ANALYTICAL STUDIES FOR NON-PERTURBATIVE QCD OF JETS AT HADRON COLLIDERS

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Abstract

Inspired by the success of analytical models for non-perturbative effects, used to investigate event shape variables at LEP and HERA, we apply them to a study of jets at hadron colliders such as the Tevatron and the LHC. We find that simple analytical estimates are able to shed considerable light on issues that could previously be tackled only through Monte-Carlo simulations, for example the role of different non-perturbative effects in various jet algorithms. In this context, we also provide testable numerical results for the commonly studied inclusive-jet p_t distribution, and we introduce new observables that could be employed to verify our calculations.

1 Introduction

With the LHC due to start later this year, there is considerable activity geared towards sharpening of theoretical and experimental tools, so as to optimize its discovery potential. A portion of this activity is directed at developing a more refined understanding of the physics of strong interactions (QCD), since they will be ubiquitous at the LHC. Since QCD has a non-perturbative aspect that is out of reach for the available tools of quantum field theory, there is an immediate challenge to the level of precision that one may hope to achieve. Inevitably, one has to deal at some level with the effects of parton hadronization, as well as with contamination from the non-perturbative underlying event that accompanies the main hard process.

In an ideal world, one may for example envisage reconstructing clear mass peaks — or other kinematic structures — for some heavy decaying particle (for instance a SUSY particle, or a Z' decaying to jets at the LHC); in the real hadron collider environment, however, these peaks will be smeared by shifts and distortions in the energy spectrum of final state jets, induced by different QCD effects, so that the signal may even be altogether washed out. The smearing effects will involve both initial and final state QCD radiation, as well as non-perturbative energy flows arising from hadronization and the underlying event. To minimise such smearing requires some understanding of the dependence of each effect on the experimental parameters involved in the study, in particular on the choice of jet-algorithm and on the choice of jet size (which is governed by a "radius" parameter R). While perturbative contributions can be obtained using Feynman graph techniques, it is less clear how to acquire information on non-perturbative effects. This is the question that we shall focus on below: we will employ analytical models [1)] that have been very successful in the context of DIS and e^+e^- event shape studies to the more complex environment of hadron collisions.

2 Non-perturbative tools for jet physics

The toolkit for non-perturbative (NP) physics of QCD jets has been thus far rather limited, comprising almost exclusively Monte Carlo (MC) studies using mostly `HERWIG` and `PYTHIA`. While MC's are indispensable tools in this and other regards, they have their own shortcomings, and a certain amount of analytical insight is thus, in our opinion, a welcome addition. For example, it is not straightforward to gain information from MC studies on the functional dependence of NP corrections on jet parameters such as radius, flavour and p_t, while this information is provided immediately by the analytical estimates we will derive. The lack of parametric information, in turn, gives rise to a lore of qualitative statements that may or may not be supported by a quantitative analysis. One may hear, for example, that the k_t algorithm [2] suffers more significantly from underlying event (UE) contamination, as compared to cone algorithms [3], which are supposed to be more significantly affected by hadronisation. We find that, if one chooses the same value of jet radius in either case, there are no differences between algorithms in a first-order calculation. For the UE, calculated to the next order [4], one sees as much variation between different cone algorithms as between cones and the k_t algorithm.

2.1 The Dokshitzer-Webber model applied to jets

We shall first examine, as an example, hadronization corrections to a jet transverse momentum p_t, and then turn to the underlying event contribution. To obtain our main analytical results for hadronization corrections, it is sufficient to use the renormalon-inspired model developed by Dokshitzer and Webber [1] (DW). This model has been widely used for QCD studies at HERA and LEP, and has been followed by several theoretical developments [5, 6, 7, 8, 9], which have firmly established its physical features in the context of our understanding of perturbative QCD. To understand our central result, it is however sufficient to use the model in its original form. In the DW model, hadronization is associated to the emission of a soft gluon with transverse momentum $k_t \sim \Lambda_{\rm QCD}$ ("gluer"). While the strong coupling associated to such an emission, $\alpha_s(k_t)$, is divergent within perturbation theory, one assumes that it can be replaced, in the infrared, by a physically meaningful infrared finite and universal coupling. One then calculates the change δp_t in the transverse momentum of a jet due to gluer emission, and one averages this change over the gluer emission probability.

In general the calculation will depend on the details of the hard process of which the triggered jet is a part. A full calculation in the threshold limit of hadronic dijet production has been reported in Ref. [10]. The calculation there reveals that the hadronization contribution is singular in the jet radius R, as $R \to 0$, *i.e.* in the limit of narrow jets. This most significant feature is in fact universal, and applies to jet production in any hard process; moreover, the leading behavior in R can be derived with a simple calculation, as we illustrate below.

