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RESEARCH

Newtonian Gravity of Antimatter and Gravitational Needs of Field Quantization: A Path to Renormalizable Gravity

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Abstract

By incorporating quantum mechanics into gravitational theory through the so-called spacetime geometrization procedure that consists in applying the principle of least action alongside the covariance of quantum mechanical motion equations, we present a model that describes the gravitational behavior of antimatter whose existence is fundamentally rooted in quantum mechanics. The gravity produced by an antimatter macroscopic body, described by continuous quantum mechanical field, shows that it produces attractive Newtonian potential on macroscopic scale. On a microscopic scale, where we cannot use the point-like mass approximation, the work shows that the Newtonian gravity includes an additional term that is inversely proportional to source mass and depending by the shape of the quantum mass density distributions.$|\psi|$. The divergence of gravitational energy for infinitesimal masses, in order to yield finite physical solutions, requires that elementary particles possess a discrete mass spectrum and that the quantization of their fields emerges as a necessary condition for the realization of the physical universe. Furthermore, the quantum mechanical contribution, induced by the energy of the quantum potential on spacetime geometry, which diverges for small masses, can possibly compensate for the divergence in quantum gravity where this contribution is not considered.

Keywords Antimatter gravity · Gravitational origin of field quantization · Gravity quantization

1 Introduction

General Relativity, a form of spacetime geometrization, is derived by utilizing two fundamental conditions: the equivalence of inertial and gravitational masses, and the principle of least action [1]. On the other hand, it is also true that the equivalence of inertial and gravitational masses corresponds to imposing the covariance of the classical equations of motion in curved spacetime In this sense, General Relativity can be conceptualized as the geometrization of classical spacetime arising from the principle of least action, together with the classical

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physics it encompasses. If, instead of the covariance of the classical motion equation, we assume the covariance of quantum mechanical motion equations (for the wavefunction and its mass distributions $|\psi|^2$) we obtain the spacetime geometry consequent to the presence of quantum bodies.

With this procedure, we obtain a curved spacetime whose geometry is determined by the quantum mass density distribution, which differs from the classical one. This essentially means that spacetime geometry is also influenced by the energy of the quantum potential, which has no equivalent in classical physics. While the contribution to gravity arising from the quantum mass density is not conceptually new compared to General Relativity, the second contribution, originating from its quantum potential, leads to new and significant outcomes. It introduces the non-local properties of quantum mechanics into gravitation and the existence of antimatter gravity.

The resulting gravity equation, structurally similar to that of General Relativity, expresses spacetime curvature on the left-hand side as a function of the energy tensor on the right-hand side, which, in turn, depends on the quantum mechanical wavefunction of the system.

The goal of this work is to derive the weak gravity limit for a macroscopic antimatter body. Here, macroscopic refers to a body of immense mass that influences large-scale structures, such as a planet on a cosmological scale.

In addition to the standard conditions imposed in General Relativity to recover weak Newtonian gravity (namely, the non-relativistic limit (see (20)) and the quasi Minkowskian condition (see (28–29))) the macroscopic condition in General Relativity, must be introduced by additionally requiring the condition of $\hbar \rightarrow 0$.

It is also important to note that, in this approach, the wavefunction fields are taken continuous. Therefore, when approaching scales relevant to particle physics, where the fundamental structure of the vacuum and field quantization become significant, the quantum mechanical gravity framework discussed here ceases to be applicable.

In cases where one seeks to describe gravity generated by elementary particles (not addressed in this work), the fields contained in the energy tensor must be quantized in the corresponding curved spacetime, where the curvature itself is a quantum operator — a particularly challenging problem.

The quantum mechanical gravity can be achieved by utilizing the Madelung hydrodynamic representation of quantum mechanics [2–4] that transforms the quantum equations (such as the Schrodinger or the Klein-Gordon or the Dirac ones) as a function of the field

$$\psi_{(x_\mu)} = |\psi_{(x_\mu)}| e^{-i\frac{S_{(x_\mu)}}{\hbar}} \tag{1}$$

in a system of two equations as a function of the real variables:$|\psi_{(x_\mu)}|$ and $\partial_\mu S = -p_\mu$.

