



The unpolarized TMD valence quark distributions of light mesons in the statistical covariant parton model approach

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Abstract We investigate the unpolarized transverse momentum dependent (TMD) structure of light mesons at low Q^2 scales in nonperturbative region of quantum chromodynamics (QCD). To this end, we calculate the unpolarized TMD valence quark distribution functions of pion and kaon based on the covariant parton model in which the flavour dependent covariant momentum distribution of quarks is needed. We apply the statistical model for this momentum distribution, for the first time, and also consider the quarks to be massive. The results of TMD valence distributions of light mesons show convenient general properties. We also obtain the integrated valence quark distributions of pion and kaon and compare our results with those of other models.

1 Introduction

Consistency of high-energy scattering processes data, for instance, Deep Inelastic Scattering (DIS) [1,2] and Drell-Yan processes (DY) [3,4] with theoretical researches, allows to explain hadron in terms of parton distribution functions (PDFs). In comparison with other known experiments, the best information on light quarks with no constraints on sea distributions, is obtained from charged lepton scattering off hadrons [5]. The PDFs of hadrons can be explored in two regions of quantum chromodynamics, perturbative and non-perturbative QCD [6–10]. Formulation of the covariant parton model (CPM), based on naive parton model, have provided a convenient framework for hadron PDFs [11–16]. There are different QCD inspired models in which PDFs are approximated using numerous parameters in polynomials [17,18] while some of them are not able to determine

flavour separation [19]. Furthermore, the efforts for offering a complete description of the polarized and unpolarized hadron structure led to introduction of a statistical way [20] which generated compatible PDFs. The other achievements obtained applying this way, established it as a successful approach [19,21–26]. The statistical approach can predict the spin and flavour features of hadronic structure which usually in global fitting are determined by adding some theoretical constraints [26].

A comprehensive picture of hadron is gained by transverse momentum dependent PDFs (TMDs) [27–32] which include polarization degree of freedom and parton transverse momentum. In order to involve these distributions, the statistical approach is extended to transverse degrees of freedom. The theoretical basis of TMDs in different twists [33–37], is provided by QCD factorization theorem from an even-chiral process measurements such as semi inclusive DIS (SIDIS) [38]. TMDs have been studied in several models like diquark spectator model [39,40], MIT bag model [41] and covariant parton model [11–16,42–46]. In spite of centralization TMD studies on simple baryons for example nucleon, interest in meson TMD distribution functions is increasing [47–49]. In particular, pion and kaon as light mesons have attracted interest in their simple valence content and the role which play as pseudoscalar bosons. Chiral quark model [50–53] and Nambu-Jona-Lasinio (NJL) model [54] are the examples of different theoretical models in which structure of mesons are inquired. The integrated parton distribution functions of light mesons have been studied in several researches which explore the nonperturbative QCD aspects at low energies [50–53,55–57]. Nevertheless, there are few works in which the TMD distribution functions of mesons have been investigated [47–49,54].

In this work, we calculate the unpolarized TMD valence quark distributions of light mesons based on the covariant parton model in nonperturbative region of QCD. In this model

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the quark covariant momentum distribution of these mesons is needed. We use the result of statistical model for this covariant momentum distribution. The organization of this paper is as follows: in Sect. 2 we explain the covariant parton model and obtain the unpolarized TMD quark distribution. The statistical model is reviewed in Sect. 3. The integrated and TMD valence quark distributions of light mesons are calculated based on the statistical covariant parton model in Sect. 4 and the obtained results for pion and kaon are given in Sects. 4.1 and 4.2, respectively. We give our conclusion in Sect. 5.

2 TMD distribution function and quark correlator

Three dimensional (3D) investigations have accoutered us to present a realistic expression for experimental observations. In CPM framework, in which there is no restriction on frames for the on-shell quarks, instead of the infinite momentum frame (IMF) that is a common reference framework, we choose the rest frame of hadron applying the light front language. As the first step for theoretical calculation, the exploration of correlation function in terms of light coordinates allows one to calculate the PDFs and TMDs in different twists. The quark correlator which is expressed by [36,46],

$$\Phi_{ij}^q(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip\xi} \langle N(P, S) | \bar{\psi}_j^q(0) \times \mathcal{W}(0, \xi; \text{path}) \psi_i^q(\xi) | N(P, S) \rangle, \quad (1)$$

