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CONFIRMATION OF ρ' (1250) IN $e^+e^- \to \pi^+\pi^-$ BY MEANS OF THE UNITARIZED ANALYTIC VMD MODEL OF THE PION FORM FACTOR

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1. Introduction

For a time there has been considerable uncertainty about the first radially excited ρ meson state. Only recently[4,5] clear and independent evidences for its existence in the form of $\rho'(1250)$ at a somewhat higher mass value from the analysis of cross section data on $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \pi^0\omega$ has been found. However, in the papers[4,5] the parametrization of corresponding cross sections in the form of a superposition of simple Breit-Wigner formulas has been taken, which must not be the most suitable, because masses and widths observed directly in an experiment need not be a true measure of the underlying resonance parameters and they can be modified significantly through interference with an appropriate nonresonant background. Therefore, in this paper, we use for a determination of the $\rho'(1250)$ parameters (besides the other adjustable parameters of the model) the more accomplished unitarized analytic VMD model[6,7], in which also contributions of the nonresonant background are incorporated in a very natural way. Moreover, instead of the only data on $e^+e^- \to \pi^+\pi^-$, the pion form factor data from the whole existing range of measured momenta $-10\,GeV^2 < t < 10\,GeV^2$ are analysed simultaneously. As a result, the occurrence of $\rho'(1250)$ resonance in $e^+e^- \to \pi^+\pi^-$ process is confirmed again and a coincidence in its mass value with that determined in [4,5] has been found. However, the values of other resonance parameters are different.

In the next section we briefly summarize the unitarized analytic VMD model of the pion form factor and also we prove that the asymptotic behaviour $\sim t^{-1}|_{t\to\infty}$ of the model has to be fixed since the most optimal asymptotic behaviour $t^{-1/2}|_{t\to\infty}$ enforced by the data in [7] contradicts the results of QCD. Section 3 is devoted to the analysis of all existing reliable pion form factor data and all adjustable parameters of the model are determined, unlike the papers [4,5], in a fitting procedure evaluating also their errors by the statistical methods[8]. Conclusions are drawn in section 4.

2. Unitarized analytic VMD model of the pion form factor

It is well-known[9] that the electromagnetic structure of the pion is totally described by one scalar function $F_\pi(t)$ depending on the four momentum transfer squared $t=-Q^2$ of the photon and directly measurable in the process $e^+e^-\to\pi^+\pi^-$ through the cross section

$$\sigma(e^+e^-\to\pi^+\pi^-)=\frac{\pi\alpha^2\beta^3}{3t}\left|F_\pi(t)+R\,e^{i\varphi}\frac{m_\omega^2}{m_\omega^2-t-im_\omega\Gamma_\omega}\right|^2 \qquad (1)$$

where

$$R=\frac{6}{\alpha m_\omega}\left(\frac{m_\omega^2}{m_\omega^2-4m_\pi^2}\right)^{3/2}\left[\Gamma(\omega\to e^+e^-)\Gamma(\omega\to\pi^+\pi^-)\right]^{\frac{1}{2}} \qquad (2)$$

α is the fine structure constant, $\beta=\left[1-\frac{4m_\pi^2}{t}\right]^{1/2}$ is the velocity of the outgoing pion and the phase φ is given through the ρ and ω mesons parameters by the expression[10]

$$\varphi=\operatorname{arc\,tg}\frac{m_\rho\Gamma_\rho}{m_\rho^2-m_\omega^2} \qquad (3)$$

There are, however, also other processes, like $\pi^-p\to e^+e^-n$ and $J/\psi\to\pi^+\pi^-$, giving the experimental behaviour of $F_\pi(t)$ for $t>0$ and the processes $e^-N\to e^-\pi N$, $\pi^-e^-\to\pi^-e^-$, from which experimental information on $F_\pi(t)$ for $t<0$ is extracted.

