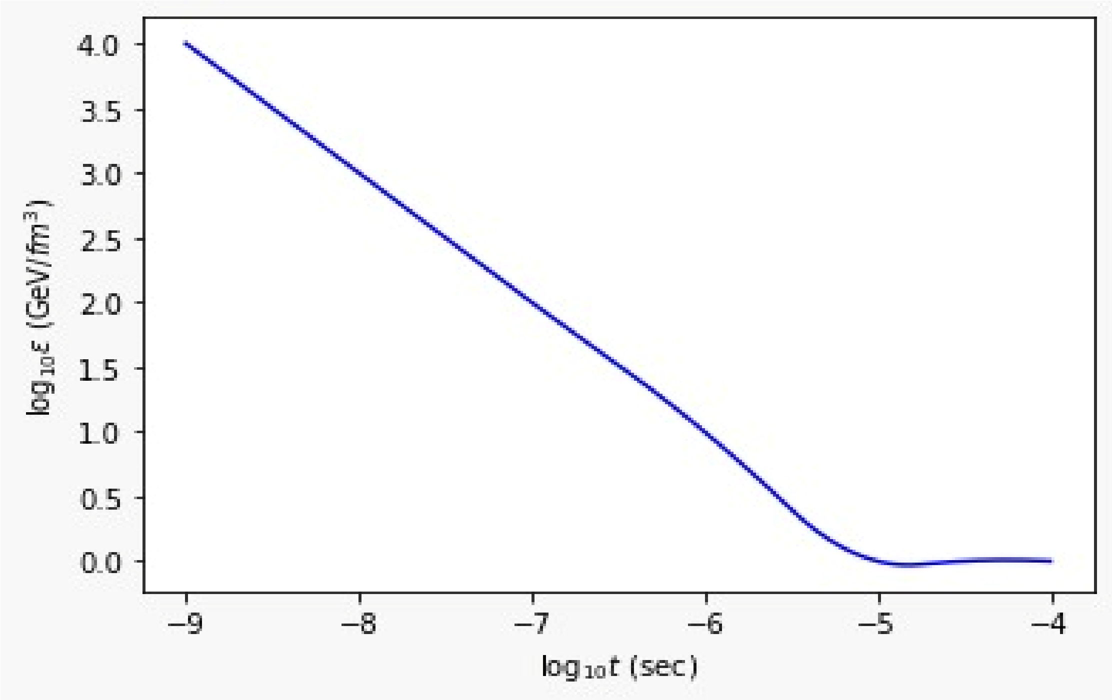

Figure 1. Plot of energy density (ε) with respect to time (t).

From the Lattice gauge theory of quantum chromodynamics, it is found that a phase shift from a hadrons to a QGP medium occurs roughly at around 150MeV [51]. From the above mentioned calculations, we obtain the graphs of the temporal evolution of ε and T, which are given in Figures 1 and 2 respectively. From the model results, we can predict the phase transition between hadronic phase and QGP phase, occurring at around 150MeV, between 10^{-5} seconds and 10^{-4} seconds. Figure 1 shows that energy density decreases continuously with time and it becomes constant just after 10^{-5} seconds. On the other hand, the time evolution of temperature in Figure 2 clearly shows that there is sudden fall of temperature in the early phase of QGP which further decreases slowly with time after 10^{-5} seconds. On critical observation, it is found that the temperature becomes constant giving a clear-cut indication that there is a phase transition occurring near about the temperature 150MeV as shown in Figure 2. Just after t=10^{-5} seconds, there is no change observed in the temperature value by which, we infer that there may be a phase transitions occur at temperature around 150MeV. Although the order of phase transition (first order or smooth crossover) is not very much clear, our model showed a transition temperature value around 150 MeV which is also in good agreement with other theoretical results. So, this information may provide us more about the order of the phase transition between these two phases of QCD. Also, our results closely match with the results of Sanches et al. [28,50].

