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Effective Quark Operator Models of $U_A(1)$ Symmetry Breaking

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Abstract

We undertake a systematic investigation of $U_A(1)$ symmetry-breaking, C-, P-, T-, and $SU_L(N_f) \times SU_R(N_f)$-invariant effective fermion operators and their consequences for pseudoscalar and scalar mesons. We construct four types of such operators that exist for any number of flavours $N_f \geq 2$, two of which can be identified with 't Hooft's interaction and the quark self-interaction leading to the Veneziano-Witten meson-interaction term. We isolate the $U_A(1)$ symmetry-breaking effect from the quark mass- and electromagnetic interaction-induced chiral symmetry-breaking effects and quantify it as the deviation from zero of $f_0^2 m_{\mathrm{U}(1)}^2 = f_{\eta'}^2 m_{\eta'}^2 + f_\eta^2 m_\eta^2 - f_K^2 \left(m_{K^+}^2 + m_{K^0}^2\right) + f_\pi^2 \left(m_{\pi^+}^2 - m_{\pi^0}^2\right)$, where m_ϕ, f_ϕ are the pseudoscalar ϕ meson mass and weak decay decay constant, respectively. Then we use Dashen's general formula to evaluate the masses and the mixing angle of isoscalar pseudoscalar mesons in the presence of the current quark masses and each one of these four types of $U_A(1)$ symmetry-breaking interactions. We find that both the 't Hooft and the Veneziano-Witten interaction push the sum of the η' and η masses squared upward and the mixing angle to negative values, in accord with empirical evidence. The other two types of $U_A(1)$ symmetry-breaking operators do not influence the pseudoscalar meson spectrum to leading order in N_C, so long as no new higher-order quark condensates are assumed. In an attempt to determine which linear combination of the 't Hooft and the Veneziano-Witten operators is responsible for the observed $U_A(1)$ symmetry breaking, we calculate the scalar meson masses in the three-flavor NJL model in the presence of either of these two interactions. Presently available data do not allow a definitive answer to that question, though they can be interpreted as favouring the 't Hooft interaction.

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I. INTRODUCTION

The $U_A(1)$ problem [1,2] can be roughly stated as the lack of agreement between the left- and right-hand side in the $U_L(3) \times U_R(3)$ symmetry mass relation $m^2_{\eta'} + m^2_\eta = 2m^2_K$. Presently almost universally accepted solution postulates an explicit breaking of the $U_A(1)$ symmetry, believed to be induced by instantons in QCD, which raises the mass of the SU(3) flavour-singlet and thus provides for the difference $m^2_{\eta'} + m^2_\eta - 2m^2_K \simeq (855\mathrm{MeV})^2$.

There is one significant exception to this universal faith: T. D. Lee has suggested that a perturbative calculation based on the anomalous triangle graphs also solves the U(1) problem [3]. This is a perfectly sound suggestion, but one that runs into certain technical difficulties: (i) the diagram in question vanishes in the chiral limit (for implications of this fact, see below); (ii) the same diagram is logarithmically divergent, thus requiring an infinite renormalization with counterterms that are not a part of the QCD Lagrangian. These two obstacles lead to a loss of predictive power. Therefore we shall not concern ourselves with this option in the rest of this paper.

Although there have been many studies of instantons' influence on the mass spectrum, the mixing angle, and decay constants [4], possible CP-violation, etc. [5] in the pseudoscalar nonet based directly on the divergence of the new anomalous axial baryon number current that includes the "topological current" in QCD, there are, to this author's knowledge, no attempts in the literature at evaluating the $U_A(1)$ symmetry-breaking effects starting from the 't Hooft-Kobayashi-Kondo-Maskawa ("'t Hooft", for short) effective interaction [6,7], with the exception of several calculations of the η' mass in effective quark models [8–11]. Moreover, there are even fewer studies of an alternative ("Veneziano-Witten") effective mechanism of $U_A(1)$ symmetry-breaking [12–14], but without indication as to if and how this dilemma can be resolved. This absence of any systematic study of $U_A(1)$ symmetry-breaking operators and their physical consequences stands in sharp contrast to the detailed studies of the chiral $SU_L(3) \times SU_R(3)$ symmetry-breaking mechanisms conducted in the late sixties and early seventies. Several symmetry-breaking models, going under the names of $(3,\bar{3}) \oplus (\bar{3},3)$, or the Gell-Mann, Oakes and Renner (GMOR) model [15], the $(8,\bar{1}) \oplus (\bar{8},1)$, $(6,\bar{6}) \oplus (\bar{6},6)$ models, etc. have been examined and the GMOR model, equivalent in this regard to QCD, was found to best fit the data [16–18].

In this paper we report our first steps in the direction of a systematic analysis of $U_A(1)$ symmetry-breaking effects among mesons. For pseudoscalar (ps) mesons this analysis is based on Dashen's current-algebraic formula relating the (would be) Goldstone meson mass to the vacuum sigma term in the theory [16]. The latter is just the negative vacuum expectation value of the double commutator of the relevant axial charge and the chiral symmetry-breaking Hamiltonian density. We show that the η, η' masses obtained in this way from the 't Hooft i nteraction coincide with those found in the Nambu–Jona-Lasinio (NJL) model calculations [10]. It so happens, however, that another independent C-,P-, and T-conserving $U_A(1)$ symmetry-breaking operator exists, which also raises the sum of η, η' masses squared and leads to the same negative mixing angle. Some indications have been given by Alkofer, Nowak, Verbaarschot and Zahed, the first paper in Ref. [14], how such an operator might arise from the instanton liquid approximation to QCD. One would like to know exactly to what extent is either of these two interactions responsible for the observed $U_A(1)$ symmetry breaking in Nature. Manifestly, no study of the pseudoscalar η, η' mesons' masses alone can resolve that issue. We offer a new test discriminating between the two interactions in effective chiral quark models. Differences arise in the *scalar* mesons spectra between models with the 't Hooft- and the Alkofer-Nowak-Verbaarschot-Zahed-Veneziano-Witten (ANVZVW) $U_A(1)$ symmetry-breaking interaction: As shown in Refs. [10,11], the former interaction leads to a mass shift within the scalar nonet that is identical in size, but opposite in sign to that found in pseudoscalars, whereas the latter does not shift the scalar meson masses at all, as we shall show below. One can find flavor-singlet scalar states in the Particle Data Group tables that fit either model, though some of the states' properties are not presently known. On the basis of this limited evidence one could argue that there is some preference for the 't Hooft model.

II. $U_A(1)$ SYMMETRY BREAKING EFFECTIVE OPERATORS

A. Classification of $U_A(1)$ symmetry-breaking operators

There are at least four C-, P-, T-, and $SU_L(N_f) \times SU_R(N_f)$ invariant, $U_A(1)$ symmetry-breaking effective fermion (quark) interactions for any $N_f \geq 2$,

$$\mathcal{L}_{\rm tH}^{(2N_f)} = \left(\frac{\kappa_{N_f}}{2^{N_f}}\right) \left[\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right] \tag{1a}$$

$$\mathcal{L}_{\rm VW}^{(4N_f)} = \left(\frac{\kappa'_{N_f}}{2^{2N_f}}\right) \left[\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) - \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right]^2 \tag{1b}$$

$$\mathcal{L}_{\rm t1}^{(2N_f)} = \left(\frac{\mu_{N_f}}{2^{N_f}}\right) \left[\det{}_f\left(\bar{\psi}\sigma_{\mu\nu}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}\sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right] \tag{1c}$$

$$\mathcal{L}_{\rm t2}^{(4N_f)} = \left(\frac{\mu'_{N_f}}{2^{2N_f}}\right) \left[\det{}_f\left(\bar{\psi}\sigma_{\mu\nu}(1+\gamma_5)\psi\right) - \det{}_f\left(\bar{\psi}\sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right]^2 \quad , \tag{1d}$$

where, for example, $\det_f\left(\bar{\psi}\sigma_{\mu\nu}(1\pm\gamma_5)\psi\right)$, with $N_f = 3$, stands for

$$\det{}_f\left(\bar{\psi}\sigma_{\mu\nu}(1\pm\gamma_5)\psi\right) = \begin{vmatrix} \bar{u}\sigma_\mu{}^\nu(1\pm\gamma_5)u & \bar{u}\sigma_\mu{}^\nu(1\pm\gamma_5)d & \bar{u}\sigma_\mu{}^\nu(1\pm\gamma_5)s \\ \bar{d}\sigma_\nu{}^\alpha(1\pm\gamma_5)u & \bar{d}\sigma_\nu{}^\alpha(1\pm\gamma_5)d & \bar{d}\sigma_\nu{}^\alpha(1\pm\gamma_5)s \\ \bar{s}\sigma_\alpha{}^\mu(1\pm\gamma_5)u & \bar{s}\sigma_\alpha{}^\mu(1\pm\gamma_5)d & \bar{s}\sigma_\alpha{}^\mu(1\pm\gamma_5)s \end{vmatrix} \quad , \tag{2}$$

and similarly for flavour determinants of the other two matrices, where there are no Lorentz indices to be contracted. Here $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, and we use Ref. [19] conventions for the space-time metric and Dirac matrices.