This transformation, for the simplest and most illustrative case of Klein-Gordon equation (KGE)

$$\psi_{;\mu}^{;\mu} = \left(g^{\mu\nu}\partial_\nu \psi\right)_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g}\left(g^{\mu\nu}\partial_\nu \psi\right) = -\frac{m^2c^2}{\hbar^2}\psi \tag{2}$$

leads to the motion equation [5]

$$g_{\mu\nu}\partial^\nu S \partial^\mu S - \hbar^2 \frac{1}{|\psi|\sqrt{-g}}\partial_\mu \sqrt{-g}\left(g^{\mu\nu}\partial_\nu |\psi|\right) - m^2c^2 = 0 \tag{3}$$

coupled to the conservation equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial q^{\mu}}\sqrt{-g}\left(g^{\mu\nu}|\psi|^{2}\frac{\partial S}{\partial q^{\nu}}\right)=0 \tag{4}$$

that gives rise to a classical-like description where the mass density $|\psi|^2$, owing the hydrodynamic impulse p_μ, is subject to the additional non-local quantum potential interaction

$$V_{qu(|\psi|)}=-\frac{\hbar^{2}}{m}\frac{1}{|\psi|\sqrt{-g}}\partial_{\mu}\sqrt{-g}\left(g^{\mu\nu}\partial_{\nu}|\psi|\right) \tag{5}$$

conceptualizing (3) in the form

$$g_{\mu\nu}p^{\nu}p^{\mu}+mV_{qu}-m^{2}c^{2}=0 \tag{6}$$

In the non-relativistic limit, where Eq. (2) reduces to the Schrödinger equation, Eq. (3) simplifies to the classical equation of motion [4], with the addition of the Madelung quantum potential. In the quantum case, analogously to the general relativity procedure, by imposing the covariance of (3) in curved spacetime we can utilize the minimum action principle to obtain the geometry of spacetime subject to the quantum physics. This approach is based on the fact that the equivalence of gravitational and inertial mass in classical General Relativity can be replaced by the condition of covariance of classical equations of motion in curved spacetime.

As shown in ref. [5, 6] minimum action condition $\delta_{QH}=0$ in the quantum hydrodynamic representation can be expressed as

$$\delta_{QH}=\frac{1}{c}\int\iiint|\psi|^{2}\sum_{k}\left(\left(\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{-g}\widetilde{L}_{(k)}}{\partial g^{\mu\nu}}-\frac{\partial}{\partial q^{\lambda}}\frac{\partial\sqrt{-g}\widetilde{L}_{(k)}}{\partial\frac{\partial g^{\mu\nu}}{\partial q^{\lambda}}}-\frac{\partial}{\partial|\psi|}\frac{\partial\sqrt{-g}\widetilde{L}_{(k)}}{\partial\frac{\partial g^{\mu\nu}}{\partial|\psi_{k}|}}\right)\right)\delta g^{\mu\nu}\right)\sqrt{-g}d\Omega \tag{7}$$

where $\widetilde{L}_{(k)}$ is the quantum hydrodynamic Lagrangian density $\widetilde{L}_{(k)}$ given in ref. [5]. Furthermore, by comprehending the contribution coming from the spacetime curvature,

$$\delta_{g}=\frac{c^{3}}{16\pi G}\int\iiint\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)\delta g^{\mu\nu}\sqrt{-g}d\Omega \tag{8}$$

the overall minimum condition

$$\delta_{QH}+\delta_{g}=\int\iiint\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}-\frac{8\pi G}{c^{4}}|\psi|^{2}\tau_{\mu\nu}\right)\delta g^{\mu\nu}\sqrt{-g}d\Omega=0 \tag{9}$$

defines the quantum gravity equation (QGE) [5, 6]

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}-\frac{8\pi G}{c^{4}}|\psi|^{2}\tau_{\mu\nu}=0 \tag{10}$$

where $\tau_{\mu\nu}$ in (10) is explicitly derived as a function of the quantum hydrodynamic Lagrangian density $\widetilde{L}_{(k)}$ in ref. [5].

