Broken Symmetries

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The term 'broken symmetry', in the sense in which I shall use it, signifies not merely an approximate symmetry, but rather an exact symmetry of the Lagrangian or Hamiltonian which is not shared by the vacuum or ground state. Theories of this type, in which the ground state is degenerate, have contributed greatly to our understanding of many non-relativistic problems. The most familiar example is the BCS model of superconductivity.[1] These impressive successes naturally led to the hope, first expressed by Nambu,[2] that similar theories might be useful in the description of some of the observed approximate symmetries of relativistic particle physics. A large number of relativistic broken symmetry models have been investigated,[3] but there remain considerable difficulties in the way of applying them to real physical problems. In this lecture, I wish to discuss some general features of these theories, and review the present situation.

It was realized at an early stage that relativistic broken symmetry models always seemed to exhibit massless particles in the excitation spectrum. This is the statement of the Goldstone theorem,[4] which was proved by Goldstone, Salam and Weinberg,[5] and by Bludman and Klein.[6] Since very few massless particles are believed to exist, this theorem would seem to drastically circumscribe the field of applicability of broken symmetry theories. Now, although the original proofs of the theorem relied on the use of relativistic invariance, they can easily be extended to cover non-relativistic cases also.[7] The theorem would then predict the existence of a branch of the excitation spectrum whose energy tends to zero in the limit of infinite wavelength. However, such excitations are known to be absent in many real superconducting systems, and it therefore became apparent that there must be some flaw in the assumptions or proof of the Goldstone theorem. Various explanations for this phenomenon were advanced, but it has now become clear that the relativistic and nonrelativistic situations do not differ in any essential respect. In either case, the zero-energy excitations predicted by the theorem may be eliminated by the presence of long-range forces. The precise way in which this happens is one of the main items I wish to discuss.

Before turning to the Goldstone theorem in its general form, it will be helpful to recall how the massless particles appear in a specific case. Let us consider Goldstone's original model[4] described by the Lagrangian density

$$L = \tfrac{1}{2}(\partial_\mu \phi_1)^2 + \tfrac{1}{2}(\partial_\mu \phi_2)^2 - V(\phi_1^2 + \phi_2^2), \qquad (1)$$

which is clearly invariant under rotations in the ϕ_1-ϕ_2 space. If V has a maximum at $\phi^2 = 0$, and a minimum at some other value, then we may expect that in the ground state the expectation value of ϕ will not be zero, but rather will be approximately equal to the value at which V has a minimum. Clearly, because of the invariance, there must be an infinitely degenerate set of ground states, with expectation values corresponding to all the points round the circle on which V has its minimum. From the equations of motion, one easily derives the consistency requirement

$$\langle \partial V/\partial \phi_1 \rangle = 0 \qquad (2)$$

which serves to fix the magnitude of $\langle \phi \rangle$. The various degenerate ground states are labelled by a phase angle α, and characterized by the expectation values

$$\langle \phi_1 \rangle = \eta \cos\alpha, \qquad \langle \phi_2 \rangle = \eta \sin\alpha,$$

with η determined by (2). To be specific, we shall choose $\alpha = 0$.

To find the qualitative features of the spectrum, one can use a Hartree-like approximation, setting

$$\phi_1 = \eta + \chi_1, \qquad \phi_2 = \chi_2, \tag{3}$$

and retaining only the quadratic terms in χ_1 and χ_2 in the Lagrangian. The one finds

$$L = \tfrac{1}{2}(\partial_\mu \chi_1)^2 - \tfrac{1}{2}m^2\chi_1{}^2 + \tfrac{1}{2}(\partial_\mu \chi_2)^2 \tag{4}$$

where m^2 is determined by the curvature of V at its minimum. Clearly, there are massive excitations corresponding to oscillations in the radial χ_1 direction, while the mass of the transverse χ_2 oscillations is zero because of the vanishing curvature of the potential surface in that direction. These modes may also be regarded as oscillations in the phase angle α. Analogous modes appear in any broken symmetry theory. For example, in a ferromagnet the analogue of the phase angle is the magnetization direction, and the corresponding modes represent spatial distortions of this direction. Their frequency should tend to zero in the infinite wavelength limit.