The most prosperous approach in the description of all these data from a global point of view, which reflects main features of the pion form factor experimental information, is the vector-meson-dominance (VMD) model

$$F_\pi(t)=\sum_v\frac{m_v^2\,(f_{v\pi\pi}/f_v)}{m_v^2-t} \qquad (4)$$

where the summation is carried out only over isovector vector mesons (further we restrict ourselves to $v=\rho,\rho',\rho''$) m_v are their masses, $f_{v\pi\pi}$ and f_v are vector-meson-pion-pion and universal vector-meson coupling constants respectively. Really, Eq.(4) takes into account the fact of the creation of various vector mesons in the reaction $e^+e^-\to\pi^+\pi^-$ explicitly. It is normalized for $t=0$ by the relation

$$\sum_v{}'(f_{v\pi\pi}/f_v)=1 \qquad (5)$$

and governs the asymptotic behaviour

$$F_\pi(t)\sim t^{-1}\big|_{t\to\pm\infty} \qquad (6a)$$

predicted[11-13] for the pion by QCD up to the logarithmic correction as follows

$$F_\pi(t)\sim\frac{64\pi^2f_\pi^2}{(11-\frac{2}{3}n_f)(-t)\ln(-t)/\Lambda^2} \qquad (6b)$$

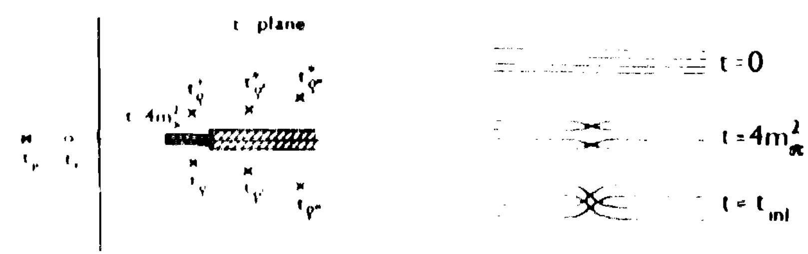
where $f_\pi=93$ MeV is the pion decay constant, n_f is the number of quark flavours and $\Lambda\approx100$ MeV is the QCD scale parameter.

Nevertheless, VMD model (4) besides these positive features has also a series of shortcomings, e.g. it acquires infinite values in the contradiction with measured cross section (1) at the vector meson positions $t=m_v^2$ and it does not respect the unitarity condition

$$\operatorname{Im}F_\pi(t)=F_\pi(t)A_1^{1*}(t)+\delta(t) \qquad (7)$$

with A_1^1 to be the P-wave isovector $\pi\pi$ scattering amplitude and with $\delta(t)$ representing higher inelastic contributions. It is also short of a smooth nonresonant background, which seems to be crucial in obtaining true values of resonance parameters from the existing experimental information on the pion form factor.

We note that all these defects of the standard VMD model are overcome by the incorporation[6,7] of the two-cut approximation of pion form factor analytic properties (see Fig. 1) into (4), which



a) b)

Fig. 1

a) Two-cut approximation of the analytic properties of the electromagnetic pion form factor;
b) structure of the corresponding four sheeted Riemann surface.

finally leads to the unitarized analytic VMD model of the electromagnetic structure of the pion. The two-cut analytic structure is generated (see Fig. 1) by $t=4m_\pi^2$ and $t=t_{ine}>4m_\pi^2$ (it will be left as a free parameter ofthe resultant model), square-root-branch points. Then utilizing the conformal mapping

$$q=[(t-4)/4]^{1/2}\ , \quad m_\pi=1 \tag{8}$$

and the inverse Zhukovsky transformation

$$W(t)=i\,\frac{[q_{ine}+q]^{1/2}-[q_{ine}-q]^{1/2}}{[q_{ine}+q]^{1/2}+[q_{ine}-q]^{1/2}}\ ,\quad q_{ine}=[(t_{ine}-4)/4]^{1/2} \tag{9}$$

leads[6,7] to

$$F_\pi^{(VMD)}[W(t)]=\left(\frac{1-W^2}{1-W_N^2}\right)^2\sum_{v}\frac{(W_N-W_{v_0})(W_N+W_{v_0})(W_N-1/W_{v_0})(W_N+1/W_{v_0})}{(W-W_{v_0})(W+W_{v_0})(W-1/W_{v_0})(W+1/W_{v_0})} \tag{10}$$

where W_{v_0} denotes the VMD pole position in the W-plane (see Fig. 2) and the asymptotic behaviour (6a) is ensured by the power "2" of the common normalized factor in front of the sum. If, further, we use