For the particular case of interest, of the macroscopically stable state (stationary energy eigenstates [7]), $\widetilde{L}_{(k)}$ reads

$$\begin{aligned}\widetilde{L}_{(k)} &= -c^2\left(\partial_t S_{(k)}\right)^{-1} g_{\mu\alpha} p^{\alpha}_{(k)} p^{\mu}_{(k)} = -c^2\left(\partial_t S_{(k)}\right)^{-1} g_{\mu\alpha}\partial^{\alpha} S_{(k)}\partial^{\mu} S_{(k)} \\ &= -c^2\frac{i\hbar}{2}\left(\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial t}\right)^{-1} g_{\mu\alpha}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q_\alpha}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q_\nu}\end{aligned} \tag{11}$$

Therefore, as shown in ref. [5], in the macroscopic weak gravity limit, (10) acquires the form

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{(k)\mu\nu} - \Lambda_Q g_{\mu\nu}\right) \tag{12}$$

where the energy tensor density $T_{(k)\mu\nu}$ reads

$$\begin{aligned}T_{(k)\mu\nu} &= |\psi_k|^2 c^2\left(\frac{\partial S_{(k)}}{\partial t}\right)^{-1}\left(\frac{\partial S_{(k)}}{\partial q^\mu}\frac{\partial S_{(k)}}{\partial q^\nu} - \left(g_{\alpha\beta}\frac{\partial S_{(k)}}{\partial q_\beta}\frac{\partial S_{(k)}}{\partial q_\alpha}\right)g_{\mu\nu}\right) \\ &= -|\psi_k|^2 c^2\frac{\hbar}{2i}\left(\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial t}\right)^{-1}\left(\begin{array}{c}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q^\mu}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q^\nu} \\ -\left(g_{\alpha\beta}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q_\beta}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q_\alpha}\right)g_{\mu\nu}\end{array}\right)\end{aligned} \tag{13}$$

where $g_{\nu\mu}$ is the metric tensor, where $g = |g_{\nu\mu}|^{-1}$ and where

$$\ln\frac{\psi_k}{\psi_k *} = -\frac{2i}{\hbar}S_{(k)} \tag{14}$$

Without loss of generality, we can assume the wavefunction ψ_k to be that of the positive energy states ψ_{+k} ($\psi_{+k} = \psi_k$), so that for the negative ones reads $\psi_{-k} = \psi_k *$. Therefore, in this case we have that $S_- = -S_+$ and

$$T_{(k)\pm\mu\nu} = \mp|\psi_k|^2 c^2\frac{\hbar}{2i}\left(\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial t}\right)^{-1}\left(\begin{array}{c}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q^\mu}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q^\nu} \\ -\left(g_{\alpha\beta}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q_\beta}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q_\alpha}\right)g_{\mu\nu}\end{array}\right) \tag{15}$$

Moreover, by employing the low-curvature limit (appropriate for Newtonian gravity) the KGE expression

$$\frac{\partial S}{\partial q^\mu}\frac{\partial S}{\partial q_\mu} = p_\mu p^\mu = \left(\frac{E^2}{c^2} - p^2\right) = m^2c^2\left(1 - \frac{V_{qu}}{mc^2}\right) \tag{16}$$

the energy-tensor (15) reads

$$T_{(k)\pm\mu\nu} = \mp|\psi_k|^2 m^2c^4\frac{\hbar}{2i}\left(\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial t}\right)^{-1}\left(\begin{array}{c}\frac{1}{m^2c^2}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q^\mu}\frac{\partial \ln[\frac{\psi_k}{\psi_k *}]}{\partial q^\nu} \\ -\left(1 - \frac{V_{qu}}{mc^2}\right)g_{\mu\nu}\end{array}\right) \tag{17}$$

