Extreme charged black hole in braneworld with cosmological constant

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Abstract

Application of the AdS/CFT correspondence to the Randall-Sundrum models may predict that there is no static solution for black holes with a radius larger than the bulk curvature scale. When the black hole has an extremal horizon, however, the correspondence suggests that the black hole can stay static. We focus on the effects of cosmological constant on the brane on such extremal brane-localized black holes. We observe that the positive cosmological constant restricts the black hole size on the brane as in ordinary four-dimensional general relativity. The maximum black hole size differs from that in four-dimensional general relativity case due to the nonlinear term in the effective Einstein equation. In the negative cosmological constant case, we obtain an implication on the Newton constant in the Karch-Randall model.

1 Randall-Sundrum model and black hole

In the Randall-Sundrum (RS) braneworld models [1–3], static solutions of black holes localized on the brane have not found yet. For this issue, the following conjecture has been proposed based on the adS/CFT correspondence [4–6]. According to the correspondence, a five-dimensional classical brane-localized black hole is dual to a four-dimensional black hole that emits the Hawking radiation. Since the latter one cannot be static due to the Hawking radiation emission, it is suggested by the duality that there is no static brane-localized black hole which is larger than the bulk curvature radius.

Here, one might realize that the adS/CFT correspondence also tells that static solutions may present when the black hole horizon is extreme [7] since the horizon temperature is zero and the Hawking radiation will not be emitted. Indeed, the authors of Ref. [7] constructed the near-horizon geometry of such extreme charged static black hole localized on the asymptotically flat brane and studied its properties.

In our study, we shall consider the near-horizon geometry of extreme charged black hole localized on the brane with non-vanishing cosmological constant to study the properties of the brane-localized black holes in more generalized settings. We also intend to reveal the non-trivial property of the gravity in the braneworld model with negative cosmological constant, the Karch-Randall model. For detailed analysis, see our paper [8].

2 Models

The model we consider in this paper is the RS braneworld model, which consists of five-dimensional asymptotically anti-de Sitter (adS) bulk spacetime and a four-dimensional brane with positive tension in it. The action of this model is given by

$$S = \frac{1}{2\kappa_5^2}\int_M d^5x\sqrt{-g}\left({}^{(5)}R + \frac{12}{l^2}\right) + \frac{1}{\kappa_5^2}\int_{\partial M} d^4x\sqrt{-h}K + \int_{\text{brane}} d^4x\sqrt{-h}\left(-\sigma - \frac{1}{2\kappa_4^2}F_{\mu\nu}F^{\mu\nu}\right), \quad (1)$$

where M is the bulk spacetime and ∂M is its outer boundary. $h_{\mu\nu}$ is the induced metric on the brane. $\kappa_5^2 = 8\pi G_5$ and $\kappa_4^2 = 8\pi G_4$ are the five and four-dimensional gravitational coupling, respectively. l is the bulk curvature radius. σ and $F_{\mu\nu}$ are the brane tension and the field strength of the Maxwell field living on the brane. K is the trace of the extrinsic curvature $K_{\mu\nu} = \frac{1}{2}\mathcal{L}_n h_{\mu\nu}$ of ∂M, where $\mathcal{L}_n$ is the Lie derivative with the unit normal vector n of the brane. We impose the Z_2-symmetry about the brane.

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3 Near-horizon geometry

We consider a static brane-localized black hole whose horizon is made to be extreme by the Maxwell field on the brane. A static black hole has constant surface gravity on its horizon. Then, when the horizon is extremal on the intersection with the brane, the whole part of the horizon in the bulk will also be extremal. For such an extremal horizon, we can take the near-horizon limit and analyze its properties. It is proved that the near-horizon geometry of a static extreme black hole can be written in a warped product of a two-dimensional Lorentzian space and a compact manifold as [9]

$$ds^2 = A(x)^2 d\Sigma^2 + g_{ab}dx^a dx^b, \tag{2}$$

where $d\Sigma^2$ is a two-dimensional Lorentzian metric M_2 of constant curvature $2k$. When the metric describes the black hole spacetime, k should be negative and then M_2 is two-dimensional AdS spacetime (adS$_2$). We also assume that $g_{ab}dx^a dx^b$ has $SO(3)$ symmetry. Choosing the coordinates $x^a = (\rho, \theta, \phi)$, the near-horizon geometry becomes

$$ds^2 = A(\rho)^2 d\Sigma^2 + d\rho^2 + R(\rho)^2 d\Omega^2, \tag{3}$$

where $d\Omega^2$ is the metric of the two-dimensional unit sphere.

We assume the horizon to be compact, which implies that $R(\rho)$ vanishes somewhere. Then, we set the "origin" of ρ as $R(\rho = 0) = 0$. We solve the bulk Einstein equation from $\rho = 0$ and the initial parameter at $\rho = 0$ is $A(0) = A_0$. The bulk equations have three free parameters $\{A_0, k, l\}$. We can set $l = 1$ without losing generality.

Assuming that the brane position is at $\rho = \rho_0$, the induced metric on the brane is defined as

$$ds^2_{\text{brane}} = |k| L_1^2 d\Sigma^2 + L_2^2 d\Omega^2, \tag{4}$$

where L_1 and L_2 are proper radii of M_2 and S^2 defined by

$$L_1^2 \equiv |k|^{-1} A(\rho_0)^2, \quad L_2^2 \equiv R(\rho_0)^2. \tag{5}$$

The induced cosmological constant Λ_4 on the brane is determined by the brane tension as

$$\Lambda_4 \equiv -\frac{3}{l^2} + \frac{\kappa_5^4 \sigma^2}{12}. \tag{6}$$

For convenience, we use the normalized tension α,

$$\alpha = \sqrt{1 + \frac{l^2 \Lambda_4}{3}} = \frac{l \kappa_5^2 \sigma}{6} = \frac{\sigma}{\sigma_{RS}}. \tag{7}$$

Moreover, we define the total charge Q on the brane

$$Q = \frac{1}{4\pi} \int_{S^2} *F. \tag{8}$$

After solving the bulk equations to $\rho = \rho_0$, we impose the Israel's junction condition on the brane which relates the bulk solution to α and Q.