The first two of Eqs. (1a),(1b) are the 't Hooft- and the Veneziano-Witten effective interactions, respectively, the third (1c) and the fourth one (1d) have not been discussed in the literature heretofore, to our best knowledge. That leaves their derivation from QCD as an open question, except in the special case $N_f = 2$ when they are related to the 't Hooft and Veneziano-Witten interactions by a Fierz tranformation. One must emphasize, however, that there is a relation ("Burgoyne identity" [10]) between Eqs. (1a) and (1b) of the form

$$\left[\mathcal{L}_{\rm tH}^{(2N_f)}\right]^2 - \left(\frac{\kappa_{N_f}^2}{\kappa'_{N_f}}\right) \mathcal{L}_{\rm VW}^{(4N_f)} = U_L(N_f) \times U_R(N_f)\text{−invariant operator} \ , \tag{3}$$

where the exact form of the operator on the right-hand side of Eq. (3) depends on the value of N_f. An analogous relation between the "tensor" operators in Eqs. (1c) and (1d) holds, as well. Moreover, for $N_f \geq 4$ there are new tensor interactions whose mathematical properties have not been explored or classified, as yet. We shall treat the two interactions in each pair [(1a), (1b)], [(1c), (1d)] as independent since the square root of an operator is ill-defined and 't Hooft actually derived the *first* power of $\mathcal{L}_{\rm tH}^{(2N_f)}$ from QCD [6].

Our normalization of the coupling constant κ_{N_f} was chosen so as to facilitate comparison with earlier papers on the subject, in particular with Ref. [20] where contact with the instanton calculus results was established, see Eq. (7). The remaining constants $\kappa'_{N_f}, \mu_{N_f}, \mu'_{N_f}$ are normalized analogously. Since these coupling constants have dimension(s) of a mass to negative integer powers it would seem natural to introduce a single energy scale M such that $\left[\kappa_{N_f}\right] = \left[\mu_{N_f}\right] = M^{(4-3N_f)}$, $\left[\kappa'_{N_f}\right] = \left[\mu'_{N_f}\right] = M^{(4-6N_f)}$, where $[a] \equiv \dim a$. We shall show in this paper that the scale M is just the cube root of the negative quark condensate, i.e., $M^3 = -\langle\bar{q}q\rangle_0$, at least for $N_f = 3$ 't Hooft and Veneziano-Witten interactions. In such a case a more natural normalization of the coupling constants would omit the additional powers of 2^{N_f} from the definitions (1a-d). Yet, that would not ensure such "renormalized" dimensionless couplings' being of $\mathcal{O}(1)$ for higher values of N_f, because the determinant structure of the interaction may yet change the overall coefficient with changing N_f. Perhaps less importantly, the odd-N_f 't Hooft interaction coupling constants, as defined above, are negative - that could be changed as well. At any rate, it seems too early to pronounce general naturalness criteria for these interactions at this point.

All of the aforementioned interactions are current quark mass independent, i.e., they do not change in the chiral limit. One can construct whole new families of $U_A(1)$ symmetry-breaking operators that vanish in the chiral limit, i.e., which break both $SU_L(N_f) \times SU_R(N_f)$ and $U_A(1)$ symmetries. For a derivation of a set of such interactions from QCD, see Ref. [21]; for a phenomenological application see Ref. [9]. We shall not investigate such operators in this paper since their effect seems equivalent, at least in second-order perturbation theory, to the combined action of the above quoted $SU_L(N_f) \times SU_R(N_f)$ chiral invariants and the current quark masses in the free fermion Lagrangian. It is important, however, not to forget that this assumption has been made, and explore the consequences of its relaxation at some later time.

B. Pseudoscalar meson mass shift due to $U_A(1)$ symmetry breaking

In this subsection we apply model-independent methods to the evaluation of pseudoscalar mesons' mass matrix. Here the only, albeit crucial, assumption is that of spontaneously broken chiral symmetry. This analysis is based on a single model-independent current-algebraic formula derived from a chiral Ward identity and due to Dashen [16], (for a straightforward derivation of this formula see e.g. [22])

$$\left(fm^2f\right)_{ab} = f_a m_{ab}^2 f_b = -\langle 0 \left| [Q_a^5, [Q_b^5, \mathcal{H}_{\chi SB}(0)]] \right| 0\rangle \ . \tag{4}$$

Here a, b are the flavour indices of the axial charges corresponding to the appropriate ps meson(s). Formula (4) describes the lowest order correction to the (vanishing) pseudoscalar meson mass squared as a consequence of chiral symmetry-breaking terms $\mathcal{H}_{\chi SB}(0)$ in the Hamiltonian density.

1. The 't Hooft interaction

A straightforward calculation using the identity

$$\left[Q_a^5, \det_f\left(\bar{\psi}(1\pm\gamma_5)\psi\right)\right] = \mp\sqrt{2N_f}\,\delta_{a0}\,\det_f\left(\bar{\psi}(1\pm\gamma_5)\psi\right) \tag{5}$$

leads to the following mass shift of the flavour-singlet meson

$$\begin{aligned} m_{00}^2(\mathrm{tH})f_0^2 &= \langle 0\left|[Q_0^5,[Q_0^5,\mathcal{L}_{\mathrm{tH}}^{(2N_f)}(0)]]\right|0\rangle \\ &= 2N_f\left(\frac{\kappa_{N_f}}{2^{N_f}}\right)\langle 0\left|\det_f\left(\bar{\psi}(1+\gamma_5)\psi\right)+\det_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right|0\rangle \\ &= 2N_f\langle 0\left|\mathcal{L}_{\mathrm{tH}}^{(2N_f)}(0)\right|0\rangle_0\ , \end{aligned} \tag{6}$$

where $\kappa_3 = -8K$ in the $N_f = 3$ case, see Ref. [10], and an integral over the instanton density $D(\rho)(>0)$ (for three colors) as follows

$$\kappa_{N_f} = \int\frac{d\rho}{\rho^5}D(\rho)\left[\frac{-4\pi^2}{N_C}\rho^3\exp\left(2\alpha\left(\frac{1}{2}\right)\right)\right]^{N_f}\ , \tag{7}$$

where ρ is the instanton size, and $\alpha\left(\frac{1}{2}\right) = 0.1458$ in the dilute instanton gas approximation, see [20]. The right-hand side (r.h.s) of Eq. (6) can be evaluated using the identity

$$\begin{aligned} \det_f\left(\bar{\psi}(1+\gamma_5)\psi\right)+\det_f\left(\bar{\psi}(1-\gamma_5)\psi\right) &= \frac{1}{6}\Big\{D_{ijk}(\bar{\psi}\boldsymbol{\lambda}_k\psi) \\ &\times\left[(\bar{\psi}\boldsymbol{\lambda}_i\psi)(\bar{\psi}\boldsymbol{\lambda}_j\psi)-3(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_j\psi)\right] \\ &+\frac{3}{2}\sqrt{6}(\bar{\psi}\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}\left[(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)^2-(\bar{\psi}\boldsymbol{\lambda}_i\psi)^2\right] \\ &+3\sqrt{6}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)(\bar{\psi}\boldsymbol{\lambda}_i\psi)\Big\}, \end{aligned} \tag{8}$$

where the summation from 0 to 8 over repeated indices is implied and D_{ijk} are the symmetric Gell-Mann SU(3) structure constants defined by

$$\{\boldsymbol{\lambda}_i,\boldsymbol{\lambda}_j\} = 2D_{ijk}\boldsymbol{\lambda}_k\ , \tag{9}$$

and extended [1] to U(3) *i.e.* the ninth generator $\boldsymbol{\lambda}_0 = \sqrt{\frac{2}{3}}\mathit{1}$ is included,

$$D_{ijk} = \begin{cases} d_{ijk}, & i,j,k\in(1,2,3,\dots,8) \\ \sqrt{\frac{2}{3}}\delta_{jk}, & i=0,\ j,\ k\in(0,1,2,\dots,8)\ , \end{cases} \tag{10}$$

as

---

[1]Our definition of D_{ijk} Eq. (10) agrees with that of d_{ijk} in Eq. (12.a.4) of B.W. Lee [22].

$$m_{00}^2(\mathrm{tH})f_0^2 = 6\langle 0\left|\mathcal{L}_{\mathrm{tH}}^{(6)}(0)\right|0\rangle$$
$$= -12K\langle 0\left|(\bar{q}q)^3\right|0\rangle + O(1/N_C)$$
$$= -12K\langle\bar{q}q\rangle_0^3 + O(1/N_C)\ , \qquad (11)$$

where we assumed that the vacuum expectation value (v.e.v.) of the operator product is saturated by the product of the individual operator v.e.v.'s, and good parity and SU(3) symmetry of the vacuum, i.e., $\langle\bar{\psi}\boldsymbol{\lambda}_3\psi\rangle_0 = \langle\bar{\psi}\boldsymbol{\lambda}_8\psi\rangle_0 = \langle\bar{\psi}\boldsymbol{\lambda}_i i\gamma_5\psi\rangle_0 = 0$, for all $i = 1,...8$. Various formal and explicit arguments about the size of corrections to the vacuum saturation hypothesis have been put forward; it is fair to say that all we know for sure is their order of magnitude as compared with the vacuum saturation contribution: they are suppressed by a factor $1/N_C$, where $N_C = 3$ is the number of colors. The symbol $O(1/N_C)$ on the right-hand side of Eq. (11) serves to remind us that we have neglected all $1/N_C$ suppressed terms, not just the corrections to the vacuum saturation hypothesis. As an example of the non-vacuum-saturation $1/N_C$ corrections may serve the set of chirally invariant $1/N_C$ corrections to the $N_f = 2$ NJL model that was calculated in Ref. [23].