We may note another feature of this theory. The conserved current corresponding to the rotational symmetry of the Lagrangian is

$$j_\mu = e(\phi_1\partial_\mu \phi_2 - \phi_2\partial_\mu\phi_1),$$

which in the approximation made above reduces to

$$j_\mu \approx e\eta\partial_\mu \chi_2 . \tag{5}$$

Thus j^0 is linear in the field variables, and its spatial integral over all space diverges. It follows that the various degenerate ground states, which are formally related by the unitary operator formed by exponentiating this integral, actually belong to different Hilbert spaces, and correspond to unitarily inequivalent representations of the algebra of field operators. This kind of behhaviour is a general feature of broken symmetry theories,[8] and is well known in the case of the BCS model.[9]

Now let us turn to the general proof of the Goldstone theorem. From the invariance of L, one can infer the existence of a current $j^\mu(x)$ satisfying the continuity equation

$$\partial_\mu j^\mu = 0. \tag{6}$$

(For the present, we consider only a one-parameter subgroup of the invariance group of the Lagrangian.) The spatial integral of j^0 over any finite volume V,

$$Q_V(t) = \int_V d^3\mathbf{x}\, j^0(t, \mathbf{x}), \tag{7}$$

must generate the transformation of the field operators within V,

$$-i\,[Q_V(t), \phi(t, \mathbf{y})] \begin{cases} = \delta\phi(t, \mathbf{y}), & \mathbf{y} \in V, \\ = 0, & \mathbf{y} \in V. \end{cases} \tag{8}$$

Note that the representations corresponding to different degenerate ground states are equivalent within any finite volume - and therefore physically equivalent. From (8), we obtain the equal-time commutation relation

$$-i\,[j^0(t, \mathbf{x}), \phi(t, \mathbf{y})] = \delta\phi(t, \mathbf{y})\ \delta^3(\mathbf{x} - \mathbf{y}). \tag{9}$$

The broken symmetry condition may be expressed by requiring that the variation of some designated operator A have a non-vanishing expectation value in the ground state, $<\delta A> \neq 0$. In practice, one may often choose A to be one of the local dynamical variables, say ϕ, and for simplicity we shall assume this to be the case. However, the proof goes through in a straightforward manner for any operator A which is a function of the field variables within a finite volume of space. Because of the assumed translational invariance of the ground state, the expectation value must be independent of position in space-time. Thus we require

$$<\delta\phi(t, \mathbf{x})> = \eta \neq 0, \tag{10}$$

which in turn implies that the expectation value of the commutator function (9) is non-zero.

Let us now examine the Fourier transform of this commutator expectation value,

$$f^{\mu}(k) = - i \int d^4x \, e^{ik\cdot x} \, <[j^{\mu}(x), \phi(0)]>,$$

which by the continuity equation (6) must satisfy

$$k_{\mu} f^{\mu}(k) = 0. \tag{11}$$

From (9) and (10), we deduce the sum rule

$$\int \frac{d\omega}{2\pi} f^0(\omega, \mathbf{k}) = \eta, \qquad (\omega = k^0) \tag{12}$$

valid for each value of $\mathbf{k}$. (If we had chosen some operator A other than a local field operator, this integral would not be independent of $\mathbf{k}$, but would still be a continuous (indeed, entire) function of $\mathbf{k}$, with the value η at $\mathbf{k} = 0$.) It is worth noting that this sum rule is sufficient to exclude the possibility of a term in f^{μ} proportional to $\delta^4(k)$, thus contradicting one of the suggested mechanisms for avoiding the conclusions of the Goldstone theorem.[10]

In a manifestly covariant theory, one must require that $f_{\mu}(k)$ be proportional to the only available vector, namely k_{μ}. Then from (11) it follows that it must also contain a factor $\delta(k^2)$. Thus, using (12), we find

$$f_{\mu}(k) = k_{\mu}[\eta\epsilon(\omega) + \xi] \, 2\pi\delta(k^2), \tag{13}$$

where ξ is arbitrary (unless we invoke time-reversal invariance). Thus there is a non-zero contribution from states with $k^2 = 0$, and therefore there must exist massless particles in the spectrum.

Even without invoking the requirement of covariance, we may take the limit $\mathbf{k} \to \mathbf{0}$ in the continuity equation,

$$\omega f^0(\omega, \mathbf{k}) - \mathbf{k}\cdot\mathbf{f}(\omega, \mathbf{k}) = 0,$$

and hence deduce that

$$\lim_{\mathbf{k}\to\mathbf{0}} f^0(\omega, \mathbf{k}) = 2\pi\eta\delta(\omega), \tag{14}$$

provided only that $\mathbf{f}(\omega, \mathbf{k})$ is non-singular in this limit. This is the non-relativistic form of the Goldstone theorem, which asserts the existence of a branch of the excitation spectrum whose energy tends to zero as $\mathbf{k} \to \mathbf{0}$ (zero energy gap).

