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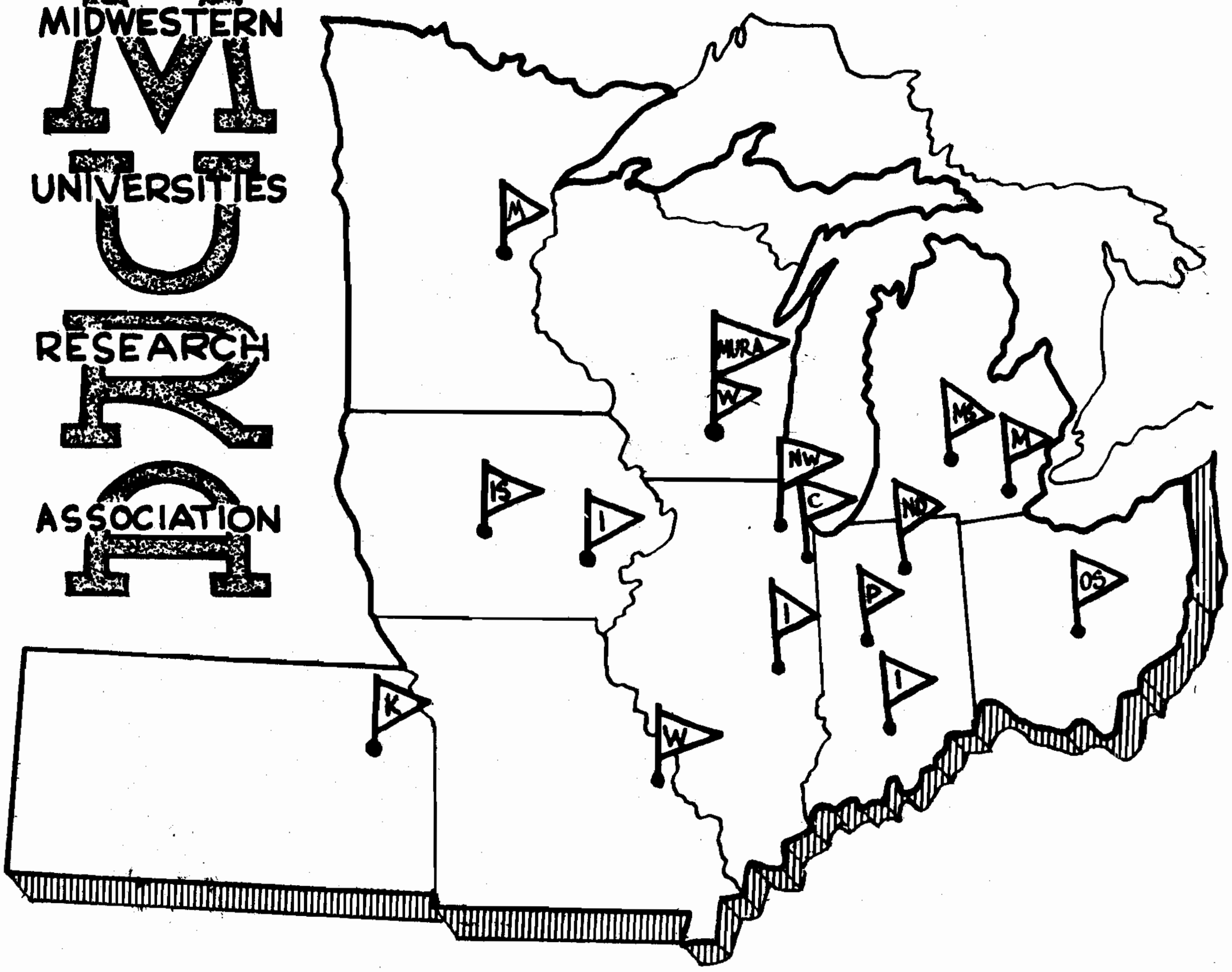

SOME CHARACTERISTICS OF THE 40 MEV TWO-WAY ELECTRON MODEL

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REPORT NUMBER 396

MURA-396
Internal

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SOME CHARACTERISTICS OF THE 40 MEV TWO-WAY ELECTRON MODEL

Lawrence W. Jones**

April 10, 1958

ABSTRACT

Specific formulae have been derived and applied to the case of 15 Bev protons in a large (180 meter radius) two-way FFAG accelerator. The purpose of this note is to apply the same methods in examining some of the properties of the beams in the 40 Mev electron model now under construction. Space charge limits at injection and full energy are estimated; beam life, clearing field, and beam-beam interactions at full energy are investigated; and electron-electron scattering counting rates are considered.