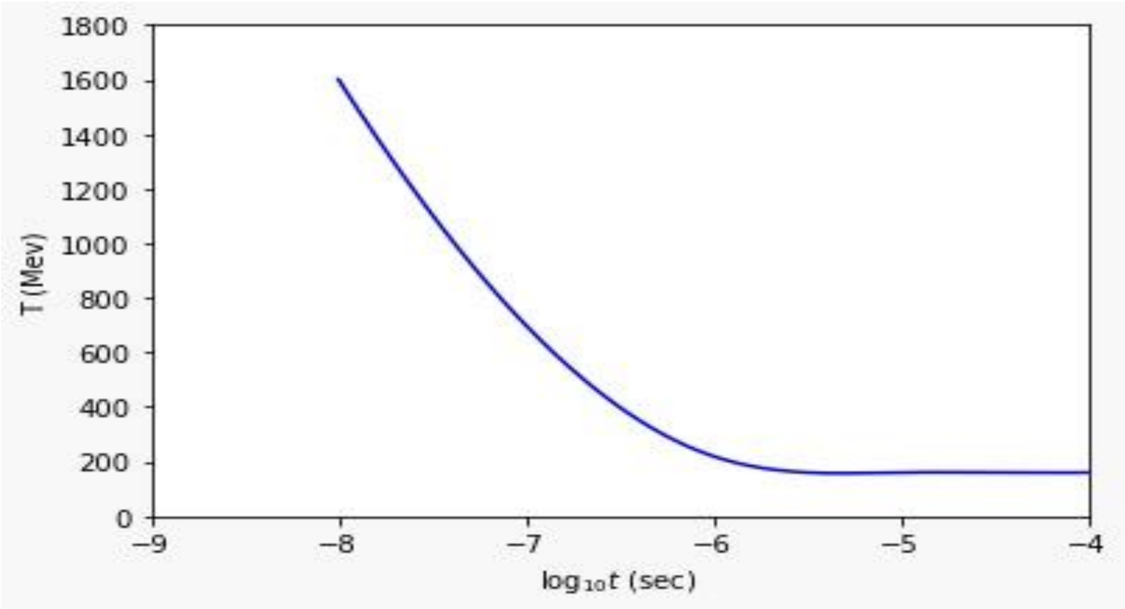

Figure 2. Plot of temperature *(T)* with respect to time (*t*).

Various approaches predict that the phase transition which occurred at the time of the big bang is an order one phase shift, but the Lattice QCD computations indicate that the cosmic phase shift for hadronization process is always a crossover phase transition even in the appearance of magnetic fields [52]. Moreover, there are several results which indicated that the shift from QGP phase into hadronic phase is an order one rather than a crossover phase transition. The results of quasi-particle model suggested the transition temperature, i.e., around 150MeV which may support us to forecast the order of phase shift. The research is still going on in this direction to predict such claims. Above all, our results are in well conformity with the other theoretical results.

The obtained theoretical results can be verified once the precise data is available from the experiments to claim the phase shift whether it is a 1st order or smooth crossover. The upcoming proposed techniques like the Laser Interferometer Space Antenna (LISA), interesting New Gravitational wave Observatory (NGO) and the Big Bang Observatory (BBO) might be an excellent source of information to guess the order of the phase shift [34]. Finally, our results with quasi-particle approach are useful to understand the behaviour of early universe expansion of QGP.

4. Conclusion

The basic models like the MIT bag model are reviewed. Due to its simplicity, several authors have been used MIT bag model although there are some discrepancies found to understand the physical picture of phase diagram of QGP. The idea of introducing the phase boundary between both the mediums using the bag model is not appropriate. Also, there is a disagreement with the lattice QCD simulation pointed out by authors [53,54,55]. However, the phase boundary separating both mediums is used as an interfacial surface instead of using the MIT bag model and this kind of difficulty can be removed by the interfacial energy through parametrization factors.