includes $\psi^q(\xi)$ that demonstrates the quark and antiquark fields as the functions of spacetime coordinates. The color gauge invariance of the correlation function is ensured by the Wilson line, $\mathcal{W}(0, \xi; \text{path})$, whose path in the spacetime depends on the considered experimental process [33,34,36]. The unpolarized T-even TMD distribution function at LO accuracy is determined by the first term of the following relation [23,43,46,58],

$$\phi^{q[\gamma^+]} = \frac{1}{2} \text{Tr} [\phi^q(x, p_T, S) \gamma^+] = f^q - \frac{\varepsilon^{jk} p_T^j S_T^k}{M} f_{1T}^{\perp q}, \quad (2)$$

in which q denotes different quark flavours, while p , x and p_T are four-momentum, longitudinal fraction and transverse component of quark momentum, respectively. $\gamma^+ = \frac{\gamma^0 + \gamma^1}{\sqrt{2}}$ and $\phi^q(x, p_T, S)$, are the gamma vector matrix [46] and the quark-quark transverse momentum dependent correlation function. We need the given quark correlator in the CPM as [46],

$$\Phi^q(p, P, S)_{ij} = 2P^0 \Theta(p^0) \delta(p^2 - m^2) \times \bar{u}_j(p) u_i(p) \mathcal{G}^q(pP), \quad (3)$$

where i and j refer to the Dirac indices. Moreover, $\bar{u}_j(p)$ and $u_i(p)$ are the Dirac spinors and m is the quark mass. The

quark correlation function includes the theta and delta functions $\Theta(P^0) \delta(p^2 - m^2)$, which ensure on-shell condition of quarks, and $\mathcal{G}^q(pP)$ as a scalar function is the covariant momentum distribution of the unpolarized quarks in different flavours. The TMD correlation function is defined by,

$$\phi^q(x, p_T, S)_{ij} = \int dp^- \int dp^+ \Phi^q(p, P, S)_{ij} \times \delta(p^+ - x P^+), \quad (4)$$

in terms of light cone coordinates (M , S and P are the mass, spin and momentum of hadron) [46]. In the rest frame, $p = (p^0, p^1, p_T)$ and $P = (M, \mathbf{0})$, using the light cone coordinates definitions, the delta function becomes $\delta(x - \frac{p^0 + p^1}{M})$ and replacing Eq. (3) in Eq. (4) one can rewrite Eq. (2) as,

$$\phi^{q[\gamma^+]} = \int \frac{\sqrt{2} dp^1}{2p^0} \mathcal{G}^q(p^0) \text{Tr} [\bar{u}(p) \gamma^+ u(p)] \times \delta\left(x - \frac{p^0 + p^1}{M}\right). \quad (5)$$

By applying the properties of tracing gamma matrix, one can obtain,

$$\phi^{q[\gamma^+]}(x, p_T, S) = f^q(x, p_T) = \int \frac{dp^1}{p^0} \mathcal{G}^q(p^0) (p^0 + p^1) \times \delta\left(x - \frac{p^0 + p^1}{M}\right), \quad (6)$$

as the unpolarized TMD distribution function in the leading twist [23,43,44]. From $p^2 = m^2$, for the massive quarks, one can find [23],

$$\begin{aligned} \tilde{p}^1 &= \frac{Mx}{2} \left[1 - \frac{p_T^2 + m^2}{(Mx)^2} \right], \\ \tilde{p}^0 &= \frac{Mx}{2} \left[1 + \frac{p_T^2 + m^2}{(Mx)^2} \right], \end{aligned} \quad (7)$$

that are the roots of the function $x - \frac{p^0 + p^1}{M}$. Equation (7) can be used for writing another form of δ function as [23,44],

$$\delta\left(x - \frac{p^0 + p^1}{M}\right) = \frac{p^0 \delta(p^1 - \tilde{p}^1)}{x}. \quad (8)$$

Inserting Eq. (8) in Eq. (6) gives [23,44],

$$f^q(x, p_T) = M \mathcal{G}^q(\tilde{p}^0). \quad (9)$$

More than different Q^2 dependent parameters in the unpolarized TMD function, mass of quark (m) is appeared in \tilde{p}^0 . We have inquired the dependence of the unpolarized TMD distribution on the mass of valence quarks.