The assumption that $\mathbf{f}$ is non-singular is evidently crucial. In fact, if we relax this assumption, then it is easy to obtain functions satisfying both (11) and (12). For example, if $\omega(\mathbf{k})$ is any given excitation spectrum, then we may take

$$\begin{aligned} f^0(\omega, \mathbf{k}) &= 2\pi\eta|\omega|\delta(\omega^2 - \omega^2(\mathbf{k})), \\ \mathbf{f}(\omega, \mathbf{k}) &= 2\pi\eta\mathbf{k}\frac{\omega^2}{\mathbf{k}^2}\,\epsilon(\omega)\delta(\omega^2 - \omega^2(\mathbf{k})). \end{aligned} \tag{15}$$

Clearly, if $\omega(\mathbf{0}) = \omega_0 \neq 0$, then $\mathbf{f}$ will be singular as $\mathbf{k} \to \mathbf{0}$.

Before discussing the conditions under which this can happen, it will be useful to rephrase the discussion in terms of coordinate space, as was done by Guralnik, Hagen and Kibble,[11] and for the non-relativistic case by Lange.[12] The condition (14) is equivalent to the time-independence of the integral

$$- i \int d^3\mathbf{x} <[j^0(t, \mathbf{x}), \phi(0, \mathbf{0})]> \tag{16}$$

Note that this integral always exists, even though the integral of j^0 itself diverges. This time independence follows from the continuity equation if and only if the surface integral

$$i \int d\mathbf{S}\cdot <[\mathbf{j}(t, \mathbf{x}), \phi(0, \mathbf{0})]> \tag{17}$$

tends to zero in the limit of infinite volume. This condition is obviously equivalent to the condition $\mathbf{k}\cdot\mathbf{f} \to 0$ as $\mathbf{k} \to \mathbf{0}$.

It is clear that we should expect the limit of (17) to vanish, except possibly in the presence of long-range forces. Indeed, it can be shown that if the forces are of limited range, then it must tend to zero, and the Goldstone theorem must apply.[13] Moreover, the examination of specific cases shows that when there are long-range Coulomb-type forces, then the condition is generally violated, and f_{μ} has a form similar to (15) with non-zero ω_0.

As a simple example which clearly exhibits this type of behaviour, let us consider the model described by the Hamiltonian

$$H = \tfrac{1}{2}\int d^3\mathbf{x}\,[\pi^2(t,\mathbf{x}) + (\nabla\phi)^2(t,\mathbf{x})] + \tfrac{1}{2}\int d^3\mathbf{x}\,d^3\mathbf{y}\,\pi(t,\mathbf{x})V(\mathbf{x}-\mathbf{y})\pi(t,\mathbf{y}). \tag{18}$$

It is invariant under the transformation $\phi(x) \rightarrow \phi(x) + c$, but clearly $\langle\phi\rangle$ is not. Thus we have a broken symmetry, albeit of a somewhat trivial kind. For this model the conserved current is given by $j^0 = -\pi$ and $\mathbf{j} = \nabla\phi$. Thus

$$-i\,[j^0(t,\mathbf{x}),\ \phi(t,\mathbf{y})] = \delta^3(\mathbf{x}-\mathbf{y}),$$

and so $\eta = 1$. The model is of course completely soluble, and it is easy to verify that f_μ has precisely the form given in (15), with $\eta = 1$. From the equations of motion, one finds the excitation spectrum

$$\omega^2(\mathbf{k}) = \mathbf{k}^2[1 + \tilde{V}(\mathbf{k})], \tag{19}$$

where $\tilde{V}(k)$ is the Fourier transform of $V(x)$. Thus $\omega(\mathbf{0}) = 0$ if and only if

$$\lim_{\mathbf{k}\rightarrow 0} \mathbf{k}^2\tilde{V}(\mathbf{k}) = 0, \tag{20}$$

or, in other words, for short-range potentials. The Coulomb potential is evidently the limiting case.