Equation (11) implies an upward mass shift of the flavour-singlet ps meson, as long as the (negative) quark condensate does not vanish $\langle\bar{\psi}\boldsymbol{\lambda}_0\psi\rangle_0 = \sqrt{\frac{2}{3}}\langle\bar{\psi}\psi\rangle_0 = \sqrt{6}\langle\bar{q}q\rangle_0 \neq 0$ and the coupling constant K is positive. As we shall show in Sec. III, the negative ps mixing angle is explained by this feature, as well. This result tells us something about the proposed "natural" mass scale M governing κ_3 via $\kappa_{N_f} \simeq M^{(4-3N_f)}$, as well. We see that there is not one, but three dimensional quantities in the new "definition" of κ_3 Eq. (11). This prevents one from a positive identification of M before further analysis reveals a connection between the left-hand side (l.h.s.) of Eq. (11) and observables.. Last, but not least, the equivalent of Eq. (11) has been derived in an explicit chiral quark model calculation employing 't Hooft's interaction [10], as we shall show in Sect. IV.A.

2. The Veneziano-Witten interaction

In the ANVZVW model we find

$$m_{00}^2(\mathrm{VW})f_0^2 = 4N_f\left(\frac{\kappa'_{N_f}}{2^{2N_f}}\right)\langle 0\left|\left(\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) - \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right)^2\right|0\rangle$$
$$+ 4N_f\left(\frac{\kappa'_{N_f}}{2^{2N_f}}\right)\langle 0\left|\left(\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right)^2\right|0\rangle\ , \qquad (12)$$

which, for $N_f = 3$, turns into

$$m_{00}^2(\mathrm{VW})f_0^2 = 12\langle 0\left|\mathcal{L}_{\mathrm{VW}}^{(12)}\right|0\rangle$$
$$+ 12\left(\frac{\kappa'_3}{2^6}\right)\langle 0\left|\left(\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right)^2\right|0\rangle\ , \qquad (13)$$

Now use the identity

$$
\begin{aligned}
\det_f\left(\bar\psi(1+\gamma_5)\psi\right) - \det_f\left(\bar\psi(1-\gamma_5)\psi\right) &= \frac{i}{6}\Big\{ D_{ijk}(\bar\psi i\gamma_5 \boldsymbol{\lambda}_k\psi) \\
&\times \left[3(\bar\psi\boldsymbol{\lambda}_i\psi)(\bar\psi\boldsymbol{\lambda}_j\psi) - (\bar\psi i\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi i\gamma_5\boldsymbol{\lambda}_j\psi)\right] \\
&+ \frac{3}{2}\sqrt{6}(\bar\psi i\gamma_5\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}\left[(\bar\psi i\gamma_5\boldsymbol{\lambda}_i\psi)^2 - (\bar\psi\boldsymbol{\lambda}_i\psi)^2\right] \\
&- 3\sqrt{6}(\bar\psi\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}(\bar\psi i\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi\boldsymbol{\lambda}_i\psi)\Big\}, \qquad (14)
\end{aligned}
$$

to show that

$$
\begin{aligned}
\langle 0\left|\mathcal{L}_{\rm VW}^{(12)}\right|0\rangle &= \left(\frac{\kappa'_3}{2^6}\right)\langle 0\left|\left[\det_f\left(\bar\psi(1+\gamma_5)\psi\right) - \det_f\left(\bar\psi(1-\gamma_5)\psi\right)\right]^2\right|0\rangle \\
&= \left(\frac{\kappa'_3}{2^6}\right)\langle 0\left|\det_f\left(\bar\psi(1+\gamma_5)\psi\right) - \det_f\left(\bar\psi(1-\gamma_5)\psi\right)\right|0\rangle^2 + O(1/N_C) \\
&= 0 + O(1/N_C)\ , \qquad (15)
\end{aligned}
$$

where we used the vacuum saturation hypothesis once again and the same comments about $1/N_C$ corrections hold as for the 't Hooft interaction result. Now use this result and Eq. (8) to derive

$$
\begin{aligned}
m_{00}^2({\rm VW})f_0^2 &= 3\left(\frac{\kappa'_3}{2^4}\right)\langle 0\left|\frac{1}{6}D_{ijk}(\bar\psi\boldsymbol{\lambda}_k\psi)(\bar\psi\boldsymbol{\lambda}_i\psi)(\bar\psi\boldsymbol{\lambda}_j\psi)\right|0\rangle^2 + O(1/N_C) \\
&= \frac{3}{4}\kappa'_3\langle\bar q q\rangle_0^6 + O(1/N_C)\ . \qquad (16)
\end{aligned}
$$

We see that, once again, the flavour-singlet pseudoscalar mass has been moved up, subject only to the now standard assumption that $\langle\bar q q\rangle_0 \neq 0$. So long as the unknown coupling constant κ' is sufficiently large, the U(1) problem will be solved in this model. In Sec. IV we shall show that in the NJL chiral quark model of Ref. [10], but with the Veneziano-Witten interaction replacing 't Hooft's one, the same result for the flavour-singlet mass, Eq. (16), is obtained. Once again the proposed "natural" mass scale M governing κ'_3 via $\kappa'_{N_f} \simeq M^{(4-6N_f)}$ turns out to be related to three different quantities. And once again we shall relegate the resolution of this question to Sect. III.C.

3. The "tensor" interactions

Due to the identities

$$
\left[Q_a^5, \det_f\left(\bar\psi\sigma_{\mu\nu}(1\pm\gamma_5)\psi\right)\right] = \mp\sqrt{2N_f}\ \delta_{a0}\ \det_f\left(\bar\psi\sigma_{\mu\nu}(1\pm\gamma_5)\psi\right) \qquad (17)
$$

the double commutators of the determinants of the "tensor" left- and right-hand chirality matrices are formally identical to those of determinants of the "scalar" left- and right-hand matrices (6),(12). For $N_f = 3$ this turns into

$$
\begin{aligned}
m_{00}^2(\mathrm{T1})f_0^2 &= \langle 0\left|[Q_0^5,[Q_0^5,\mathcal{L}_{t1}^{(2N_f)}(0)]]\right|0\rangle \\
&= 6\left(\frac{\mu_3}{2^2}\right)\langle 0\left|\left(\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1+\gamma_5)\psi\right)+\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right)\right|0\rangle \\
&= 2N_f\langle 0\left|\mathcal{L}_{t1}^{(2N_f)}(0)\right|0\rangle \ . \qquad (18)
\end{aligned}
$$

Now use the identity

$$
\begin{aligned}
\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1+\gamma_5)\psi\right)+\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1-\gamma_5)\psi\right) &= \frac{1}{6}\Big\{D_{ijk}(\bar\psi\sigma_\mu{}^\nu\boldsymbol{\lambda}_k\psi)\Big[(\bar\psi\sigma_\nu{}^\alpha\boldsymbol{\lambda}_i\psi)(\bar\psi\sigma_\alpha{}^\mu\boldsymbol{\lambda}_j\psi) \\
&\quad -3(\bar\psi i\sigma_\nu{}^\alpha\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi i\sigma_\alpha{}^\mu\gamma_5\boldsymbol{\lambda}_j\psi)\Big] \\
&\quad +\frac{3}{2}\sqrt{6}(\bar\psi\sigma_\mu{}^\nu\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}\Big[(\bar\psi i\sigma_\nu{}^\alpha\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi i\sigma_\alpha{}^\mu\gamma_5\boldsymbol{\lambda}_i\psi) \\
&\quad -(\bar\psi\sigma^\alpha{}_\nu\boldsymbol{\lambda}_i\psi)(\bar\psi\sigma_\alpha{}^\mu\boldsymbol{\lambda}_i\psi)\Big] \\
&\quad +3\sqrt{6}(\bar\psi i\sigma_\mu{}^\nu\gamma_5\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}(\bar\psi i\sigma_\nu{}^\alpha\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi\sigma_\alpha{}^\mu\boldsymbol{\lambda}_i\psi)\Big\}, \qquad (19)
\end{aligned}
$$

to evaluate the right-hand side in Eq. (18). It is clear that we cannot use the vacuum saturation hypothesis here since Lorentz invariance demands that $\langle 0\left|(\bar\psi\sigma_\mu{}^\nu\lambda^a\psi)\right|0\rangle = 0$, for arbitrary flavour matrix λ^a. Heretofore no one has considered condensates such as $\langle 0\left|(\bar\psi\sigma_\mu{}^\nu\psi)(\bar\psi\sigma_\nu{}^\alpha\psi)(\bar\psi\sigma_\alpha{}^\mu\psi)\right|0\rangle$ in print; moreover, such condensates certainly do not exist in the leading order in $1/N_C$ approximate solution to the chiral quark model employed in Sec. IV of this paper. Hence we do *not* expect a shift of the flavour-singlet mass to leading order in $1/N_C$, i.e.,