In the classical derivation of Newtonian potential as weak limit of General Relativity, $m|\psi|^2$ represents the classical mass distribution density into the spacetime. Here, as long as we describe macroscopic quantum bodies, it represents the quantum mechanical mass density defined by the body wavefunction. When we talk about weak gravity of macroscopic antimatter body, we want to find the Newtonian gravity of a body such as, for instance, an antimatter Earth. To obtain such a result we have to introduce the not-relativistic condition (see (20) below), absence of spin and the macroscopic limit of $\hbar \to 0$.

Furthermore, since when we talk about matter with refers to bodies with not null rest mass, we are actually considering fermions and we should consider the more complex gravity produced by quantum mechanical fermion bi-spinor wavefunction [6].

To simplify the problem, we observe that in the non-relativistic limit, the Dirac equation reduces to the Pauli equation, which, in absence of external magnetic field and disregarding the spin-spin interaction of particles for the determination of the spacetime gravity, further leads to the Schrödinger equation [8]. Therefore, in the case of macroscopic matter or antimatter that appears macroscopically spinless regardless of the spin of its elementary constituents, the non-relativistic limit of the KGE (2) can be applied to bodies with nonzero rest mass in the low-velocity regime.

In this context, the wavefunction (2) can be used in (17) and (12). Moreover, to define the antimatter wavefunction relative to the matter one, we consider, as exemplifying case, that of a charged KGE state (not necessarily electric charge). When the matter wavefunction is defined accordingly $\psi_{m+} = \frac{\psi+\psi*}{\sqrt{2}}$, the antimatter wavefunction takes the form $\psi_{m-} = \frac{\psi-\psi*}{\sqrt{2}}$ [9].

Consequently, performing the (non-relativistic) weak gravity limit of (12) with quantum mechanical mass densities $\left|\psi_{m+}\right|^2$ and $\left|\psi_{m-}\right|^2$ we obtain the Newtonian forces for both matter and antimatter states.

To derive the macroscopic (classical) Newtonian gravity of matter and antimatter, the limit $\hbar \to 0$ leads to the identities [4]

$$lim_{\hbar\to 0}\Lambda_Q = 0 \tag{18}$$

and

$$lim_{\hbar\to 0}V_{qu} = 0 \tag{19}$$

Moreover, the conditions for the establishing of non-relativistic limit ($c \to \infty$) gives

$$\gamma \cong 1 \tag{20}$$

leading to the identity

$$\frac{2i}{\hbar}\frac{\partial \ln[\frac{\psi_k}{\psi_k*}]}{\partial t} = \frac{\partial S_+}{\partial t} = -E = -\gamma mc^2 \cong -mc^2 \tag{21}$$

and

$$\frac{\partial S_-}{\partial t} = E = \gamma mc^2 \cong mc^2 \tag{22}$$

that introduce into (17) leads to

$$\begin{aligned} T_{(k)\pm\mu\nu} &= |\psi_{\pm k}|^2 mc^2\left(\frac{1}{m^2c^2}\frac{\partial \ln[\frac{\psi_k}{\psi_k*}]}{\partial q^\mu}\frac{\partial \ln[\frac{\psi_k}{\psi_k*}]}{\partial q^\nu} - g_{\mu\nu}\right) \\ &= |\psi_{\pm k}|^2 mc^2\left(\left(\frac{1}{mc}\right)^2 p_\mu p_\nu - g_{\mu\nu}\right) \\ &= |\psi_{\pm k}|^2 mc^2\left(u_\mu u_\nu - g_{\mu\nu}\right) \end{aligned} \tag{23}$$

where $u_\mu = \frac{p_\mu}{mc}$ is the velocity field.

