Electrical Conductivity tensor at finite vorticity of non-relativistic matter

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The medium created in heavy ion collision (HIC) experiments contains deconfined states of quarks and gluons collectively called Quark-Gluon plasma(QGP). The QGP formed behaves like a fluid, and one can fit experimental observables by solving the equation of dissipative fluid dynamics. The equation of the dissipative fluid dynamics takes the transport coefficients of the QGP as inputs. In off-central or peripheral HIC, a huge magnetic field as well as angular velocity can be created [1, 2]. The Lorentz forces on charged particles due to the magnetic field and Coriolis force on massive particles due to angular velocity have quite similarities in transport phenomena. To describe the transport phenomena of QGP, Boltzmann transport equation (BTE) in relaxation time approximation (RTA) provides a microscopic expression of electrical conductivity. External force on the matter like Lorentz force or Coriolis force has to be put in the force term of BTE, for which the expression of electrical conductivity can be modified. Here, we have calculated the electrical conductivity by including Coriolis force in the force term of the Boltzmann transport equation. The calculation has been performed by exploring the similarity between the Coriolis force and the Lorentz force. As a beginning level step, we will do a non-relativistic calculation. For full calculation details one can see [3].

We know that the microscopic expression of electrical current density given by kinetic theory is [5]:

$$j_i = qg\int \frac{d^3\vec{p}}{(2\pi)^3} v_i \, \delta f \ , \tag{1}$$

where q is charge on the particles, δf is small perturbation to equilibrium distribution function, j_i is electrical current density in the medium, and g is the degeneracy associated with a particle. The macroscopic expression of current density j_i is given by $j_i = \sigma_{ij}\tilde{E}_j$, where σ_{ij} is the conductivity tensor. Our aim will be to express δf of Eq. (1) in terms of $\tilde{E}_j$, for which we will take help from the BTE. Now, we will write down the BTE with Coriolis force and electric field as force terms:

$$[q\vec{\tilde{E}} + 2m(\vec{v}\times\vec{\Omega})]\cdot\frac{\partial f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c} \ . \tag{2}$$

where τ_c is the relaxation time, $\vec{p} = m\vec{v}$ is particle momentum and f_0 will be equilibrium distribution function like Bose-Einstein distribution for bosons and Fermi-Dirac distribution for fermions.

Keeping the leading contribution f_0 for the electric field and δf for Coriolis force, we get

$$q\vec{\tilde{E}}\cdot\frac{\partial f^0}{\partial \vec{p}} + 2m(\vec{v}\times\vec{\Omega})\cdot\frac{\partial \delta f}{\partial \vec{p}} = -\frac{\delta f}{\tau_c}. \tag{3}$$

Let us assume that, $\delta f = -\vec{p}\cdot\vec{F}\big(\frac{\partial f^0}{\partial E}\big)$ with $\vec{F} = \alpha\hat{e} + \beta\hat{\omega} + \gamma(\hat{e}\times\hat{\omega})$, where $\hat{e}$ and $\hat{\omega}$ are the unit vectors along electric field($\vec{\tilde{E}}$) and angular velocity $\vec{\Omega}$. α, β, γ are unknown constants, which would be determined from Eq. (3). Upon substituting the assumed form of δf in Eq. (3), one obtains the following values for the unknown coefficients:

$$\alpha = \frac{\tau_c\big(\frac{q\tilde{E}}{m}\big)}{1+\big(\frac{\tau_c}{\tau_\Omega}\big)^2} \ , \qquad \gamma = \frac{\tau_c\big(\frac{\tau_c}{\tau_\Omega}\big)\big(\frac{q\tilde{E}}{m}\big)}{1+\big(\frac{\tau_c}{\tau_\Omega}\big)^2} \ ,$$

$$\beta = \frac{\tau_c\big(\frac{\tau_c}{\tau_\Omega}\big)^2(\hat{\omega}.\hat{e})\big(\frac{q\tilde{E}}{m}\big)}{1+\big(\frac{\tau_c}{\tau_\Omega}\big)^2} \ , \tag{4}$$

where we defined $\tau_\Omega \equiv \frac{1}{2\Omega}$. These values obtained in Eq. (4) can be used to get the expression of δf as:

$$\delta f = -\frac{\partial f^0}{\partial E}\left(\frac{q\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\right)\left[\delta_{jl} + \left(\frac{\tau_c}{\tau_\Omega}\right)^2 \omega_j\omega_l + \left(\frac{\tau_c}{\tau_\Omega}\right)\epsilon_{ljk}\omega_k\right]\tilde{E}_j v_l \ . \tag{5}$$

Now we can substitute Eq.(5) in Eq.(1) to get the expression of current density as follows:

$$j_i = -q^2 g \int \frac{d^3p}{(2\pi)^3}\frac{\partial f^0}{\partial E}\left(\frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\right)\frac{v^2}{3} \left[\delta_{ij} + \left(\frac{\tau_c}{\tau_\Omega}\right)^2 \omega_j\omega_i + \left(\frac{\tau_c}{\tau_\Omega}\right)\epsilon_{ijk}\omega_k\right]\tilde{E}_j \ . \tag{6}$$

Upon comparison of Eq. (6) with the macroscopic version of Ohm's law, i.e., $j_i = \sigma_{ij}\tilde{E}_j$ one obtain the conductivity tensor σ_{ij} as:

$$\sigma_{ij} = -gq^2 \int \frac{d^3p}{(2\pi)^3}\frac{\partial f^0}{\partial E}\left(\frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}\right)\frac{v^2}{3} \left[\delta_{ij} + \left(\frac{\tau_c}{\tau_\Omega}\right)^2 \omega_j\omega_i + \left(\frac{\tau_c}{\tau_\Omega}\right)\epsilon_{ijk}\omega_k\right] . \tag{7}$$

Further simplification of Eq. (7) leads to,

$$\sigma_{ij} = \sigma_0\delta_{ij} + \sigma_1\epsilon_{ijk}\omega_k + \sigma_2\omega_i\omega_j \ , \tag{8}$$

where σ_n can be written in terms of fermi functions as:

$$\sigma_n = \frac{\sqrt{m\pi}gq^2}{2\pi^2\sqrt{2}}\frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)^n}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2}T^{\frac{3}{2}}f_{3/2}(A) \ . \tag{9}$$

For angular velocity in the z-direction, i.e., $\vec{\Omega} = \Omega\hat{k}$, the conductivity tensor can be written in the following matrix form:

$$[\sigma] = \begin{pmatrix} \sigma_\perp & \sigma_\times & 0 \\ -\sigma_\times & \sigma_\perp & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix} \ , \tag{10}$$

where we have defined $\sigma_0 \equiv \sigma_\perp$, $\sigma_0 + \sigma_2 \equiv \sigma_{||}$, and $\sigma_1 \equiv \sigma_\times$.

In summary, we calculated the conductivity matrix for a system under finite rotation by only including the Coriolis force term in the BTE.To keep things simple, we approximated the collision kernel by RTA. We saw that keeping the Coriolis force as an external force in BTE brings anisotropy into the system. The anisotropic nature of the conductivity in the presence of external forces is not new and has already been studied in the literature for systems in the presence of magnetic fields [4]. One can find a similarity in mathematical steps between RTA based electrical conductivity calculation due to Lorentz force for finite magnetic field and Coriolis force for finite angular velocity. Present work may be considered as beginning step to understand the role of Coriolis force in anisotropic conductivity tensors for non-relativistic matter, which is planned to extend for relativistic matter and those formalism can be applicable for QGP study.

References

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