$$
m_{00}^2(\mathrm{T1})f_0^2 = 0 + O(1/N_C)\ , \qquad (20)
$$

in all models with this kind of $U_A(1)$ symmetry-breaking interaction. The same holds for

$$
\begin{aligned}
m_{00}^2(\mathrm{T2})f_0^2 &= \langle 0\left|[Q_0^5,[Q_0^5,\mathcal{L}_{t2}^{(12)}(0)]]\right|0\rangle \\
&= 12\left(\frac{\mu'_3}{2^2}\right)\langle 0\left|\left(\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1+\gamma_5)\psi\right)-\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right)^2\right|0\rangle \\
&\quad + 12\left(\frac{\mu'_3}{2^6}\right)\langle 0\left|\left(\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1+\gamma_5)\psi\right)+\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right)^2\right|0\rangle \\
&= 0 + O(1/N_C)\ , \qquad (21)
\end{aligned}
$$

which follows from the tensor analog of Eq. (14)

$$
\begin{aligned}
\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1+\gamma_5)\psi\right)-\det{}_f\left(\bar\psi\sigma_{\mu\nu}(1-\gamma_5)\psi\right) &= \frac{i}{6}\Big\{D_{ijk}(\bar\psi i\sigma_\mu{}^\nu\gamma_5\boldsymbol{\lambda}_k\psi)\Big[3(\bar\psi\sigma_\nu{}^\alpha\boldsymbol{\lambda}_i\psi) \\
&\quad \times(\bar\psi\sigma_\alpha{}^\mu\boldsymbol{\lambda}_j\psi)-(\bar\psi i\sigma_\nu{}^\alpha\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi i\sigma_\alpha{}^\mu\gamma_5\boldsymbol{\lambda}_j\psi)\Big] \\
&\quad +\frac{3}{2}\sqrt{6}(\bar\psi i\sigma_\mu{}^\nu\gamma_5\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}\Big[(\bar\psi i\sigma_\nu{}^\alpha\gamma_5\boldsymbol{\lambda}_i\psi) \\
&\quad \times(\bar\psi i\sigma_\alpha{}^\mu\gamma_5\boldsymbol{\lambda}_i\psi)-(\bar\psi\sigma_\nu{}^\alpha\boldsymbol{\lambda}_i\psi)(\bar\psi\sigma_\alpha{}^\mu\boldsymbol{\lambda}_i\psi)\Big] \\
&\quad -3\sqrt{6}(\bar\psi\sigma_\mu{}^\nu\boldsymbol{\lambda}_0\psi)\sum_{i=1}^{8}(\bar\psi i\sigma_\nu{}^\alpha\gamma_5\boldsymbol{\lambda}_i\psi)(\bar\psi\sigma_\alpha{}^\mu\boldsymbol{\lambda}_i\psi)\Big\}. \qquad (22)
\end{aligned}
$$

In the special case of two flavors, one can define the Fierz transformation of the quartic tensor self-interaction, which is $1/N_C$ suppressed. That "new" interaction is nothing but the 't Hooft interaction for $N_f = 2$. It is not clear how to extend the Fierz transformation to $2N_f$-point fermion self-interactions when $N_f \geq 3$.

III. PSEUDOSCALAR MESON MASSES AND MIXING ANGLE

A. Preliminaries

In this section we incorporate our results form Sect. II into the Gell-Mann-Oakes-Renner (GMOR) relations [15] for the flavour-singlet ps meson and the off-diagonal elements of the mass matrix. That allows us to express the unknown coupling constant of the $U_A(1)$ symmetry-breaking effective interaction in terms of observables, which, in turn, leads to a formula for the ps mixing angle $\theta_{\rm ps}$ expressed as a function of well-known masses of the ps meson and of their less well-known weak decay constants. We discuss the role of the uncertainties in our knowledge of ps decay constants in the determination of the ps mixing angle.

Formula (4) describes the lowest order correction to the otherwise vanishing pseudoscalar meson mass squared as a consequence of chiral symmetry-breaking terms $\mathcal{H}_{\chi SB}(0)$ in the Hamiltonian density. There are three known sources of chiral $U_L(3) \times U_R(3)$ symmetry breaking in QCD: (i) current quark masses, (ii) electro-weak interactions, (iii) $U_A(1)$ symmetry-breaking effective interaction. The first two have been dealt with long ago [16], and the third was the subject of our Sect. II. When one inserts the current quark mass Hamiltonian into Eq. (4), one finds the celebrated GMOR relations

$$m_\pi^2(\text{mech}) f_\pi^2 = -\left[m_u^0 \langle \bar{u}u \rangle_0 + m_d^0 \langle \bar{d}d \rangle_0\right] \tag{23a}$$

$$m_{K^\pm}^2(\text{mech}) f_K^2 = -\left[m_u^0 \langle \bar{u}u \rangle_0 + m_s^0 \langle \bar{s}s \rangle_0\right] \tag{23b}$$

$$m_{K^0}^2(\text{mech}) f_K^2 = -\left[m_d^0 \langle \bar{d}d \rangle_0 + m_s^0 \langle \bar{s}s \rangle_0\right] \tag{23c}$$

between the "mechanical" pseudoscalar mass $m_\phi(\text{mech})$ and the decay constant f_ϕ on one hand and the current quark mass m_q^0 and the respective quark condensate $\langle \bar{q}q \rangle_0$ on the other. Here $\langle \bar{q}q \rangle_0 \equiv \langle 0 | \bar{q}(x) q(x) | 0 \rangle$ is the interacting vacuum expectation value of the *local* product of two Heisenberg fermion fields. This object is related to the trace of the *exact* quark propagator via $\langle 0 | \bar{q}q | 0 \rangle = -\mathrm{i} \lim_{y \to 0^+} \mathrm{Tr} S_F(y)$, which, is a function of the constituent quark mass: A non-zero value of the condensate is a sign of a nonvanishing effective ("constituent") quark mass. Equations (23a-c) are easily solved for $m_u^0 \langle \bar{u}u \rangle_0$, $m_d^0 \langle \bar{d}d \rangle_0$ and $m_s^0 \langle \bar{s}s \rangle_0$, thus allowing the determination of the current quark mass ratios. That, however, requires the knowledge of the "mechanical" $m_{\rm mech}(\phi)$, or equivalently of the EM part $m_{\rm EM}(\phi)$ of the observed ps meson mass $m(\phi)$, since

$$m_\phi^2 = m_\phi^2(\text{mech}) + m_\phi^2(\text{EM}).$$

This is where Dashen's theorem enters.

Dashen applied his mass formula (4) to the EM interaction as a source of chiral symmetry breaking to derive his celebrated theorem [16]

$$m^2_{\pi^0}(\mathrm{EM}) = m^2_{K^0}(\mathrm{EM}) = m^2_{\eta}(\mathrm{EM}) = m^2_{\eta'}(\mathrm{EM}) = 0 \tag{24a}$$

$$m^2_{\pi^\pm}(\mathrm{EM}) = m^2_{K^\pm}(\mathrm{EM}) = O(\alpha) \quad \alpha \simeq 1/137 \ . \tag{24b}$$

It is important to remember that these results were derived as a small correction to the chiral limit, and that, strictly speaking, they are not valid in a situation where the chiral symmetry is broken, e.g. by the current quark masses to begin with. This is true in particular when the "initial" chiral symmetry breaking is not small, such as in the case(s) when the strange quark is present. Then the $O(m^0\alpha)$ cross-terms become non-negligible. Corrections of this "mixed" kind to Dashen's theorem for the neutral kaon(s) are a subject of lively investigation, see references in [24], precisely because they are model dependent. They have been calculated in the NJL model [25], but only with two flavours, i.e., for charged pions. The approximations made in that calculation are not readily extendable to kaons due to the much larger kaon mass. Henceforth we shall disregard them.