Therefore, from (23) the weak gravity limit of QGE reads

$$R_{\pm\nu\mu} - \frac{1}{2} g_{\nu\mu} R_{\pm\alpha}{}^{\alpha} = \frac{8\pi G}{c^2} m |\psi_{\pm(k)}|^2 u_\mu u_\nu \tag{24}$$

leading to the trace identity

$$R_{\pm\alpha}{}^{\alpha} - \frac{1}{2} \delta_\alpha{}^{\alpha} R_{\pm\alpha}{}^{\alpha} = -R_{\pm\alpha}{}^{\alpha} = \frac{8\pi G}{c^2} m |\psi_\pm|^2 u_\alpha u^\alpha \tag{25}$$

Thus, in the particles reference system where

$$u_\mu = (1,0,0,0) \tag{26}$$

it follows that

$$R_{\pm 0}{}^{0} = \frac{8\pi G}{c^2} m |\psi_{\pm k}|^2 = \frac{8\pi G}{c^2} m |\psi_k|^2 = \frac{8\pi G}{c^2} m |\psi_k *|^2 \tag{27}$$

and therefore

$$R_0{}^0 = \frac{8\pi G}{c^2} m |\psi_k|^2 = \frac{8\pi G}{c^2} m |\psi_k *|^2 = \frac{8\pi G}{c^2} m \left| \frac{\psi_{m+} \pm \psi_{m-}}{\sqrt{2}} \right|^2 \tag{28}$$

2 The Newtonian Potential for Macroscopic Point-like Antimatter

Given the Newtonian gravitational potential ϕ, as a function of the component g_{00} of the metric tensor [10]

$$\frac{2\phi}{c^2} = g_{00} - 1 \tag{29}$$

whose trace, at zero order, reads

$$g_{\alpha\alpha} \cong -2 \tag{30}$$

leading to the identity

$$R_0{}^0 = \frac{4\pi G}{c^2} m |\psi|^2 = R_{00} = \frac{\partial \Gamma^{\alpha}{}_{00}}{\partial q^\alpha} \approx -\frac{1}{2} \partial_\alpha \left(g_{\gamma\gamma} \partial_\alpha g_{00} \right) = \frac{1}{c^2} \partial_\alpha \partial_\alpha \phi \tag{31}$$

it follows that

$$\partial_\alpha \partial_\alpha \phi = 4\pi G m \left| \frac{\psi_{m+} \pm \psi_{m-}}{\sqrt{2}} \right|^2 \tag{32}$$

Given that in (31) ψ_{m+} and ψ_{m-} describe the quantum mechanical mass density distribution of matter and antimatter, respectively, in order to obtain the macroscopic limit of the Newtonian potential, we have to introduce the macroscopic condition where the particle mass (with its quantum mechanical mass density distribution) is collapsed in a macroscopically punctual domain. In this case the typical inter-particles distance is much bigger than both the physical length of the quantum configuration of mass distribution and the range of interaction of the quantum potential [7]. In this limit, we can describe the puctual mass density of the point-like particle and the antiparticle, located in R_{m+} and R_{m-}, respectively, by the spatial densities

$$|\psi_{m+}|^2 = \delta^{(3)}(r - R_{m+}) \tag{33}$$

$$|\psi_{m-}|^2 = \delta^{(3)}(r - R_{m-}) \tag{34}$$

and, therefore, the total normalized matter and antimatter density reads

$$\frac{1}{2}|\psi_{m+} \pm \psi_{m-}|^2 = \frac{\delta^{(3)}(r - R_{m+}) + \delta^{(3)}(r - R_{m-})}{2} \tag{35}$$

with the normalized condition

$$\iiint \delta^{(3)}(r - R_{m\pm}) d\mathrm{V} = 1 \tag{36}$$

that by (31) leads to

$$\begin{aligned} &\iiint dV \partial_\alpha \partial_\alpha \phi = 2\pi G m \iiint \delta^{(3)}(r - R_{m+}) + \delta^{(3)}(r - R_{m-}) dV = 4\pi G m \\ &= \oiint \partial_\alpha \phi \cdot dS^\alpha = \int_0^{4\pi} \frac{\partial \phi}{\partial r} \cdot \left(r - R_{m\pm}\right)^2 d\Omega = \frac{\partial \phi}{\partial r} 4\pi \left(r - R_{m\pm}\right)^2 \end{aligned} \tag{37}$$