Another mismatch results with lattice simulations can be resolved using thermal masses in quasi-particle model [55,56,57]. The quasi-particle model provides an opportunity to determine the EoS of QGP in early universe expansion and its related properties. The work related to the QGP is highlighted in view of the early universe evolution. We have studied the plots of energy density (ε) against time (t) in Figure 1 and temperature (T) against time (t) in Figure 2 for understanding the evolution of the early universe. It is observed from the figures, at the time between 10^{-5} sec to 10^{-4} sec, a phase shift from QGP to hadrons is expected at about 150MeV. We have also

discussed that the earlier theoretical works have predicted that hadronization is an order one phase transition, whereas the Lattice QCD calculations have articulated that it is a crossover phase shift. It shows that the order of phase shift is a matter for further discussion and works along this line are still in progress [31].
From our current work, we have confirmed that the QGP-hadron phase transition is actually a reality that may exist at temperature ~ 150MeV although it is still a challenging problem to predict the phase shift order. We infer that our results obtained from quasi-particle model are promising in studying the equations of state of QGP and are in good agreement with Sanches et al. [50]. This work needs further calculations with the consideration of factors like chemical potential, magnetic field etc. which may help us to compute the EoS in a more accurate way and also enable us to predict the exact phase shift order.

5. Further Work
To study the early universe expansion of QGP, our future work will provide a more clear picture on using other important scales like chemical potential, magnetic field, etc. in addition to thermal quark mass and temperature. These different scales may be important in the initial phase of early universe expansion and cannot be ignored to describe the interesting features of QGP medium. This work needs more investigation for deeper insights to explore the early universe expansion.

Acknowledgments
Authors would like to extend their gratitude to the Department of Physics, Deshbandhu College, University of Delhi for the constant support and encouragement to pursue further research in the field of High Energy Physics.

References
[1] Aoki Y, Endrodi G, Fodor Z, Katz S D and Szabo K K 2006 *Nature D* **443** 675.
[2] Aoki Y, Fodor Z, Katz S D and Szabo K K 2006 *Phys. Lett. B* **643** 46.
[3] Karsch F and the RBC-Bielefeld Collaboration 2007 *J. Phys. G* **34** S627.
[4] Meyer-Ortmanns H, Reisz T 2006 *World Sci. Lect. Notes Phys*. **77** 226.
[5] Fodor Z and Katz S D 2002 *JHEP* **0203** 014.
[6] Stephanov M, Rajagopal K and Shuryak E 1998 *Phys. Rev. Lett.* **81** 4816.
[7] Boeckel T and Schaffner-Bielich J 2012 *Phys. Rev. D* **85** 103506.
[8] Agasian N O and Fedorov S M 2008 *Phys. Lett. B* **663** 445.
[9] Tsue Y, Providencia J, Providencia C and Yamamura M 2010 *Prog. Theor. Phys.* **123(6)** 1013.
[10] Mukherjee S, Mustafa M G and Ray R 2007 *Phys. Rev. D.* **75** 094015.
[11] Turner M S, Weinberg E J and Widrow L M 1992 *Phys. Rev. D* **46** 2384.
[12] Kisslinger L S, Walawalkar S and Johnson M B 2005 *Phys. Rev. D* **71** 065017.
[13] Lugones G and Horvath J E 2004 *Phys. Rev. D* **69** 063509.
[14] Lai X Y and Xu R X 2010 *J. Cosmol. Astropart. Phys.* **05** 028.
[15] Witten E 1984 *Phys. Rev. D* **30** 272.
[16] Boyanovsky D, de Vega H J and Schwarz D J 2006 *Ann. Rev. Nucl. Part. Sci.* **56** 441.
[17] Farhi E and Jaffe R L 1984 *Phys. Rev. D* **30** 2379.
[18] Chandra and Goyal A 2000 *Phys. Rev. D* **62** 063505.
[19] Boeckel T and Schaffner-Bielich J 2010 *Phys. Rev. Lett.* **105** 041301.
[20] Schettler S, Boeckel T and Schaffner-Bielich J 2011 *Prog. Part. Nucl. Phys.* **66** 266.
[21] Caprini C, Durrer R and Xavier S 2010 *Phys. Rev. D* **82** 063511.
[22] Schettler S, Boeckel T and Schaffner-Bielich J 2011 *Phys. Rev. D* **83** 064030.
[23] Layek B, Sanyal S and Srivastava A M 2001 *Phys. Rev. D.* **63** 083512.
[24] Mocsy A and Sorensen P 2010 *Phys. Lett. B* **690** 135.
[25] Somorendro S S, Gosain D S, Kumar Y and Jha A K 2010 *Pram. J. Phys.* **74(1)** 27.
[26] Friedman A 1999 *Gen. Rel. and Grav.* **31(12)** 1991.
[27] Stephani H, Kramer D, MacCallum M, Hoenselaers C and Herlt E 2009 *Cambridge university press*.
[28] Sanches S M *et al.* 2015 *JoP: Conf. Ser.* **630** 012028.
[29] Boeckel T, Schettler S and Schaffner-Bielich J 2011 *Prog. Part. and Nucl. Phys.* **66(2)** 266.
[30] Chodos A J, Jaffe R L, Johnson K, Thorn C B and Weisskopf V F 1974 *Phys. Rev. D.* **9(12)** 3471.