3 Statistical model

From Eq. (9) one can find the importance of covariant quark distribution in momentum space. This function in different models explains how the longitudinal and transversal degrees of freedom are distributed. In this paper we apply the model which is parametrized based on the idea that a hadron can be regarded as a partonic gas in a finite volume and this gaseous system is in a thermal equilibrium [5, 22, 25]. Due to the Pauli exclusion principle, individual treatment of valence and sea fermions can be expressed in terms of Fermi-Dirac distribution. In this regard we use the following unpolarized covariant distribution,

$$\mathcal{G}^q(p^0) = \frac{g_q V}{(2\pi)^3 \left[\exp\left(\frac{p^0 - \mu_q}{T}\right) + 1 \right]}, \quad (10)$$

that is introduced in the statistical model [22]. The physical parameters include: color-spin degeneracy $g_q = 6$ for three light quarks (u, d, s), volume (V) and temperature (T) which have the same value for all flavours. μ denotes chemical potential which satisfies the relation $\mu_{\bar{q}} = -\mu_q$ for quark and antiquark based on the chiral essence of QCD. For each flavour of quarks and antiquarks in hadrons, inserting the fermion distribution of Eq. (10) in Eq. (9), we will get the unpolarized TMD quark distribution:

$$f^q(x, p_T) = \frac{M g_q V}{(2\pi)^3 \left[\exp\left(\frac{\frac{Mx}{2} \left[1 + \frac{p_T^2 + m^2}{(Mx)^2} \right] - \mu_q}{T}\right) + 1 \right]}. \quad (11)$$

It should be pointed that the parameters in Eq. (11) such as volume and chemical potential are Q^2 dependent, also using Eq. (7), the explicit dependences of TMD quark distribution function on x and p_T are given. Now we are able to calculate the unpolarized TMD quark distribution functions of hadrons using Eq. (11).

4 Unpolarized integrated and TMD valence distributions of light mesons

In this section we focus on the light pseudoscalar mesons which play an essential role in the description of the structure of nucleons and nuclei and also in the realization of confinement within the standard model due to their responsibility for the dynamical chiral symmetry breaking. We study the unpolarized TMD structure of these light mesons, pion and kaon, at low Q^2 scales in nonperturbative region of QCD. In this region the contributions of sea quarks and gluon to the structure of mesons can be neglected.

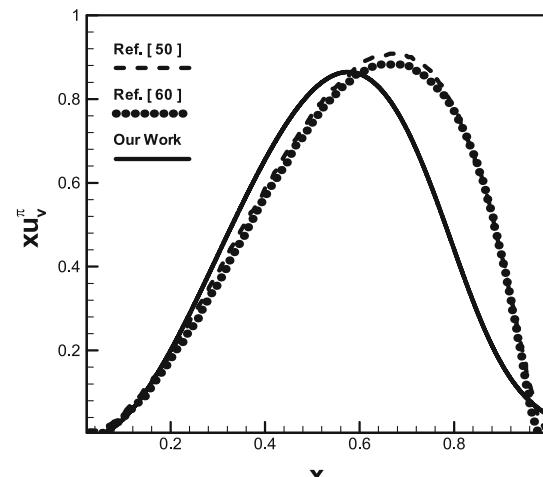


Fig. 1 The valence quark density of pion. The result of our work has been compared with those of Refs. [50, 60]

4.1 Pion

In comparison with other mesons, pion ($u\bar{d}$) has the smallest mass and plays a dominant role in mediating nuclear force [59]. Because of the small difference between u and \bar{d} masses, we assume that each of these two valence quarks has the same contribution in the structure of pion. We should note that we use the notation $f^q(x, p_T) \equiv q(x, p_T)$ in this section.

In the first step of our calculations, we obtain the up valence quark distribution of pion ($u_v^\pi(x)$) by integrating Eq. (11) over transverse momentum. Then by considering this fact that this valence distribution should satisfy the following sum rules,

$$\int_0^1 dx \ u_v^\pi(x) = \int_0^1 dx \ \bar{d}_v^\pi(x) = 1, \\ \int_0^1 dx \ x[u_v^\pi(x) + \bar{d}_v^\pi(x)] = 1, \quad (12)$$

we determine the numerical values of some parameters within Eq. (11). We take the pion mass $M^\pi = 140$ MeV and the effective mass of u quark in pion $m_u^\pi = 30$ MeV.