Furthermore, if we consider each matter and antimatter body of equal mass m_B (with total mass of system $m = 2m_B$) and we bring to infinity the matter body respect to the antimatter one so that the integration is done on an infinite domain that do not contain its mass distribution $\delta^{(3)}(r - R_{m+})$, it follows that

$$\frac{\partial \phi}{\partial r} 4\pi \left(r - R_{m-}\right)^2 = 4\pi G m_B \iiint \delta^{(3)}(r - R_{m-}) d\mathrm{V} = 4\pi G m_B \tag{38}$$

and

$$\frac{\partial \phi}{\partial r} 4\pi \left(r - R_{m+}\right)^2 = 4\pi G m_B \iiint \delta^{(3)}(r - R_{m+}) d\mathrm{V} = 4\pi G m_B \tag{39}$$

and, by integration, that

$$\phi_{m\pm} = -G \frac{m_B}{|r - R_{m\pm}|} \tag{40}$$

showing that the Newtonian gravity of macroscopically punctual antimatter is equal to that of matter (in empty space without the test mass).

3 Newtonian Gravity at Short Distance for Quantum Mechanical Antimatter body

The results (35,38–40) hold as long as the localization of the wave function produces a mass distribution that, at distances much greater than the characteristic length scale of the quantum mass density of particles, can be satisfactorily approximated by a point-like domain and the wave functions of matter and antimatter do not overlap (see (35)). In this case, the fundamental condition for recovering Newtonian gravity is that.$\hbar \rightarrow 0$ and that $lim_{\hbar \rightarrow 0} V_{qu} = 0$ so that the quantum potential energy can be disregarded. At very short distances, when the characteristic length of the system is comparable to the spatial extent of the mass distributions, the gravitational interaction is influenced by the effective shape of the quantum mass density distribution $|\psi_{\pm}|^2$ with wavefunctions overlapping. In this case, since the quantum mass density configuration becomes relevant, we must remove the condition of $\hbar \rightarrow 0$ [4]. By doing so, we derive the gravitational potential of a particle coming from the spacetime curvature induced by its quantum mass distribution $|\psi_{\pm}|^2$ through the gravitational equations

$$\begin{aligned} R_{\nu\mu} - \tfrac{1}{2} g_{\nu\mu} R_{\alpha}{}^{\alpha} &= \tfrac{8\pi G}{c^4} \tfrac{mc^2 |\psi_{\pm}|^2}{\gamma} \begin{pmatrix} \left(\sqrt{1 - \tfrac{V_{qu}}{mc^2}} - 1 \right) g_{\mu\nu} - \Lambda_{Q\pm} g_{\mu\nu} \\ + \sqrt{1 - \tfrac{V_{qu}}{mc^2}}^{-1} \left(\tfrac{\hbar}{2mc} \right)^2 \partial_{\mu} ln[\tfrac{\psi_{\pm}}{\psi_{\pm}*}] \partial^{\lambda} ln[\tfrac{\psi_{\pm}}{\psi_{\pm}*}] g_{\lambda\nu} \end{pmatrix} \\ &= \tfrac{8\pi G}{c^4} \tfrac{mc^2 |\psi_{\pm}|^2}{\gamma} \begin{pmatrix} \left(\sqrt{1 - \tfrac{V_{qu}}{mc^2}} - 1 \right) g_{\mu\nu} - \Lambda_{Q\pm} g_{\mu\nu} \\ + \sqrt{1 - \tfrac{V_{qu}}{mc^2}}^{-1} \left(\tfrac{\hbar}{2mc} \right)^2 p_{\mu\pm} p^{\lambda}{}_{\pm} g_{\lambda\nu} \end{pmatrix} \end{aligned} \tag{41}$$

