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A metric for Planck Stars derived from Gravity in Asymptotic Safety

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Abstract. The Asymptotically Safe Gravity (ASG) framework suggests a "running" Newtonian coupling constant, which depends on two free parameters $\tilde{\omega}$ and γ. The new black hole metrics inferred from such a "running" gravitational constant naturally match with a Schwarzschild metric at large radial coordinate. By further imposing the matching with the Donoghue quantum corrections to the Schwarzschild field, we find a negative value of the $\tilde{\omega}$ parameter, and this leads to a not yet explored black hole metric, which surprisingly turns out to describe the so-called Planck stars.

1. Introduction

Many models in quantum gravity (see e.g. [1, 2, 3, 4, 5] and references therein) share the property that the fundamental parameters entering the action (Newton's constant, electromagnetic coupling, cosmological constant etc), are considered scale dependent quantities. Actually, scale dependence at the level of the effective action is a generic feature of ordinary quantum field theory. In gravity theories then the scale dependence is expected to modify the horizon, the thermodynamics, the quasinormal modes spectra of classical black hole backgrounds [6, 7, 8, 9, 10]. Among the mentioned approaches based on scale-dependent gravity, which go beyond classical GR, we can find also a particular method usually known as "improved" asymptotically safe (AS) gravity [11, 12, 13]. For renormalization group (RG)-improved black hole metrics, cosmologies, and inflationary models from asymptotic safety, the reader can usefully consult e.g. Refs. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. In the ASG scenario usually the beta function for the gravitational coupling is integrated in order to compute the Newton's constant G as a function of some energy or momentum scale k. The "running" Newton's constant $G(k)$ so constructed is then inserted into the classical black hole solution and an "improved" lapse function is obtained, which is considered to automatically include, in this way, the quantum gravity effects. Clearly, in this approach it is essential to establish a link between the energy scale k and the radial coordinate r, so that we can write $G(k(r)) \equiv G(r)$. Only after this step is done, the improved black hole metric can be considered complete and useful. Those extended

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solutions, inspired by the asymptotic safety program, are expected to modify the classical black hole solutions by incorporating quantum features. However, different modified black hole metrics, in particular metrics also not affected by the central singularity, can be found, e.g, in Refs. [27, 28, 29], although those examples are not directly connected with the ASG program.

On the other hand, by reformulating General Relativity as an effective quantum field theory of gravity at low energies, John Donoghue and other authors [30, 31, 32, 33, 34, 35, 36, 37, 38] have, along the years, established a solid prediction of the quantum corrections to the Schwarzschild field, and therefore to the Newtonian potential, at least at the first order in $\hbar$. By comparing the effective Newtonian potential predicted by the ASG approach with the one computed in the framework of GR as an effective QFT, we arrive to establish, for the first time, a *negative* value for the parameter $\tilde{\omega}$, unlike previous early predictions (see Refs. [11, 22]). This in turn leads directly, without further assumptions, to a specific metric which is able to describe the principal features of the so called Planck stars. It is remakable that, while Planck stars were introduced in Ref. [39] on the grounds of plausible quite general physical considerations, here on the contrary they appear as an almost unavoidable consequence of a metric obtained from the Asymptotically Safe Gravity approach.

We work in units where $c = k_B = 1$, and ℓ_P is the Planck length defined as $G\hbar = \ell_P^2$. Then of course the Planck mass m_P satisfies $2Gm_P = \ell_P$ and $\hbar = 2m_P\ell_P$.

2. Black hole metrics from Running Newtonian coupling

So, according to the literature on the topic [11, 40, 41], the basic idea of the AS gravity approach, in order to obtain the renormalization-improved, classical Newtonian or general relativistic solutions is to replace everywhere the numerical Newton constant G with the 'running constant' $G(r)$, whose explicit form is given in Ref. [11] as

$$G(r) = \frac{Gr^3}{r^3 + \tilde{\omega}G\hbar\,(r + \gamma GM)}\,, \tag{1}$$

where, in accordance with our conventions, $c = k_B = 1$ and we retained $\hbar$. Here $\tilde{\omega}$ and γ are dimensionless numerical parameters, whose concrete value will be discussed later.