In Fig. 1 we have presented the result of our calculations for valence quark density of pion, $x u_v^\pi(x)$. We have compared our result with those of Refs. [50, 60] at $Q_0^2 = 0.15$ GeV 2 and $Q_0^2 = 0.11$ GeV 2 , respectively. It is found that the result of our work is in good agreement with those of these references.

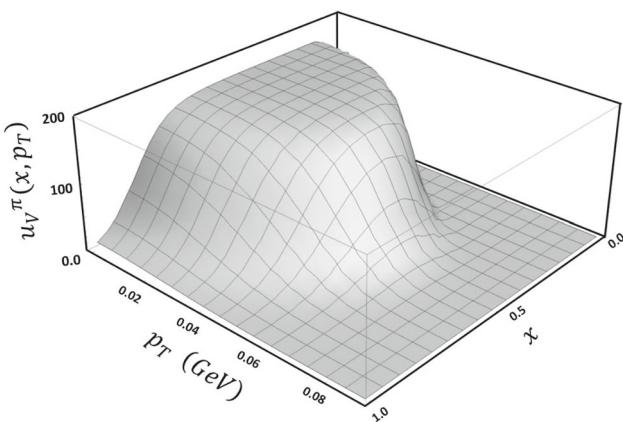
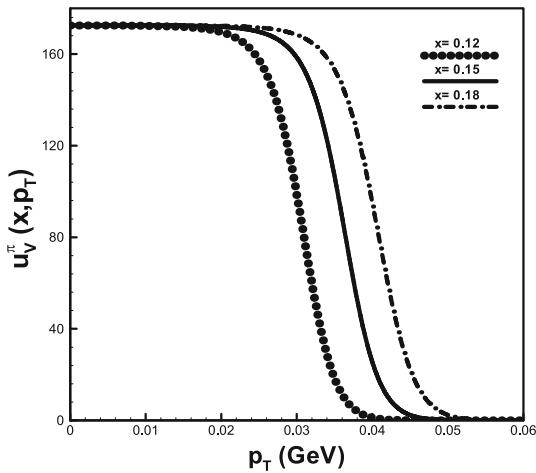
We have also calculated the Mellin moments of u_v^π based on the following relation,

$$\mathcal{M}_q^n = \int_0^1 dx \ x^n q(x), \quad (13)$$

for $n = 2, n = 3$ and $n = 4$. The results have been listed in Table 1.

Table 1 The n -th order moments of u_v^π .

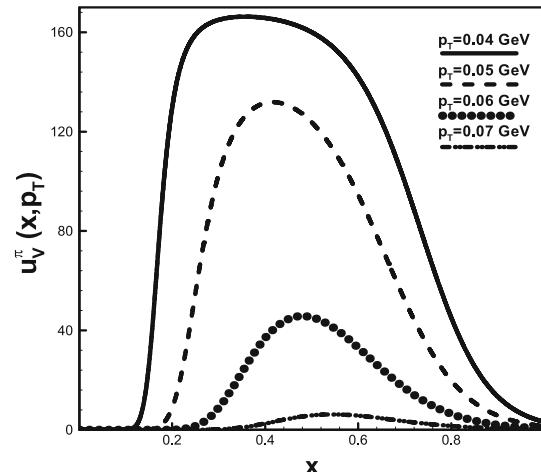
\mathcal{M}^n	$n = 2$	3	4
Our work	0.261	0.161	0.107

**Fig. 2** Three dimensional representation of $u_v^\pi(x, p_T)$ **Fig. 3** p_T -distribution of $u_v^\pi(x, p_T)$ at three values of x

In further step of our calculations, we have obtained the TMD valence quark distribution of pion based on the statistical covariant parton model (Eq. (11)). In Fig. 2, we have displayed the three dimensional representation of u_v^π with respect to x and p_T . Such 3D representation can provide us an appropriate insight into the general properties of TMD distribution function.