AEC Research and Development Report

*Research supported by the Atomic Energy Commission, Contract No. AEC AT(11-1)-384

**University of Michigan

I. Space Charge Limits

The space charge limit for a beam of cylindrical cross section is

$$I = \frac{10 m_o c^2}{e} (\nu_z \Delta\nu) \left(\frac{a}{R}\right)^2 \beta^3 \gamma^3 \quad {}^{1)}$$
$$= 1.7 \times 10^4 \; \nu_z \Delta\nu \left(\frac{a}{R}\right)^2 \beta^3 \gamma^3. \tag{1.1}$$

For the parameters of the 40 Mev model[2] (two-way operation)

ν_z = 5.3

R = 125 cm.

E = 100 kev

β = .55

γ = 1.2

The injection aperture (inside the vacuum tank) may be about 6.5 cm, so that the maximum vertical oscillation amplitude, a, would be about $\pm$ 2.5 cm (averaged over a sector). These figures give

I = 2.6 amperes*

corresponding to 7.8×10^{11} electrons.

At maximum energy, adiabatic damping reduces the beam amplitude by the square root of the field ratios:

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*It may be noted that this current is greater than that of MURA-285, as a larger oscillation amplitude and larger ν_z (corresponding to two-way operation) are used. A comparable improvement factor may be expected here due to A.G. space charge effects.

1) MURA Proposal for a High Energy Research Facility (I.7.1), except note $m_o = 9.1 \times 10^{-28}$ g
$I_o = \frac{10 m_o c^2}{e}$: $e = 4.8 \times 10^{-10}$ e.s.u., $c = 3 \times 10^{10}$ cm/sec, I_o amperes.

2) MURA-375

$$\frac{a_2}{a_1} = \sqrt{\frac{B_1}{B_2}} \quad ; \quad \frac{B_1}{B_2} = \left(\frac{R_1}{R_2}\right)^k = (1.64)^{9.3} \cong 100, \quad ^{2)} \tag{1.2}$$

$$\frac{a_2}{a_1} \cong 10.$$

Therefore the beam damps to $\pm$ 0.25 cm at full energy. At 39 Mev (γ = 76), formula (1.1) gives a space charge limit of

$I = 1.55 \times 10^4$ amperes.

If there is complete neutralization, this is reduced by γ^2, to

$I = 2.7$ amperes.

If the beam grows (due to detuning, gas scattering, etc.) to $\pm$ 1 cm,

$I = 43$ amperes

with complete neutralization.

II. Stacked Beam

A. Clearing Field

The rf handling to achieve stacked currents of 50 to 100 amperes has been treated elsewhere (L. Johnston and E. Rowe; MURA Minutes # 61, February 10, 1958). In the following it will be assumed that 100 amperes have been stacked. The effects are all less for smaller currents.

The clearing field required for a relativistic beam is

$$E = \frac{I}{2\pi\epsilon_o c a}$$

$$E = \frac{60\, I}{a} \tag{2.1}$$

where E is volts/cm, I is in amperes, and a is in centimeters.

For a 100 ampere beam in a $\pm$ .25 cm tube,

$$E = 24,000 \text{ volts/cm}.$$

If the beam "relaxes" to $\pm$ 1 cm,

$$E = 6,000 \text{ volts/cm}.$$

This clearing field is equivalent to a horizontal magnetic field given by

$$B_r = \frac{E_z}{300} = \left\{ \begin{matrix} 80 \\ 20 \end{matrix} \right\} \text{ gauss.}$$

This in turn is equivalent to a vertical magnet displacement of

$$\Delta z = B_r \Big/ \frac{kB}{R} \qquad (2.2)$$

For $k = 9.3$, $B = 5,000$ gauss, $R = 200$ cm; $kB/R = 230$ gauss/cm.

The equivalent magnet displacement for the two amplitudes is

$$\Delta z = \left\{ \begin{matrix} 3.5 \\ .9 \end{matrix} \right\} \text{mm.}$$

If the clearing field is reversed in successive magnets, a vertical equilibrium orbit scallop will result given by[3)]

$$A = \frac{f\,R}{N^2} \qquad (2.3)$$

$$f = \frac{B_R\ C}{B_{max}} .$$

For $B_{max} = 5,000$ gauss, $C = 8$, $N = 16$, $R = 200$,

$$A = \left\{ \begin{matrix} 1 \\ .25 \end{matrix} \right\} \text{mm.}$$

B. Multiple Scattering

Enoch[4)] estimated the beam life due to multiple Coulomb scattering in terms

3) Reference 1) page 102

4) MURA Minutes #16, January 28, 1957

of a parameter $\nu_z \frac{a}{R}$. A portion of his table is reproduced below:

γ \ $\nu_z A/R$	.03	.01
50	3.4	0.53
100	12	1.74

beam lifetime (seconds)

gas pressure: 1.4×10^{-6} mm Hg (Nitrogen).

For $\nu_z = 5.3$, $a = .25$ cm, $R = 200$ cm, $\nu_z A/R = .0067$.