B. $U_A(1)$ symmetry breaking

Next we turn to the calculation of the principal $U_A(1)$ symmetry-breaking effects using the 't Hooft interaction. This leads to the following (mass f_ϕ)^2 (sub-)matrix

$$\left(fm^2f\right)_{ab} = \begin{pmatrix} m^2_{00}f^2_0 & m^2_{08}f_0f_8 \\ m^2_{08}f_0f_8 & m^2_{88}f^2_8 \end{pmatrix} \ , \tag{25}$$

(the GMOR relations (23a-c) for the pions and kaons are unchanged to leading order)

$$\begin{aligned} m^2_{00}f^2_0 &= f^2_0 m^2_{\mathrm{U}(1)} \\ &\quad - \frac{2}{3}\left[m^0_u\langle\bar{u}u\rangle_0 + m^0_d\langle\bar{d}d\rangle_0 + m^0_s\langle\bar{s}s\rangle_0\right] \end{aligned} \tag{26a}$$

$$m^2_{08}f_0f_8 = -\frac{\sqrt{2}}{3}\left[m^0_u\langle\bar{u}u\rangle_0 + m^0_d\langle\bar{d}d\rangle_0 - 2m^0_s\langle\bar{s}s\rangle_0\right] \tag{26b}$$

$$m^2_{88}f^2_8 = -\frac{1}{3}\left[m^0_u\langle\bar{u}u\rangle_0 + m^0_d\langle\bar{d}d\rangle_0 + 4m^0_s\langle\bar{s}s\rangle_0\right] \ , \tag{26c}$$

which can be written in terms of known ps meson masses using the solutions $m^0_q\langle\bar{q}q\rangle_0$ to Eqs. (23a-c), as follows

$$\begin{aligned} m^2_{00}f^2_0 &= f^2_0 m^2_{\mathrm{U}(1)} \\ &\quad + \frac{1}{3}\left[m^2_{\mathrm{mech}}(\pi)f^2_\pi + \left(m^2_{\mathrm{mech}}(K^\pm) + m^2_{\mathrm{mech}}(K^0)\right)f^2_K\right] \end{aligned} \tag{27a}$$

$$m^2_{08}f_0f_8 = \frac{\sqrt{2}}{3}\left[2m^2_{\mathrm{mech}}(\pi)f^2_\pi - \left(m^2_{\mathrm{mech}}(K^\pm) + m^2_{\mathrm{mech}}(K^0)\right)f^2_K\right] \tag{27b}$$

$$m^2_{88}f^2_8 = \frac{1}{3}\left[-m^2_{\mathrm{mech}}(\pi)f^2_\pi + 2\left(m^2_{\mathrm{mech}}(K^\pm) + m^2_{\mathrm{mech}}(K^0)\right)f^2_K\right] \ , \tag{27c}$$

which can be further rewritten in terms of observed meson masses and decay constants using Dashen's theorem (24a,b). The masses of the two ps mesons that contain an admixture of the flavor-singlet (ninth) ps state are further shifted by the $U_A(1)$ symmetry-breaking interaction.

The mass matrix (25) is diagonalized by the rotation (in the (0-8) flavor plane) matrix

$$R = \begin{pmatrix} \cos\theta_{\rm ps} & -\sin\theta_{\rm ps} \\ \sin\theta_{\rm ps} & \cos\theta_{\rm ps} \end{pmatrix} , \tag{28}$$

where

$$\begin{aligned} \tan 2\theta_{\rm ps} &= \frac{\left(2\sqrt{2}/3\right)\Delta^2_{\rm ps}}{f_8^2 m_{88}^2 - f_0^2 m_{00}^2} \\ &= \frac{\left(2\sqrt{2}/3\right)\Delta^2_{\rm ps}}{(1/3)\,\Delta^2_{\rm ps} - f_0^2 m^2_{\rm U(1)}} , \end{aligned} \tag{29}$$

and

$$\Delta^2_{\rm ps} = f_K^2\left(m^2_{K^0} + m^2_{K^+}\right) - f_\pi^2\left(m^2_{\pi^0} + m^2_{\pi^+}\right) \tag{30a}$$

$$f_0^2 m^2_{\rm U(1)} = f_{\eta'}^2 m^2_{\eta'} + f_\eta^2 m^2_\eta - f_K^2\left(m^2_{K^+} + m^2_{K^0}\right) + f_\pi^2\left(m^2_{\pi^+} - m^2_{\pi^0}\right) , \tag{30b}$$

where we have also taken into account the minuscule EM correction for completeness' sake. We disregarded the $O(m^0\alpha)$ cross-terms, however. The quantity $f_0^2 m^2_{\rm U(1)}$ defined in Eq. (30b) is also known in the literature as the topological susceptibility [12].

This completes our formal manipulations – all objects of interest are expressed in terms of observables. We are now ready to evaluate several key ingredients of the present model(s) and compare them with theoretical predictions, where available. In the process we shall also make two self-consistency checks.

C. Results and Discussion

The $U_A(1)$ symmetry-breaking mass $m_{\rm U(1)}$ was evaluated as 855 MeV in Ref. [10], where it was called $m_{\rm tH}$, assuming equality of all pseudoscalar decay constants, which is a fair approximation to the model used there, but not nearly as good in Nature: The two well-known ps decay constants are $f_\pi = 93$ MeV, $f_K = 113$ MeV. The η, η' decay constants are substantially more uncertain: Older estimates placed them at $f_\eta = f_{\eta'} = 110 \pm 10$ MeV leading to $m_{\rm U(1)} = 830 \pm 60$ MeV, whereas PDG96 [26] quote $f_\eta = 93 \pm 9$ MeV, $f_{\eta'} = 83 \pm 7$ MeV leading to $m_{\rm U(1)} = 600 \pm 135$ MeV, where $f_0 = 88 \pm 5$ MeV was used. The former set of numbers is based on older analysis of experimental data [27] and various theoretical calculations of f_η and $f_{\eta'}$, whereas the latter set is based on two recent "direct" measurements, see p. 320 in PDG96 [26]. The two experiments are in agreement with each other, and their results for the ratios f_0/f_π and f_8/f_π are consistent with older estimates. But, their absolute values are roughly 10% smaller than the standard estimates. For example the overall scale is set by the neutral pion decay constant which is evaluated as $f_{\pi^0} = 84 \pm 3$ MeV, which is more than two standard deviations (2σ) away from the conventional value. For this reason one might, perhaps, consider the second set as a tentative one.

Both the 't Hooft and the Veneziano-Witten model predict a nonvanishing value of $f_0^2 m^2_{\rm U(1)}$, given by Eq. (11) and Eq. (16), respectively, as long as their respective coupling constants are non-zero. The said coupling constants can then be adjusted so as to fit the right-hand side of Eq. (30b). This procedure amounts to little more than a phenomenological description of experience, although 't Hooft's model actually predicts the (very) wide range of values $(1.4\ \mathrm{GeV}^{-1})^5 \leq |\kappa_3| \leq (6.8\ \mathrm{GeV}^{-1})^5$ for the (negative) coupling constant $\kappa_3 = -K/8$ in the dilute instanton gas approximation [20]. The whole range of the phenomenologically extracted $\kappa_3 \simeq -(4 \pm 2\ \mathrm{GeV}^{-1})^5$, easily fits within the bounds of the above prediction. Roughly one half of the uncertainty in the "empirical" value of κ_3 is due to the uncertainty in the quark condensate which was taken to be $\langle \bar{q}q \rangle_0 = -(250 \pm 50\ \mathrm{MeV})^3$. Hence we may say that the mass scale M determining κ_3 via $[\kappa_3] = M^{4-3N_f}$ is given by $M^3 = -\langle \bar{q}q \rangle_0$. This is perhaps somewhat fortuitous since it depends on $f_0^2 m_{\mathrm{U}(1)}^2$ falling within the range $(250 \pm 50\ \mathrm{MeV})^4$ which it does: $f_0^2 m_{\mathrm{U}(1)}^2 \simeq (260 \pm 40\mathrm{MeV})^4$. We suspect that a calculation of κ_3 can be extended to the instanton liquid approximation [28], although we are not aware of anyone having carried it out as of the time of writing. What has been done instead, by Alkofer et al. [14], also under the name of instanton liquid approximation, is a calculation of the Veneziano-Witten effective interaction. Though they do not present their results in terms of a coupling constant equivalent to κ_3', but rather in terms of ps meson masses and the quark condensate, we can nevertheless translate the latter information into $\kappa_3' \simeq (4.2\ \mathrm{GeV}^{-1})^{14}$. The empirically extracted value is $\kappa_3' \simeq (4 \pm 1\ \mathrm{GeV}^{-1})^{14}$, where the (huge) "error" band is again dominated by the uncertainty in the quark condensate. A better way of estimating the "quality" of the theoretical prediction is the comparison of the calculated value of the $U_A(1)$ symmetry-breaking mass $m_{\mathrm{U}(1)} = 1077$ MeV versus its "empirical" value of 855 MeV. The reader is once again advised to recall the spread induced in the latter number by the uncertainties in the ps meson decay constants.

It is manifest from Eq. (29) that the explicit breaking of the $U_A(1)$ symmetry is essential to the exact value of the $\eta - \eta'$ mixing angle. Choosing one or the other parameter set for $m_{\mathrm{U}(1)}$ and the η, η' decay constants, one finds $\theta_{\mathrm{ps}} = -(25 \pm 10)$ deg, or $\theta_{\mathrm{ps}} = (5 \pm 37)$ deg, respectively. This ought to be compared with $\theta_{\mathrm{ps}} = -18$ deg obtained from Eq. (29) under the assumption of SU(3) symmetric, i.e., equal ps decay constants. There are, of course, other independent measures of θ_{ps}, e.g., from the ps $\to 2\gamma$ decays, which yield - 20 deg, see p. 100 in Ref. [26]. We see that in all of the cases discussed our extracted values are consistent with the ps $\to 2\gamma$ number. The final word on the subject of ps mixing angle will have to wait until the η and η' decay constants are better known. This was our first consistency check.