The unpolarized TMD valence distribution of pion has been plotted in Fig. 3 with respect to transverse momentum (p_T) at three fixed values of x ; $x = 0.12, 0.15$ and 0.18 . As can be found from this figure the width of p_T -distribution is dependent on x -value. This width increases by increasing the value of x .

Finally, we have depicted the x -distribution of u_v^π at four values of transverse momentum, $p_T = 0.04$ GeV,

**Fig. 4** TMD valence quark distribution of pion with respect to x at $p_T = 0.04$ GeV, 0.05 GeV, 0.06 GeV, and 0.07 GeV

0.05 GeV, 0.06 GeV and 0.07 GeV in Fig. 4. It is found that the probability of finding the valence quarks in pion decreases by increasing p_T value as expected.

4.2 Kaon

In this subsection we probe the TMD structure of kaon as a simple bound state of a valence up quark (u) and a valence strange antiquark (\bar{s}). Since kaon has a heavier strange quark inside, studying its structure is interesting for indicating the importance of mass especially in nonperturbative region of QCD [59].

In similar to what we have done for pion, we have first computed the integrated valence distributions of kaon, $u_v^K(x)$ and $\bar{s}_v^K(x)$, by integrating Eq. (11) over p_T at low Q_0^2 scale. Then by using the sum rules of this meson as,

$$\int_0^1 dx u_v^K(x) = \int_0^1 dx \bar{s}_v^K(x) = 1, \\ \int_0^1 dx x[u_v^K(x) + \bar{s}_v^K(x)] = 1, \quad (14)$$

we have determined the values of some needed parameters of Eq. (11). It should be noted that the valence quarks have the dominant role in the structure of kaon at low energy scales and we take the kaon mass $M^K = 494$ MeV and the effective masses of u quark and s antiquark in kaon $m_u^K = 151$ MeV and $m_{\bar{s}}^K = 188$ MeV.

We have plotted the results of valence densities of kaon in Fig. 5. In this figure we have compared our results by those of Refs. [50,52]. As can be seen in Fig. 5 the results of our work have the shifts to smaller values of x in comparison with those of Refs. [50,52]. We have also calculated the Mellin moments of u_v^K and \bar{s}_v^K and presented the results in Tables 2

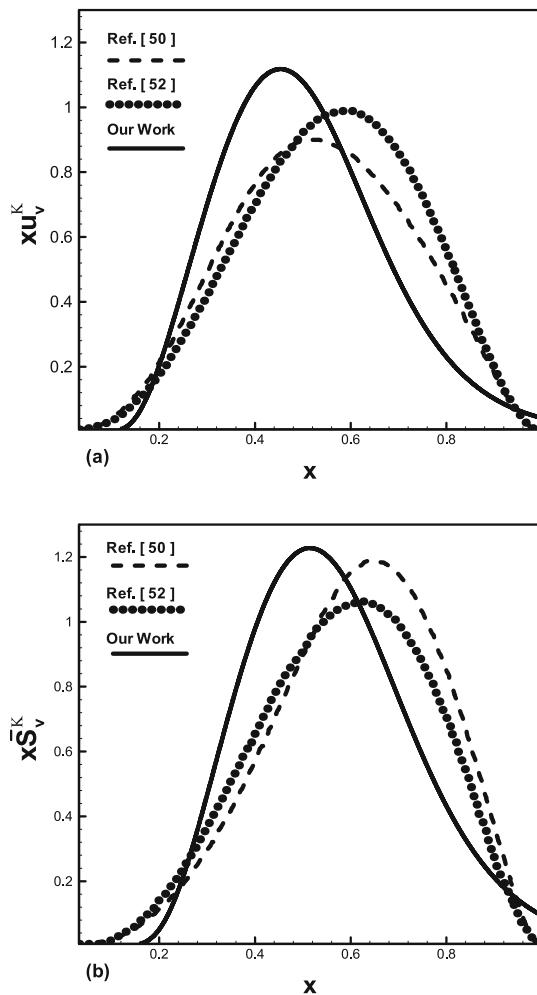


Fig. 5 The valence densities of kaon: **a** $xu_v^K(x)$ and **b** $x\bar{s}_v^K(x)$

Table 2 The n-th order moments of u_v^K .