The line element for the spherically symmetric, Lorentzian metric preserves the usual form, that is

$$\mathrm{d}s^2 = F(r)\mathrm{d}t^2 - F(r)^{-1}\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2, \tag{2}$$

where r is the radial coordinate, and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$ is the line element of the unit two-sphere. But now, according to the above prescriptions, the lapse function $F(r)$ of our ASG improved Schwarzschild geometry reads

$$F(r) = 1 - \frac{2MG(r)}{r} = 1 - \frac{2GMr^2}{r^3 + \tilde{\omega}G\hbar(r + \gamma GM)}\,, \tag{3}$$

with $G(r)$ given by (1) and M the mass of the black hole. Of course, we suppose $\tilde{\omega} \neq 0$, otherwise we would go back to the standard Schwarzschild metric. Two very important limiting cases should be considered.
The first corresponds to the low energy scales ($r \to \infty$, or $k \to 0$), which implies

$$F(r \to \infty) \simeq 1 - \frac{2GM}{r}, \tag{4}$$

so the standard Schwarzschild metric at large distances is recovered, and this behavior is independent from the values of $\tilde{\omega}$ and γ.

The second limit corresponds to the high energy scales ($r \to 0$, or $k \to \infty$). Here we have to distinguish two subcases.
If $\gamma \neq 0$, then

$$F(r \to 0) \simeq 1 - \frac{2r^2}{\tilde{\omega}\gamma G\hbar}\,, \tag{5}$$

and thus the lapse function corresponds to a deSitter ($\tilde{\omega}\gamma > 0$) or an Anti-deSitter ($\tilde{\omega}\gamma < 0$) core of our metric, depending on the sign of $\tilde{\omega}\gamma$.
If $\gamma = 0$, then

$$F(r) = 1 - \frac{2GMr}{r^2 + \tilde{\omega}G\hbar}\,, \tag{6}$$

and therefore

$$F(r \to 0) \simeq 1 - \frac{2Mr}{\tilde{\omega}\hbar}\,, \tag{7}$$

so in this case we have a conic singularity at the origin. Clearly, the presence of $\hbar$ signals the quantum character of the correction that the ASG approach gives to the core of the standard Schwarzschild metric. In both cases the central Schwarzschild singularity has disappeared.

3. Values of the parameters $\tilde{\omega}$ and γ

As we said in Sec.II, the ASG-improved Newtonian potential can be obtained from the standard Newton formula

$$V(r) = -\frac{GMm}{r} \tag{8}$$

by simply replacing the experimentally observed Newton constant G with the running coupling $G(r)$ given in Eq.(1). Thus we get

$$V^{ASG}(r) = -\frac{G(r)Mm}{r} = -\frac{G\,M\,m\,r^2}{r^3 + \tilde{\omega}\,G\,\hbar\,(r + \gamma GM)}\,, \tag{9}$$

which can be expanded for large r as

$$V^{ASG}(r) = -\frac{GMm}{r}\left[1 - \frac{\tilde{\omega}G\hbar}{r^2} - \frac{\gamma\tilde{\omega}G^2\hbar M}{r^3} + \mathcal{O}\left(\frac{G^2\hbar^2}{r^4}\right)\right]. \tag{10}$$

Clearly, the corrections to the standard Newtonian potential predicted by the ASG approach are all of quantum nature. This is suggested by the presence of $\hbar$ in each term of correction. In fact, there are no correction terms of classical origin, coming from some kind of post-Newtonian approximation.

On the other hand, corrections of quantum origin to the classical Newtonian potential have been elaborated by several researchers [30, 31, 32, 33, 34, 35, 36, 37, 38] in the last three decades or so. In particular, it was pointed out by Donoghue [31, 34] that the standard perturbative quantization of Einstein gravity leads to a well defined, finite prediction for the leading large distance quantum correction to Newtonian potential. The numerical coefficients of the quantum expansion have undergone a certain evolution over the years [32, 33], but the result today accepted by the community [34, 35] reads

$$V^{QGR}(r) \;=\; -\frac{GMm}{r}\left[1 \;+\; \frac{41}{10\pi}\frac{G\hbar}{r^2} \;+\; \ldots\right]. \tag{11}$$

This is an expansion at first order in $\hbar$, where the first correction term represents a genuine quantum correction proportional to $\hbar$.