Here, for sake of completeness, we also re-consider the contribution that can come from the quantum pressure term $-\Lambda_{Q\pm} g_{\mu\nu}$ where the quintessence-like term $\Lambda_{Q\pm}$, given in ref. [5], reduces to small constants, Λ_{Q+} and Λ_{Q-} (for matter and antimatter respectively) in quasi-Minkowskian spacetime approximation [11]. Therefore, Eq. (41) for the non-relativistic case, where $\frac{V_{qu}}{mc^2} \ll 1$, leads to

$$R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R_{\alpha}{}^{\alpha} = \frac{8\pi G}{c^4} \frac{mc^2 |\psi_{\pm}|^2}{\gamma} \begin{pmatrix} -\left(\frac{V_{qu}}{2mc^2} + \Lambda_{Q\pm} \right) g_{\mu\nu} \\ + \left(1 + \frac{V_{qu}}{2mc^2} \right) u_{\mu\pm} u_{\nu\pm} \end{pmatrix} \tag{42}$$

Furthermore, by using the identity

$$-\left(\frac{V_{qu}}{2mc^2} + \Lambda_{Q\pm} \right) \left(\delta_{\alpha}^{\alpha} - u_{\alpha} u^{\alpha} \right) = -3 \left(\frac{V_{qu}}{2mc^2} + \Lambda_{Q\pm} \right) \tag{43}$$

it follows that

$$-R_{\alpha}{}^{\alpha} = \frac{8\pi G}{c^4} mc^2 |\psi_{\pm}|^2 \left(1 - 3 \left(\frac{V_{qu}}{2mc^2} + \Lambda_{Q\pm} \right) \right) \tag{44}$$

and, by utilizing (37), that the gravitational force $\partial_r \phi$ reads

$$\partial_r \phi_{\pm} = \frac{Gm}{\left(r - R_{\pm} \right)^2} \int_0^V |\psi_{\pm}|^2 \left(1 - 3 \left(\frac{V_{qu}}{2mc^2} + \Lambda_{Q\pm} \right) \right) d^3 V \tag{45}$$

where, we can recognize classical and quantum contributions that read, respectively,

$$\partial_r \phi_{\pm Class} = \frac{Gm}{\left(r - R_{\pm}\right)^2} \int_0^{(r-R)} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} |\psi_{\pm}|^2_{(r-R,\vartheta,\varphi)} \left(r - R_{\pm}\right)^2 d(r-R) \cos\vartheta d\vartheta d\varphi \tag{46}$$

$$\partial_r \phi_{\pm Q} = \frac{3}{2} \frac{\hbar^2}{mc^2} \frac{G}{\left(r - R_{\pm}\right)^2} \int_0^{(r-R)} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} |\psi_{\pm}|_{(r-R,\vartheta,\varphi)} \left(\partial_\mu \partial^\mu |\psi_{\pm}| + 2mc^2 |\psi_{\pm}| \Lambda_{Q\pm}\right) (r-R)^2 d(r-R) \cos\vartheta d\vartheta d\varphi \tag{47}$$

From (46) we can acknowledge the classical Newtonian gravity is the same for matter and antimatter bodies, while the quantum contribution (47) may differ by the values of Λ_{Q+} for matter and Λ_{Q-} for antimatter.