\mathcal{M}^n	$n = 2$	3	4
Ref. [61]	0.269	0.173	0.120
Refs. [61, 62]	0.244	0.146	0.093
Our work	0.229	0.126	0.076

Table 3 The n-th order moments of \bar{s}_v^K .

\mathcal{M}^n	$n = 2$	3	4
Our work	0.289	0.175	0.114

and 3, respectively. Our results for the moments of u_v^K have been compared with those of Refs. [61, 62].

In further step, we have calculated the TMD valence distributions of kaon using Eq. (11) and given the 3D representations of these TMD distributions in Fig. 6.

The TMD distributions of u_v^K and \bar{s}_v^K have been plotted in Fig. 7 with respect to p_T at three values of x ; $x = 0.12$, 0.15

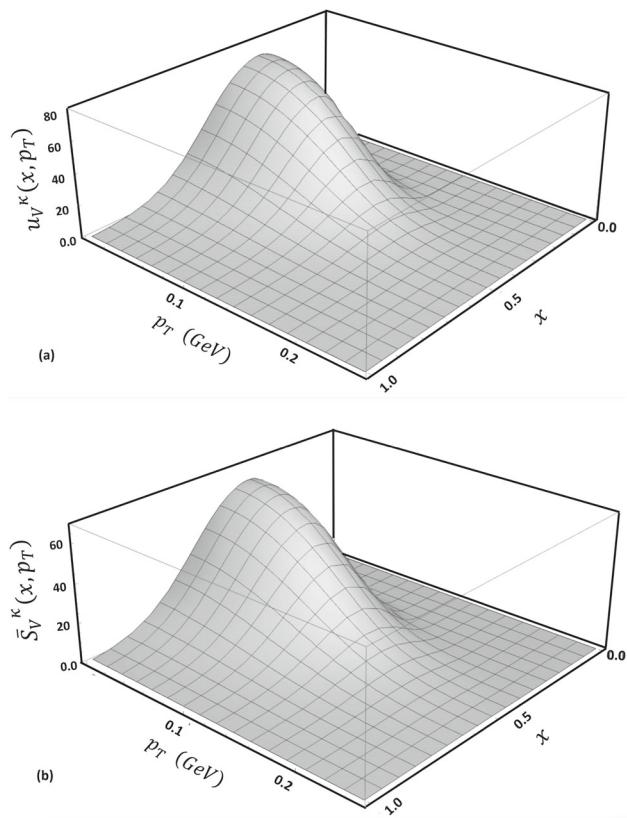


Fig. 6 Three dimensional representations of **a** $u_v^K(x, p_T)$ and **b** $\bar{s}_v^K(x, p_T)$

and 0.18. It is found that while the width of TMD distribution increases by increasing the x -value, the probability for finding valence quark and antiquark in kaon also increases. Furthermore, by comparing Figs. 2, 6 and also Figs. 3, 7, we can find the essential effect of meson mass on the results of TMD distributions (Eq. (11)).

Finally, we have displayed the x -distribution of $u_v^K(x, p_T)$ and $\bar{s}_v^K(x, p_T)$ at $p_T = 0.05 \text{ GeV}$, 0.1 GeV , 0.15 GeV and 0.2 GeV in Fig. 8. By comparing Figs. 4 and 8, one can see that the probability of finding valence quark and antiquark approach to zero within $p_T > 0.07 \text{ GeV}$ and $p_T > 0.2 \text{ GeV}$ for pion and kaon, respectively. This difference is due to the mass difference between these mesons.

5 Conclusion

Studying the structure of light mesons, pion and kaon, is of great interest particularly in nonperturbative QCD because of their role as Goldstone bosons on chiral symmetry breaking in this region. Probing the integrated PDFs of pion and kaon have been done in several works but the investigation of the transverse momentum dependence of their structure is poor yet. We have studied the unpolarized TMD valence quark