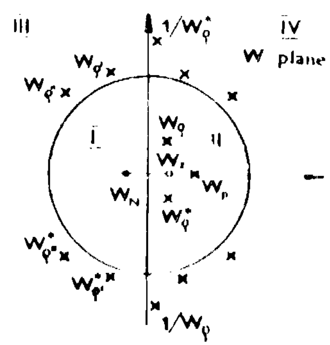


Fig. 2
Result of the mapping of the four-sheeted Riemann surface onto one W-plane

inequalities[7] for the masses of vector mesons, $m_\rho^2 < t_{ine}$ and $m_{\rho'}^2, m_{\rho''}^2 > t_{ine}$, introduce the nonzero values of vector meson Γ_v widths by means of the changes

$$q_{v_0}=[(m_v^2-4)/4]^{1/2}\to q_v=\{[(m_v-i\Gamma_v/2)^2-4]/4\}^{1/2}$$
$$W_{v_0}(q_{v_0})\to W_v(q_v) \tag{11}$$

and take into account the left-hand cut contribution[14,15] by a normalized factor consisting of one pole and one zero on the positive real axis (see Fig. 2) inside the unit circle in W-plane, which corresponds to the negative real axis of the second Riemann sheet in - variable, we finally obtain the unitarized analytic VMD model of the pion form factor

$$F_\pi[W(t)]=\left(\frac{1-W^2}{1-W_N^2}\right)^2\frac{(W-W_z)(W_N-W_p)}{(W-W_p)(W_N-W_z)}\ \times$$
$$\times\left\{\frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}(f_{\rho\pi\pi}/f_\rho)\,+\right. \tag{12}$$
$$\left.+\sum_{v=\rho',\rho''}\frac{(W_N-W_v)(W_N-W_v^*)(W_N+W_v)(W_N+W_v^*)}{(W-W_v)(W-W_v^*)(W+W_v)(W+W_v^*)}(f_{v\pi\pi}/f_v)\right\}$$

defined on a four-sheeted Riemann surface where ρ -poles (always at complex conjugate points) are placed on the second and fourth sheets; and ρ',ρ'' -poles, on the third and fourth sheets. In the next section this model will be used for the analysis of the pion form factor data to determine the resonance parameters $m_v, \Gamma_v, f_{v\pi\pi}/f_v$ $(v=\rho,\rho',\rho'')$. However, before we draw our attention to the asymptotic behaviour of (12).

Though in principle the change of the power of the normalized factor in front of the sum in (12) to an arbitrary positive integer as follows

$$\left(\frac{1-W^2}{1-W_N^2}\right)^2\to\left(\frac{1-W^2}{1-W_N^2}\right)^M \tag{13}$$

leads to the generalization[6,7] of the pion form factor asymptotic behaviour of the form

$$F_\pi(t)\sim t^{-\frac{M}{2}}\Big|_{|t|\to\infty} \tag{14}$$

without any violation of positive features of the model (12) reached by the incorporation of the analyticity into (4), further we prove that only positive even M is allowed in (14) in accordance with (6b).

Really, the analytic continuation of (6b) to the upper boundary of the pion form factor cut on the positive real axis t by means of the substitution $t\to t\,e^{i\pi}$

$$t \to t e^{-i\pi} \tag{15}$$

leads to the expressions

$$Re\, F_{\pi}(t) \underset{t\to\infty}{\sim} - \frac{64\pi^2 f_{\pi}^2}{(11-\frac{2}{3}n_f)\, t \ln t/\Lambda^2} \tag{16a}$$

and

$$Jm\, F_{\pi}(t) \underset{t\to\infty}{\sim} - \frac{\pi\, 64\pi^2 f_{\pi}^2}{(11-\frac{2}{3}n_f)\, t \ln^2 t/\Lambda^2} \tag{16b}$$

respectively, from which one can see immediately that in the asymptotic region of the pion form factor $Re\, F_{\pi}(t)$ dominates the $Jm F_{\pi}(t)$.