Consider the emission of a soft gluon from a hard parton (say a quark to be definite), such that the gluon is not recombined with the quark jet. We will work in the collinear approximation, which is sufficient to reproduce the leading small-R behaviour. If the transverse momentum of the quark jet was p_t

before gluon emission, it becomes zp_t after the emission, with z the fraction of the initial quark momentum carried by the final quark, so that in the soft limit $z \to 1$. The change in p_t induced by gluon emission is then $\delta p_t = (z-1)p_t$. Averaging this over phase space with the appropriate probability distribution leads to [1]

$$\langle \delta p_t \rangle = p_t \int \frac{d\theta^2}{\theta^2} \int dz\,(z-1)\,P_{qq}(z)\,\frac{\alpha_s\left(\theta z(1-z)p_t\right)}{2\pi}\,\Theta\left(\theta - R\right)\,. \qquad (1)$$

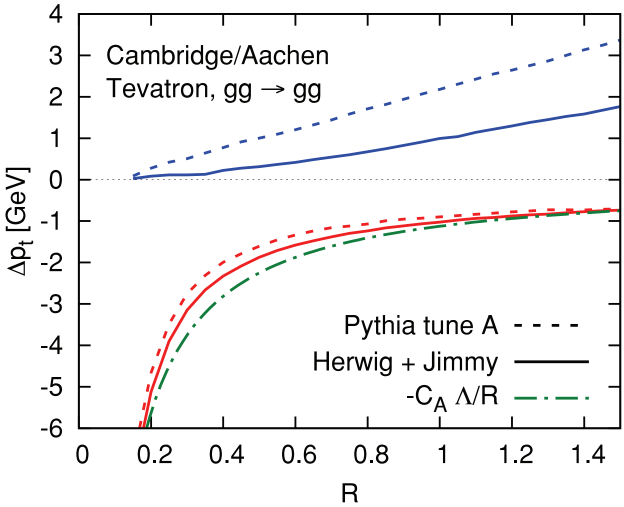

Figure 1: Hadronisation (negative) and underlying event (positive) contributions to jet p_t, as functions of the jet radius R, for gluon jets at the Tevatron.

In the perturbative regime Eq. (1) gives a $\log R$ behaviour, which is a reflection of the collinear enhancement. To evaluate non-perturbative contributions we change variable to $k_t = z(1-z)\theta p_t$, we insert the soft limit of the splitting function $P_{qq} = 2/(1-z)$, and we substitute to the coupling its non-perturbative modification $\delta\alpha_s$, corresponding to 'gluer' emission. We then integrate over θ

[1] The condition that the gluon not be recombined with the jet reduces to $\theta > R$ in the soft limit for all the commonly used jet algorithms.

and z, which gives

$$\langle \delta p_t \rangle^h = -\frac{4}{R} C_F \int \frac{dk_t}{k_t} k_t \frac{\delta\alpha_s(k_t)}{2\pi}, \tag{2}$$

where $\delta\alpha_s$ is the non-perturbative QCD coupling minus its perturbative counterpart, and it is non-vanishing only in the infrared region, $0 < k_t < \mu_I$, with μ_I an infrared matching scale conventionally taken to be $\mu_I = 2$ GeV. The value of the integral of $\delta\alpha_s(k_t)$ cannot be computed, but it can be extracted from event shape variables, under the assumption of universality. We arrive then at a simple result for the p_t shift of a quark jet, which amounts to $\approx -0.5/R$ GeV. For a gluon jet the corresponding result is obtained by replacing C_F with C_A in Eq. (2).

The behaviour of underlying event contributions to the same observable, on the other hand, is regular, and vanishes like R^2 as $R \to 0$, in stark contrast with Eq. (2). This result is natural since the underlying event is disentangled from the dynamics of the jet, which serves merely as a receptacle for soft radiation from partons uncorrelated with the hard scattering. Assuming a uniform rapidity distribution for the soft radiation gives a contribution to δp_t proportional to the jet area [4], with a functional dependence on R given by $R J_1(R) = R^2 + \mathcal{O}(R^4)$.

We have compared our expectations for the R dependence with Monte Carlo event generators, and the results are shown in Fig. 1. One observes that the $1/R$ hadronization correction is in good agreement with the event generators `HERWIG` and `PYTHIA`, in both shape and normalization, over virtually the full range of R studied. In contrast, while the underlying event varies with R as expected, its normalisation is different depending on the event generator model. We also emphasize that very similar results are obtained with all commonly used jet algorithms, so that we have displayed just the Cambridge/Aachen [11] algorithm. We conclude that by varying R it is possible to enhance or reduce the sensitivity to one non-perturbative effect or the other, as desired, which leads to the possibility of isolating and testing individually the different sources of non-perturbative contributions to jets at hadron colliders. We note finally that the size of the underlying event contribution, unlike that of hadronisation, is not under theoretical control, and is different for `HERWIG` and `PYTHIA` at Tevatron energies. Further work is needed to obtain a less ambiguous picture for this component of NP physics.