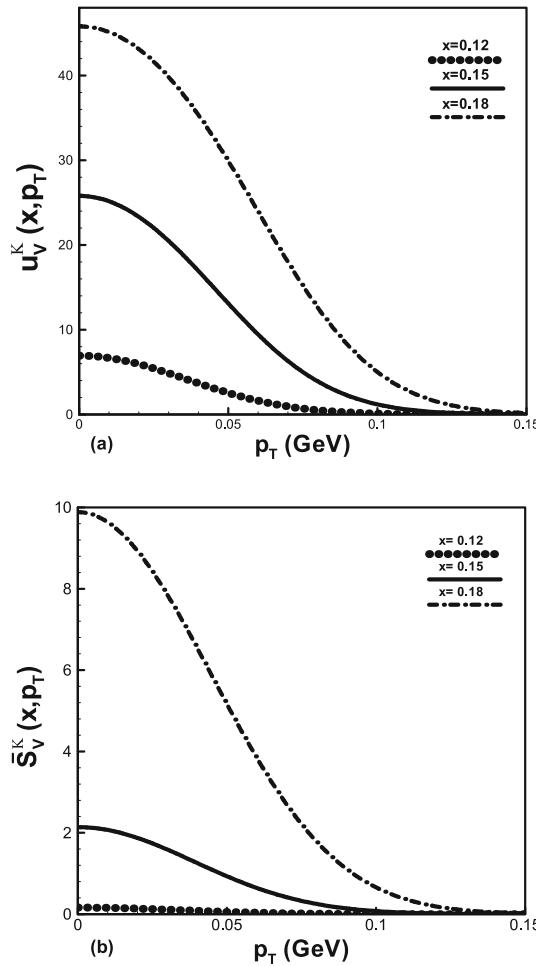


Fig. 7 The p_T -distribution of **a** $u_v^K(x, p_T)$ and **b** $\bar{s}_v^K(x, p_T)$ at $x = 0.12, 0.15$ and 0.18

distributions of pion and kaon at low Q^2 scales in noneperturbative QCD. We have linked the covariant parton model and the statistical model to calculate these TMD distributions, for the first time. In covariant parton model the quarks are often considered to be massless but we have regarded them massive in this article.

The unpolarized integrated and TMD valence distributions of pion and kaon have been obtained and the results have been given in previous section. The results of integrated PDFs are in good agreement with the results of Refs. [50, 52, 60] and those of TMD ones show appropriate general properties. The differences between some properties of TMD results for pion and kaon are due to the mass difference between these two mesons and it shows the important effect of meson mass (M) on the unpolarized TMD distribution function (please see Eq. (11)).

We hope to study the polarized TMDs of light mesons and report the results in future.

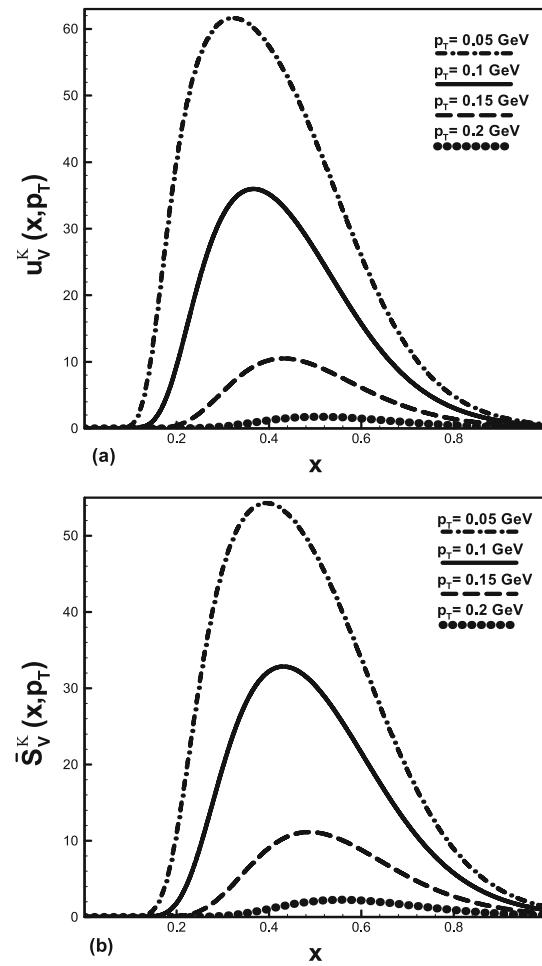


Fig. 8 The x -distribution of **a** $u_v^K(x, p_T)$ and **b** $\bar{s}_v^K(x, p_T)$ at $p_T = 0.05$ GeV, 0.1 GeV, 0.15 GeV and 0.2 GeV

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The needed data can be found in the text and references of this paper.]

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