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The comparison of the two expansions (10) and (11) allows us to fix the parameter $\tilde{\omega}$, which results to be

$$\tilde{\omega} = -\frac{41}{10\pi}\,. \tag{12}$$

The ASG parameter γ cannot be fixed by these considerations. To this aim, we refer the reader to the arguments originally developed in Ref.[11], and then taken up also by other authors (e.g.[22]). Those classical general relativistic arguments have to do with the correct identification of the infrared cutoff, and they fix $\gamma = 9/2$. Different kind of considerations, based on the generalized uncertainty principle (see Ref. [40]; see also Refs. [42]), lead to the value $\gamma = 0$. In this paper we will assume always $\gamma \geq 0$, and in some specific cases we shall comment on the special value $\gamma = 0$. However, most of the results will be qualitatively the same for all $\gamma \geq 0$.

We should here emphasize that the *sign* of the first order correction term in $\hbar$ in the expansion (11) is crucial for the physics of ASG-improved black hole. For example, Duff, in his first calculation [30] of 1974 obtained an expansion of the same kind of (11), with a *positive* coefficient of the $\hbar$ term, and therefore a negative $\tilde{\omega}$. In order to fix the $\tilde{\omega}$ parameter, many authors of ASG papers, even quite recently, (e.g.[11, 22, 23]), rely instead on the early calculations performed by Donoghue and others [31, 32] in the period 1994-1995, where the $\hbar$ coefficient obtained was negative. As a direct consequence, they get a *positive* value of $\tilde{\omega}$. This of course has the nice consequence of a black hole metric without singularity, where in particular the central singularity is wiped out, in favour of a De Sitter or an Anti de Sitter core, as can be easily inferred from Eqs.(3)(5), and is also widely discussed in the above References. However, during the years, the analytical techniques used in GR as an effective QFT have been refined, and the results now accepted by the community are those expressed, initially, in Refs.[34, 35], and then confirmed in Refs.[36], as well as in the very recent Ref.[37]. All these results coherently point to a *positive* $\hbar$ coefficient in the expansion (11), and hence a *negative* value of $\tilde{\omega}$. This fact has deep consequences on the structure of the black hole metric (3), as we will see in the next Sections [1].

4. Study of the new ASG-improved Schwarzschild metric

The key information obtained in the previous Section is that $\tilde{\omega}$ is negative. This, as we will see, represents a major change in respect to others modified (but regular) Schwarzschild metrics present in literature [7, 10, 11, 28, 29]. Instead, some contact with our results can be found in Ref. [44], although there the authors don't deal with ASG models. So, according to the previous

[1] Before proceeding further, we should mention the ongoing lively debate among the communities working on the Effective Quantum Field Theory (EQFT) approach, and those on the ASG approach. The focus is about "if" and "to what extent" the results obtained on ASG-improved metrics by the ASG community can be compared with the results on the quantum corrected metrics obtained by the EQFT community (see e.g. Refs. [38, 43]). Here, we cannot of course enter in details. However, within the Asymptotic Safety literature the parameter $\tilde{\omega}$ is usually considered positive. In fact, considering (see e.g. Refs. [11, 40]) the running Newton's coupling $G(k) = G/(1 + \tilde{\omega}Gk^2/\hbar)$, where k is the energy or momentum scale, we see that $dG/dk < 0$ only if $\tilde{\omega} > 0$. With an $\tilde{\omega} < 0$, $G(k)$ diverges for $k \sim k_{Planck}$, and this behavior of $G(k)$ looks, in principle, outside the spirit of ASG. On the other hand, while the calculations of the EQFT community point nowadays firmly towards a positive coefficient of $\hbar$, there are still doubts about the comparability of those results with the ASG RG-improved metrics, and therefore on the possibility to infer from that comparison a negative value for $\tilde{\omega}$. In particular, it would seem that the two different frameworks effectively consider, and implement, different classes of Feynman diagrams in order to get their final metrics expressions. And we know that, for example, including or not including certain subclasses of diagrams could change the sign of the $\tilde{\omega}$ parameter. Thus, many people get the overall feeling that for the time being the available calculations are not yet able to definitely clarify the situation. This however looks to us as an even stronger motivation to explore the physical consequences of a (still well possible) *negative* $\tilde{\omega}$ parameter.