Now we prove that the latter comes true in (12) only under the assumption that M is a positive even number.

Really, for $t \to \pm\infty$ on the first Riemann sheet, $W \to -1$ (see Fig. 2), then all terms in the sum in (12) are, in the limit $t\to\infty$, constant and the pion form factor asymptotic behaviour is ensured only by the normalized factor (13) placed in front of the sum in (12). As a consequence, taking into account also the change (13), Eq.(12) very near to the point $W=-1$ can be rewritten into the form

$$F_{\pi}[W(t)] = (1-W)^M (1+W)^M C(t) = (1+W)^M B(t) \tag{17}$$

where $Jm\, B(t) \to 0$ for $t\to\infty$. Moreover, for $t_{ine} < t < \infty$ $|W(t)| = 1$ i.e. $W(t) = e^{i\varphi(t)}$, where $Re\, e^{i\varphi(t)} \to -1$ as soon as $\varphi(t)\to\pi$. So, the relation (17) in the asymptotic region takes the following form

$$F_{\pi}[W(t)] \simeq (i \sin\varphi(t))^M B(t) \tag{18}$$

from which it is evident that

a) if M is even, then $F_{\pi}[W(t)]$ for $t\to\pm\infty$ is dominated by $Re\, F_{\pi}(t)$;

b) if M is odd, then $F_{\pi}[W(t)]$ for $t\to\pm\infty$ is dominated by $Jm\, F_{\pi}(t)$.

In order to respect the consequence of the perturbative QCD given by (16a,b), one has to take M in (12) to be a positive even integer. Since the asymptotic behaviour (6a) is right behind $t^{1/2}|_{t\to\infty}$ taken in /7/ to be nearest to the form (6b), one has to fix $M=2$ in (12).

Consequently, one cannot take the resonance parameters in /7/ to be true because they have been determined in a fitting procedure of the pion form factor data by means of (12) with a wrong asymptotic behaviour.

3. Analysis of all existing reliable pion form factor data

The unitarized analytic VMD model (12) constructed in the previous section depends on the following 12 adjustable parameters

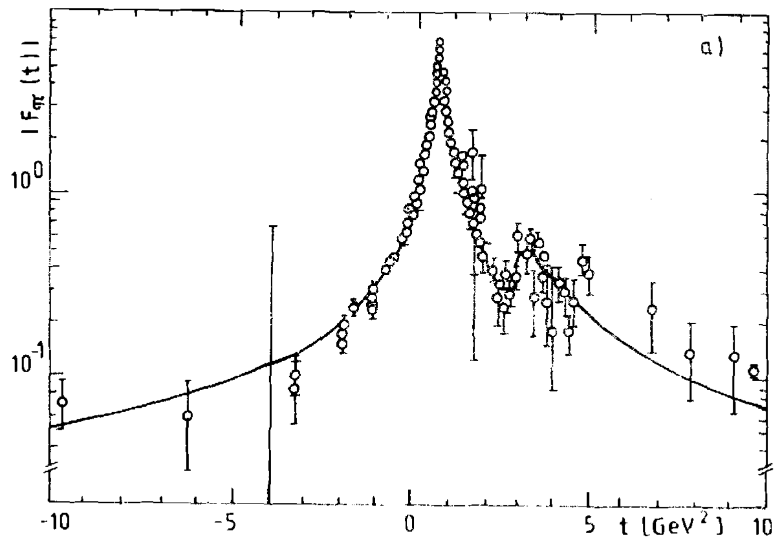
$$W_z,\ W_p,\ t_{ine},\ m_v,\ \Gamma_v,\ f_{v\pi\pi}/f_v \quad (v=\rho,\rho',\rho'') \tag{19}$$

with the clear physical meaning. Their number is reduced to 11 by equation (5) conserved also by the model (12) which is a consequence of a normalization of the pion form factor to the pion electric charge taken to be equal to one. In the analysis of data we choose the ratio of coupling constants $f_{\rho\pi\pi}/f_{\rho}$ of ρ-meson to be expressed by means of (5) through the coupling rations of the other resonances contained in the model (12).