This example shows that in the presence of long-range forces the surface integral (17) need not tend asymptotically to zero, and (16) need not be time-independent. In fact, when ω_0 differs from zero, (16) oscillates at the frequency ω_0. This result is easy to understand physically. The Goldstone bosons correspond, as we have seen, to oscillations in the phase parameter which labels the various degenerate ground states. For example, in the case of the ferromagnet, which is particularly easy to visualize, they correspond to oscillations in the magnetization direction. Now let us consider the introduction of a distortion of this parameter which varies slowly with position. If the forces are of finite range, this requires essentially no energy, and will lead to oscillations of nearly zero frequency. This is not the case, however, in the presence of long-range forces. Then, any small part of the system is affected not merely by its immediate surroundings but by distant regions as well. Thus even a distortion which varies very slowly with position requires a finite amount of energy, and the oscillation frequency will tend to a finite infinite-wavelength limit. Moreover, even an initially localized disturbance will produce long-range effects, leading to a finite oscillating flux across any large sphere.

This particular model is very similar to the well known example of the condensed Bose gas at finite density ρ. This is a broken symmetry theory in which the particle number symmetry is broken by setting $\langle\psi\rangle = \rho^{\frac{1}{2}} e^{i\alpha}$. As before, the parameter α labels the various degenerate ground states. At zero temperature, one can use the Bogoliubov approximation,[14] which consists in writing $\psi = \langle\psi\rangle + \psi'$, and neglecting terms of higher than second degree in ψ', just as we did for the Goldstone model. The frequency spectrum is then given by

$$\omega^2(\mathbf{k}) = \frac{\mathbf{k}^2}{2m}\left[\frac{\mathbf{k}^2}{2m} + 2\rho\tilde{V}(\mathbf{k})\right]. \tag{21}$$

Once again, we see that the condition for a zero energy gap is precisely (20).

Now let us ask whether an analogous effect can occur in a relativistic theory. At first sight, one might think the answer should be no, because (15) is obviously not covariant. However, it was pointed out by Higgs,[15] and by Guralnik, Hagen and Kibble,[11] that this argument does not necessarily apply to theories involving gauge fields, since for such theories it is necessary to use the radiation gauge in which manifest covariance is destroyed. Moreover, it is precisely in this case, relying on the analogy of nonrelativistic theories, that one should expect the Goldstone theorem to fail, since relativistically long-range forces are described by gauge fields. In fact, it is easy to construct examples of theories of this type which do not possess massless particles. For instance, let us regard the original Goldstone model as describing the two real components of a charged field, and introduce the usual electromagnetic coupling. We then have

$$L = \tfrac{1}{2}(\partial_\mu\phi_1 - eA_\mu\phi_2)^2 + \tfrac{1}{2}(\partial_\mu\phi_2 + eA_\mu\phi_1)^2 - V(\phi_1^2 + \phi_2^2) - \tfrac{1}{4}(\partial_\nu A_\mu - \partial_\mu A_\nu)^2. \tag{22}$$

Making the same substitution (3) as before, and again retaining only the quadratic terms, we find

$$L = \tfrac{1}{2}(\partial_\mu\chi_1)^2 - \tfrac{1}{2}m^2\chi_1^2 + \tfrac{1}{2}(\partial_\mu\chi_2 + e\eta A_\mu)^2 - \tfrac{1}{4}(\partial_\nu A_\mu - \partial_\mu A_\nu)^2$$

Introducing a new field

$$B_\mu = A_\mu + \frac{1}{e\eta}\partial_\mu\chi_2, \tag{23}$$

we may write this Langrangian as

$$L = \tfrac{1}{2}(\partial_\mu \chi_1)^2 - \tfrac{1}{2}m^2\chi_1{}^2 - \tfrac{1}{4}(\partial_\nu B_\mu - \partial_\mu B_\nu)^2 + \tfrac{1}{2}e^2\eta^2 B_\mu{}^2. \tag{24}$$

As before, we have a massive scalar field corresponding to the radial oscillations. However, the massless photon field has combined with the massless Goldstone boson to produce a massive vector excitation. This is precisely the type of behaviour which was shown by Anderson[16] to be responsible for the nonzero frequency of the plasmon modes in a superconducting electron gas.

One obvious question needs to be answered. It would of course be possible to treat this model in a manifestly covariant fashion by using the Lorentz gauge. Then the Goldstone theorem must apply, and one might ask what has happened to the massless particles it predicts. Indeed, if the appropriate modification of the Lagrangian (22) is made, then in the same approximation we find not only the massive fields of (24), but also a pair of massless scalar fields, one of which appears with a sign corresponding to a negative metric in Hilbert space. These fields are purely guage parts, with vanishing matrix elements between physical states. Thus, although the theorem allows us to deduce the existence of massless states in the Hilbert space, it does not allow us to conclude that they correspond to physical particles.