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section, we consider here the case

$$\tilde{\omega} < 0 \;\Rightarrow\; \tilde{\omega} = -|\tilde{\omega}|\,; \qquad\qquad \gamma > 0\,. \tag{13}$$

The lapse function (3) can therefore be written as

$$F(r) = 1 - \frac{2GMr^2}{r^3 - |\tilde{\omega}|G\hbar(r + \gamma GM)}\,. \tag{14}$$

While for $\tilde{\omega} > 0$ the lapse (3) is regular everywhere when $r > 0$ (see e.g. Ref. [11]), here, with $\tilde{\omega} < 0$, the scenario is very different. First, we notice that the behavior of $F(r)$ at $r \to \infty$ remains that described by Eq.(4), namely a standard Schwarzschild metric for large r. At $r \to 0$ we have an Anti-DeSitter core, namely $F(r) \simeq 1 + 2r^2/(|\tilde{\omega}|\gamma G\hbar)$. But now the denominator $D(r)$ appearing in (14) can develop zeros. Luckly, a simple analysis is sufficient to clarify the situation (for details, see Ref. [41]).

In fact, it is possible to draw a general graph (Fig.1) of the lapse function $F(r)$ in the region $r > 0$, valid for any $\gamma > 0$ and of course $M > 0$. As we see, there is always one single positive zero $r = r_h$ of $F(r)$, which is the horizon of the ASG-improved black hole metric (14). There is also an essential singularity at $r = r_0 > 0$. The most relevant difference with the standard Schwarzschild metric is the fact that the essential (ineliminable) singularity is at $r_0 > 0$, instead of being at $r_0 = 0$. It is also clear that it is always $r_0 < r_h$, for any $M > 0$. So the singularity is always protected by the event horizon. The singularity is never naked, in full accordance with the Cosmic Censorship Conjecture.

The asymptotic behavior of the horizon r_h and of the singularity r_0 in the physical relevant limits of large M and small M can be summarized as follows (again, see Ref. [41] for details).

4.1. ***Horizon*** r_h

The behavior of the horizon for large M, $M \to \infty$, is

$$r_h \;\simeq\; 2GM \;+\; \frac{(2+\gamma)|\tilde{\omega}|\hbar}{4M}\,, \tag{15}$$

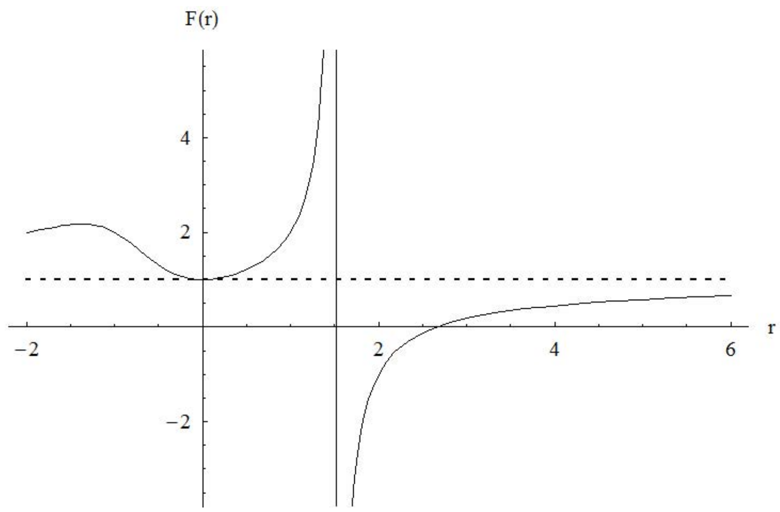

Figure 1. Lapse function $F(r)$: physical region for $r > 0$, singularity at $r = r_0$, horizon at $r = r_h$, where $F(r_h) = 0$.