Note that so far we have considered only one, the trace, of two independent invariants of the mass matrix (25) under the rotation Eq. (28). The second invariant is the mass matrix determinant, which leads to the so-called Schwinger sum rule [29]

$$\left(m_{\eta'}^2 + m_\eta^2\right)\left(4m_K^2 - m_\pi^2\right) - 3m_{\eta'}^2 m_\eta^2 = 8m_K^2\left(m_K^2 - m_\pi^2\right) + 3m_\pi^4 \ . \tag{31}$$

If one evaluates the left- and the right-hand sides of Eq. (31), one finds .344 GeV^4 vs. .447 GeV^4., i.e., a discrepancy of 23 %. If we evaluate the determinant of the fm^2f matrix Eq. (25), Schwinger's sum rule (31) turns into the *identity*

$$\frac{1}{3}\left(4f_K^2 m_K^2 - f_\pi^2 m_\pi^2\right)\left[f_0^2 m_{\mathrm{U}(1)}^2 + \frac{1}{3}\left(2f_K^2 m_K^2 + f_\pi^2 m_\pi^2\right)\right] = f_{\eta'}^2 m_{\eta'}^2 f_\eta^2 m_\eta^2 + \frac{8}{9}\left(f_K^2 m_K^2 - f_\pi^2 m_\pi^2\right)^2 . \tag{32}$$

This is our second consistency check.

These two examples (self-consistency checks) illustrate the range of variation in two observables of interest due to the inclusion of the ps meson decay constants into mass formulas based on Dashen's equation (4). In the following we make an extended comment on the effects of SU(3) symmetry-breaking in the ps decay constants on the current quark mass ratios.

Weinberg included both the quark mass terms (26a-c) and the EM corrections, while neglecting the SU(3) symmetry-breaking in the ps decay constants, in the pseudoscalar mass GMOR relations [30] which led him to the now widely accepted current quark mass ratios [2]

$$\frac{m_d^0}{m_u^0} = \frac{m^2(\pi^\pm) - m^2(K^\pm) + m^2(K^0)}{2m^2(\pi^0) - m^2(\pi^\pm) + m^2(K^\pm) - m^2(K^0)} = 1.80 \tag{33a}$$

$$\frac{m_s^0}{m_d^0} = \frac{m^2(K^\pm) + m^2(K^0) - m^2(\pi^\pm)}{m^2(\pi^\pm) - m^2(K^\pm) + m^2(K^0)} = 20.1 \tag{33b}$$

and, with additional assumptions, to the absolute values of current quark masses. The justification for setting $f_\pi = f_K$ is that the difference would lead to higher-order (in the current quark masses) corrections, which can be neglected in the leading-order approximation. This statement makes another tacit assumption, however: that the expansion of f_π, f_K is an analytic one. This assumption has been proven incorrect in the meantime in chiral perturbation theoretic calculations. Then the following question arises: is it "better" to calculate these symmetry-breaking corrections, or to take them from experiment? The same comments hold for the quark condensates, which are not observable, however.

Note that it appears as inconsistent to quote these numbers to three significant figures, as is commonly done, because terms of $\mathcal{O}(m_q^0\alpha)$ and higher, were neglected in this analysis, and they are likely to contribute at the 1% level. Such "mixed" term corrections are enhanced in the ratio: as an illustration of this point remember that the inclusion of the model-independent lowest-order EM corrections changes the m_d^0/m_u^0 ratio by about 15% (see the first footnote on p.188 in [30] and p. 270 of [31]), substantially higher than the nominal estimate of $1\% \sim \mathcal{O}(\alpha)$.

The SU(3) symmetry-breaking differences between the kaon and pion decay constants need not be a source of uncertainty, for they are observable and have been measured to at least two, and arguably to three significant figures as f_π = 93 MeV, f_K = 113 MeV [26]. [3] Quark condensates, on the other hand, are not observable, so one needs theory to divine their ratios. Whereas $\langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0$ certainly seems a reasonable assumption, $\langle\bar{u}u\rangle_0 = \langle\bar{s}s\rangle_0$ and $\langle\bar{d}d\rangle_0 = \langle\bar{s}s\rangle_0$ are very likely subject to significant corrections. It ought to be clear that for this reason the second (s/d) current quark mass ratio is far less reliable than the first (d/u) one. Inclusion of the ps decay constants leads to

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[2]It is perhaps interesting to note that Nambu obtained these ratios a few years earlier [32], but not knowing Dashen's theorem [16] neglected the EM effects which led him to somewhat different results – see below.

[3]Even the exact value of the charged pion decay constant is a subject of controversy: Holstein [33] claims 92.4 MeV after separation of EM radiative corrections, whereas PDG96 claims 93.3 MeV.

$$\frac{m_d^0}{m_u^0} = \frac{f_\pi^2 m^2(\pi^\pm) - f_K^2\left(m^2(K^\pm) - m^2(K^0)\right)}{f_\pi^2\left(2m^2(\pi^0) - m^2(\pi^\pm)\right) + f_K^2\left(m^2(K^\pm) - m^2(K^0)\right)} = 2.28 \tag{34a}$$

$$\frac{m_s^0}{m_d^0} = \frac{f_K^2\left(m^2(K^\pm) + m^2(K^0)\right) - f_\pi^2 m^2(\pi^\pm)}{f_\pi^2 m^2(\pi^\pm) - f_K^2\left(m^2(K^\pm) - m^2(K^0)\right)} = 27.9 \tag{34b}$$

These numbers ought to be also compared with the "canonical" values shown in Eqs. (33a,b): the differences are striking. To be sure, there is little surprise in the change of the s/d ratio, since the relevant quark condensates are certainly not equal and the uncertainties are expected to be large. In the u/d case, however, two essentially identical condensates cancel in the ratio, causing the surprisingly large shift from 1.80 to 2.28. This ought to be compared with the latest re-evaluation of the current quark mass ratios including the state-of-the-art corrections leads to $m_d^0/m_u^0 = 1.82 \pm 0.14$, $m_s^0/m_d^0 = 18.9 \pm 0.8$ [24]: The discrepancy is greater than three standard deviations (3σ) in the supposedly reliable case of u/d ratios and even bigger for the s/d ratio! When quoting the current quark mass ratios, it is clear that one ought not only specify the estimated uncertainties, but all of one's assumptions as well. Moreover, the said uncertainties have to be assigned more liberally. We would guess the theoretical uncertainty in the u/d ratio as the difference between the central values in this and Leutwyler's analysis, for example, and perhaps even larger for the s/d ratio.

Returning now to the main line of argument, we have shown that the $U_A(1)$ symmetry-breaking term leads to a particularly large (on the scale of ps meson masses) symmetry breaking mass $m_{\mathrm{U(1)}} \simeq 855$ MeV which leads us to believe that its cross terms with the current mass and/or the EM Hamiltonian might also be rather large. We shall not attempt an evaluation of these cross terms, which would be model-dependent, in the present paper, but rather point out their existence, which has hitherto been neglected, to the best of our knowledge. This leads us to conclude that the current quark mass ratios can be determined in a model-independent way up to at most two, but more likely only to one significant figure.

We have seen that the two types of U(1) symmetry breaking are indistiguishable as far as the pseudoscalar meson spectrum is concerned. Hence we are forced to look for other observables which might discriminate between them. One such set of observables was identified in Refs. [10,11]: the isoscalar scalar meson mass spectrum, but only the 't Hooft interaction case was examined there. In the following we shall examine the scalar meson spectrum with the Veneziano-Witten interactions in the hope that it will distinguish between the two models.

IV. EFFECTIVE THREE-FLAVOR CHIRAL QUARK MODELS

In the following we shall use an effective chiral field theory of quarks and spinless mesons with a non-trivial ground state characterized by a finite quark condensate and various effective $U_A(1)$ symmetry-breaking interactions, following Nambu and Jona-Lasinio (NJL) [34]. This model has turned out to be a reliable laboratory for testing the lightest spinless meson mass relations induced by $U_A(1)$ symmetry-breaking, as is best seen from the comparison between the NJL model results [10] and a confining potential model's predictions [11]. The close agreement of the spectra is the best, albeit *ex post facto* justification of the NJL model.

A. 't Hooft interaction

The following is to serve as a proof of the claim made in Sect. II.B.1 that explicit calculation in the NJL model agrees with the general result Eq. (11), as well as reminder of results pertaining to the scalar meson sector. The flavour-singlet meson mass shift due to the 't Hooft interaction in the NJL model has been established in Ref. [10] as

$$m^2_{\mathrm{U}(1)}(N_f = 3) = \left(\frac{3g^2_{\eta qq} G_2}{G_1^2}\right) + \mathcal{O}(1/N_C^2) \ . \tag{35}$$

Now use the definitions

$$G_1 = 2G \tag{36}$$
$$G_2 = -\frac{1}{3} K \langle \bar{\psi}\psi \rangle_0 = -K\langle \bar{q}q\rangle_0 \ . \tag{37}$$

and the gap equation

$$\begin{aligned} m_q &= m_q^0 - 4G\langle \bar{q}q\rangle_0 + 2K\langle \bar{q}q\rangle_0^2 \\ &\simeq -4G\langle \bar{q}q\rangle_0 + O(1/N_C) \ . \end{aligned} \tag{38}$$

This leads to the following result

$$\begin{aligned} m^2_{\mathrm{U}(1)}(N_f = 3) &= g^2_{\eta qq}\left(\frac{12 G_2}{G^2}\right) + \mathcal{O}(1/N_C^2) \\ &= g^2_{\eta qq}\left(\frac{-12K\langle \bar{q}q\rangle_0}{G^2}\right) + \mathcal{O}(1/N_C^2) \\ &= g^2_{\eta qq}\left(\frac{-12K\langle \bar{q}q\rangle_0^3}{m^2}\right) + \mathcal{O}(1/N_C^2) \\ &= -12K\langle \bar{q}q\rangle_0^3 f_\eta^{-2} + \mathcal{O}(1/N_C^2) \ , \end{aligned} \tag{39}$$

which is in agreement with the general result (11), as anounced earlier.