It is worth noting that the quantum contribution (47) becomes larger smaller the particle mass m leading to the asymptotic expression for infinitesimal mass

$$\lim_{m \to 0} \partial_r \phi_{\pm Q} \approx \frac{3}{2} \frac{\hbar^2}{mc^2} \frac{G}{\left(r - R_{\pm}\right)^2} \left(\int_0^{(r-R)} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} |\psi_{\pm}|_{(r-R,\vartheta,\varphi)} \partial_\mu \partial^\mu |\psi_{\pm}| (r-R)^2 d(r-R) \cos\vartheta d\vartheta d\varphi \right) \tag{48}$$

that diverges for $m \to 0$..

It is straightforward to note that the quantum contribution (47) to the gravitational field, arises from quantum potential energy (5), that, for weak Newtonian gravity, takes the form:

$$V_{qu(|\psi|)} \cong -\frac{\hbar^2}{m} \frac{1}{|\psi|} \partial^\mu \partial_\mu |\psi| \tag{49}$$

being characterized by the term $\frac{\hbar^2}{m}$ inversely proportional to the particle mass.

Since the gravitational term (47) diverges as the particle mass approaches zero, it is important to highlight that, in order to have finite gravitational energies, the mass of particles cannot decrease continuously to zero but must be quantized with minimum values as supported by the quantum field theory.

Up to now, we have derived the gravitational contribution of a spinless quantum-mechanical body that is sufficiently macroscopic to be described by continuous quantum wavefunction fields. In this approximation, the result neglects all contributions to Newtonian gravity at the sub-microscopic scale of elementary particles, which arise from the quantized nature of their fields.

If the quantization of gravity based on classical General Relativity encounters insurmountable difficulties in obtaining a renormalizable theory that eliminates divergences, then the quantization of quantum-mechanical gravity, introducing a new opposite diverging contribution, could lead to a renormalizable theory of quantum gravity. In this framework, field quantization would emerge as a necessary physical condition to ensure that gravity remains finite even for extremely small masses

Finally, since in general, when matter and antimatter are brought to a finite distance (and there is overlap between their wavefunctions) so that

$$|\psi_{m+} \pm \psi_{m-}|^2 \neq |\psi_{m+}|^2 + |\psi_{m-}|^2 \tag{50}$$

the gravity includes an additional contribution beyond (46, 47).

This contribution is similar to that in classical General Relativity, where the gravitational interaction can be defined as originating from a potential independent of the mass of neighboring bodies only to first order of approximation. If we consider the quantum nature of the bodies, the superposition of their quantum wavefunctions introduces a perturbative contribution not linearly depends on the presence and state of neighboring mass.

From this standpoint, it is worth noting that, once again, this could provide the basis for an experimental test by measuring the gravitational deviation from Newtonian law produced when matter atoms approach their corresponding antimatter counterparts.

4 Conclusion

By employing quantum-mechanical spacetime geometrization, which describes gravity in spacetime also accounting for the energy of the Madelung quantum potential energy, it is possible, in the weak gravity limit, to derive the Newtonian potential for massive bodies based on their quantum mechanical physics, including antimatter. This work demonstrates that, on macroscopic scales relevant to planetary and cosmological problems, the Newtonian gravity of antimatter is identical to that of matter.

Furthermore, the theory reveals additional weak gravity contributions arising directly from the quantum nature of bodies, which depend on the configuration of quantum mass density distributions. The contribution to gravity, stemming from the energy of the quantum potential, becomes significant as the mass of a body approaches zero. This suggests that if elementary particle masses would have a continuous spectrum down to zero, the gravitational potential energy would diverge.

This insight implies that field quantization, which leads to a discrete spectrum of elementary particle masses, may be a necessary condition for obtaining finite physical solutions. Additionally, the quantization of quantum-mechanical gravity, by introducing a possibly opposing diverging gravitational contribution, could lead to a renormalizable theory of quantum gravity. This aligns with the idea that the quantized nature of reality is a fundamental and necessary property of the universe in order to obtain its finite physical solution.

Author Contribution The author developed the theoretical formulation of the gravitational model, wrote the paper, and derived the discussion and conclusions of the work.

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Declarations

Conflicts of interest The authors declare no competing interests.

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