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so the usual Schwarzschild expression for the horizon is recovered in the large M limit.

We can also investigate the behavior of the horizon in the small M limit.

We find

$$r_h \simeq \sqrt{|\tilde{\omega}|G\hbar} + \left(1+\frac{\gamma}{2}\right)GM\,. \tag{16}$$

Essentially, $r_h \geq \ell_P$, which sounds reasonable, since any length below Planck length is physically meaningless, namely unobservable.

4.2. ***Singularity*** r_0

As we have seen, the singularity of the metric (14) is located at the only positive root $r = r_0$ of the equation

$$r^3 - |\tilde{\omega}|G\hbar r - \gamma|\tilde{\omega}|G^2\hbar M = 0\,. \tag{17}$$

The behavior of the radial coordinate $r = r_0$ of the singularity for small M is

$$r_0 \simeq \sqrt{|\tilde{\omega}|G\hbar} + \frac{\gamma}{2}GM\,, \tag{18}$$

so again $r_0 \geq \ell_P$.

Instead, for $M \to \infty$ the situation is much more interesting. The large M limit of the singularity equation (17) yields for the behavior of the root r_0

$$r_0 \simeq (\gamma|\tilde{\omega}|G^2\hbar M)^{1/3} + \frac{|\tilde{\omega}|G\hbar}{3(\gamma|\tilde{\omega}|G^2\hbar M)^{1/3}}\,. \tag{19}$$

An important physical consideration can now be stated. As already suggested by the initial analysis, the singularity results to be always protected by the horizon. Namely, by comparing Eqs.(15), (19) for large M, or instead by comparing Eqs.(16), (18) in the small M limit, we always have

$$r_0 < r_h\,, \tag{20}$$

so there are no naked singularities.

For sake of clarity, in Fig.2 the reader can find the plots of the mass function $M(r_h)$ for the horizon (red dashed line)

$$GM(r_h) = \left.\frac{r^3 - |\tilde{\omega}|\ell_P^2 r}{2r^2 + |\tilde{\omega}|\ell_P^2\gamma}\right|_{r=r_h}\,, \tag{21}$$

the mass function $M(r_0)$ for the singularity (blue dot-dashed line)

$$GM(r_0) = \left.\frac{r^3 - |\tilde{\omega}|\ell_P^2 r}{|\tilde{\omega}|\ell_P^2\gamma}\right|_{r=r_0}\,, \tag{22}$$

and the standard Schwarzschild horizon $GM = r_{SCH}/2$ (green solid line). Any horizontal line (black dashed) representing an arbitrary $M > 0$ intersects first the blue line and then the red line, namely $r_0(M) < r_h(M)$ for any $M > 0$. Notice that both the horizon and the singularity mass functions have a simple zero at $r = r_c = \sqrt{|\tilde{\omega}|}\,\ell_P$ with $r_c^2 = |\tilde{\omega}|\,\ell_P^2$.

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5. A metric for the Planck stars

It is of the greatest interest to examine the core of the black hole metric we obtained. For masses M larger than the Planck mass m_p, $M \gg m_p$, the central hard singularity is located at $r = r_0$, and it has, in fact, a finite positive size $r_0 \simeq (\gamma|\tilde{\omega}|G^2\hbar M)^{1/3} > 0$, contrary to what happens in the standard Schwarzschild black hole, where the singularity is point-like. Observe that this finite size is completely of quantum origin: in fact, $r_0 \to 0$ if we take the classical limit $\hbar \to 0$.