To take into account also the isospin violating $\omega\to\pi^+\pi^-$ decay leading to the so-called ρ-ω interference effect in $e^+e^-\to\pi^+\pi^-$, we correct the pion form factor by adding (see the cross section (1)) the Breit-Wigner form of the ω-meson multiplied by a complex ρ-ω interference amplitude $Re^{i\varphi}$ where R is defined by the relation (2) and the phase φ is expressed through ρ and ω meson parameters by the relation (3). So, finally there are 12 free parameters to be determined in a fitting procedure of data affixing the amplitude to the previous 11 adjustable parameters of the model (12) and taking the values of ω (783) resonance parameters from the Review of Particle Properties /16/.

Most of the used in the analysis pion form factor data are compiled in /17/. But there are also the most recently obtained data /18,19/ which in the combination with the previous ones given all together 293 experimental points. Nevertheless, taking into account the recent results /20/ on a direct compatibility check of the space-like region data obtained from electroproduction processes with the most reliable ones obtained from $e^+e^-\to\pi^+\pi^-$, we find /20/ 5 doubtful electroproduction pion form factor data with a corresponding partial $\chi^2 > 10$. They are excluded from further analysis.

The most optimal description (see Figs. 3a,b) of the remaining 288 reliable pion form factor experimental points from the range of momenta $-9.770\ GeV^2 < t < 9.579\ GeV^2$ is achieved with $\chi^2/ndf = 382/276$ and the following values of parameters:



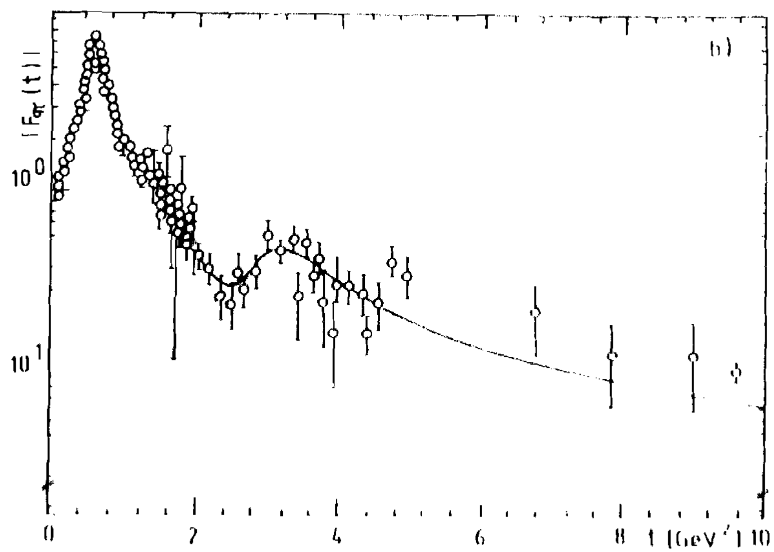


Fig. 3.

a) Comparison of the prediction of the model (12) with the values of parameters (20) and (21) with all existing reliable pion-form-factor data from the range of momenta $-9.770\ \mathrm{GeV}^2 \le t \le 9.579\ \mathrm{GeV}^2$.
b) Exhibition of ρ (770), ρ' (1250) and ρ''(1600) contributions to the pion form factor behaviour in detail.

$$t_{in} = 1.3 \pm 0.1\ \mathrm{GeV}^2$$

$$\begin{array}{llllll}
m_\rho & = 762 \pm 3\ \mathrm{MeV} & \Gamma_\rho & = 143 \pm 5\ \mathrm{MeV} & & \\
m_{\rho'} & = 1422 \pm 90\ \mathrm{MeV} & \Gamma_{\rho'} & = 685 \pm 170\ \mathrm{MeV} & f_{\rho'\pi\pi}/f_{\rho'} & = -0.18 \pm 0.02 \\
m_{\rho''} & = 1682 \pm 58\ \mathrm{MeV} & \Gamma_{\rho''} & = 402 \pm 81\ \mathrm{MeV} & f_{\rho''\pi\pi}/f_{\rho''} & = +0.16 \pm 0.01 \\
W_z & = 0.46 \pm 0.16 & W_p & = 0.64 \pm 0.25 & R & = 0.0142 \pm 0.0037
\end{array} \tag{20}$$