It is interesting to verify the expected behaviour of the expectation values of commutators for this model. In the approximation made above, the conserved current is simply

$$j_\mu = e\eta(\partial_\mu \chi_2 + e\eta A_\mu) = e^2\eta^2 B_\mu. \tag{25}$$

The continuity equation thus reduces to $\partial_\mu B^\mu = 0$, which is a simple consequence of the field equations. The commutators may be evaluated by using the radiation-gauge commutation relations of ϕ_1, ϕ_2, and A_μ. However, a simpler method is to express χ_2 in terms of B_μ. Using the radiation gauge condition $\nabla\cdot\mathbf{A} = 0$, we find from (23) that

$$\chi_2 = - e\eta\nabla^{-2}\nabla_k B^k. \tag{26}$$

The commutator function of the fields B_μ is

$$[B_\mu(x),\ B_\nu(y)] = - i \left(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{e^2\eta^2}\right) \Delta(x - y;\ e^2\eta^2). \tag{27}$$

Hence, as expected, we find that the commutator of χ_2 with j^0 has the local structure

$$[j^0(x),\ \chi_2(0)] = i\partial_0\Delta(x;\ e^2\eta^2), \tag{28}$$

while that with j^k has the acausal form

$$[j_k(x),\ \chi_2(0)] = i(e^2\eta^2\nabla^{-2} - 1)\nabla_k\Delta(x;\ e^2\eta^2). \tag{29}$$

Taking the Fourier transform, we arrive precisely at the structure (15) with $\omega^2(\mathbf{k}) = \mathbf{k}^2 + e^2\eta^2$, and η there set equal to unity.

The main conclusion, therefore, of this discussion is that in both relativistic and nonrelativistic theories the criterion for applicability of the Goldstone theorem is the same - the absence of long-range Coulomb-type forces. A broken symmetry theory with only short-range forces must exhibit an energy spectrum extending down to zero, but when long-range forces are included it need not do so.

It is a rather paradoxical feature of the relativistic broken-symmetry theories that in order to avoid the appearance of massless particles in the excitation spectrum, it is necessary to insert fields of zero bare mass in the original Lagrangian. Even in a theory with unbroken symmetry, the introduction of Yang-Mills fields[17] need not necessarily lead to the appearance of massless vector bosons in the spectrum, as has been demonstrated by Schwinger.[18] However the possibility that they might occur was a rather undesirable feature of the theory. But as Anderson remarked,[16] the problems posed by the vanishing masses of the Yang-Mills bosons and the Goldstone bosons can in suitable circumstances cancel each other out. Instead of the massless particles we might expect with either separately, the introduction of both together can lead instead to the appearance of massive vector particles.

Clearly, in view of this very suggestive result, it is interesting to ask whether the approximate symmetries of relativistic particle physics could be explained along these lines. When we consider a multi-dimensional symmetry group, such as SU(3), we do not in general want to break all of its subgroups. There will remain some residual subgroup of unbroken symmetries. In this situation, we cannot avoid the appearance of massless particles altogether. In order to remove the massless Goldstone bosons corresponding to the broken components of the symmetry, we have to couple the corresponding currents to vector gauge fields. For the broken components, these fields will acquire a mass, but for the unbroken components the mass will remain zero. At first sight, this might seem to be not unreasonable. For SU(3), for example, the only subgroup which is completely unbroken is that generated by the electric charge. Thus, of the octet of gauge vector fields, we should expect just one to be massless - the electromagnetic field. Unfortunately, however, a closer examination quickly reveals that this idea is subject to very grave difficulties. For, if we neglect electromagnetic and weak interactions, then the unbroken symmetry is the $SU(2) \times U(1)$ isotopic spin and hypercharge group. In this approximation, four of the eight vector fields would be massless. Moreover, even including electromagnetic symmetry-breaking, we have a $U(1) \times U(1)$ group generated by the third component of the isospin and the hypercharge, and therefore two massless fields. Thus, although four of the vector bosons might reasonably be expected to have large masses, we should expect two to have masses of purely electromagnetic origin, and one to have a mass due only to the weak symmetry breaking. It hardly needs to be said that masses of the order of magnitude these arguments would suggest are quite as badly in disagreement with observation as strictly zero masses would be.