4 Results

4.1 Positive cosmological constant case: $\alpha > 1$

In $\alpha > 1$ case, positive cosmological constant is induced on the brane and the brane geometry becomes asymptotically de Sitter spacetime. We observed a restriction on the black hole size in the sense that the black hole size $R(\rho_0)$ has an upper bound which depends on α.

The size of the black hole horizon in de Sitter spacetime is known to be restricted by the cosmological constant in the ordinary general relativity. From our result, we can confirm that the same restriction holds even in the braneworld setup.

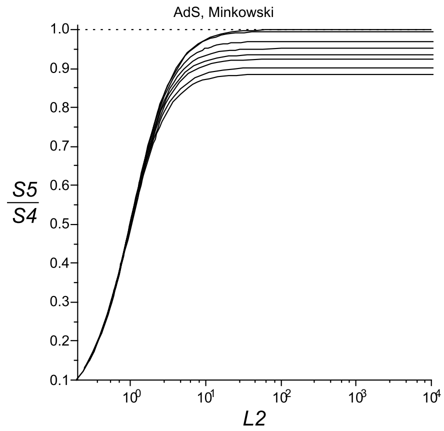

Figure 1: $L_2 = R(\rho_0)$ dependence of the entropy ratio S_5/S_4 for the brane solutions which are asymptotically adS($\alpha < 1$) or flat($\alpha = 1$). Each lines correspond to $\alpha = 1, 0.9995, 0.995, 0.99, 0.985, 0.98, 0.97$ and 0.96 from the top.

Comparing the braneworld upper limit $\alpha_{\max}^{\rm BW}$ with the upper limit $\alpha_{\max}^{\rm 4D}$ in the ordinary four-dimensional general relativity, we can see that

$$\alpha_{\max}^{\rm BW} > \alpha_{\max}^{\rm 4D} = \sqrt{1 + \frac{1}{6L_2^2}} \tag{9}$$

is satisfied. It tells us that the restriction on the black hole size is weaker in the braneworld model. The value of $\alpha_{\max}^{\rm BW}$ is given by $k = 0$ solution.

The difference in the cosmological constant is evaluated as

$$\Lambda_4^{\rm 4D} - \Lambda_4^{\rm BW} = 3\Big((\alpha_{\max}^{\rm 4D})^2 - (\alpha_{\max}^{\rm BW})^2\Big) \simeq -\frac{1}{16L_2^4}. \tag{10}$$

This, in fact, is consistent with the value which can be read off from the effective Einstein equation on the brane in [10] at the leading order.

4.2 Negative cosmological constant case: $\alpha < 1$

In the anti-de Sitter brane case, we observed discrepancies between the near-horizon geometry of the brane-localized black hole from that of the four-dimensional extreme adS-RN black hole. We found that the adS$_2$ radius and the charge are smaller than those of four-dimensional adS-RN black holes, and confirmed that those discrepancies vanish in the flat brane limit ($\alpha \to 1$).

In the large black hole limit,

$$\frac{L_1^2}{{L_{1(\rm 4D)}}^2} = 1 - \frac{3}{2}(1-\alpha) + \mathcal{O}\big((1-\alpha)^2\log(1-\alpha)\big), \tag{11}$$

$$\frac{Q^2}{{Q_{\rm 4D}}^2} = \frac{G_4}{G_5}(1 + 2(1-\alpha)\log(1-\alpha) + \mathcal{O}(1-\alpha)). \tag{12}$$

We also calculated the five and four-dimensional black hole entropies. In the large black hole limit,

$$\frac{S_5}{S_4} = \frac{G_4}{G_5}\frac{A_5}{A_4} = \frac{G_4}{G_5}\Big(1 + 2\,(1-\alpha)\log\,(1-\alpha) + \mathcal{O}\,(1-\alpha)\Big). \tag{13}$$

When the black hole is much larger than the AdS curvature radius, the two entropies are expected to coincide by AdS/CFT correspondence. Then, Eq. (13) may imply the change in G_4 as

$$\begin{aligned}\frac{G_5}{G_4} &= 1 + 2(1-\alpha)\log(1-\alpha) + \cdots \\ &= 1 + \frac{l^2}{L^2}\log\left(\frac{2l^2}{L^2}\right) + \cdots .\end{aligned} \tag{14}$$

This is interesting because the charge ratio (Eq. (12)) also becomes unity at the leading order in such G_4 choise.

In the Karch-Randall model, it is suggested by Ref. [11] that the four-dimensional gravity weakens as $G_4/G_5 \approx 1 - \mathcal{O}\left(l^2/L^3\right)\mathcal{R}$ for $L \lesssim \mathcal{R} \lesssim L^3/l^2$, where $\mathcal{R}$ is the separation of two gravitating objects, due to small four-dimensional graviton mass. However, the formula for G_4/G_5 we proposed in this paper, Eq. (14), has a different form from it. It may be peculiar that our formula is independent of the black hole size, while the formula of Ref. [11] depends on propagation distance $\mathcal{R}$ of the gravity. It will be interesting to investigate whether these two formulae are compatible or not.

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