But most importantly, if we presume that the whole collapsing mass M is concentrated into the central hard sphere of radius r_0, then we can compute the (non covariant) volume of this sphere, and hence the density of this matter (as seen by an observer at infinity), which will result to be finite, and precisely

$$r_0 = (\gamma|\tilde{\omega}|\ell_P^2 GM)^{1/3} \;=\; (\gamma|\tilde{\omega}|/2)^{\frac{1}{3}}\;\ell_P\left(\frac{M}{m_p}\right)^{\frac{1}{3}} \qquad \Rightarrow \qquad \varrho = \frac{M}{V_{core}} = \frac{3}{2\pi}\frac{m_p}{\gamma|\tilde{\omega}|\ell_P^3} \simeq \frac{\varrho_{Planck}}{2\gamma|\tilde{\omega}|}, \tag{23}$$

where we used the definitions $G\hbar = \ell_P^2$, $2Gm_p = \ell_P$, and

$$\varrho_{Planck} = \frac{m_p}{\ell_P^3}\,. \tag{24}$$

So the central hard kernel of our black hole results to have a *finite* size, and a density of the order of the Planck density. These are exactly the characteristics of the so called Planck stars, first proposed in Ref. [39], on the ground of general qualitative considerations. Many of the general properties described in [39] (see also Ref. [45]) can now be repeated for our black hole. The finite positive size of the central core, being of pure quantum origin, is presumably due to the action of the Heisenberg uncertainty principle, which prevents matter to be arbitrarily concentrated into a geometrical point of size zero. The central kernel can presumably keep trace of all the information swallowed by the black hole: we see here a possible way out of the information paradox. Of course, all the above considerations make sense only for $\gamma > 0$ strictly.

The original proposal [39] contains a certain amount of qualitative considerations, including an educated guess on the form of the metric able to describe a Planck star. Such metric was

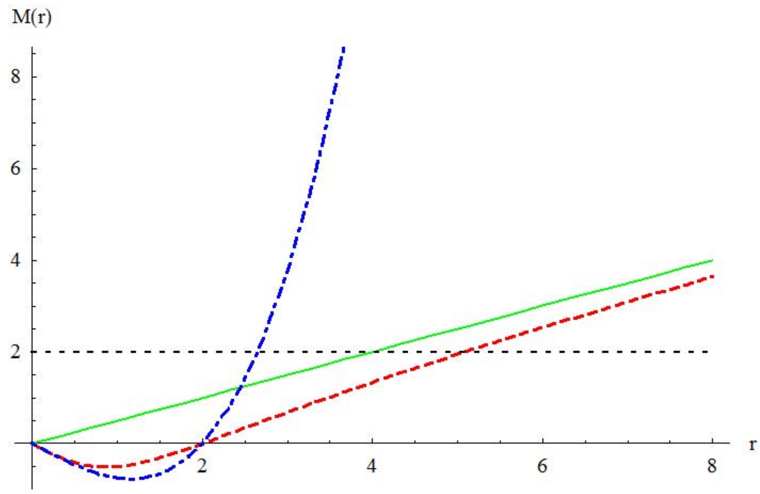

Figure 2. Horizon mass function $M(r_h)$ (red dashed line), singularity mass function $M(r_0)$ (blue dot-dashed line), and Schwarzschild mass function (green solid line). The horizontal black dashed line represents an arbitrary $M > 0$, and always intersects blue and red lines at $r_0(M) < r_h(M)$, namely there are no naked singularities.

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initially chosen to be the Hayward metric [28]

$$F(r) = 1 - \frac{2GMr^2}{r^3 + 2GML^2} \tag{25}$$

where L is a parameter with dimensions of a length. No particularly compelling argument, from the physical point of view, was exhibited for that choice, with the exception, perhaps, that the Hayward metric is a well known example of singularity-free metric. For large M the metric (25) develops two horizons, one inner

$$r_- \simeq L + \frac{L^2}{4GM}, \tag{26}$$

and one outer

$$r_+ \simeq 2GM - \frac{L^2}{2GM}. \tag{27}$$

However, no specific indication is contained in the metric (25) about the size of the hard kernel of a Planck star. And certainly not of a hard kernel with a size increasing with M, as instead Eq.(23) suggests (to be compared with (26)). Even worse, the Hayward metric *per se* is unable to mimic the well established quantum correction to the Newtonian potential [34] that occurs at low energies. This is due to the lack of a term $1/r^3$ in the expansion of the metric (25). The authors of Ref. [46] found a smart way to cure this shortcoming, but at the price of introducing a further metric function $H(r)$, determined through a bunch of additional constraints, so that their "modified Hayward" metric now reads