For $a = 1$ cm, $\nu_z A/R = .027$ cm. With γ of about 76 and a pressure of 10^{-6} mm Hg, the beam life might be expected to be

$$\tau \cong 1 \text{ second for } a = 0.25 \text{ cm},$$

$$\tau \cong 7 \text{ seconds for } a = 1 \text{ cm}.$$

If the beam were not clipped at small amplitude, it would be expected to gas scatter to a centimeter amplitude or larger, although radiation damping might decrease this build up. It therefore seems appropriate to carry figures for the beams of both large and small dimensions.

C. Rf Knockout

It is possible to tune the new model so that no primary knockout frequencies are crossed. It is of interest, nevertheless, to note the degree of amplitude build up to be expected if a knockout frequency lies within the rf range (for example if ν_x has a quarter integral value).

The radial build up amplitude for one passage through the knockout frequency is given by[5)]

$$X = \frac{V\,R}{2\,E_{f_o}\,\beta^2\,(k+1)} \sqrt{1 + \frac{(2\,(k+1)\,\tan\delta)^2}{\nu_x^{\,2}}} \left(\frac{f_o}{\sqrt{df/dt}}\right) \qquad (2.4)$$

5) Reference 1) formula (I.3.19). Note factor 1/2 in reference (see also MURA-133, MURA-260)

where V is the cavity voltage, E the total energy, f_o the particle frequency, and df/dt the rate of modulation. This latter may be found from

$$\frac{df}{dt} = \left(\frac{df}{dR}\right)\left(\frac{dR}{dE}\right)\left(\frac{dE}{d\,(\text{turn})}\right)\left(\frac{d\,(\text{turn})}{dt}\right)$$

where $\dfrac{dE}{d\,(\text{turn})} = V\Gamma$

$$\frac{d\,(\text{turn})}{dt} = f$$

$$\frac{df}{dR} = -\frac{f}{R}$$

$$\frac{dR}{dE} = \frac{R}{kE_f},$$

assuming completely relativistic particles, and operation on the fundamental.

Thus:

$$\frac{df}{dt} = -\frac{V\Gamma f^2}{kE_f} \tag{2.5}$$

so that

$$\frac{f_o}{\sqrt{df/dt}} = \frac{f_o}{f}\sqrt{\frac{kE_f}{\Gamma V}} \tag{2.6}$$

For V = 500 volts,

$\Gamma = .6$

$E_{f_o} = 40$ Mev

$E_f = 3$ Mev

$f/f_o = 1.25$

$\delta = \dfrac{AN}{R} = 24.5^o$, $\left(\dfrac{A}{R} = 0.027\right.$: Reference 2),

formula (2.4) gives

X = .05 cm.

The amplitude of oscillation will build up in a random-walk manner as the square root of the number of passes, so that after 25 stacks, X will be 0.25 cm, and this will add in quadrature to the existing amplitudes. Thus the radial build-up of oscillation amplitudes can be comparable to the build-up due to multiple scattering at 10^{-6} mm Hg if the resonance frequency is crossed and if of order 100 bunches per second are accelerated.

D. Beam-Beam Interactions

When two intense, circulating beams intersect in every straight section, two effects may occur. They may cause vertical detuning of both beams, and they may displace each other vertically. The detuning effect is given by[6)]

$$\Delta \nu_z = -\frac{4 I N R}{\nu_z 10 \pi B \rho \, a \tan\alpha} . \tag{2.7}$$

For N = 16, I = 100 amperes, $\alpha = 2\delta = 49^{\circ}$,

$$\Delta \nu_z = .204, \quad a = 0.25 \text{ cm}$$

$$\Delta \nu_z = .051, \quad a = 1 \text{ cm.}$$

It is interesting that the effect is much smaller here than in the 15 Bev proton accelerator.

The vertical displacement effect might be observed if the two beams miss completely in one straight section. One such interaction would give rise to a closed orbit oscillation of amplitude given by

$$z = \frac{8 \pi I R}{10 B \rho \nu_z \tan\alpha} . \tag{2.8}$$

For the numbers above, with I = 100 amperes,

6) Reference 1) formula III. 8. 2. Note factor R.

$$Z = 0.064 \text{ cm}.$$

If such effects were felt in all 32 straight sections (unlikely) this might be increased by $\sqrt{32}$ or 5.65 to 0.36 cm, still a negligible figure.