A generalization of the Goldstone model which illustrates this behaviour of the masses has been discussed by Higgs. We consider an octet of scalar fields ϕ_i, possibly with some SU(3)-invariant self-interaction, and an octet of gauge vector fields $A_{i\mu}$ coupled to the corresponding conserved currents. The symmetry may be broken by requiring a non-vanishing expectation value for ϕ_i. By an appropriate SU(3) transformation we can arrange that the only non-zero components are $\langle\phi_8\rangle$ and $\langle\phi_3\rangle$. These may be taken to represent the medium strong and electromagnetic symmetry breaking respectively. (The effect of weak symmetry breaking is not included in this model.) The vector mesons then separate into three doublets with squared masses proportional to $|\langle\phi_3\rangle|^2$ and $|\langle\frac{3}{2}\phi_8 \pm \frac{1}{2}\phi_3\rangle|^2$, while the two corresponding to unbroken components, $A_{8\mu}$ and $A_{3\mu}$, are massless.

It appears therefore that massless particles in one guise or another present a formidable obstacle to the interpretation of the approximate isospin and SU(3) symmetries as broken symmetries of the type discussed here. At present, the only way round the difficulty would seem to be to invoke Schwinger's argument, and suppose that the unwanted massless vector bosons are absent for some dynamical reason. This, however, does not get us very far, since as yet we do not understand the dynamics.

Before concluding, I ought to mention the possibilities of broken symmetry theories in which the symmetry is a space-time symmetry. There is no difficulty whatsoever associated with the breaking of discrete symmetries such as parity or charge conjugation. For example, if the Lagrangian were CP invariant, the observed violation of CP could be interpreted as being due to the non-invariance of the vacuum under this transformation. In this case, however, we would have only two vacua, rather than an infinite number, and there would be no Goldstone bosons. The non-invariance of the vacuum could be expressed by requiring a nonzero expectation value for some field odd under CP. Such an interpretation might perhaps throw some light on the structure of the theory, although it is formally rather empty, since any CP-violating theory can readily be embedded in a broken symmetry theory of this type.

Much more interesting is the possibility of breaking the symmetries associated with the continuous Lorentz transformations. This is obviously a rather dangerous thing to do, because, unlike isospin or SU(3), the Poincaré group represents, so far as we know, an exact symmetry group. It is, however, possible to break this symmetry without necessarily violating the observed Lorentz invariance. This can be shown explicitly for the case of the Bjorken model,[19] in which the electromagnetic interaction is replaced by a direct point interaction of the form $g_0 j^\mu(x) j_\mu(x)$. The Lorentz invariance is broken by requiring $\langle j_\mu\rangle = \eta_\mu \neq 0$. The theory is formally equivalent, as was shown by Guralnik[20], to ordinary electrodynamics in the presence of a constant external potential $A_\mu^{ext} = g_0\eta_\mu$. The fact that this theory is not in contradiction with observed Lorentz invariance is essentially an expression of the unobservability of a constant potential. As in previous cases, the representation based on different values of η_μ are mathematically inequivalent, but physically indistinguishable. It is interesting to note that one can carry over to this model the previous interpretation of the Goldstone bosons. In this case, because the symmetry-breaking parameter is a vector, the Goldstone bosons have vectorial character, and are of course simply photons. They may be identified with oscillations in the symmetry-breaking parameter η_μ. In other words, the photons are to be regarded as oscillations in the external field about its vacuum value $g_0\eta_\mu$.

It is important to note that this particular theory has a number of unusual features which conspire in a rather remarkable way to avoid any contradiction with Lorentz invariance. In general, one could hardly expect this to be true, and the scope for theories in which rotational symmetry is broken must be very strictly limited. Indeed, it would not be unreasonable to say that this theory is consistent only because it can be reformulated in a manner which makes no reference to symmetry breaking.[21]

Our conclusions, then, are perhaps somewhat pessimistic, at least in regard to possible applications of broken symmetry theories. The photon, if we wish, can be regarded as a kind of Goldstone boson, but whether this is useful is not altogether clear. In most ordinary broken symmetry theories, the scalar Goldstone bosons are usually an embarrassment. They can be eliminated by introducing long-range forces, represented by gauge vector fields, but then these fields must be coupled to every component of the current associated with the symmetry group, and the massless fields coupled to unbroken components remain to plague us. However, at least we now know where we stand in relation to the Goldstone theorem, which has been divested of its aura of mysticism, and instead has taken root in the solid ground of nonrelativistic physics.

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