Next we remind the reader that the sum rule

$$m_\eta^2 + m_{\eta'}^2 - m_{K^+}^2 - m_{K^0}^2 = m^2_{K_0^{*+}} + m^2_{K_0^{*0}} - m^2_{f_0} - m^2_{f_0'} \ . \tag{40}$$

relating the ps and scalar meson masses has been derived in Ref. [10], and equivalent results were found in a different model in Ref. [11], as a primary effect of the 't Hooft $U_A(1)$ symmetry breaking interaction. The same result was also found by L. Burakovsky [35] on apparently different grounds. The derivation of Eq. (40) shown in Ref. [10] is based on a calculation of scalar and pseudoscalar $q\bar{q}$ states' masses using the Bethe-Salpeter equation and the three-flavor NJL Lagrangian including the 't Hooft interaction.

The sum rule (40) shifts the masses of the physical iso-singlet scalar states f_0, f_0' from their simple quark model positions. The masses of other members of the ps and scalar octets are unchanged. In particular, the ordering of the meson masses in the octet, specifically the ordering of the isovector scalar a_0 presently placed at 1450 MeV - up from 1320 MeV - by the 96 Particle Data Group [26] and of the scalar kaon $K_0^*(1430)$ is completely independent of any $U_A(1)$ symmetry breaking and/or mixing with other states due to their nonvanishing isospin and strangeness, respectively. The ordering of these two states is governed by the strange - up/down quark mass difference in accord with the (simplest) quark model. In that light it is clear that the new mass assignment for a_0 places it outside of the $q\bar{q}$ octet. On the other hand, the old assignment fits perfectly.

Assuming that the well-established $f_0(1500)$ is one of the two isoscalar scalar states, the sum rule (40) predicts the mass of the other. That second scalar state mass is 1000 MeV to an accuracy of about 5% if the lefthand side (l.h.s.) of the sum rule is taken to be 830 ± 60 MeV, or 200-300 MeV higher with the l.h.s. at 600 ± 135 MeV. Since there are *two* iso-singlet scalar states f_0 in the Particle Data tables [26] with their mass(es) very close to 1 GeV, the $f_0(980)$ and ("Pennington's") $f_0(\varepsilon(1000))$, one is presented with an unexpected choice. Pennington's $f_0(\varepsilon(1000))$ was chosen in Ref. [10] on account of its large width as demanded by the model used there, and $f_0(980)$ was chosen in Ref. [11] so as to conform with the predictions of that model. It ought to be kept in mind that neither of these two models were unitary as of the time of writing, and new kinds of phenomena, such as images, or reflections of poles on unphysical sheets of the coupled channel scattering amplitudes near the $K\bar{K}$ threshold have been claimed to arise as a consequence of a proper unitarization [36,37,26]. This means that one "bare" $q\bar{q}$ state can appear as two observed resonances. It is not clear, however, if that situation applies to the two states at 1 GeV. Manifestly, much more work will have to be done before one can claim understanding of this problem. The case of $f_0(1500)$ is in much better agreement with theory: Ritter et al. [11] have recently explained the puzzling absence of $K\bar{K}$ pairs from the $f_0(1500)$ two-body decay products as a consequence of the 't Hooft interaction [4]. This explanation depends crucially on the scalar mixing angle θ_s being small and *positive*, where

$$\tan 2\theta_s = \frac{\left(4\sqrt{2}/3\right)\left(m_{K_0^*}^2 - m_{a_0}^2\right)}{m_{U(1)}^2 + (2/3)\left(m_{K_0^*}^2 - m_{a_0}^2\right)} \ . \tag{41}$$

It is hence clear that the said condition is met only when the correct quark model ordering of the a_0, K_0^* states takes place, i.e., when $m_{K_0^*} > m_{a_0}$. In view of these facts and of the discussion earlier in this subsection, one is lead to the conclusion that $a_0(1450)$ cannot be a member of the scalar $q\bar{q}$ octet in this model.

B. Veneziano-Witten interaction

We shall start from an $N_f = 3$ NJL Lagrangian

[4]In this regard I would like to correct Eqs. (46a,b) in Ref. [10], where the contributions of the 't Hooft interaction to the effective s-ps-ps couplings, e.g. $g_{f_0\pi\pi}^{\rm tH} = -\sin\theta_s m_{\rm tH}^2 \left(2\sqrt{6}f_\pi\right)^{-1}$ and $g_{f_0'\pi\pi}^{\rm tH} = \cos\theta_s m_{\rm tH}^2 \left(2\sqrt{6}f_\pi\right)^{-1}$, were inadvertently omitted and the quark-loop contribution ought to be divided by 4. The numerical results remain unchanged, however.

$$\mathcal{L}^{(12)}_{\rm NJL} = \bar{\psi}[i\partial\!\!\!/ - m^0]\psi + G\sum_{i=0}^{8}[(\bar{\psi}\boldsymbol{\lambda}_i\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)^2]$$
$$+ K'\left[\det\left(\bar{\psi}(1+\gamma_5)\psi\right) - \det\left(\bar{\psi}(1-\gamma_5)\psi\right)\right]^2 , \qquad (42)$$

consisting of the free quark Lagrangian and the $U(3)_L \times U(3)_R$ symmetric quartic self-interaction terms (the first line), the same as in Eq.(23) of Ref. [10], and the $U(1)_A$ symmetry-breaking Veneziano-Witten (VW) determinant interaction term (second line), which is now of 12th order in the quark fields. There are at present no readily available nonperturbative methods in the literature, to the present author's knowledge, for a direct approach to the 12-point operator in Eq. (42). Therefore we proceed to construct an "effective mean-field quartic self-interaction Lagrangian" $\mathcal{L}^{(4)}_{\rm eff}$ from Eq. (12) following the procedure employed on the$N_f = 3$ 't Hooft interaction in Refs. [8,10,38]. We shall closely follow the method used in Ref. [10]. That procedure leads to consistent chiral dynamics in the sense that the Goldstone theorem and other chiral Ward-Takahashi identities pertainin to the ps octet remain intact in the chiral limit. The procedure that turns the 12th-order interaction into a 4th-order one can be characterised in several apparently different ways: (a) By "averaging" of the interaction over the ground state ("vacuum") of the system, analogous to making the mean-field approximation in statistical mechanics, reduces the number of fermi fields left in the interaction by two and multiplies the appropriate coupling constant by one power of the quark condensate at a time. Only the leading terms in the $1/N_C$ expansion are kept, and of those only the vacuum expectation values of scalar, flavour-matrix-diagonal operators are non-zero, all others vanish. One must be careful to properly count the allowed possibilities when the original interaction term is a higher power of a single Dirac bilinear. Four repetitions of this step reduce the 12-point Lagrangian to a 4-point one. This procedure is the same as the so-called "linearization of the equations of motion" method used in quantum many-body physics, see in particular Sect. 3.2. in Hatsuda and Kunihiro [38]. (b) Mathematically the above is completely equivalent to taking a quark and an antiquark external line and closing them into a loop using Feynman rules for the Lagrangian (42) in all possible ways while taking into account the proper symmetry number of the diagram, e.g. see Fig. (1). After closing up eight of 12 external lines one ends up with a four-fermi interaction. Thus we find in the SU(3)-symmetric limit, i.e., with $\langle\bar{\psi}\boldsymbol{\lambda}_0\psi\rangle_0 = \sqrt{\frac{2}{3}}\langle\bar{\psi}\psi\rangle_0 \neq 0$; $\langle\bar{\psi}\boldsymbol{\lambda}_3\psi\rangle_0 = \langle\bar{\psi}\boldsymbol{\lambda}_8\psi\rangle_0 = 0$, the following effective four-point interaction Lagrangian

$$\mathcal{L}^{(4)}_{\rm effVW} = -4K'\langle 0\Big|\left(\bar{\psi}\boldsymbol{\lambda}_0\psi\right)^2\Big\{\frac{1}{2\sqrt{6}}(\bar{\psi}\boldsymbol{\lambda}_0\psi)(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_0\psi)$$
$$+\frac{1}{2\sqrt{6}}\sum_{i,j=1}^{8}D_{ij0}(\bar{\psi}\boldsymbol{\lambda}_i\psi)(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_j\psi)$$
$$-\frac{1}{2}\sqrt{\frac{3}{2}}\sum_{i=1}^{8}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i(\bar{\psi}\boldsymbol{\lambda}_i\psi)\Big\}^2\Big|0\rangle + \mathcal{O}(1/N_C). \qquad (43)$$