III. Electron-Electron Scattering Experiments

The cross section for electron-electron elastic scattering may be most simply estimated from the radius of the Coulomb potential barrier due to a point electronic charge.

$$r = \frac{q}{V_{e.s.u.}} = \frac{4.8 \times 10^{-10} \times 300}{V \text{ electron volts}} . \tag{3.1}$$

Since the 39 Mev electrons collide at an angle of 49^o, the center-of-mass energy is given by

$$V = E_{cm} \doteq 39 \times 2 \cos 24.5^o = 71 \text{ Mev},$$

$$r \cong 2 \times 10^{-15} \text{ cm},$$

$$\sigma_e = \pi r^2 \cong 10^{-29} \text{ cm}^2.$$

Alternatively, Rutherford scattering gives $\sigma_e = \left(\frac{e}{2E}\right)^2 \frac{1}{\sin^4(\theta/2)}$ which agrees within a factor of π at $\theta = 90^o$ with the above expression. For a large proton accelerator, the interaction density N, the interactions per unit length NA, and the total number of interactions per straight section n are all of interest. Here, for electrons beams of order a centimeter or less cross section and intersection angles of almost 50^o, n is probably the most relevant figure.

$$N = \frac{2 J^2 \sigma_e \cos\alpha}{q^2 c} , \quad 7) \tag{3.2}$$

$$n = \frac{2 I^2 \sigma_e}{2 q^2 c\, a \tan\alpha} , \tag{3.3}$$

7) Reference 1) formula II.2.1; note that here the $\cos\alpha$ factor is included as it is not negligible. If the beam is cylindrical rather than rectangular, J is increased by $4/\pi$

where $\frac{2\,\sigma_e}{q^2\, c}$ = .078 for this cross section. For I = 100 amperes, N and $\mathcal{N}$ are given below corresponding to both values of a.

	N $\frac{\text{interactions}}{\text{cm}^3\ \text{sec}}$	$\mathcal{N}$ $\frac{\text{interactions}}{\text{sec}}$
a = .25 cm	2060	1360
a = 1.0 cm	130	340

It may be noted again that if only $\mathcal{N}$ is important for experiments, the radial width of the beam is not a factor, hence rf handling is less critical.

The elastic scattering rate on residual gas may be estimated as above, where σ is now increased by Z^2 of the gas molecules and another factor of 4 because the energy is now only that of one electron (39 Mev). Thus for Z = 7,

$$\sigma_g \cong 2 \times 10^{-27}\ \text{cm}^2.$$

The interaction rate with residual gas per centimeter of length is given by

$$\mathcal{M} = \frac{I\, \eta\, \sigma_g}{q} \quad . \tag{3.4}$$

For nitrogen gas at 10^{-6} mm Hg, $\eta \cong 6 \times 10^{10}$ atoms per cubic centimeter. Therefore at this pressure

$$\mathcal{M} = 750\ I$$

for a beam going one way, or

$$2\,\mathcal{M} = 1500\ I$$

for both beams. At 100 amperes,

$$2\,\mathcal{M} = 1.5 \times 10^5\ /\ \text{cm sec}.$$

As an example of an experiment to detect electron-electron scattering, consider two Cerenkov detectors, each subtending a solid angle Ω from the interaction region and connected in a coincidence circuit with a resolving time, τ, of 10^{-7} seconds. The singles counting rate in each counter is given by

$$C = 2 m l \Omega \tag{3.5}$$

where l is the length of the orbit "seen" by (i.e. unshielded from) the counters. The accidental coincidence rate is given by

$$A = C^2 \tau = (2 m l \Omega)^2 \tau \tag{3.6}$$

The real coincidence rate (for properly oriented counters) is

$$R = 2 n \Omega \tag{3.7}$$

The factor of 2 is present since two electrons result from each interaction. The counts to background coincidence rate is then

$$\frac{R}{A} = \frac{n}{2 m^2 l^2 \Omega} \tag{3.8}$$

$$= \frac{\sigma_e}{2 c a \tan \alpha \, \eta^2 \sigma_g^2 l^2 \Omega \tau},$$

where it is interesting to note that I cancels out! Of course for small currents both rates may be inordinately small and uninteresting. For the numbers that have been considered, the following table summarizes the counting rates.

I = 100 amperes, l = 10 cm, $\tau = 10^{-7}$ seconds		C	A	R	R/A
a = .25 cm	$\Omega = 10^{-3}$	1.5×10^3	2.25×10^{-1}	2.72	12
	$\Omega = 10^{-4}$	1.5×10^2	2.25×10^{-3}	2.72×10^{-1}	120
a = 1 cm	$\Omega = 10^{-3}$	1.5×10^3	2.25×10^{-1}	6.8×10^{-1}	3
	$\Omega = 10^{-4}$	1.5×10^2	2.25×10^{-3}	6.8×10^{-2}	30

It is therefore concluded that it will be not difficult to detect and measure electron-electron scattering with this machine. The coincidence counting rates may lie in the range of from one to one hundred per minute depending on geometry and beam. If the stacked current falls to ten amperes, all counting rates will drop by a factor of 100. A radiation length in iron is about one inch, so that electrons will come through the 1/8" vacuum chamber with little alte r a tion. Thin walls in the chamber over one or two straight sections would make possible more quantitative experiments.