[31] Florkowski W 2011 *Nucl. Phys. A.* **853(1)** 173.
[32] Einstein A and Rosen N 1937. *Jour. of Frank. Inst.* **223(1)** 43.
[33] Weber J 1960 *Phys. Rev.* **117(1)** 306.
[34] Roque V M and Lugones G 2013 *Phys. Rev. D.* **87(8)** 083516.
[35] Caprini C 2015 *JoP: Conf. Ser.* **610(1)** 012004.
[36] Green S R and Wald R M 2014 *Clas. and Quan. Grav.* **31(23)** 234003.
[37] Johnson K 1975 *Acta Phys. Pol. B.* **6(12)** 8.
[38] Strottman D 1979 *Phys. Rev. D* **20(3)** 748.
[39] Kumar Y 2016 *JoP: Conf. Ser.* **668** 012110.
[40] Kumar Y and Singh S S 2017 *EPJ Web Conf.* **137** 13008.
[41] Kumar Y and Singh S S 2012 *Can. J. Phys.* **90** 955.
[42] Jain P and Kumar Y 2014 *J. Mod. Phys.* **5** 686.
[43] Singh S S and Kumar Y 2013 *J. Mod. Phys.* **4** 582.
[44] Kumar Y and Singh S S 2013 *ISRN High E. Phys.* **2013** 156747.
[45] Mardor I and Svetitsky B 1991 *Phys. Rev. D* **44** 878.
[46] Weyl H 1911 *Nachr. Akad. Wiss Gottingen* **110**.
[47] Solfrank J *et al.* 1997 *Phys. Rev. C* **55** 392.
[48] Shuryak E 2004 *Prog. Part. Nucl. Phys.* **53** 273.
[49] Kumar Y 2019 *JPS Conf. Proc.* **26** 024028.
[50] Sanches Jr S M, Navarra F S and Fogaca D A 2015 *Nucl. Phys. A.* **937** 1.
[51] Borsanyi S, Fodor Z, Hoelbling C, Katz S D, Krieg S, Ratti C and Szabo K K 2010 *JHEP* **9** 1.
[52] Ray A and Sanyal S 2013 *Phys. Lett. B* **726(1-3)** 83.
[53] Engels J, Fingberg J, Karsch F, Miller D, Weber M 1990 *Phys. Lett. B.* **252** 625.
[54] Shukla P and Mohanty A K 2001 *Phys. Rev. C* **64** 054910.
[55] Peshier A, Kampfer B, Pavlenko O P and Soff G 1994 *Phys. Lett. B* **337** 235.
[56] Gorenstein M I and Yang S N 1995 *Phys. Rev. D* **52** 5206.
[57] Goloviznin V and Satz H 1993 *Z. Phys. C* **57** 671.