$$ds^2 = -\,H(r)F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega^2\,, \quad \text{with} \quad H(r) = 1 - \frac{\beta GM\alpha}{\alpha r^3 + \beta GM}\,, \tag{28}$$

where β is a parameter that in Ref.[46] plays the rôle of our $|\tilde{\omega}|$. The above metric [2] finally contains the $1/r^3$ term necessary to mimic the Donoghue modified Newtonian potential [34] for large r.

Although smart and working, the above solution is undeniably contorted and intricate. On the contrary, within the formalism of the Renormalization Group, the mathematical structure of the metric is dictated by the general properties of the AS Gravity, and its lapse function (14) results clearly simpler than the above product $H(r)F(r)$. The ASG metric already contains the right terms to match, at large distances, the quantum corrected Newtonian potential. Moreover, and this is quite astonishing, by simply imposing that match, the final form of the metric is uniquely fixed, and it automatically displays the size of the central hard kernel of the Planck star.

6. Conclusions

In this paper we have derived an exact value of the parameter $\tilde{\omega}$ characterizing the spherically symmetric metric suggested by the Asymptotically Safe Gravity approach. The result has been obtained by comparing the corrected Newtonian potential computed through ASG, with the analog correction suggested by the Donoghue approach to GR as a low energy effective QFT. The decisive novelty in respect to the previously computed values of $\tilde{\omega}$ is that we get a *negative* value of $\tilde{\omega}$, and this because we used the more recent results [34, 35, 36] of Donoghue, Khriplovich, and collaborators.

[2] A further requirement imposed by the authors on the function $H(r)$ is that $H(r)$ should allow for a time delay between a clock sitting at the center of the collapsed object ($r = 0$), and a clock at infinity (put a clock at $r = 0$ is in principle conceivable, just because the Hayward metric is regular at $r = 0$). To get this, authors demand that $H(r = 0) = 1 - \alpha$. They justify this further request by saying that "is a physically unmotivated restriction" to leave $H(0) = 1$. In any case, we do not have this kind of problem with the ASG metric (14), since the center $r = 0$ cannot be reached, being protected by the singularity at $r = r_0 > 0$.

The fact $\tilde{\omega} < 0$ completely changes the geometry of the ASG "improved" black hole metric. Previously unexplored aspects of this metric have been studied, the most relevant one being the presence of a finite-size singularity at the core of the black hole. Surprisingly, the size of this "black kernel" results to be exactly what needed to describe the so called Planck stars. These objects were introduced years ago on the basis of semi-qualitative arguments [39], while in our context they appear as a natural mathematical consequence of the ASG metric with a negative $\tilde{\omega}$ parameter (see also e.g. [47]).

It is worth mentioning that the phenomenology of these objects could be quite rich, and presents both astrophysical and cosmological signatures, in particular in the realm of (primordial) black hole evaporation [48]. As a Planck star evaporates, with a hard kernel of finite positive size, then the final explosion may occur at "macroscopic" scale, namely at a much bigger scale than the Planck scale. So, Planck star explosions could be naturally associated with some of the measured short gamma-ray bursts (SGRBs) [49]. In Ref. [50] authors estimated that several short gamma-ray bursts per day, around 10 MeV, with isotropic distribution, can be expected coming from a region of a few hundred light years around us. On the other hand, also fast radio bursts, strong signals with millisecond duration, which are probably of extragalactic origin, have been shown in Ref. [51] to have wavelengths not far from the expected size of the exploding hole.

On the theoretical side, further investigations, aimed to better understand Penrose diagrams, energy conditions, singularity theorems, quasi-normal modes, as well as a statistical interpretation of entropy and information paradox, related to this kind of metrics are currently being carried out.

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