Simplify this further using Eq.(10)

$$\mathcal{L}^{(4)}_{\rm effVW} = -\frac{K'}{6}\langle\left(\bar{\psi}\boldsymbol{\lambda}_0\psi\right)^2\Big\{(\bar{\psi}\boldsymbol{\lambda}_0\psi)(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_0\psi)$$

$$-2\sum_{i=1}^{8}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)(\bar{\psi}\boldsymbol{\lambda}_i\psi)\Big\}^2\rangle_0 + \mathcal{O}(1/N_C)$$

$$= -\frac{2K'}{27}\langle\bar{\psi}\psi\rangle_0^4(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_0\psi) + \mathcal{O}(1/N_C). \quad (44)$$

and then insert the result Eq. (44) into Eq. (42) to find

$$\mathcal{L}_{\rm NJL}^{(4)} = \bar{\psi}[i\partial\!\!\!/ - m^0]\psi + \left[K_0^{(-)}(\bar{\psi}\boldsymbol{\lambda}_0\psi)^2 + \sum_{i=1}^{8}K_i^{(+)}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_i\psi)^2\right] + \left[K_0^{(+)}(\bar{\psi}i\gamma_5\boldsymbol{\lambda}_0\psi)^2 + \sum_{i=1}^{8}K_i^{(-)}(\bar{\psi}\boldsymbol{\lambda}_i\psi)^2\right] \quad , \quad (45)$$

where

$$K_0^{(-)} = K_i^{(\pm)} = G \; , \quad i = 1, \ldots, 8 \; ; \quad (46a)$$

$$K_0^{(+)} = G - 6K'\langle\bar{q}q\rangle_0^4 \; , \; , \quad (46b)$$

where the quark condensates are defined as follows

$$\langle\bar{q}q\rangle_0 = -iN_C{\rm tr}S_F^q(x,x) = -4iN_C\int\frac{d^4p}{(2\pi)^4}\frac{m_q}{p^2 - m_q^2 + i\varepsilon}, \; q = u, d, s \;\; ; \quad (47a)$$

$$\langle\bar{\psi}\psi\rangle_0 = \langle\bar{u}u\rangle_0 + \langle\bar{d}d\rangle_0 + \langle\bar{s}s\rangle_0 = -iN_C{\rm tr}S_F(x,x) \quad . \quad (47b)$$

Eq. (45) is just the "effective quartic Lagrangian" in the exact SU(3), for the VW interaction (42). The SU(3) gap equations now read

$$m_q = m_q^0 - 4G\langle\bar{q}q\rangle_0 \quad . \quad (48)$$

The meson masses are read off from the poles of their propagators, which in turn are constrained by the gap Eq. (38). The reader will easily convince himself that the effective Lagrangian Eq. (45) preserves the Goldstone theorem for the pseudoscalar meson octet in the chiral limit. We use Eq.(45) to derive the η' meson mass. where, To leading order in N_C we find the following relations between the meson masses

$$m_{\rm U(1)}^2(N_f = 3) = m_{\eta'}^2 = -12g_{\eta qq}^2\left(\frac{K'}{mK_0^{(+)}}\right)\langle\bar{q}q\rangle_0^5 + \mathcal{O}(1/N_C^2) = 48f_{\eta qq}^{-2}K'\langle\bar{q}q\rangle_0^6 + \mathcal{O}(1/N_C^2) \; , \quad (49)$$

in agreement with the general result Eq. (16). Upon introducing explicit chiral symmetry breaking in the form of non-zero current quark masses m_i^0, we find the standard GMOR relation (26a) correction to Eq. (48), as well. The remaining GMOR relations (26b,c) are independent of $U_A(1)$ symmetry breaking and are well-established in the NJL model.

Next we seek scalar states in the PDG96 tables [26] that fit predictions of this model. It turns out that, in the absence of flavour singlet-octet mass splitting in the scalar sector, the flavour-singlet scalar mesons mix ideally, as can be seen from Eq. (41), but with $m^2_{\mathrm{U}(1)}$ omitted from the denominator, and one finds one non-strange and one purely strange state, split roughly by two strange-nonstrange quark mass differences, i.e., normally by about 300 MeV. This ideal mixing is independent of the $a_0 - K_0^*$ ordering and predicts that the lower (nonstrange) state be degenerate with the isovector scalar mesons. That means that it ought to be at 1320 MeV according to this model, or at 1450 MeV according to PDG96. Curiously, there is an f_0 state at 1370 MeV. Then the heavy scalar meson ought to be near 1600 MeV. The only candidate state in the vicinity is the familiar $f_0(1500)$, at least 100 MeV below the prediction and with a puzzling absence, for an $s\bar{s}$ state, of the $K\bar{K}$ decay mode which has already prompted suggestions that it is not an ordinary $q\bar{q}$ octet member, as the Veneziano-Witten model predicts. This evidence and the apparent success of the 't Hooft model at explaining the $f_0(1500)$ decay pattern [11] seem to rule out the Veneziano-Witten model, though it would certainly not harm if the decays of the $f_0(1370)$ and the mass of the a_0 were better established before the definitive verdict. It must be stated that, independently of other details of the VW model, it does *not* allow an isovector state scalar other than around 1300 MeV.

C. Tensor interactions

A straightforward application of the "tensor" operators (1c,d) in conjunction with the $U_L(3) \times U_R(3)$ symmetric NJL Lagrangian readily leads to the conclusion that *neither pseudoscalar nor scalar* meson masses are affected by it, to leading order in $1/N_C$. The ps and s meson spectra are unaffected by either of these $U_A(1)$ symmetry-breaking operators, to leading order in $1/N_C$.

Thus we confirm in explicit model calculations the general results pertaining to these two interactions based on Dashen's double commutator relation (Sect. II.B.3). The scalar meson sector is unaffected by these interactions and hence raises doubts about the proper identification of the a_0 meson. Nontrivial consequences of these two operators are yet to be found. They are to be sought among the properties of antisymmetric tensor mesons - an entirely unexplored field, at least within the realm of NJL-like models. Of course there is a connection between antisymmetric tensor fields and spinless (Klein-Gordon) ones, as first pointed out by Kalb and Ramond [39], though this connection is very difficult to see from the point of view of explicit model calculations, such as the present one. The said connection can be gleaned in the special case of two-flavours where the Fierz rearrangement of the 't Hooft interaction

$$\mathcal{F}\left[\mathcal{L}_{\mathrm{tH}}^{(4)}\left(N_f = 2\right)\right] = \frac{1}{2}\mathcal{L}_{\mathrm{tH}}^{(4)}\left(N_f = 2\right) + \frac{1}{4}\mathcal{L}_{\mathrm{t1}}^{(4)}\left(N_f = 2\right) \ , \tag{50}$$

$$\tag{51}$$

where we have set $\kappa_{N_f=2} = \mu_{N_f=2}$. This equals a linear combination of the 't Hooft- and the (linear) tensor U(1) symmetry-breaking interactions. Since the "fierzing" of an interaction in the NJL model corresponds to the addition of the Fock self-energy, which is an $1/N_C$ correction to the Hartree self-energy, we conclude that the Kalb-Ramond relation is to be sought among the $1/N_C$ corrections to the present tensor interaction model(s). The $1/N_C$

corrections to NJL-type of models [23] form a topic far beyond the scope of this paper; therefore we stop the discussion at this point.

V. CONCLUSIONS

In summary, we have investigated the scalar and pseudoscalar meson mass spectra for four different $U_A(1)$ symmetry-breaking interactions using the model-independent Dashen ps mass formula and explicit calculations in the three-flavor version of the appropriately extended NJL model. We have found perfect agreement, to leading order in $1/N_C$, between the general and the specific model calculation results for the ps masses, which leads us to believe that the NJL model used here is a reliable one in these kinds of calculations. Then we use our model calculations in search of other observables sensitive to $U_A(1)$ symmetry breaking interactions. The flavour-singlet *scalar* meson masses were identified in Refs. [10,11] as one such observable sensitive to the presence of the 't Hooft interaction. Scalar meson states in agreement with the masses predicted by this model interaction have been found. Their definitive identification will have to await a better decay analysis, however.

An analogous analysis of the Veneziano-Witten $U_A(1)$ symmetry-breaking interaction showed no response in the scalar meson sector. Scalar states can be found in the latest Particle Data Group tables [26] that are in agreement with the predictions of this model. Their decay properties have not been measured as yet, so we cannot make a definitive statement about their viability in this instance either.

Perhaps the most surprising result of this work is that two of the four interactions examined do *not* shift either the pseudoscalar or the scalar flavour-singlet meson masses, to leading order in $1/N_C$, despite their $U_A(1)$ symmetry-breaking nature. This leaves the (following) scenario open where the "true" $U_A(1)$ symmetry-breaking force in Nature is $N_C = 3$ times larger than previously thought, albeit it manifests itself only in an $1/N_C$ suppressed form, at least in the observables studied here.

One prediction all four of these models share is that $a_0(1450)$ cannot be a $q\bar{q}$ state.

In conclusion, it is clear that our study has opened more questions than it has answered. More work is called for.

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FIGURES

FIG. 1. An "elementary" 't Hooft six-point vertex (a), and an effective four-point interaction produced from (a) by closing a quark and an anti-quark external line into a loop (b). In order to complete the effective quartic interaction Lagrangian one must include all of such "closures". The construction of an effective Lagrangian for the Veneziano-Witten interaction proceeds analogously, the main difference being that there are 12 external lines (for $N_f = 3$) to begin with so that it takes four closed loops to reduce it to a quartic interaction (see text).

(a) (b)