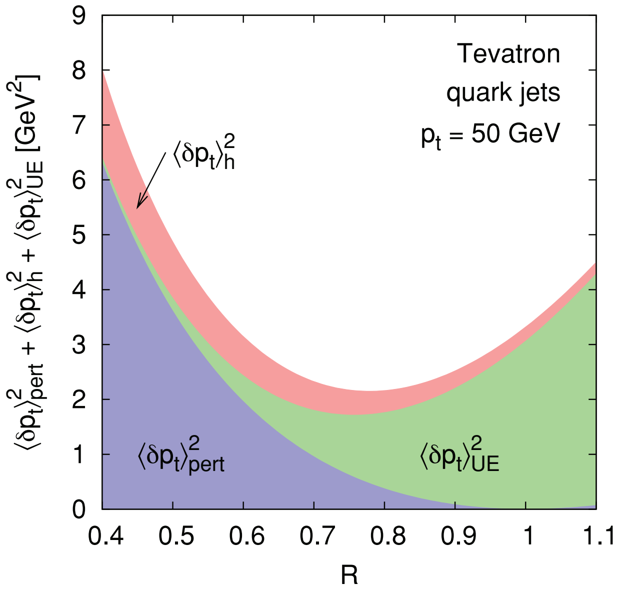

Figure 2: The dispersion of jet p_t as a function of jet radius, due to perturbative and non-perturbative QCD effects, for 50 GeV quark jets at the Tevatron. The minimum value for the total dispersion corresponds to the best value of R if one wishes to minimize all QCD effects.

3 Experimental tests and applications

We briefly present here some experimental avenues to corroborate and exploit the results mentioned above. A fuller account is available in our article [10). One idea that emerges from computing the different R dependencies of perturbative and non-perturbative QCD effects is that of optimal values of R for studies involving jets. In the sort of study we mentioned before, aiming at the reconstruction of the mass of a heavy particle decaying to jets, we would like to minimise the dispersion on jet p_t due to all QCD effects (perturbative and non-perturbative). A detailed study of this dispersion would require a knowledge of correlations between different physical effects, which is not available with current tools. To get a qualitative understanding, one may approximate the true dispersion with the uncorrelated sum

$$\langle \delta p_t^2 \rangle = \langle \delta p_t \rangle_h^2 + \langle \delta p_t \rangle_{\mathrm{UE}}^2 + \langle \delta p_t \rangle_{\mathrm{PT}}^2 \,. \tag{3}$$

Each term in the sum has a characteristic R dependence at small R, with the perturbative piece varying as $\log R$, the hadronisation correction as $-1/R$, and the underlying event contribution as $RJ_1(R)$. The result is plotted for 50 GeV quark jets at the Tevatron in Fig. 2, where we displayed the dispersion due to each effect separately, as well as the approximate total dispersion, whose minimum corresponds to the optimal R.

While Fig. 2 reflects what could be achieved with current knowledge about the R dependence, it should not be taken too literally as far as the precise value of the optimal R is concerned, since we neglected correlations, and furthermore we have oversimplified the perturbative piece, retaining only the leading small R behaviour. The general features of Fig. 2 are however robust, since they follow from the different parametric dependence on R of the various physical effects. From our studies we are also able to predict how the optimal R may change with a change of jet parameters such as "flavour" or p_t. As might be expected, a gluon jet favours a larger R value than a quark jet, and likewise the optimal R rises in a predictable manner with increasing jet p_t (see [10)] for details).

For QCD studies, involving, say, the determination of α_s from jet observables, one may again search for an optimal R: in this case however one should seek to minimize only the non-perturbative contributions. One finds [10)] that the optimal R, in this case, is proportional to the cube root of the ratio of the characteristic scales for hadronization and underlying event.

Various direct experimental tests can be carried out to check our predictions. In this regard one may for example study inclusive jets at HERA, where the steeply falling p_t spectrum would be approximately shifted by the $1/R$ hadronisation effect. Hence a study of inclusive jets with variable R would provide a valuable opportunity to confirm our results. Similarly studies at the Tevatron could lead to a direct determination from data of the scale of the underlying event, addressing the current disagreement between the MC models of `HERWIG` and `PYTHIA`. It is also possible to define operationally, and measure directly as a function of R, the change in the jet p_t due to nonperturbative effects as one changes the jet algorithm or the jet parameters; this definition can be implemented in Monte Carlo studies and could be useful to determine the non-perturbative scales associated with hadronization and underlying event.

To conclude, we would like to emphasise the role of simple analytical studies, which are however well grounded in the technology of perturbative QCD, in order to obtain information about complex non-perturbative properties of jets. This information, reflected for example in the dependence on the jet radius of various jet observables, ought to be of use in carrying out precision studies involving jets at current and future colliders. We would especially like to emphasize the importance of maintaining flexibility in the choice of jet algorithm and jet parameters, since our results show that choices that may be very useful for one class of studies may lead to poor results for other cases.

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