The values of $f_{\rho\pi\pi}/f_\rho$ and φ calculated from (5) and (3) respectively are the following:

$$f_{\rho\pi\pi}/f_\rho = 1.02 \pm 0.02 \quad \text{and} \quad \varphi = 106.3° \pm 0.4° \tag{21}$$

The errors of all parameters in (20) were determined by looking for the change of χ^2 corresponding to 68.3% of a confidence level for 12 free parameters [8].

The existing identity between the pion form factor phase and the ρ-wave isovector $\pi\pi$ phase shift $\delta_1^1(t)$ in the elastic region (practically up to almost 1 GeV²) following from the pion form factor unitarity condition enables in principle to verify to what extent the model (12) and the unitarity condition hold. The comparison of the calculated from (12) with existing data [21] on $\delta_1^1(t)$ in Fig.4

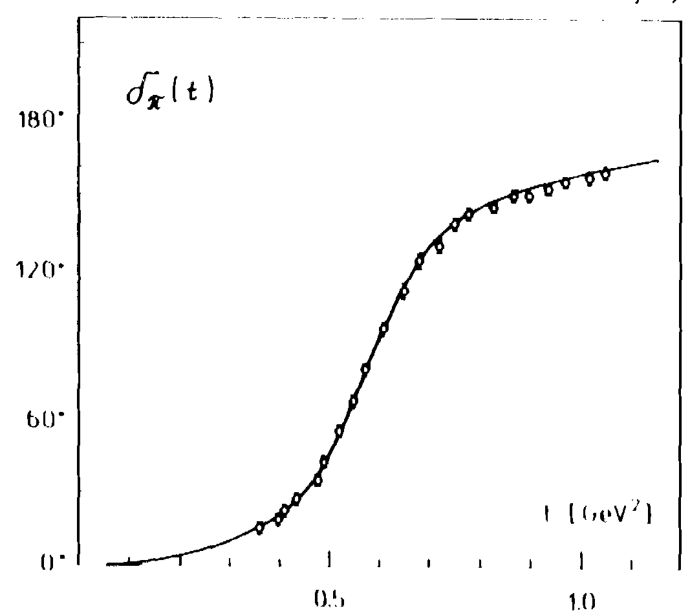


Fig. 4. The predicted pion form factor phase $\delta_\pi(t)$ from (12) and its comparison with the ρ-wave isovector $\delta_1^1(t)$ phase shift data [21].

shows that our unitarized analytic pion form factor model (12) with the values of parameters (20) and (21) obeys the unitarity condition.

4. Conclusion

The unitarized analytic pion form factor model (12) was constructed, which (see Fig. 3a) is able to reproduce all existing pion form factor data. A clear occurrence of all three isovector vector mesons $\rho(770)$, $\rho'(1250)$ and $\rho''(1600)$ is demonstrated in Fig. 3b. The evaluated mass of $\rho'(1250)$, $m_{\rho'} = 1422 \pm 90$ MeV, coincides with the value obtained recently from the process[4] $e^+e^- \to \pi^+\pi^-$ and independently from the process[5] $e^+e^- \to \pi^0\omega$. The parametrization of the corresponding cross sections was made only in the form of a superposition of Breit-Wigner formulas.

There are two points (see Fig. 3a) around 5 GeV^2 and the last time-like region experimental point obtained from J/ψ decay, which are not reproduced by our pion form factor model (12). They could be considered as indications for the existence of the $\rho'''(2150)$ resonance, the inclusion of which into (12) could lead even to a better agreement with the data. However, to do a definite conclusion about the ρ''' (2150) in $e^+e^- \to \pi^+\pi^-$ more precise and dense data are required in the region of this resonance. More precise data in the region of other resonances included into the model (12) need also required in order to diminish errors of most of the parameters (20).

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